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# Efficient Synthesis of Sensor Deception Attacks Using Observation Equivalence-Based Abstraction

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**Abstract:** This paper investigates the synthesis of successful sensor deception attack functions in supervisory control using abstraction methods to reduce computational complexity. In sensor deception attacks, an attacker hijacks a subset of the sensors of the *plant* and feeds incorrect information to the *supervisor* with the intent on causing damage to the supervised system. The attacker is successful if its attack causes damage to the system and it is not identified by an intrusion detection module. The existence test and the synthesis method of successful sensor deception attack functions are computationally expensive, specifically in partially observed systems. For this reason, we leverage results on abstraction methods to reduce the computational effort in solving these problems. Namely, we introduce an equivalence relation called *restricted observation equivalence*, that is used to abstract the original system before calculating attack functions. Based on this equivalence relation we prove that the existence of successful attack functions in the abstracted supervised system guarantees the existence of successful attack functions in the unabstracted supervised system and vice versa. Moreover, successful attack functions synthesized from the abstracted system can be exactly mapped to successful attack functions on the unabstracted system, thereby providing a complete solution to the attack synthesis problem.

*Keywords:* Automaton, Deception Attacks, Abstraction, Supervisory Control Theory.

## 1. INTRODUCTION

Over the past decades, human dependability on cyber-physical systems has rapidly increased. Usually, these systems are highly complex, and appear in settings that are safety critical, where small failures may result in huge financial and/or human losses. These failures may be caused by external attacks that manipulate the sensor measurements received by the controller or the supervisor. This class of attacks is called sensor deception attacks.

Prior work on sensor deception attacks in the field of Discrete Event Systems (DES) focuses on characterizing successful attack strategies for *fixed supervisors* (Meira-Góes *et al.*, 2017; Su, 2018; Meira-Góes *et al.*, 2019*a*; Meira-Góes *et al.*, 2019*c*), on designing intrusion detection modules for *fixed supervisors* (Thorsley and Teneketzis, 2006; Carvalho *et al.*, 2018; Lima *et al.*, 2019), or on designing robust supervisors against sensor deception attacks (Wakaiki *et al.*, 2018; Su, 2018; Meira-Góes *et al.*, 2019*d*; Meira-Góes *et al.*, 2019*b*); see (Rashidinejad *et al.*, 2019) for a review in this area. These works mostly focus on the effect of the attacker on the supervisory control framework. Formally, the attacker is modelled by an attack function that edits the behavior generated by the system and feeds this edited behavior to the supervisor. However, these recent results do not provide computationally efficient methods, especially in the case of partially observed systems. For more information on the supervisory control framework under sensor deception attacks see (Meira-Góes *et al.*, 2017; Meira-Góes *et al.*, 2019*a*).

This work focuses on synthesizing successful sensor deception attack strategies for *fixed supervisors* in an efficient manner. The work of (Meira-Góes *et al.*, 2019*a*) provides a methodology to solve this problem based on a game-graph structure called the All Insertion-Deletion Attack (AIDA) structure. The AIDA structure captures the interaction between the supervisor and the environment (which includes the system and the attacker). Based on the AIDA, a successful attack strategy, if one exists, can be obtained.

The construction of the AIDA is based on the discrete model of the system and the supervisor that contains admissible control decisions and differentiates normal/abnormal behavior. Usually the discrete model of a cyber-physical system is complex and as a result the computation of the AIDA could be potentially costly. To mitigate this issue, in this paper an abstraction method is investigated to reduce the size of the system before calculating the AIDA. The proposed abstraction method

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is based on observation equivalence, which is a well-known abstraction method (Milner, 1989). Observation equivalence considers states as equivalent if they have the same future behavior. Observation equivalence however, can not be used to abstract a system *before* calculating the AIDA and some adjustments are necessary. Abstraction-based observation equivalence was used in (Zhang and Zamani, 2017; Mohajerani and Lafortune, 2019; Mohajerani *et al.*, 2019) in opacity setting.

This paper introduces a *restricted version of observation equivalence*, which is used to abstract the system before calculating the AIDA. Since the abstraction method reduces the size of the system by merging some states the computational complexity of the AIDA can be reduced. Moreover, the abstraction method can be calculated in polynomial time and it guarantees that successful attack functions synthesized from the abstracted system can be exactly mapped to successful attack functions on the original system.

The presentation of our results is organized as follows. Sect. 2 gives a brief background on modeling and the All Insertion-Deletion Attack structure. Next, Sect. 3 explains the abstraction method that is used in this paper and shows how it is leveraged for synthesizing successful attack functions. Finally, some concluding remarks are given in Sect. 4.

## 2. SYSTEM AND ATTACK MODEL

### 2.1 Supervisory Control System Model

Discrete system behaviors can be modeled by deterministic or nondeterministic automata.

*Definition 1.* A (nondeterministic) finite-state automaton is a tuple $G = \langle \Sigma, Q, \rightarrow, x^\circ \rangle$, where $\Sigma$ is a finite set of events, $Q$ is a finite set of states, $\rightarrow \subseteq Q \times \Sigma \times Q$ is the *state transition relation*, and $x^\circ \subseteq Q$ is the *initial state*. $G$ is *deterministic*, if $x \xrightarrow{\sigma} y_1$ and $x \xrightarrow{\sigma} y_2$ always implies $y_1 = y_2$.

$\Sigma^*$ is the set of all finite traces of events from $\Sigma$, including the *empty trace* $\epsilon$.

The transition relation is written in infix notation $x \xrightarrow{\sigma} y$, and is extended to strings in $\Sigma^*$ by letting $x \xrightarrow{\epsilon} x$ for all $x \in Q$, and $x \xrightarrow{t\sigma} z$ if $x \xrightarrow{t} y$ and $y \xrightarrow{\sigma} z$ for some $y \in Q$. Furthermore, $x \xrightarrow{t}$ means that $x \xrightarrow{t} y$ for some $y \in Q$, and $x \rightarrow y$ means that $x \xrightarrow{t} y$ for some $t \in \Sigma^*$. These notations also apply to state sets, $X \xrightarrow{t} Y$ for $X, Y \subseteq Q$ means that $x \xrightarrow{t} y$ for some $x \in X$ and $y \in Y$, and to automata, $G \xrightarrow{t}$ means that $x^\circ \xrightarrow{t}$, etc.

The *language* of an automaton $G$ is $\mathcal{L}(G) = \{ s \in \Sigma^* \mid G \xrightarrow{s} \}$. In addition, for $q \in Q$ let $\Gamma_G(q) = \{e \in \Sigma | q \xrightarrow{e}\}$ be the set of active events at state $q$. By a slight abuse of notation, we write $\Gamma_G(S) = \cup_{q \in S} \Gamma_G(q)$ for $S \subseteq Q$.

One common automaton operation is the *quotient* modulo an equivalence relation on the state set.

*Definition 2.* Let $Z$ be a set. A relation $\sim \subseteq Z \times Z$ is called an *equivalence relation* on $Z$ if it is reflexive, symmetric, and transitive. Given an equivalence relation $\sim$ on $Z$, the *equivalence class* of $z \in Z$ is $[z] = \{ z' \in Z \mid z \sim z' \}$, and $\tilde{Z} = \{ [z] \mid z \in Z \}$ is the set of all equivalence classes modulo $\sim$.

*Definition 3.* Let $G = \langle \Sigma, Q, \rightarrow, x^\circ \rangle$ be an automaton and let $\sim \subseteq Q \times Q$ be an equivalence relation. The *quotient automaton* of $G$ modulo $\sim$ is

$$\tilde{G} = \langle \Sigma, \tilde{Q}, \rightarrow/\sim, \tilde{x}^\circ \rangle \ , \tag{1}$$

where $\rightarrow/\sim = \{ ([x], \sigma, [y]) \mid x \xrightarrow{\sigma} y \}$ and $\tilde{x}^\circ = [x^\circ]$.

In supervisory control theory, an automaton $G$, called the plant, models the uncontrolled behavior of a system. This automaton is controlled by a supervisor $S_P$ that dynamically enables and disables the controllable events such that it enforces some safety property on $G$. The limited actuation capabilities of $G$ are modeled by a partition of the event set $\Sigma = \Sigma_c \cup \Sigma_{uc}$, where $\Sigma_{uc}$ is the set of uncontrollable events and $\Sigma_c$ is the set of controllable events. In the notation of the theory of supervisory control of DES initiated in (Ramadge and Wonham, 1987), the resulting controlled behavior is a new DES denoted by $S_P/G$ and resulting in the closed-loop language $\mathcal{L}(S_P/G)$, which is defined in the usual manner (Cassandras and Lafortune, 2008). The set of admissible control decisions is defined as $\mathbf{\Gamma} = \{\gamma \subseteq \Sigma \mid \Sigma_{uc} \subseteq \gamma\}$, where admissibility guarantees that a control decision never disables uncontrollable events.

In addition, due to the limited sensing capabilities of $G$, the event set is also partitioned into $\Sigma = \Sigma_o \cup \Sigma_{uo}$, where $\Sigma_o$ is the set of observable events and $\Sigma_{uo}$ is the set of unobservable events. Based on this second partition, the *projection* function $P : \Sigma^* \rightarrow \Sigma_o^*$ is defined as $P(\epsilon) = \epsilon$, and for $s \in \Sigma^*$, $e \in \Sigma$ then $P(se) = P(s)e$ if $e \in \Sigma_o$ or $P(se) = P(s)$ if $e \in \Sigma_{uo}$. The inverse projection $P_o^{-1} : \Sigma_o^* \rightarrow 2^{\Sigma^*}$ is defined as $P_o^{-1}(t) = \{s \in \Sigma^* | P(s) = t\}$. For brevity, $p \overset{s}{\Rightarrow} q$, with $s \in \Sigma_o^*$, denotes the existence of a string $t \in \Sigma^*$ such that $P(t) = s$ and $p \xrightarrow{t} q$. Similarly, $p \Rightarrow q$ means there exists $t \in \Sigma_{uo}^*$ such that $p \xrightarrow{t} q$.

Formally, a partial observation supervisor is a function $S_P : P(\mathcal{L}(G)) \rightarrow \mathbf{\Gamma}$. Without loss of generality, we assume that $S_P$ is realized (i.e., encoded) as a deterministic automaton $R = (Q_R, \Sigma, \rightarrow_R, q_0)$, see (Cassandras and Lafortune, 2008). Based on supervisor $R$, we construct a supervisor $R_{dead}$ that is equivalent to $R$ but captures $P(\mathcal{L}(R/G))$ and also detects the language $P(\mathcal{L}(G)) \setminus P(\mathcal{L}(R/G))$. Formally, $R_{dead} = (Q_{R_{dead}} = Q_R \cup \{dead\}, \Sigma, \rightarrow_d, q_0)$ is a copy of $R$ augmented with a deadlock state called *dead* that is only reached via strings in $P(\mathcal{L}(G)) \setminus P(\mathcal{L}(R/G))$.

The supervisor $R_{dead}$ allows us to merge an intrusion detection mechanism with supervisor $R$, where strings that reach the *dead* state are detectable.

For convenience, we define two operators that are used in this paper together with some useful notation. The *unobservable reach* of the subset of states $S \subseteq Q$ under the subset of events $\gamma \subseteq \Sigma$ is given by:

$$UR_\gamma(S) := \{x \in Q | (\exists u \in S)(\exists s \in (\Sigma_{uo} \cap \gamma)^* \text{ s.t. } u \xrightarrow{s} x)\} \tag{2}$$

The *observable reach* (or next states) of the subset of states $S \subseteq Q$ given the execution of the observable event $e \in \Sigma_o$ is defined as:

$$NX_e(S) := \{x \in Q | \exists u \in S \text{ s.t. } u \xrightarrow{e} x\} \tag{3}$$

For any string $s \in \Sigma^*$, let $s^i$ denote the prefix of $s$ with the first $i$ events, and let $e_s^i$ be the $i^{th}$ event of $s$, so that $s^i = e_s^1 \dots e_s^i$; by convention, $s^0 = \epsilon$. We define $\bar{s}$ as the set of prefixes of string $s \in \Sigma^*$. Lastly, we denote by $\mathbb{N}$, $\mathbb{N}^+$, and $\mathbb{N}^n = \{0, \dots, n\}$ the sets of natural numbers, positive natural numbers, and natural numbers bounded by $n$, respectively.

### *2.2 The Insertion-Deletion Attack Structure*

The problem introduced in (Meira-Góes *et al.*, 2017; Meira-Góes *et al.*, 2019*a*) is to synthesize an attack function such that a critical state is reachable and the state *dead* is unreachable in the attacked controlled behavior.

In (Meira-Góes *et al.*, 2019*a*), the framework of supervisory control under sensor deception attacks is introduced, where an attacker is modelled by an attack function. We assume that $Q$ contains a set of *critical states* defined as $Q_{crit} \subset Q$, which are never reached when $S_P$ controls $G$ and no attacker is present. The problem of synthesizing "successful" attack functions is posed, where successful means an attacker that causes the system to reach $Q_{crit}$ without being detected. A two-player structure called the All-Insertion-Deletion Attack structure (AIDA) is introduced in (Meira-Góes *et al.*, 2019*a*) as a general solution methodology to synthesize stealthy deception attacks against a fixed supervisor $R_{dead}$. The AIDA captures the interaction between the supervisor and the environment that is defined by the plant executions and attacker actions.

Namely, the AIDA enumerates all deception attack actions and all possible plant executions based on the control decisions of the supervisor. Thus, we define the *pair* $IS \in 2^Q \times Q_{R_{dead}}$ to be the information state, and $I = 2^Q \times Q_{R_{dead}}$ the set of all information states. As defined, an $IS$ embeds the necessary information for either the supervisor or the environment to make a decision.

In order to construct the AIDA the set of compromised events $\Sigma_a \subseteq \Sigma_o$ needs to be introduced. This event set specifies the events that the attacker can edit. To identify the attacker actions, let $\Sigma^i = \{e_i | e \in \Sigma_a\}$ and $\Sigma^d = \{e_d | e \in \Sigma_a\}$ be the sets of inserted and deleted events, respectively. These events sets clearly identify the attacker actions at the communication channel between the plant $G$ and the supervisor $R_{dead}$. Note that the subscripts are introduced for convenience and the supervisor receives the actual event execution in $G$. Let $\Sigma_m = \Sigma \cup \Sigma^i \cup \Sigma^d$ be the complete event set.

We define $\mathcal{M}(e_i) = \mathcal{M}(e_d) = \mathcal{M}(e) = e$ for $e \in \Sigma$, $P^G(e) = \mathcal{M}(e)$ for $e \in \Sigma \cup \Sigma^d$ and $P^G(e) = \epsilon$ for $e \in \Sigma^i$, and $P^S(e) = \mathcal{M}(e)$ for $e \in \Sigma \cup \Sigma^i$ and $P^S(e) = \epsilon$ for $e \in \Sigma^d$. The mask $\mathcal{M}$ removes subscripts, when present, from events in $\Sigma_m$, the projection $P^G$ projects an event in $\Sigma_m$ to its actual event execution in $G$, and the projection $P^S$ projects an event in $\Sigma_m$ to its event observation by $R_{dead}$.

For simplicity, in Def. 4 $\mu_d$ is considered as the transition function of $R_{dead}$, where $\mu_d(q, e) = p$ is the same as $q \xrightarrow{e}_d p$.

*Definition 4.* (Meira-Góes *et al.*, 2019*a*) An All-Insertion-Deletion Attack structure (AIDA) $A$ w.r.t. $G$, $\Sigma_a$, and $R_{dead}$, is defined as

$$A = (Q_S, Q_E, h_{SE}, h_{ES}, \Sigma_m, y_0) \tag{4}$$

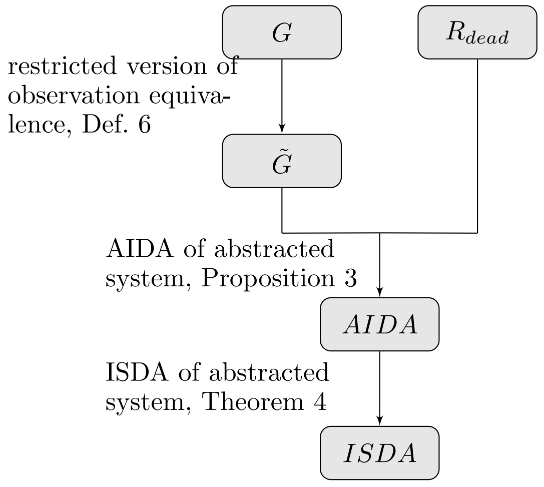
where:

- $Q_S \subseteq I$ is the set of $S$-states, where S stands for Supervisor and where each $S$-state is of the form $y = (I_G(y), I_S(y))$, where $I_G(y)$ and $I_S(y)$ denote the state estimate of the plant and the state of the supervisor, respectively;
- $Q_E \subseteq I$ is the set of $E$-states, where E stands for Environment; each $E$-state is defined in the same way as in the $S$-states case;
- $h_{SE} : Q_S \times \mathbf{\Gamma} \to Q_E$ is the partial transition function from $S$-states to $E$-states, defined for $y \in Q_S$ and $\gamma = \Gamma_{R_{dead}}(I_S(y))$ as:
$$h_{SE}(y, \gamma) := (UR_\gamma(I_G(y)), I_S(y)) \tag{5}$$
- $h_{ES} : Q_E \times (\Sigma_o \cup \Sigma^i \cup \Sigma^d) \to Q_S$ is the partial transition function from $E$-states to $S$-states. It is defined as follows: for any $y \in Q_S$, $(z_G, z_S) \in Q_E$ and $e \in \Sigma_o \cup \Sigma^i \cup \Sigma^d$, $h_{ES}((z_G, z_S), e) := y$ where:
$$y = \begin{cases} (NX_e(z_G), \mu_d(z_S, P^S(e))) & \text{if} \\ & e \in \Gamma_{R_{dead}}(z_S) \cap \Gamma_G(z_G) \\ (z_G, \mu_d(z_S, P^S(e))) & \text{if } e \in \Sigma^i \text{ and} \\ & \mathcal{M}(e) \in \Gamma_{R_{dead}}(z_S) \\ (NX_{P^G(e)}(z_G), z_S) & \text{if } e \in \Sigma^d \text{ and} \\ & \mathcal{M}(e) \in \Gamma_{R_{dead}}(z_S) \cap \Gamma_G(z_G) \end{cases} \tag{6}$$
- $y_0 := (\{x^\circ\}, q_0) \in Q_S$ is the initial state.

An $S$-state is an $IS$ where the supervisor issues its control decision and an $E$-state is an $IS$ at which the environment (system or attacker) selects one among the observable events to occur. A transition from an $S$-state to an $E$-state represents the updated unobservable reach in $G$'s state estimate together with the current supervisor state. On the other hand, a transition from an $E$-state to an $S$-state represents the "observable reach" immediately following the execution of the observable event by the environment. In this case, both the system's state estimate and the supervisor's state are updated. However, these updates depend on the type of event generated by the environment: (i) true system event unaltered by the attacker; (ii) (fictitious) event insertion by the attacker; or (iii) deletion by the attacker of an event just executed by the system. Thus, the transition rules are split into three cases as defined by Equation (6).

Definition 4 also defines a construction algorithm of the AIDA, where starting from the initial state $y_0$ a breadth first search is performed to compute all states and transitions in the AIDA. Since the goal of the attacker is to reach any state in $Q_{crit}$, no transitions are defined in $E$-states $z \in Q_E$ that satisfy $I_G(z) \subseteq Q_{crit}$.

After calculating the AIDA of a system a pruning process that removes non-stealthy attacks from the AIDA is applied. This pruning process can be mapped to a supervisory control problem, as explained in (Meira-Góes *et al.*, 2019*a*). Specifically, any event $e \in \Sigma_a \cup \Sigma^i \cup \Sigma^d$ is considered controllable and any event $e \in \Sigma_o \setminus \Sigma_a$ is uncontrollable. The plant is AIDA $A$ and the specification $A_{trim}$ is obtained by removing all states of the AIDA where the supervisor $R_{dead}$ reaches a *dead* state, $y = (S, dead)$. The pruning process in (Meira-Góes *et al.*, 2019*a*) is a fixed-point algorithm that removes states from $A_{trim}$ iteratively until convergence. The pruned AIDA, where only



G
$R_{dead}$
restricted version of observation equivalence, Def. 6
$\tilde{G}$
AIDA of abstracted system, Proposition 3
AIDA
ISDA of abstracted system, Theorem 4
ISDA

Fig. 1. The steps of calculating the abstracted ISDA of system $G$.

stealthy attack functions are present, is denoted as the Interruptible Stealthy Deceptive Attack (ISDA) structure.

## 3. ABSTRACTED INSERTION-DELETION ATTACK STRUCTURE

The objective of this paper is to calculate an abstracted ISDA with less states and the same interruptible and stealthy attacks as the original ISDA. In this section the proposed algorithm to calculate the abstracted ISDA is described. Fig. 1 gives an overview of the methodology to construct the abstraction-based ISDA. The input to the algorithm is a nondeterministic automaton $G$ and its supervisor $R_{dead}$. In the proposed algorithm the nondeterministic system $G$ is first abstracted using a restricted version of *observation equivalence* before the AIDA is calculated. This leads to an AIDA with less states compared to the original AIDA. Finally, using the abstracted AIDA, we show that the abstracted ISDA contains the same interruptible and stealthy attack strategies as the original ISDA.

In the following, first, the restricted version of observation equivalence is defined. Next, it is shown that the abstracted AIDA and the original AIDA contain the same attack strategies.

### *3.1 Observation Equivalence*

This section presents an abstraction method that can be used to abstract a nondeterministc automaton before calculating the AIDA of the system. The abstraction method is based on *observation equivalence*, which is computationally efficient and can be calculated in polynomial-time.

Observation equivalence or weak bisimulation is a widely-used notion of abstraction that merges states with the same future behavior when unobservable events are disregarded.

*Definition 5.* (Milner, 1989) Let $G = \langle \Sigma, Q, \rightarrow, x^\circ \rangle$, be a nondeterministic automaton. An equivalence relation $\approx\, \subseteq Q \times Q$ is called an *observation equivalence* on $G$, if the following holds for all $x_1, x_2 \in Q$ such that $x_1 \approx x_2$: if $x_1 \overset{s}{\Rightarrow} y_1$ for some $s \in \Sigma^*$, then there exists $y_2 \in Q$ such that $x_2 \overset{s}{\Rightarrow} y_2$, and $y_1 \approx y_2$.

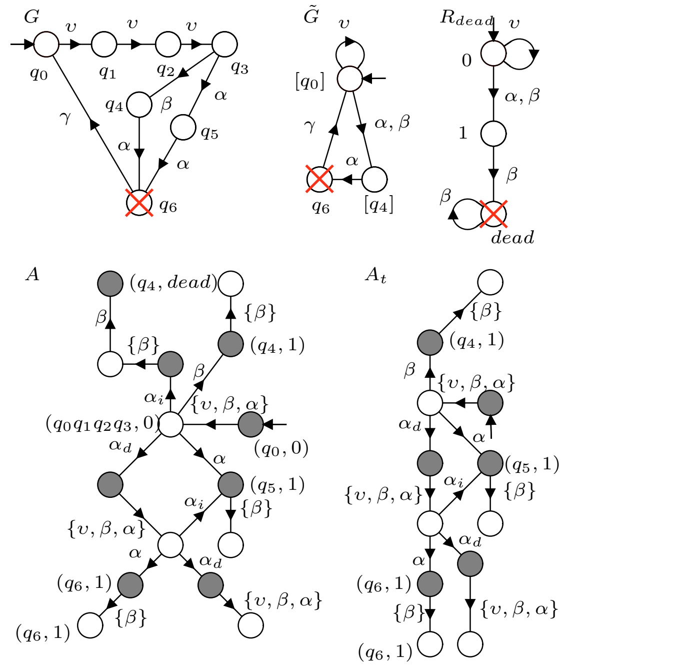


Fig. 2. Automata of Example 1.

In order to be able to use observation equivalence in the attack framework of this paper, we need to consider the critical states.

*Definition 6.* Let $G = \langle \Sigma, Q, \rightarrow, x^\circ \rangle$ be a non-deterministic automaton. An equivalence relation $\sim\, \subseteq Q \times Q$ is called *restricted version of observation equivalence* on $G$, if the following holds for all $x_1, x_2 \in Q$ such that $x_1 \sim x_2$:

- if $x_1 \overset{\sigma}{\rightarrow} y_1$ and $\sigma \in \Sigma_o$ then $x_2 \overset{\sigma}{\rightarrow} y_2$ and $y_1 \sim y_2$
- if $x_1 \overset{\sigma}{\rightarrow} y_1$ and $\sigma \in \Sigma_{uo}$ then $x_2 \overset{\sigma}{\Rightarrow} y_2$ and $y_1 \sim y_2$
- $x_1 \in Q_{crit}$ if and only if $x_2 \in Q_{crit}$

In the restricted version of observation equivalence, similar to observation equivalence, two states are equivalent if they have the same future behavior. Moreover, the restricted version of observation equivalence requires the equivalent states to have the same critical status, i.e., either all are critical or none of them are critical states.

*Example 1.* Consider the system $G$ with the set of critical states $Q_{crit} = \{q_6\}$, $\Sigma_{uc} = \{\beta\}$, $\Sigma_{uo} = \{v\}$ and the compromised event set $\Sigma_a = \{\alpha\}$ . Automaton $G$ and $R_{dead}$ are shown in Fig. 1. The figure also shows the original AIDA of the system, $A$. In the figure the initial states are marked by an arrow pointing into them, the $S$-states of $A$ are shaded grey and the critical states of $G$ and the *dead*-state of the supervisor $R_{dead}$ are crossed out. The initial state of $A$ is $(q_0, 0)$. The active event of $R_{dead}$ at state 0 is $\Gamma_{R_{dead}}(0) = \{v, \beta, \alpha\}$ and $UR_{\{v,\beta,\alpha\}} = \{q_0, q_1, q_2, q_3\}$. Thus, event $\{v, \beta, \alpha\}$ is executable at the initial state of $A$ leading the AIDA to the $E$-state $(q_0q_1q_2q_3, 0)$. Now consider event $\beta$, which is an active event at state $q_3$ of $G$. As $\beta \in (\Gamma_{R_{dead}}(z_S) \cap \Gamma_G(z_G))$ it holds that $\big(NX_e(\{q_0q_1q_2q_3\}), \mu_d(0, \beta)\big) = (q_4, 1)$ is an $S$-state of AIDA $A$ and it is reached from state $(q_0q_1q_2q_3, 0)$ by executing $\beta$. The whole structure is interpreted in a similar way. To obtain the ISDA, we must eliminate the non-stealthy strategies. Applying the pruning process previously described in Sect. 2, we obtain the ISDA $A_t$

depicted in Fig. 2. State $q_6$ is reachable in $A_t$, which implies that there exists a successful stealthy attack function.

In the framework considered in this paper, the system automaton $G$ is abstracted by applying the restricted version of observation equivalence before the construction of the AIDA. States $q_0$, $q_1$ and $q_2$ are equivalent as $q_i \Rightarrow q_3$ for $i = 0, 1, 2$. Moreover, states $q_4$ and $q_5$ are also equivalent as $q_4 \overset{\alpha}{\to} q_6$ and $q_5 \overset{\alpha}{\to} q_6$. Merging equivalent states results in automaton $\tilde{G}$ shown in Fig. 1. In the figure the critical states of $\tilde{G}$ are crossed out.

### *3.2 AIDA with Abstracted Model*

In the previous section, we have introduced the restricted version of observation equivalence as a mean to abstract a system before calculating the AIDA of the system. In this section, we prove that the abstracted AIDA and the original AIDA contain the same attack strategies if the restricted version of observation equivalence is used to abstract the system.

In the following, Proposition 3 establishes that the abstracted AIDA and the original AIDA contain the same attack strategies. Using the results from Proposition 3, Theorem 4 proves that the abstracted ISDA embeds all possible interruptible stealthy insertion-deletion attack strategies. First Lemmas 1 and 2 provide auxiliary results to prove Proposition 3.

*Lemma 1.* Let $G = \langle \Sigma, Q, \to, x^\circ \rangle$ and let $\sim$ be the restricted version of observation equivalence. Let $\tilde{G}$ be the quotient automaton of $G$ by applying $\sim$. Consider $S \subseteq Q$ and $\tilde{S} \subseteq \tilde{Q}$ such that $x \in S$ if and only if $[x] \in \tilde{S}$, where $x \in [x]$. Then $y \in UR_\gamma(S)$ if and only if $[y] \in UR_\gamma(\tilde{S})$, where $y \in [y]$.

**Proof.** ($\Rightarrow$) Assume that $y \in UR_\gamma(S)$. Based on (2), it holds that there exists $t \in (\Sigma_{uo} \cap \gamma)^*$ such that $x \overset{t}{\to} y$. Based on Def. 6 it holds that there exists $t' \in (\Sigma_{uo} \cap \gamma)^*$ such that $[x] \overset{t}{\to} [y]$ and $y \in [y]$.

($\Leftarrow$) A similar argument as above holds.

□

Lemma 1 proves that the unobservable reach of the two subsets $S$ and $\tilde{S}$, where the states of $\tilde{S}$ are restricted observation equivalent to the states of $S$, are also restricted observation equivalent.

*Lemma 2.* Let $G = \langle \Sigma, Q, \to, x^\circ \rangle$ and let $\sim$ be the restricted version of observation equivalence on $G$. Consider $S \subseteq Q$ and $\tilde{S} \subseteq \tilde{Q}$ such that $x \in S$ if and only if $[x] \in \tilde{S}$, where $x \in [x]$. Then $y \in NX_e(S)$ if and only if $[y] \in NX_e(\tilde{S})$, where $y \in [y]$.

**Proof.** ($\Rightarrow$) Assume that $y \in NX_e(S)$. Then based on (3) it holds that there exists $x \overset{e}{\to} y$. Based on Def. 6 it holds that $[x] \overset{e}{\to} [y]$ and $y \in [y]$.

($\Leftarrow$) A similar argument as above holds.

□

Similarly to Lemma 1, Lemma 2 proves that the observable reach of the two subsets that contain restricted observation equivalent states are also restricted observation equivalent.

*Proposition 3.* Let $G = \langle \Sigma, Q, \to, x^\circ \rangle$ and let $\sim$ be the restricted version of observation equivalence on $G$. Let $A$ be the AIDA with respect to $G$, $\Sigma_a$ and $R_{dead}$ and $\tilde{A}$ be the AIDA with respect to $\tilde{G}$, $\Sigma_a$ and $R_{dead}$. Then $A \overset{\omega}{\to} p$ if and only if $\tilde{A} \overset{\omega}{\to} \tilde{p}$ such that:

(i) $p$ is an $S$-state, $E$-state, if and only if $\tilde{p}$ is an $S$-state, $E$-State,
(ii) $I_G(p) \subseteq Q_{crit}$ if and only if $I_{\tilde{G}}(\tilde{p}) \subseteq \tilde{Q}_{crit}$,
(iii) $I_S(p) = dead$ if and only $I_S(\tilde{p}) = dead.$

**Proof.** We show that a transition is defined in $A$ if and only if the same transition is defined in $\tilde{A}$. It is shown by induction on $n \geq 0$ that $y_0 \overset{\omega}{\to} p_n$ in $A$ if and only if $\tilde{y}_0 \overset{\omega}{\to} \tilde{p}_n$ in $\tilde{A}$. Note that $p_n$ or $\tilde{p}_n$ consist of $S$-states and $E$-states. Since there is no reduction on $R_{dead}$ it holds that $I_S(p_n) = I_S(\tilde{p}_n)$.

*Base case:* ($\Rightarrow$) Let $\tilde{x}^\circ$ be the initial state of $\tilde{G}$ and $q^\circ$ be the initial state of $R_{dead}$. Then the initial state of $A$ is $y_0 = (\{x^\circ\}, q^\circ)$ and the initial state of $\tilde{A}$ is $\tilde{y}_0 = (\{\tilde{x}^\circ\}, q^\circ)$ and they both are $S$-states. First assume that $y_0 \overset{\gamma_0}{\to} z_0$ in $A$, which is an $h_{SE}$ transition. We need to show that there exists $\tilde{y}_0 \overset{\gamma_0}{\to} \tilde{z}_0$ in $\tilde{A}$. Based on Def. 4 it holds that $I_G(z_0) = UR_{\gamma_0}(\{x^\circ\})$, $I_S(z_0) = I_S(y_0) = q^\circ$ and $\gamma_0 = \Gamma_{R_{dead}}(I_S(y_0)) = \Gamma_{R_{dead}}(q^\circ)$. Since $x^\circ \in \tilde{x}^\circ$ it means that $v \in I_G(y_0)$ if and only if there exists $[v] \in I_{\tilde{G}}(\tilde{y}_0)$. Based on Lemma 1 it holds that $u \in UR_{\gamma_0}(\{x^\circ\})$ if and only if there exists $[u] \in UR_{\gamma_0}(\{\tilde{x}^\circ\})$. This means that there exists $I_{\tilde{G}}(\tilde{z}_0) = UR_{\gamma_0}(\{\tilde{x}^\circ\})$, where for every $u \in I_G(z_0)$ there exists a $[u] \in I_{\tilde{G}}(\tilde{z}_0)$. Since $u \in [u]$ it follows from Def. 6 that $[u] \in \tilde{Q}_{crit}$ if $u \in Q_{crit}$, which implies that $I_{\tilde{G}}(\tilde{z}_0) \subseteq \tilde{Q}_{crit}$ if $I_G(z_0) \subseteq Q_{crit}$. Thus, $\tilde{y}_0 \overset{\gamma_0}{\to} \tilde{z}_0$ in $\tilde{A}$, where $I_{\tilde{G}}(\tilde{z}_0) = UR_{\gamma_0}(\{\tilde{x}^\circ\})$, $I_S(\tilde{z}_0) = I_S(\tilde{y}_0) = q^\circ$ and $\gamma_0 = \Gamma_{R_{dead}}(I_S(\tilde{y}_0)) = \Gamma_{R_{dead}}(q^\circ)$.

($\Leftarrow$) Now assume that $\tilde{y}_0 \overset{\gamma_0}{\to} \tilde{z}_0$ in $\tilde{A}$. A similar argument as above holds.

*Inductive step:* Assume that the claim holds for some $n \geq 0$, i.e, $y^0 \overset{\omega}{\to} p_n$ in $A$ if and only if $\tilde{y}^0 \overset{\omega}{\to} \tilde{p}_n$ in $\tilde{A}$, $p_n$ is an $S$-state, $E$-state, if and only if $\tilde{p}_n$ is an $S$-state, $E$-state and $I_G(p_n) \subseteq Q_{crit}$ if and only if $I_{\tilde{G}}(\tilde{p}_n) \subseteq \tilde{Q}_{crit}$, $I_S(p_n) = dead$ if and only $I_S(\tilde{p}_n) = dead$ and $v \in I_G(p_n)$ if and only if there exists $[v] \in I_{\tilde{G}}(\tilde{p}_n)$.

($\Rightarrow$) Now it must be shown that if $p_n \overset{\gamma_n}{\to} p_{n+1}$ in $A$ then $\tilde{p}_n \overset{\gamma_n}{\to} \tilde{p}_{n+1}$ in $\tilde{A}$ such that $I_G(p_{n+1}) \subseteq Q_{crit}$ if and only if $I_{\tilde{G}}(\tilde{p}_{n+1}) \subseteq \tilde{Q}_{crit}$ and $I_S(p_{n+1}) = dead$ if and only $I_S(\tilde{p}_{n+1}) = dead.$ Note that since based on the inductive assumption $p_n$ is an $S$-state, $E$-state, if and only if $\tilde{p}_n$ is an $S$-state, $E$-state, it is clear that $p_{n+1}$ is an $S$-state, $E$-state, if and only if $\tilde{p}_{n+1}$ is an $S$-state, $E$-state.

Consider the following two possibilities:

- $p_n \in Q_S$, which implies that $p_n \overset{\gamma_n}{\to} p_{n+1}$ is an $h_{SE}$ transition. Then based on Def. 4 it holds that $I_G(p_{n+1}) = UR_{\gamma_n}(I_G(p_n))$, $I_S(p_{n+1}) = I_S(p_n)$ and $\gamma_n = \Gamma_{R_{dead}}(I_S(p_n))$. Since based on inductive assumption it holds that $v \in I_G(p_n)$ if and only if there exists $[v] \in I_{\tilde{G}}(\tilde{p}_n)$ it follows from Lemma 1 that $u \in UR_{\gamma_n}(I_G(p_n))$ if and only if there exists $[u] \in UR_{\gamma_n}(I_{\tilde{G}}(\tilde{p}_n))$. This means there exists $I_{\tilde{G}}(\tilde{p}_{n+1}) = UR_{\gamma_n}(I_{\tilde{G}}(\tilde{p}_n))$, where for every $u \in$

$I_G(p_{n+1})$ there exists an $[u] \in I_{\tilde{G}}(\tilde{p}_{n+1})$. Since $u \in [u]$ it holds that $I_G(p_{n+1}) \subseteq Q_{crit}$ if and only if $I_{\tilde{G}}(\tilde{p}_{n+1}) \subseteq \tilde{Q}_{crit}$. Thus, $\tilde{p}_n \xrightarrow{\gamma_n} \tilde{p}_{n+1}$ is in $\tilde{A}$, where $I_{\tilde{G}}(\tilde{p}_{n+1}) = UR_{\gamma_n}(I_{\tilde{G}}(\tilde{p}_n))$, $I_S(\tilde{p}_{n+1}) = I_S(\tilde{p}_n)$ and $\gamma_n = \Gamma_{R_{dead}}(I_S(\tilde{p}_n))$.

- $p_n \in Q_E$, which implies that $p_n \xrightarrow{\gamma_n} p_{n+1}$ is an $h_{ES}$. Then $\gamma_n = e$ and there are three cases:
  - $e \in \Sigma_o \cap \Gamma_{R_{dead}}(I_S(p_n)) \cap \Gamma_G(I_G(p_n))$. Based on Def. 4 it holds that $p_{n+1} = (NX_e(I_G(p_n)), \mu_d(I_S(p_n), e))$. Since based on inductive assumption it holds that $v \in I_G(p_n)$ if and only if there exists $[v] \in I_{\tilde{G}}(\tilde{p}_n)$ it follows from Lemma 2 that there exists $NX_e(I_{\tilde{G}}(\tilde{p}_n))$ such that $u \in NX_e(I_G(p_n))$ if and only if $[u] \in NX_e(I_{\tilde{G}}(\tilde{p}_n))$. Then, based on Def. 6 it holds that $I_G(p_{n+1}) \subseteq Q_{crit}$ if and only if $I_{\tilde{G}}(\tilde{p}_{n+1}) \subseteq \tilde{Q}_{crit}$. Thus, $\tilde{p}_n \xrightarrow{e} \tilde{p}_{n+1}$ in $\tilde{A}$, where $\tilde{p}_{n+1} = (NX_e(I_{\tilde{G}}\ (\tilde{p}_n)), \mu_d(I_S(\tilde{p}_n), e))$.
  - $e \in \Sigma^i$ and $M_e(e) \in \Sigma_a \cap \Gamma_{R_{dead}}(I_G(p_n))$. Based on Def. 4 it holds that $p_{n+1} = (I_G(p_n), \mu_d(I_S(p_n), P^S(e)))$, and $\tilde{p}_{n+1} = (I_{\tilde{G}}(\tilde{p}_n), \mu_d(I_S(\tilde{p}_n), P^S(e)))$.
  - $e \in \Sigma^d$ and $M_e(e) \in \Sigma_a \cap \Gamma_{R_{dead}}(I_G(z)) \cap \Gamma_G(I_G(z))$. Note that $P^G(e) = P^{\tilde{G}}(e)$. Based on Def. 4 it holds that $p_{n+1} = (NX_{P^G(e)}(I_G(p_n)), I_S(p_n))$. Since based on inductive assumption it holds that $v \in I_G(p_n)$ if and only if there exists $[v] \in I_{\tilde{G}}(\tilde{p}_n)$ it follows from Lemma 2 that there exists $NX_{P^G(e)}(I_{\tilde{G}}(\tilde{p}_n))$ such that $u \in NX_{P^G(e)}(I_G\ (p_n))$ if and only if $[u] \in NX_{P^G(e)}(I_{\tilde{G}}(\tilde{p}_n))$. Then based on Def. 6 it holds that $I_G(p_{n+1}) \subseteq Q_{crit}$ if and only if $I_{\tilde{G}}(\tilde{p}_{n+1}) \subseteq \tilde{Q}_{crit}$. Thus, $\tilde{p}_n \xrightarrow{e} \tilde{p}_{n+1}$ in $\tilde{A}$, where $\tilde{p}_{n+1} = (NX_{P^G(e)}\ (I_G(\tilde{p}_n)), I_S(\tilde{p}_n))$.

($\Leftarrow$) It must be shown that if $\tilde{p}_n \xrightarrow{\gamma_n} \tilde{p}_{n+1}$ in $\tilde{A}$ then $p_n \xrightarrow{\gamma_n} p_{n+1}$ in $A$. A similar argument as above holds. □

Proposition 3 proves that a system can be abstracted using the restricted version of observation equivalence before calculating the AIDA and the abstracted AIDA and the original AIDA have the same structure.

*Example 2.* Consider the system $G$ and its corresponding AIDA $A$ shown in Fig. 2. As it was shown in Example 1 automaton $G$ can be abstracted using the restricted version of observation equivalence. Using $\tilde{G}$ and $R_{dead}$ the abstracted AIDA $\tilde{A}$, shown in Fig. 3, can be calculated, which has 2 states less than the original AIDA $A$. Automata $\tilde{G}$, $R_{dead}$ and $A$ are shown in Fig. 2.

After constructing the abstracted AIDA $\tilde{A}$, the non-stealthy interruptible attack strategies must be removed via the pruning process. The following theorem is the main contribution of the paper and it proves that successful attack functions synthesized from the abstracted system can be exactly mapped to successful attack functions on the original system. To establish this result, the theorem proves that the original AIDA $A$ and the abstracted AIDA $\tilde{A}$ are bisimilar and the pruning process removes from $A$ and $\tilde{A}$ bisimilar states. Consequently, the abstracted ISDA $A_t$ and the original ISDA $\tilde{A}_t$ are bisimilar and have the same structure.

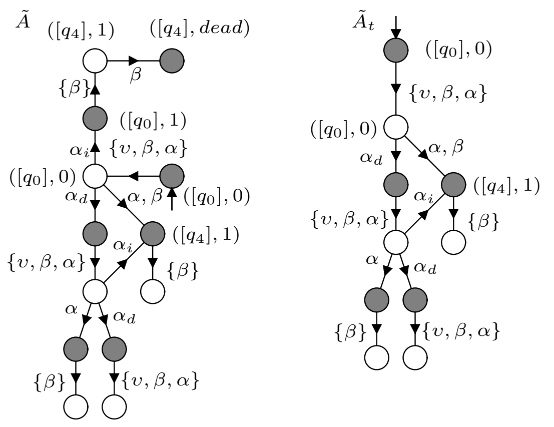


Fig. 3. Automata of Examples 2 and 3. The initial states are marked by an arrow pointing into them and the $S$-states of $\tilde{A}$ and $\tilde{A}_t$ are shaded grey.

*Theorem 4.* Let $G = \langle \Sigma, Q, \rightarrow, x^\circ \rangle$ and let $\sim$ be the restricted version of observation equivalence on $G$. Let $\tilde{A}_t$ be the ISDA with respect to $\tilde{G}$, $\Sigma_a$ and $R_{dead}$. Then $\tilde{A}_t$ embeds all possible interruptible stealthy insertion-deletion attack strategies.

**Proof.** To prove the theorem we show the following:

(i) the AIDA of the original system and the abstracted system are bisimilar,
(ii) the specification of the original system and the abstracted system are bisimilar,
(iii) the pruning process removes from $A$ and $\tilde{A}$ bisimilar states.

(i): Let $A$ be the AIDA based on $\Sigma_a$ and $R_{dead}$ and $G$ and $\tilde{A}$ be the AIDA based on $\Sigma_a$, $R_{dead}$ and $\tilde{G}$. It was shown in Proposition 3 that $A \xrightarrow{\omega}$ if and only $\tilde{A} \xrightarrow{\omega}$, which implies that $A$ and $\tilde{A}$ are bisimilar.

(ii): Let $A_{trim}$ and $\tilde{A}_{trim}$ be the specifications obtained by removing states of $A$ and $\tilde{A}$, where $I_S(p) = dead$ and $I_S(\tilde{p}) = dead$. Based on Proposition 3 it holds that the $A \xrightarrow{\omega} p$ if and only if $\tilde{A} \xrightarrow{\omega} \tilde{p}$ and $I_S(p) = dead$ if and only $I_S(\tilde{p}) = dead$, which implies that the specifications $A_{trim}$ and $\tilde{A}_{trim}$ are bisimilar.

(iii): It has been proven in (Mohajerani *et al.*, 2014) that the synthesis procedure that removes uncontrollable states from two deterministic bisimilar automata produces bisimilar supervisors. This proves that the uncontrollable states removed from $A$ and $\tilde{A}$ are bisimilar. The pruning process in (Meira-Góes *et al.*, 2019*a*) is a fixed-point algorithm that also removes all the $E$-states $q \in Q_{A^i}$ such that $e \in \Gamma_A(q)$ but both $e$ and $e_d$ are absent from $\Gamma_{A^i}(q)$, where $A^i$ is an intermediate result in the fixed-point pruning process. Now we need to prove that bisimilar $E$-states are removed from $A$ and $\tilde{A}$. This is shown by induction on the iteration steps.

*Base case:* $i = 0$.

($\Rightarrow$) Assume that $q \in Q_{A^0}$. Based on Proposition 3 it holds that if $A \rightarrow q$ then $\tilde{A} \rightarrow \tilde{q}$, which implies that $\tilde{q} \in \tilde{Q}_{\tilde{A}^0}$.

($\Leftarrow$) Assume that $\tilde{q} \in \tilde{Q}_{\tilde{A}^0}$. A similar argument as above holds.

*Inductive steps:* Assume that the claim holds at iteration $n$, i.e, $q \in Q_{A^n}$ if and only if $\tilde{q} \in \tilde{Q}_{\tilde{A}^n}$. Now we need to show that the claim holds for iteration $n+1$. First let $q \in Q_{A^{n+1}}$, which implies that $e \in \Gamma_A(q)$ and $e \in \Gamma_{A^n}(q)$ or $e_d \in \Gamma_{A^n}(q)$. From Proposition 3 it holds that if $A^{n+1} \to q \xrightarrow{e}$ then $\tilde{A}^{n+1} \to \tilde{q} \xrightarrow{e}$, which implies that $e \in \Gamma_{\tilde{A}(\tilde{q})}$. A similar argument as above holds for $A_{trim}$ and $\tilde{A}_{trim}$, which means that $e \in \Gamma_{\tilde{A}_{trim}}(\tilde{q})$ or $e_d \in \Gamma_{\tilde{A}_{trim}}(\tilde{q})$. This proves that $\tilde{q} \in \tilde{Q}_{\tilde{A}^{n+1}}$ if $q \in Q_{A^{n+1}}$. Now assume $\tilde{q} \in \tilde{Q}_{\tilde{A}^{+1}}$. A similar argument as above holds.

This means that $A_t \xrightarrow{\omega} p$ if and only if $\tilde{A}_t \xrightarrow{\omega} \tilde{p}$, which implies that $\tilde{A}_t$ embeds all possible interruptible stealthy insertion-deletion attack strategies. □

*Example 3.* Continuing Example 2, the corresponding abstracted AIDA $\tilde{A}$ is shown in Fig. 3. To calculate the abstracted ISDA, the pruning process explained in Sect. 2 needs be applied on $\tilde{A}$. First, state $([q_4], dead)$ is removed producing specification $\tilde{A}_{trim}$, which makes $E$-state $([q_4], 1)$ and then $S$-state $([q_0], 1)$ uncontrollable. This results in abstracted ISDA $\tilde{A}_t$, shown in Fig. 3, with 2 less states compared to the original $A_t$, shown in Fig. 2.

## 4. CONCLUSION

In this paper, we investigate abstraction-based attack function synthesis when the attacker feeds incorrect information to the supervisor in order to damage the supervised system. To synthesize successful stealthy sensor deception attacks, the All-Insertion-Deletion Attack structure (AIDA) is constructed. Calculating the AIDA can be computationally expensive, specifically in partially observed systems. To mitigate the computational complexity of constructing the AIDA an abstraction method called *restricted version of observation equivalence* is introduced that can abstract the system by merging some states before constructing the AIDA. We prove that the abstracted AIDA provides a complete solution to the attack synthesis problem. It would be of interest to investigate the abstraction methods not only for the system also for the supervisor. Moreover, developing abstraction-based AIDA-like structures for modular systems consisting of interacting subsystems is also of interest.

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