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Sharing Asymmetric Tail Risk
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Abstract

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1 Introduction

The Global Financial Crisis, the sovereign risk crisis in the euro-area, the early effects of the looming Climate Change and more recently the COVID-19 pandemic have progressively exposed the lack of resilience of the global economy to large financial and macroeconomic distress and disasters. Policymakers around the world are bracing for a new global environment with heightened tail risk—the risk of rare but disruptive events. Agents' perceptions of tail risk may hinder economic recovery from large disturbances (Baker et al., 2020), or even weigh on long-term growth prospects (Kozlowski et al., 2020). While the recent large crises have a strong global component, it has become increasingly clear that regions and countries have a very different exposure to disaster shocks. Global crises and tail events transmit quite asymmetrically across borders, widening the international divide in wealth and welfare. Hence tail risk, even when associated to global disturbances, raises the value of international risk sharing, achievable either through capital market development and integration, or through institutional arrangements.

In the context of heightened perception of tail risk, risk sharing arrangements at global level are seen as highly desirable as they give regions and countries opportunities for smoothing consumption and moderate the costs of adjustment to adverse shocks. However, insuring disaster risk has potentially significant macroeconomic and financial implications. For any given distribution of fundamentals, going from a low to a high degree of risk insurance changes the equilibrium valuation of national assets. Any change in asset pricing in turn translates into an equilibrium adjustment in relative wealth and demand, possibly leading to a re-allocation of labor and production across regions and countries. This means that insurance may also affect trade and the international prices of goods.

In this paper, we study the joint financial, macroeconomic and welfare implications of enhancing risk sharing in the presence of disaster risk. In the tradition of open macroeconomics, we focus on GDP fluctuations as the fundamental source of macroeconomic risk. Drawing on financial theory and asset pricing, we bring forward the analysis of kurtosis and skewness, in addition to variance, in the distribution of the variable underlying macroeconomic risk. Relative to the literature, thus, we explicitly account for cross-border heterogeneity in both second and higher moments of the distribution of national GDP. For analytical clarity and tractability, we focus our analysis by contrasting the extreme cases of financial autarky (no role for insurance via financial markets) to complete markets (perfect insurance).[1] We carry out our analysis

[1]This guarantees that, in the absence of economic distortions, trade in financial assets will unam-

both analytically and numerically, setting parameters based on evidence on the GDP distribution across countries.

To motivate our analysis, we present a set of stylized facts on the variance, kurtosis and the skewness of output for a large sample of countries. We show that, first, that there is substantial heterogeneity in these moments across countries. Second, the variance of output is positively correlated with kurtosis but negatively correlated to skewness. As one would expect, especially in light of the past decades of data, higher volatility of output is associated with higher frequency of large downturns.

Our theoretical contribution is twofold. First, we offer a novel decomposition of the gains from risk sharing into a "smoothing effect" (SE) and a "level effect" (LE). The former captures welfare gains from risk diversification in terms of the distribution of marginal utility growth. The latter synthesizes welfare gains or losses through the average consumption of goods and leisure. These in turn materialize via interrelated channels. The relative wealth channel works via the revaluation of a country assets, including physical, financial and human capital, at the equilibrium prices with perfect insurance (relative to imperfect insurance). Asset prices reflect any adjustment not only in the equilibrium discount factor, but also in the (average) international price of a country's output —the good price channel—, associated to the equilibrium reallocation of production and demand. We specifically highlight the terms of trade as a novel channel by which perfect insurance may affect social welfare: for a given relative wealth, the country experiencing an improvement in its average terms of trade gains in higher consumption and lower labor effort. An important advantage of our decomposition consists of clarifying how varying the relative riskiness of national GDPs may move the smoothing and level effects (SE and LE) in opposite directions. Riskier countries gain in terms of smoother consumption and labor, but lose out in terms of average consumption and labor effort. Our decomposition maps these movements into aggregate quantities and prices. Intuitively, these movements are the general equilibrium analog of a premium paid or received by a country to benefit from or offer macro insurance.

Second, we derive analytically and numerically the independent and joint contribution of different moments of the GDP distribution to a country's gains from risk sharing, by different channels. We show that smoothing and level effects tend to compensate each other with asymmetries in second moments (volatility of income)–with substantial macroeconomic adjustment to risk sharing via relative wealth and terms of trade movements. With fat tails asymmetric to the left (third and fourth moments) in

biguously bring about positive welfare gains.

the distribution of GDP, the gains from risk sharing are instead dominated by improvement in consumption smoothing of the riskier country. Relative safe countries gain in terms of higher average asset prices and better average terms of trade. Model simulations accounting for the correlation across moments based on country-pair comparison corroborate the empirical relevance of both the smoothing and the level components of the gains from risk sharing. Overall, tail risk enhances the relative gains from capital market integration of countries that are more exposed to it–our empirically-motivated exercise suggests that riskier countries have a potential relative gains advantage for riskier up to 10 percentage points of the total gains from risk sharing. This relative advantage however results from different contributions from the SE and LE components of the gains across countries. In relative terms, the consumption smoothing component can contribute up to a 15 percentage point advantage for riskier countries. The level effect component produces relative gains and losses up to 10% of the total gains from risk sharing.

We carry out our study, both analytically and numerically, using perturbation methods.[2] For the purpose of studying tail risk, these methods are preferable to alternative (global) methods, as they naturally yield a decomposition of the solution in the higher moments of the data generating process. As a methodological contribution to the literature, we spell out a theoretically consistent algorithm applicable in general equilibrium models. The algorithm yields an efficient solution to the problem of solving for the initial distribution of wealth under complete markets, using perturbation. The usefulness of this contribution is best appreciated in light of a widespread practice in the literature, consisting of omitting the specification of the budget constraint under the assumption of complete markets. This practice is not necessarily consequential when countries are assumed to be sufficiently symmetrical in economic structure and distribution of disturbances. It becomes problematic in exercises realistically allowing for large asymmetries in structure and risk across borders, as omitting the budget constraint from the solution algorithm rules out by construction the level effect component driving the gains from risk sharing.[3]

Analytical tractability allows us to explore in detail how the structure of the economy translate the properties of the fundamental process driving output into income risk. We provide insight on non-loglinearities in the economic structure that drive the transmission of tail risk, and specifically discuss how income risk varies with the intra- and intertemporal elasticity of substitution. For some parameterizations of the model,

[2]See e.g. Holmes (1995), Judd (1998), Schmitt-Grohé and Uribe (2007), Lombardo and Sutherland (2007), Kim et al. (2008), Andreasen et al. (2018), and Lombardo and Uhlig (2018).

[3]It is easy to produce examples where a model solved using this practice erroneously predicts welfare losses for a country when moving from autarky to complete markets.

standard in the literature, we are able to derive sharp and instructive propositions. One instance is the proposition of "equal gains" from risk sharing, applied to symmetric countries differing only in the volatility of output: asymmetries in risk do not translate into differences in welfare gains, but only affect the composition of these gains. The riskier country gains more in consumption smoothing, the other benefits from higher average consumption.

Our decomposition of the gains from risk sharing into smoothing and level effects has policy relevance. In the policy literature, the gains from risk sharing are typically assessed in terms of consumption smoothing only (e.g. Viñals (2015), Constâncio (2016)), i.e. it focuses only on the first leg of our decomposition. Consistently, most of the empirical evidence and indicators of risk sharing published in policy reports are based on measures of consumption volatility and cross country correlations of consumption.[4] Relying on these indicators to assess international risk diversification raises a number of logical issues. As shown in our analysis, the SE is only one component of the total gains from diversification: an assessment of these gains based on these indicators is at best incomplete. When a relatively safe and a relatively risky country integrate their capital markets, it is plausible that the safe country mainly gains in term of higher average wealth and consumption, and despite diversification may even loose out in terms of consumption volatility. By no means this implies that the safe country derives no gain from asset trade, as the wealth and consumption level effects of asset revaluation at the new equilibrium price would more than compensate losses in smoothing, if any. Vice versa, the risky country is likely to lose out in terms of relative wealth and consumption. The "implicit transfer" via asset revaluation and terms of trade adjustment is typically disregarded in policy debate, arguably because it is difficult to quantify. Yet, it is a key channel through which countries benefit from the integration of frictionless financial markets.[5] This channel cannot be ignored in policy debates on the pros and cons of capital market integration among countries differing in their risk profile and economic features.

Our analysis draws on a long standing body of literature highlighting the effects of higher moments on asset prices and risk premia. Early on, Samuelson (1970) already warned against limiting the analysis of optimal portfolio choices to first and

[4]The empirical literature is large. See for example Obstfeld (1994) and the literature review in Kose et al. (2009). One popular approach consists of testing directly the consumption risk-sharing condition, which predicts a perfect correlation of consumption growth of two economies trading in complete financial markets. Another popular approach measures the correlation between domestic consumption growth and domestic output growth: the more a country is globally financially integrated, the less domestic consumption depends on idiosyncratic domestic disturbances.

[5]This point is apparent in the theoretical literature, starting from textbook treatments (e.g. Ljungqvist and Sargent, 2012), and including recent research (e.g. Engel, 2016; Coeurdacier et al., 2019).

second moments (mean-variance models), which in his view can be justified only in limiting cases. In general, higher moments co-determine the values of assets and the degree of hedging they can provide. Ingersoll (1975) emphasizes that skewness plays a role in determining efficient portfolio frontiers, in a way conceptually similar to the role of second moments. Indeed, Kraus and Litzenberger (1976) extend the CAPM to include the third moment, showing that such an extension substantially improves the empirical fit of the model. Harvey and Siddique (2000) and Smith (2007) argue that conditional systematic co-skewness of returns helps addressing the empirical puzzle of the failure of the market beta to explain the cross section of expected returns. More recently, Bekaert and Engstrom (2017) build a consumption-based asset pricing model that can generate skewness of consumption growth as well as the negative correlation of such skewness with the option-implied volatility, as observed in the data.[6] Hence, from an international finance perspective, differences in higher moments of cross-country returns can have important implication for relative asset prices. Finally Fang and Lai (1997) extend the analysis of asset pricing to the fourth moment. They show that investors seek compensation for higher variance and kurtosis, while willing to forgo compensation for higher skewness.

At the same time, our paper follows the tradition of international macroeconomics, stressing that not only relative asset price, but also relative good prices are key to understanding cross-border aggregate risk (eg Cole and Obstfeld, 1991).[7] A direction of research combining insight from finance and open macro to study tail risk is still relatively unexplored in the international macroeconomics literature,[8], but can be expected to become dominant in light of recent crisis (see the growing body of contribution on "growth at risk", e.g. Adrian et al. (2019).)

Our paper speaks directly to the large theoretical and empirical literature on the gains and extent of risk sharing across countries, e.g. Lewis (1996), van Wincoop (1994), van Wincoop (1999), Athanasoulis and van Wincoop (2000) and Lewis and Liu (2015). Our analysis is also in line with recent literature, stressing that, to the extent that cross-border insurance allows countries to reduce their reliance on precautionary saving, it may also lead to significant reallocation of capital across borders (a point discussed by Gourinchas and Jeanne (2006), Coeurdacier et al. (2019)).

[6]Guvenen et al. (2018) show that idiosyncratic income fluctuations display non-Gaussian features. They show that skewness and kurtosis affect the welfare costs of incomplete insurance.

[7]In our analysis, asset and good prices together drive the gains from risk sharing at different "orders of risk". We will specifically detail how GDP volatility translates into income risk, as a function of risk aversion and trade elasticity, i.e. the degree of substitution between domestic and foreign goods.

[8]The international macroeconomics literature has limited the analysis of risk to first and second moments (e.g. Obstfeld and Rogoff, 2000). This is particularly the case for the vast literature that uses second-order approximations to evaluate optimal policies.

The rest of the paper is organized as follows. Section 2 presents stylized facts on the distribution of output growth, showing that variance, skewness and kurtosis (of GDP) are substantially heterogeneous across borders. Section 3 specifies the model. Section 4 presents higher order solution to the model and Section 5 discusses allocations under complete markets. Section 6 carries out an analytical decomposition of the gains from risk sharing by moment and channels. Section 7 presents and discusses our numerical results. Section 8 concludes.

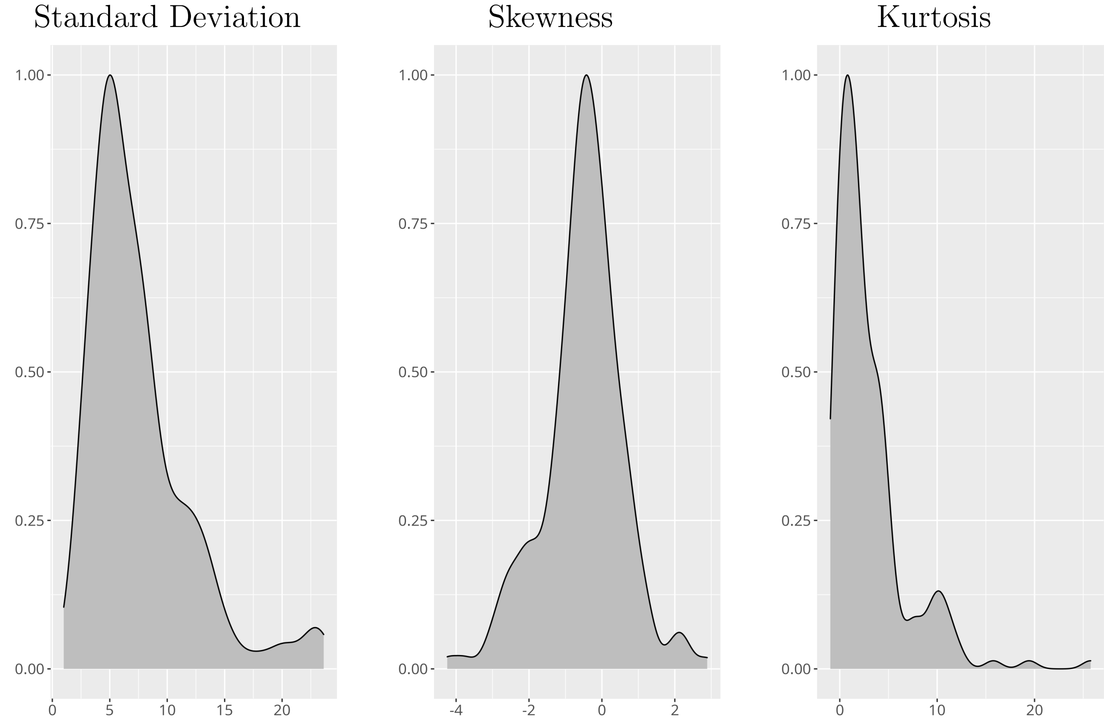
2 Volatility and fat tails in the distribution of output: a cross country analysis

International macroeconomic and finance has long focused on output volatility as the main source of macroeconomic risk driving asset prices and motivating portfolio diversification and risk sharing arrangement across borders (see for example Uribe and Schmitt-Grohé (2017)). The global financial crisis (GFC) and more recently the COVID-19 pandemic together with rising concerns about climate change have brought forward the need to improve our understanding of fat-tail and especially left-tail risks.

To set the stage of our study, we present basic stylized facts about the cross-country joint distribution of output. Figure 1 shows the cross-section density function of the three moments of the first difference of log per-capita real GDP (PWT, 9.1). A first notable result is heterogeneity across countries: all the moments appear to be quite disperse and with a pronounced long tail. A relatively small fraction of countries displays an exorbitant volatility. Moreover, for skewness, very large negative values are much more likely than large positive ones. Table 1 shows percentiles of the distributions. Differences in moments are considerable, even leaving out the extreme tails of the distribution. A country in the 95% of the distribution would have about 97% of the total variance if combined with a country in the lowest 5%, and about 85% of the total variance if combined with the median country. For skewness and kurtosis the 90% intervals are $(-2.53, 1.06)$ and $(-0.43, 10.28)$ respectively. Appendix D shows the three moments of interest for the full list of countries.

The cross-country heterogeneity highlighted by Figure 1 and Table 1 motivates our key question. Holding constant the global economy exposure to the risk of extreme adverse realization of output, how would countries with a more negatively skewed distribution (in the data, skewness for Mexico is -2.289) benefit from sharing aggregate

Figure 1: Distribution of moments across countries



Source: Penn World Table (9.1). GDP per capita at 2011 constant prices; 1960-2017; 156 countries.

output risk with countries with a lower or no exposure to tail risk (in the data, some countries even have a positively skewed distribution of output, as is the case for Ireland, with a skewness of 1.876)?

The second and higher moments of the distribution are not uncorrelated. We illustrate a second important empirical regularity using the simple cross-sectional scatter plot displayed in Figure 2. The two panels report the (log) standard deviation of per-capita GDP growth (first difference of log GDP per capita) on the horizontal axis and the skewness (top panel) and excess kurtosis (bottom panel) of per-capita GDP growth. As shown in the figure, the skewness and kurtosis of per-capita GDP growth are correlated with the standard deviation. In related work (Corsetti et al., 2021), we derive formal results, using both cross section analysis and panel regressions (See also Bekaert and Popov, 2019).

The key takeaway from this section is straightforward. First, there is considerable heterogeneity in all moments of output distribution. Second, in the data, adverse macroeconomic tail risk tends to be associated with higher volatility of output. A conjecture, recently revived by the literature (Kozlowski et al., 2020), is that

Figure 2: GDP-growth: Skewness and kurtosis against standard deviation.

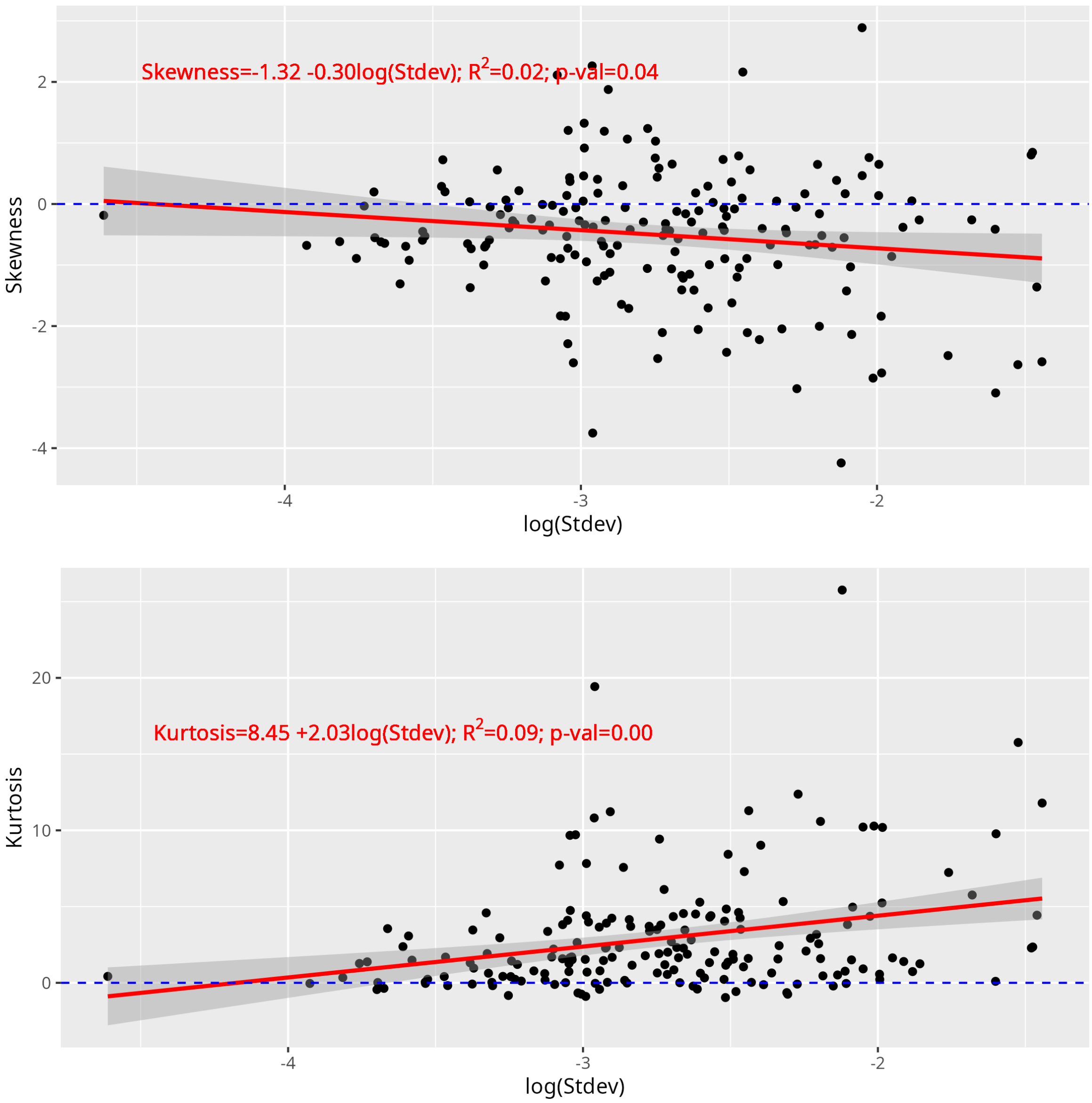


Table 1: Distribution of higher moments of GDP growth

Percentile	Standard Deviation	Skewness	Excess Kurtosis
5%	2.71	-2.53	-0.43
25%	4.70	-1.00	0.43
50%	6.45	-0.42	1.66
75%	9.03	0.09	4.03
95%	15.59	1.06	10.28

Source: Penn World Table (9.1). GDP per capita at 2011 constant prices; 1960-2017; 156 countries.

large shocks, such as COVID-19, may alter the way firms, households and government perceive the risk distribution when taking decisions, possibly exacerbating both heterogeneity and correlation across moments. The observed distribution of moments and their correlation are of course the result of structural factors and policies. Throughout our analysis, however, we will take them as exogenous, consistent with the specific aim of this paper, to inspect the determinants and channels of the gains from sharing macroeconomic risk.

3 Model

Our baseline model is a canonical, discrete time, two-country, frictionless model with differentiated goods and home bias in consumption. In each period the state of the economy consists of a realization of the stochastic total-factor-productivity (TFP) processes and a distribution of financial assets. The two countries, Home (H) and Foreign (F), are heterogeneous in size and distribution of their TFP processes—while symmetric in all other parameters. Foreign variables are denoted by a superscript *. World population size is normalized to unity, with country H population set to $n \in (0, 1)$.

3.1 Firms

Each country produces a differentiated good. In each country an infinite number of firms operate in perfectly competitive markets using a Cobb-Douglas technology in capital (K_t) and labor (L_t), subject to an exogenous stochastic process for TFP. To account for common and country specific component of TFP, we posit that each country's TFP is a geometric average of two underlying stochastic processes, D_t and

D_t^* (discussed further below), with different weights. We write the overall production functions as follows:

$$Y_t = D_t^{\iota} D_t^{*1-\iota} K_t^{\alpha} L_t^{1-\alpha} \tag{3.1}$$
$$Y_t^* = D_t^{1-\iota^*} D_t^{*\iota^*} K_t^{*\alpha} L_t^{*1-\alpha}, \tag{3.2}$$

where $\iota, \iota^*, \alpha \in (0,1)$. Note that we write D_t and D_t^* for convenience. One can think of these processes as sector- or technology-specific, i.e. they don't have a "national" connotation. National TFP nonetheless depends on the country-specific mix adopted by domestic firms.[9] Goods markets are competitive. For tractability, we also posit that the aggregate capital stock is constant throughout the analysis.

In each period the representative firm rents capital at rate $r_{K,t}$ and hires workers at the real wage w_t (in units of the consumption basket) from households to solve the following problem:

$$\min_{K_t, L_t} r_{K,t} K_t + w_t L_t \tag{3.3}$$

subject to the production function (3.1).

The associated factor demands satisfy the following conditions:

$$K_t = p_{H,t} \alpha \frac{Y_t}{r_{K,t}}; \qquad L_t = p_{H,t} (1-\alpha) \frac{Y_t}{w_t}. \tag{3.4}$$

where $p_{H,t}$ is the price of domestic output relative to the consumer price index.

3.2 Consumer Problem

The representative agent in country H consumes a bundle of domestic and foreign goods C_t, trades in units of capital at price $P_{K,t}$, supplies labor L_t at a wage w_t and capital K_t at rate $r_{K,t}$, to firms, and trades in Arrow-Debreu securities A_{t+1}, at the price $\Lambda_{t+1|t}$, that pay one unit of consumption in period $t+1$. Capital must be purchased one

[9] The sectoral composition of output can imply strong cross-country commonalities in TFP disturbances. For example shocks hitting the IT sector (new processors, microchips shortages etc.), the financial sector (new financial products, new payment systems, etc.) or the automobile sector (new engines, new pollution standards, etc.) can simultaneously hit different countries, albeit to different extents. This domestic diversification, generating cross-country commonalities, has strong implications for international risk sharing. Our model of TFP is meant to capture this considerations if only in a stylized way.

period in advance. All prices are in units of the consumption basket. Consumers thus solve the following problem:

$$\max_{C_t, A_{t+1}, K_{t+1}, L_t} \sum_{t=0}^{\infty} E_0 \delta^t U\left(C_t, L_t\right) \tag{3.5}$$

subject to the individual budget constraint:

$$C_t + E_t \Lambda_{t+1|t} A_{t+1} + P_{K,t} K_{t+1} = r_{K,t} K_t + w_t L_t + A_t + P_{K,t} K_t. \tag{3.6}$$

We consider two specifications of preferences, the standard CRRA form, where

$$U\left(C_t, L_t\right) := \frac{C_t^{1-\rho} - 1}{1-\rho} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \tag{3.7}$$

and GHH preferences (Greenwood et al., 1988), where

$$U\left(C_t, L_t\right) := \left(C_t - \chi \frac{L_t^{1+\varphi}}{1+\varphi}\right)^{1-\rho} (1-\rho)^{-1}. \tag{3.8}$$

In either case, $\rho > 0$ is the degree of risk aversion, and $\varphi > 0$ is the inverse Frisch elasticity of labor supply. One reason for using these specifications is comparability: both are ubiquitous in the literature. Another reason is that, unlike CRRA preferences, GHH preferences eliminate the wealth effect on labor supply. The comparison will be useful in contrasting demand and supply effects driving the terms of trade.[10]

Total consumption is a CES function of domestic and foreign goods, i.e.

$$C_t = \left(\nu^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta-1}{\theta}} \tag{3.9}$$

where $C_{H,t}$ $C_{F,t}$ denote consumption of Home and Foreign goods, respectively, $\theta > 0$ is the trade elasticity and $\nu \in (0,1)$ is a function of relative size of countries, and the degree of openness, $\lambda \in (0,1)$, such that $(1-\nu) = (1-n)\lambda$. λ thus measures the degree of home bias in consumption. Similarly, for the foreign representative agent we have the following preferences:

$$C_t^* = \left(\nu^{*\frac{1}{\theta}} C_{H,t}^{*}{}^{\frac{\theta-1}{\theta}} + (1-\nu^*)^{\frac{1}{\theta}} C_{F,t}^{*}{}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta-1}{\theta}} \tag{3.10}$$

[10]It should be noted that he empirical literature finds significant income effects in labor supply estimations (e.g. Attanasio et al., 2018)

with $\nu^* = n\lambda$. The relative prices associated with the above preferences obey the following relationships:

$$1 = \nu p_{H,t}^{1-\theta} + (1-\nu){p_{F,t}}^{1-\theta} \tag{3.11}$$

$$Q_t^{1-\theta} = \nu^* {p_{H,t}}^{1-\theta} + (1-\nu^*){p_{F,t}}^{1-\theta} \tag{3.12}$$

with $Q_t = \frac{S_t P_t^*}{P_t}$ the real exchange rate. Total demand for home and foreign goods are:

$$Y_t = p_{H,t}^{-\theta}(\nu C_t + \nu^* \frac{1-n}{n} Q_t^{\theta} C_t^*) \tag{3.13}$$

$$Y_t^* = p_{F,t}^{-\theta}(\frac{(1-\nu)n}{1-n} C_t + (1-\nu^*) Q_t^{\theta} C_t^*). \tag{3.14}$$

The first order conditions are:

$$C_t : U_C(C_t, L_t) - \zeta_t = 0 \tag{3.15a}$$

$$L_t : U_L(C_t, L_t) + w_t \zeta_t = 0 \tag{3.15b}$$

$$A_{t+1} : \delta\zeta_t - \zeta_{t-1}\Lambda_{t|t-1} = 0 \tag{3.15c}$$

$$K_{t+1} : P_{K,t} = E_t \delta \frac{\zeta_{t+1}}{\zeta_t}\left(r_{k,t+1} + P_{K,t+1}\right), \tag{3.15d}$$

where ζ_t is the Lagrange multiplier on the budget constraint, and $\Lambda_{t|t-1}$ is the stochastic discount factor (SDF).

International trade in Arrow-Debreu securities implies the following risk-sharing condition:

$$\Lambda_{t|t-1}^* = \frac{Q_t}{Q_{t-1}} \Lambda_{t|t-1}. \tag{3.16}$$

The definition of equilibrium is standard and omitted to save space.

3.3 Productivity and Income Risk

Countries are heterogenous in terms of the aggregate TFP process, depending on the country-specific mix of technologies, ι and ι^*. These differences ultimately drive the relative income risk faced by residents in each country.

For each technology, the stochastic process driving TFP follow an AR(1) process in logs:[11]

$$lnD_t = (1 - \varphi_D)\, ln\bar{D} + \varphi_D lnD_{t-1} + \omega\sigma_D\varepsilon_{D,t}; \tag{3.17}$$

where the parameter $\varphi_D \in (0,1)$ measures the persistence of the TFP process, σ_D is the standard deviation of the serially-uncorrelated exogenous innovation $\varepsilon_{D,t}$, ω is the perturbation parameter (identical across countries) such that if $\omega = 0$ the model is deterministic, and a bar over variables indicate the deterministic steady state value. An analogous process drives D^*.[12]

The probability distribution of $\varepsilon_{D,t}$ plays a central role in the analysis. For analytical clarity, we will carry out most of our analysis under the assumption that the *moments* of $log(D_t)$ and $log(D_t^*)$ are mutually independent.

Assumption 1 (Probability distribution)**.**

The probability distribution of the innovation $\varepsilon_{D,t}$ is characterized by the following moments:

$$E\left(\varepsilon_{D,t}\right) = 0 \qquad \text{(Mean)}$$

$$E\left(\varepsilon^i_{D,t}\varepsilon^j_{D^*,t}\right) = 0;\ i,j \in \mathbb{N} \qquad \text{(Cross-Moments)}$$

$$E\left(\varepsilon^2_{D,t}\right) = \Gamma\gamma \qquad \text{(Variance)}$$

$$E\left(\varepsilon^3_{D,t}\right) = \phi \qquad \text{(Skewness)}$$

$$E\left(\varepsilon^4_{D,t}\right) = \eta \qquad \text{(Kurtosis)}$$

$$E\left(\varepsilon^m_{D,t}\right) = \begin{cases} (m-1)!!\,(\Gamma\gamma)^{\frac{m}{2}} & \textit{if } m \textit{ is even AND } m > 4 \\ 0 & \textit{if } m \textit{ is odd AND } m > 4 \end{cases} \qquad \text{(m-th Moment)}$$

$$\Gamma := E\left(\varepsilon^2_{D,t}\right) + E\left(\varepsilon^2_{D^*,t}\right)$$

The m-th moment assumption coincides with the moments of a Gaussian dis-

[11]The specification of the stochastic process in the log of TFP has implications that we discuss later in the paper. This assumption is ubiquitous in the macro literature and we adopt it as it greatly simplifies the analysis.

[12]As we discuss further below, in this paper we use perturbation methods as described in Holmes (1995) and Lombardo and Uhlig (2018).

tribution. We will carry out our analysis up to the fourth order, allowing for departures from Gaussianity.

Note that, by our assumption above, the distributions of $log(D_t)$ and $log(D_t^*)$ are mean-preserving. If this were not the case higher moments would affect the mean. In this sense, our assumption will help us clarify the specific role of each moment in the distribution of TFP in determining asset prices and the way a country gains from risk sharing.[13]

The uncertainty generated by unexpected realizations of $\varepsilon_{D,t}$ is the source of risk for the households. There are various measures of risk in the literature. A common approach consists of distinguishing between the quantity of risk, depending on the covariance between the SDF and the return on the risky asset, and the price of risk, depending on the mean of the SDF, e.g. Cochrane (2009). This approach may not be ideal when higher moments play an important role, as pointed out by Harvey and Siddique (2000). We prefer to adopt an intuitive and simple measure, using the relative price of risky domestic assets under complete markets.

Namely, we assess relative risk across borders comparing the equilibrium value of each country's income stream at the complete-markets state price.

Definition 1 ((Inverse) Measure of Relative Risk)**.** Relative risk is defined as a difference between home and foreign asset prices under complete markets:

$$\mathcal{R}^R := \left(P_{K,t}^{cm} - P_{K^*,t}^{cm}\right). \tag{3.18}$$

If $\mathcal{R}^R > 0$, then the Home country is safer than the Foreign. This measure is obviously not country-specific (i.e., it is the same for Home and Foreign residents), since under complete markets the stochastic discount factor is equalized across all agents, independently of where they reside.

We should note here that, in equilibrium, uncertainty about productivity translates into uncertainty about income depending on the equilibrium realization of goods prices. This in turn depends on structural parameters of the model, in particular on trade elasticity θ and risk aversion ρ. In what follows we will discuss how relative risk depends on the combination of these parameters.

[13] It will still be possible to gauge the effect of non mean-preserving spreads, i.e. effects of higher moments on the first moment, by modelling how higher moments of TFP impinge on its mean value. Specifically, one could draw on the empirical evidence in Section 2 to specify the interdependence among moments parametrically.

4 Higher Order Solution of the Model

We solve our model, both analytically and numerically, using perturbation methods.[14] In doing so, we contribute to the literature a theoretically consistent algorithm generally applicable in general equilibrium models. In this section, we first motivate and lay out our solution method. Next we discuss the non-loglinearities that, in our model economy, are key in determining the equilibrium allocation of risk.

4.1 Solution Method

We adopt a perturbation method in alternative to global methods, used by other contributions in the literature—see e.g., the related paper by Coeurdacier et al. (2019). There are two reasons for our choice. First, perturbation methods naturally yield a decomposition of the solution in the various higher moments of the data generating process. This allows us to relate our solution to variance, kurtosis and skewness in the distribution of fundamentals in a clean way. Second, as already mentioned, a novel contribution of our paper consists of showing how to address the problem of solving for the initial distribution of wealth under complete markets, using perturbation methods at any order of approximation.

Following Lombardo and Uhlig (2018), we represent all the variables in our model as functions of the loading parameter for the exogenous stochastic process (ω), e.g. $C_t = C(t;\omega)$. We then take higher order series expansions of the model with respect to ω around the risk-less equilibrium ($\omega = 0$). The resulting system of stochastic difference equations is recursively linear. Standard solution methods can then be applied recursively to solve for the rational-expectation equilibrium.

As customary in the literature, we begin by observing that the risk-sharing condition (3.16) can be solved backward to yield:

$$\log \zeta_t - \log \zeta_t^* + \log Q_t = \log \zeta_0 - \log \zeta_0^* + \log Q_0 := \rho \log \kappa. \tag{4.1}$$

where the subscript 0 indicates the time in which the risk-sharing agreement is decided and implemented for the first time. The time-invariant variable κ is the risk-sharing "constant", ubiquitous in the open-macroeconomics literature under complete markets

[14] See e.g. Holmes (1995), Judd (1998), Schmitt-Grohé and Uribe (2007), Lombardo and Sutherland (2007), Kim et al. (2008), Andreasen et al. (2018), and Lombardo and Uhlig (2018).

(e.g. Chari et al., 2002).[15] This constant reflects the endogenous initial distribution of wealth under complete markets, which, as we work out in the rest of the paper, is a function of equilibrium assets and goods prices, as well as of the quantities produced.

The initial distribution of wealth under a full set of period-by-period state contingent (Arrow) securities is pinned down by a condition on the initial distribution of Arrow securities across borders. Usually this is set equal to zero, de facto imposing that the risk-sharing agreement is first started from a position of zero net foreign assets, see (see, e.g. Ljungqvist and Sargent, 2012, Ch 8)). This implies that solving forward the budget constraint (3.6) and imposing the transversality conditions must satisfy:

$$A_0 = E_0 \sum_{t=0}^{\infty} \Lambda_{t|0} \left(C_t - p_{H,t} Y_t \right) = 0. \tag{4.2}$$

Upon adding the initial condition constraint (4.2) to the system of equations representing our model, we can solve for all the endogenous variables of the model, including κ. In general this requires solving for a fixed-point. Below we show how to approach the problem using perturbation methods.

Take the $m - order$ series-expansion of κ around $\omega = 0$

$$\log \kappa \left(\omega \right) \approx \kappa^{(0)} + \kappa^{(1)} \omega + \ldots + \kappa^{(m)} \frac{1}{m!} \omega^m \tag{4.3}$$

where $\kappa^{(m)} := \left. \frac{\partial^m \log \kappa}{\partial \omega^m} \right|_{\omega=0}$. At each order m, solve for $\kappa^{(m)}$ and proceed recursively, starting from $\kappa^{(0)} = \bar{\kappa}$ and $\kappa^{(1)} = 0$ (as certainty equivalence holds at first order). By way of example, a second order expansion implies that

$$\widetilde{\kappa} := \log \kappa(\omega) - \log \kappa^{(0)} \approx \frac{1}{2} \kappa^{(2)} \tag{4.4}$$

where *wlog* we set $\omega = 1$.[16]

Our solution algorithm (whether applied analytically or numerically) proceeds as follows

1. Expand to the order of interest the system of equations constituting the model;

[15] For non-separable preferences or recursive preferences, e.g. à la Bansal and Yaron (2004), a similar decomposition can be obtained. Details can be obtained from the authors on request.

[16] The accuracy of the approximation clearly depends on the size of ω. Nevertheless, we can normalize this to 1 and scale appropriately the standard deviation of the underlying shocks, *wlog*.

2. Find the RE solution for all variables as a function of $\widetilde{\kappa}$;

3. Use the appropriate series expansion of condition (4.2) to solve for $\widetilde{\kappa}$.

For higher orders of approximation this algorithm can be used recursively starting from lower orders to build the solution for higher orders, i.e. to construct a solution for each of the variables of the model with the same structure as in equation (4.3).

In Appendix A, we detail an additional step which is particularly useful for numerical solutions of DSGE models of any size, e.g. using Dynare (Juillard, 1996).

4.2 Economic Structure and the Transmission of Output Risk

In economic models, higher order moments (in the stochastic processes of productivity) would matter for risk sharing, even if the economy could be reduced to a system of log-linear equation—this is because the welfare function itself is typically not loglinear. In general, however, higher moments can and do play a key role in determining the equilibrium prices and allocation via different mechanisms. It is instructive to map and briefly review the non-loglinearities in our model, to gain insight on which part of the economy actively shapes the transmission of asymmetric risk. Throughout this section, to ease exposition, we assume that the TFP process has no common component across countries, i.e., $\iota = \iota^* = 1$.

To start with, we note that the only equations in our model that are loglinear for any parameter values are the labor supply equations and the production function. In particular, we take the ratio of Home and Foreign labor supply (equation 3.15b and Foreign counterpart), using the production function (equation 3.1 and Foreign counterpart). Denoting the terms of trade as $\tau_t := \dfrac{p_{F,t}}{p_{H,t}}$, we obtain an expression linking relative consumption to a weighted geometric average of relative output, relative productivity and relative prices:

$$Q\left(\frac{C_t^*}{C_t}\right)^{j\rho} = \left(\frac{Y_t}{Y_t^*}\right)^{\frac{\varphi+\alpha}{(1-\alpha)}} \left(\frac{D_t^*}{D_t}\right)^{\frac{\varphi+1}{(1-\alpha)}} \tau_t, \tag{4.5}$$

where $j = 1$ with CRRA preferences and $j = 0$ with GHH preferences.

All other equations are generally non-loglinear, although some become loglinear in special cases. We organize our discussion distinguishing between non-loglinearities that are/are not independent of the international financial arrangements. The first (common to financial autarky and complete markets) arise from the demand aggregators ((3.13) and (3.14)) and the price aggregators ((3.11) and (3.12)). The second pertain to the financial structure, and are thus embedded in the budget constraints (3.6).

4.2.1 Non-loglinearities in Price Indexes and Production Functions

Among the loglinearities that are independent of the financial regime, a first one arises from the price indexes (equations 3.11 and 3.12). In general, taking the ratio of equations (3.11) and (3.12) gives a nonlinear relation between the real exchange rate and terms of trade:

$$Q_t^{\theta-1} = \frac{\nu\tau^{\theta-1} + 1 - \nu}{\nu^*\tau^{\theta-1} + 1 - \nu^*} \tag{4.6}$$

Focusing on equal-size economies, a fourth order approximation of equation (4.6) ($n = \frac{1}{2}$) can be written as:

$$\widetilde{Q}_t = (1-\lambda)\widetilde{\tau}_t - \frac{1}{6}(1-\lambda)(\theta-1)^2\lambda\left(1-\frac{\lambda}{2}\right)\widetilde{\tau}_t^3 \tag{4.7}$$

This shows that, for equal-size economies, up to fourth order of approximation the relationship between the mean log of the real exchange rate and the mean log of the terms of trade is affected by the skewness, but not by the kurtosis, of the terms of trade. For economies of unequal size, also the variance and kurtosis of terms of trade come into play. One may nonetheless note that the expressions above become loglinear (only) when $\theta = 1$, that is, under Cobb-Douglas consumption aggregator, and when $\lambda = 1$, that is, under Purchasing Power Parity (PPP).[17]

Turning to aggregate demand, the ratio of equations (3.13) and (3.14) yields:

$$\frac{Y_t}{Y_t^*} = \tau_t^\theta \frac{\nu + (1-\nu)Q_t^\theta \frac{C_t^*}{C_t}}{\frac{(1-\nu)n}{1-n} + (1 - \frac{n}{1-n}(1-\nu))Q_t^\theta \frac{C_t^*}{C_t}}, \tag{4.8}$$

which becomes exactly loglinear in the special case of PPP—in which case relative output is proportional to the terms of trade. In the general case (failing PPP), a fourth order approximation of equation (4.8) (again) for the case of equal-size economies,

[17] In the cafse of PPP, the real exchange rate is constant and equation 3.12 becomes redundant.

yields:

$$\widetilde{Y}_t - \widetilde{Y}_t^* = \theta \widetilde{\tau}_t + \widetilde{X}_t, \tag{4.9}$$

where

$$\widetilde{X}_t = -(1-\lambda)(\widetilde{r}_t + \widetilde{Q}_t(\theta - 1)) + \frac{1}{6}(1-\lambda)\lambda\left(1 - \frac{\lambda}{2}\right)\left(\widetilde{r}_t + \widetilde{Q}_t(\theta - 1)\right)^3 \tag{4.10}$$

and where $\widetilde{r}_t$ is relative consumption expressed in equivalent units, that is,

$$\widetilde{r}_t = \widetilde{C}_t^* - \widetilde{C}_t + \widetilde{Q}_t. \tag{4.11}$$

Up to fourth order of approximation, for equal-size economies, the relationship between the mean log of relative output and terms of trade is affected only by the mean and skewness of log relative consumption adjusted for the real exchange rate. Variance and kurtosis come into play if the economies differ in size.

The set of equations considered so far characterizes the solution for relative prices and cross-country ratios—the real exchange rate and the terms of trade (equation 4.6) as well as *relative* aggregate demand (equation 4.8). When solving for the *level* of variables, further non-loglinearities arise from equation (3.13) (or 3.14), even under $\theta = 1$ and PPP.

4.2.2 Non-loglinearities in the Budget Constraint

The non-loglinearity that is arguably most consequential for welfare and allocation, however, is the one arising from alternative specifications of financial markets—specifically, from the budget constraint (3.6). To fully appreciate this point, note that under autarky, the ratio of home and foreign household's budget constraints implies that relative consumption at each point in time is determined by relative output:

$$\frac{Q_t C_t^*}{C_t} = \tau_t \frac{Y_t^*}{Y_t}. \tag{4.12}$$

This expression is loglinear, meaning that it will not come into play in the way order moments of the distribution will contribute to allocations. This is in sharp contrast to the case of complete markets, in which case the budget constraint (3.6) cannot be reduced to a simple loglinear equation. As discussed in detail below, this difference is crucial in understanding how risk-sharing, or lack thereof, impinges on allocations.

It should be stressed that failing to account for the budget constraint causes models to yield a number of puzzling results (i.e., welfare being higher in autarky than under complete markets) that become apparent in the presence of significant heterogeneity across regions and countries. Our analysis thus warns against the common practice of writing quantitative models under complete markets omitting the budget constraint.

5 Asset Prices, Terms of Trade and Consumption

Under complete markets, because of the non-linearities arising from the budget constraint, risk drives relative wealth and consumption by impinging on assets and goods prices. In this section, we provide analytical insight on the transmission mechanism.

We start by showing how higher (co-)moments affect asset prices. Imposing the transversality condition $\lim_{i\to\infty} \Lambda_{t+i|t} P_{K,t+i} = 0$, the price of domestic productive asset (3.15d) can be written as follows:

$$P_{K,t} = E_t \sum_{i=1}^{\infty} \Lambda_{t+i|t} r_{K,t+i}. \tag{5.1}$$

where this price is increasing in the comovement between the SDF and the net return. Then, by taking the fourth-order series expansion of this expression around $\bar{D} = \bar{D}^* = 1$ and $\bar{\Lambda} = \delta = \bar{R}_k^{-1} = \bar{R}_k^{*-1}$ we can further write

$$\dot{P}_{K,t} = E_t \frac{1-\delta}{\delta} \sum_{i=1}^{\infty} \delta^i \left[\dot{r}_{K,t+i} + \dot{\Lambda}_{t+i|t} + \mathcal{P}_{H,t+i} \right]. \tag{5.2}$$

where $\dot{P}_{K,t} = 1 + \widetilde{P}_{K,t} + \frac{1}{2}\widetilde{P}_{K,t}^2 + \frac{1}{6}\widetilde{P}_{K,t}^3 + \frac{1}{24}\widetilde{P}_{K,t}^4$.[18] In line with this, we can express the home household's "premium" on home returns as follows:

$$\begin{aligned} \mathcal{P}_{H,t+i} :=& E_t \widetilde{r}_{k,t+i} \widetilde{\Lambda}_{t+i|t} + \\ &+ E_t \frac{1}{2} \left[\widetilde{r}_{k,t+i}^2 \widetilde{\Lambda}_{t+i|t} + \widetilde{r}_{k,t+i} \widetilde{\Lambda}_{t+i|t}^2 \right] + \\ &+ E_t \frac{1}{4} \widetilde{r}_{k,t+i}^2 \widetilde{\Lambda}_{t+i|t}^2 + E_t \frac{1}{6} \left[\widetilde{r}_{k,t+i}^3 \widetilde{\Lambda}_{t+i|t} + \widetilde{r}_{k,t+i} \widetilde{\Lambda}_{t+i|t}^3 \right] \end{aligned} \tag{5.3}$$

[18] We collect these terms together as the higher orders are due to the log-expansion and not to intrinsic non-loglinearities of the model.

This premium characterizes the "riskiness" of the asset, depending on how its return comoves with its valuation (the SDF)—$\mathcal{P}_{H,t+i}$ is indeed related to the "beta" measure of risk discussed in the financial literature (e.g. Cochrane, 2009). The "beta" measure depends on the covariance between the SDF and the gross return on the risky asset: our premium spells out how the riskiness of an asset depends on the various co-moments between the SDF and its net return. Specifically, the first line of equation (5.3) corresponds to the second-order premium and involves the "covariance" between the SDF and the net return. The second line corresponds to the third-order premium and involves the "coskewness" between these variables. The third line corresponds to the fourth-order premium and involves the "cokurtosis" of the same variables.[19]

Asymmetries in risk and size affect asset prices, average terms of trade (reflecting relative demand and supply of national outputs), average real exchange rates and average consumption (reflecting relative wealth). The equilibrium link between these variable is complex, but becomes tractable in some special cases, e.g., under symmetric preferences, implying PPP, or log utility in consumption. To build intuition, the following proposition states a useful didactic result under PPP.[20]

Proposition 1. *Under complete markets and power utility in consumption, holding PPP, relative consumption is equal to the relative value of a country's current and future output—assessed at the equilibrium good and asset prices:*

$$\kappa = \frac{p_{F,0}Y_0^* + P_{K^*,0}^*}{p_{H,0}Y_0 + P_{K,0}}. \tag{5.4}$$

Proof. With complete markets and power utility in consumption, it is easy to see that, holding PPP, per equations (4.1) and the aggregate resource constraint, equation (4.2) implies

$$\mu := \frac{1}{n + (1-n)\kappa} = \frac{E_0 \sum_{t=0}^{\infty} \Lambda_{t|0} p_{H,t} Y_t}{E_0 \sum_{t=0}^{\infty} \Lambda_{t|0} Y_{w,t}} = \frac{p_{H,0}Y_0 + P_{K,0}}{E_0 \sum_{t=0}^{\infty} \Lambda_{t|0} Y_{w,t}} \tag{5.5}$$

where μ is the share of the value of Home output in world output, current and future, the latter expressed in Home consumption units: $Y_{w,t} := np_{H,t}Y_t + (1-n)p_{F,t}Y_t^*$, whereas we have used equation (5.1) to express the numerator in terms of asset prices

[19]It may be noted that these are not central moments. We use this terminology for simplicity and directness.

[20]The proposition generalizes the result in Obstfeld and Rogoff (1996) assuming one world homogeneous good.

and current (time 0) income (equivalent current output evaluated at its time 0 good price).[21] A similar expression (as (5.5)) characterizes the Foreign country, making use of the fact that $C_t^* = \frac{1-n\mu}{1-n} Y_{w,t}$. (5.4), directly follows taking the ratio of equation (5.5) and its foreign counterpart. □

The proposition establishes that, under PPP, relative consumption moves with relative financial wealth, reflecting the relative valuation of country-specific assets—which are claims to the income generated by the domestic (current ad future) production of national goods valued at their equilibrium prices (terms of trade). Note that, from equation (4.8), we also know that the output stream will be endogenous, since in equilibrium prices and wealth differences will impinge on relative labor supply across border.

Relaxing PPP, a closed form solution for κ can only be derived for the special case of log utility, $\rho = 1$. In general, the exchange rate will drive an optimal wedge between marginal utility across borders. The risk-sharing condition (4.1) together with the aggregate resource constraint will imply:

$$C_t = \left(n + (1-n) Q_t^{1-\frac{1}{\rho}} \kappa\right)^{-1} Y_{w,t} = \mu_{Q,t} Y_{w,t} \tag{5.6}$$

The share of a country consumption in global output depends not only on κ, but also on the real exchange rate. For the Home country, the time-zero condition of the budget constraint (4.2) becomes

$$\frac{p_{H,0} Y_0 + P_{K,0}}{E_0 \sum_{t=0}^{\infty} \mu_{Q,t} \Lambda_{t|0} Y_{w,t}} = 1, \tag{5.7}$$

while its foreign counterpart (converted in Home consumption units) is

$$\frac{p_{F,0} Y_0^* + Q_0 P_{K^*,0}^*}{Q_0 E_0 \sum_{t=0}^{\infty} \frac{1-n\mu_{Q,t}}{1-n} \Lambda_{t|0} Q_t^{-1} Y_{w,t}} = 1, \tag{5.8}$$

This expression calls attention to the fact that, when capital market integration moves the economy from financial autarky to perfect risk sharing, repricing of goods and assets can be expected to result in significant changes in relative wealth,

[21] This equivalence follows in the version of the model holding the aggregate capital stock fixed. More in general the numerator of equation (5.5) would be related to the price of a claim to the Home country income. The denominator would be the price of a claim to global income.

impinging on relative consumption and labor supply. In the next sections we will articulate this point, resorting to higher-order perturbation methods to fully characterize the model solution in terms of higher moments.

6 The Relative Gains from Risk Sharing (RGRS): a decomposition into smoothing and level effects

Having described how higher moments drive the valuation effects of capital market integration, in this section we focus on how these effects concur to determine the relative welfare gains across countries. One question often asked in the literature is whether "riskier" countries gain more from integrating their financial markets with safer ones. This question provides a good angle to analyze which specific moments and structural parameters determine the relative riskiness of a country, and drive its relative gains from risk sharing.

We start by defining the relative gains from perfect risk sharing, $RGRS$, in terms of relative welfare changes from financial autarky.

Definition 2 ($RGRS$)**.**

$$RGRS := E_0 \sum_{t=0}^{\infty} \delta^t \left[\left(\widehat{U}_t^{cm} - \widehat{U}_t^{au} \right) - \left(\widehat{U}_t^{*\,cm} - \widehat{U}_t^{*\,au} \right) \right] + \mathcal{O}\left(\omega^5\right),$$

where $\widehat{U}_t = U_t - \bar{U}$, cm denotes complete markets and au denotes autarky.

The RGRS can be decomposed into a "level effect" (LE) and a "smoothing effect" (SE) of moving from autarky to complete markets. These effects are defined as follows:

Definition 3 (Level and level effect)**.** LE corresponds to the linear term of the 4th order approximation of RGRS:

$$LE := E_0 \sum_{t=0}^{\infty} \delta^t \bar{C} \bar{U}_C \left[\left(\widehat{C}_t^{cm} - \widehat{C}_t^{au} \right) - \left(\widehat{C}_t^{*\,cm} - \widehat{C}_t^{*\,au} \right) \right] + \mathcal{O}\left(\omega^5\right),$$

SE summarizes all higher moments of the 4th order approximation of RGRS:

$$SE := RGRS - LE.$$

For the sake of analytical tractability, we will focus on two equal-size countries ($n = 0.5$) with identical preferences, so that PPP holds. We will show that our main conclusions go through also when PPP doesn't hold.

Recall that, under the simplifying assumptions of PPP, mean-preserving distributions and symmetry of initial steady state endowments, mean consumption under autarky is identical across countries. Therefore, LE is simply the difference between home and foreign consumption levels under complete markets.

We now state two propositions that establish, for equal-size countries under PPP (a) a simple mapping of RGRS into asset prices under complete markets and autarky, and (b) how the overall RGRS are driven by asymmetries in the distribution of output.

Proposition 2 (RGRS and asset prices)**.** *Assume that: aggregate capital is fixed (and normalized to 1); $\alpha = 1$; $\bar{Y} = \bar{Y}^*$; consumption preferences are identical across countries ($\lambda = 1$ and PPP holds) and that countries have equal size ($n = \frac{1}{2}$). Then, we have that*

$$E_0 \sum_{t=0}^{\infty} \delta^t \left[\widehat{U}_t^{cm} - \widehat{U}_t^{*\,cm} \right] = E(P_{K,t}^{cm} - P_{K^*,t}^{*,cm}) = \mathcal{R}^R$$

and

$$E_0 \sum_{t=0}^{\infty} \delta^t \left[\widehat{U}_t^{au} - \widehat{U}_t^{*\,au} \right] = \frac{E(P_{K,t}^{aut} - P_{K^*,t}^{*,aut})}{(1-\rho)}$$

Proof. The proof follows from direct calculation. □

Remarkably, for equal-size countries under PPP, our measure of relative risk $\mathcal{R}^R$ is also a measure of the relative welfare under complete markets. Safer countries invariably gain more from perfect risk sharing. Relative risk translates partly into asymmetries in consumption levels, partly into asymmetries in the volatility and thickness of the tails of the distribution of consumption. While we will analyse how these two effects shape the gains below, we can anticipate here that the relative gains to the safer country accrue mostly in terms of average consumption.

The second part of the proposition shows that an analogous condition holds under autarky. Barring trade in assets, relative welfare is proportional to relative asset prices. Since under our assumptions the level component of consumption is identical under autarky, the difference in asset prices reflects exclusively the volatility and thickness of the distribution of consumption. Not surprisingly, in this case safer countries have higher welfare than riskier countries because residents enjoy a smoother consumption.

The following proposition shows how RGRS depends on different moments of the TFP distribution, under the same assumptions as for the previous propositions.

Proposition 3 (Distribution of Gains)**.** *Assume that: aggregate capital is fixed (and normalized to 1);* $\alpha = 1$ *;* $\bar{Y} = \bar{Y}^*$*; consumption preferences are identical across countries (*$\lambda = 1$ *and PPP holds) and that countries have equal size (*$n = \frac{1}{2}$*). Then, for given total variance* $\Gamma := \gamma + \gamma^*$ *and total kurtosis* $N := \eta + \eta^*$ *we have that*

$$sign\left(\frac{\partial RGRS}{\partial\left(\gamma-\gamma^*\right)^2}\right) = sign\left(\left(\theta-1\right)\left(\rho-1\right)\left(\iota-\iota^*\right)\left(\iota-\left(1-\iota^*\right)\right)\right) \tag{6.1a}$$

$$sign\left(\frac{\partial RGRS}{\partial\left(\phi-\phi^*\right)}\right) = -sign\left(\left(\theta-1\right)\left(\iota-\left(1-\iota^*\right)\right)\right), \tag{6.1b}$$

$$sign\left(\frac{\partial RGRS}{\partial\left(\eta-\eta^*\right)}\right) = sign\left(\left(\theta-1\right)\left(\rho-1\right)\left(\iota-\left(1-\iota^*\right)\right)\right). \tag{6.1c}$$

Proof. Propositions 2 and 3 are proved by direct calculation of the solution of the model. In particular, solving the model to fourth-order of accuracy under the assumptions of Proposition 3 yields

$$\begin{aligned} RGRS = &-\frac{\delta(\theta-1)^3\rho(\rho+1)\left(\iota-\left(1-\iota^*\right)\right)^3}{96(1-\delta)\theta^3}\times \\ &\left\{3\left(\iota-\iota^*\right)(\rho-1)\Gamma^2\left(\left(\gamma-\gamma^*\right)^2-1\right)\right. \\ &\left.+4\left(\phi-\phi^*\right)-2\left(\rho-1\right)\left(\left(\eta-\eta^*\right)+\left(\iota-\iota^*\right)N\right)\right\} \end{aligned} \tag{6.2}$$

where $N := \eta + \eta^*$. □

We start by noting a remarkable "equal-gain" result established by the proposition. As long as $\iota = \iota^*$, the first equation in Proposition 6.1 simplifies to

$$\left(\frac{\partial RGRS}{\partial \left(\gamma - \gamma^*\right)^2}\right) = 0$$

The benefits from risk sharing are always symmetric for countries with symmetric size, preferences and technology, but differing in the volatility of their output.[22] The relevance of this result lies in the fact that, if only for a special case, it clearly illustrates the interplay of the level and smoothing channels in determining the RGRS. Specifically, up to a fourth order of approximation, differences in the volatility of output do not impinge at all on the relative welfare gains from complete markets relative to autarky: the welfare improvement is independent of the relative riskiness of the national income process. It follows that, varying relative output volatility, any gains in terms of consumption smoothing are exactly offset by losses in average consumption (and vice-versa).

The equal gains result above provides a useful benchmark against which to assess the general case away from symmetry. Henceforth, for clarity of exposition, we assume that $\iota > 1 - \iota^*$, that is, each country has a higher intensity in one technology. When the stochastic properties of the two technologies differ, the result differ depending on ρ and θ. Provided that $\rho > 1$ and $\theta > 1$, the gains from risk sharing are larger for the country whose production has a relatively stronger bias in 'own' technology, proportionately to the square of the aggregate variance Γ. This effect is partially compensated by the square of relative variances. Note that, since $\gamma \in (0, 1)$, then $(\gamma - \gamma^*)^2 \leq 1$, so that this compensation is typically only partial.

For $\theta > 1$, when countries differ in terms of the frequency of very negative or positive realization of TFP (skewness), the country whose income is more exposed to negative events will tend to benefit more from risk sharing. Because of the benefit of insuring (low-probability but) large realizations of income, the gains in consumption smoothing exceed the equilibrium losses in terms of the level effect. By the same token, the country whose income distribution is characterized by fatter tails (excess kurtosis) and thus prone to extreme events will tend to benefit more from risk sharing, unless consumption preferences are sub-logarithmic. Note that, for aggregate kurtosis (N) there is a further effect on the RGRS: the country with a relatively stronger technological 'bias' will gain relatively more, the larger aggregate kurtosis is.

Proposition 3 highlights that the gains from risk sharing cannot be correctly

[22] RGRS are also zero, trivially, when technologies are identical, i.e. $\iota = 1 - \iota^*$. In this case, shocks are global and there are no gains from risk sharing.

understood without a comprehensive analysis of the different channels through which they materialize. The gains accruing from relative consumption levels are usually inversely related to the gains accruing from smoothing. Which channel prevails, smoothing vs level, varies with the distribution of the shock, and structural parameters of the economy.

The proposition also establishes the way higher skewness impinges on the relative gains from risk sharing depends exclusively on θ, a parameter that also matters at the fourth order. Conversely, ρ (together with θ) drives the relative gains from risk sharing only at even orders. Below we expand on the analysis of the different role of the (static) elasticity of substitution θ between goods and risk aversion ρ in shaping tail risk sharing at different orders of accuracy.

6.1 Trade Elasticity

The key to understand the role of θ is that higher relative *output* volatility, skewness and kurtosis do not mechanically translate into higher *income* volatility, skewness and kurtosis—the mapping depends on the equilibrium response of relative good prices to output shock. This in turn depends on the elasticity of substitution between domestic and foreign goods, θ.

This point is best appreciated in light of the literature pioneered by Cole and Obstfeld (1991), Corsetti and Pesenti (2001) and Corsetti et al. (2008). In their well known contribution, Cole and Obstfeld (1991) show that in the limiting case of a unit elasticity of substitution, relative prices and relative output move opposite to each other in the same proportion—hence output fluctuations cannot cause any variation in relative incomes. Under PPP, using equation (4.8), relative non-financial income can be written as

$$\frac{p_{H,t}Y_t}{p_{F,t}Y_t^*} = \tau^{\theta-1}. \tag{6.3}$$

If $\theta = 1$ non-financial income is always identical across countries, independently of shocks to the supply of output. Indeed, under autarky, per equation (4.12) and using again equation (4.8) we have that

$$\widetilde{C}_t^{au} - \widetilde{C}_t^{*,au} = \frac{\theta - 1}{\theta}\left(\widetilde{Y}_t - \widetilde{Y}_t^*\right). \tag{6.4}$$

With $\theta = 1$, consumption is equalized across countries. Introducing complete markets in this environment would be irrelevant because domestic income and wealth are already perfectly correlated across countries. Relative price adjustment makes contingent transfers (providing insurance) redundant: relative welfare is identical across countries whether or not markets are complete. In light of our definition 1, risk is also identical, independently of the different stochastic properties of the TFP processes.

Conversely, a θ above or below 1 affects how the relative moments of output translate into the relative moments of income via the equilibrium adjustment in the relative price of goods. Intuitively, an elasticity above unity implies that prices still move opposite relative to a country output, but less than proportionally relative to quantities. Hence, higher output volatility translates into higher income volatility. On the contrary, with an elasticity below 1, prices move more than proportionally to any change in relative output. With perfect insurance, higher quantity volatility translates into lower income volatility.[23] In general equilibrium in turn, the relative price adjustment will affect relative wealth, the ratio of the present discounted value of income, and therefore the relative adjustment in the relative risk (Definition 1).

6.2 Risk Aversion

In our Proposition 3 above, risk aversion plays a role in determining the RGRS from asymmetries in variance and kurtosis, but not from asymmetries in skewness. To gain insight into this result, and more in general on how risk aversion influences the different channels through which countries benefit from mutual insurance of macroeconomic risk, it is instructive to focus on the case $\theta \to \infty$, so that home and foreign goods are perfect substitutes and there is no adjustment in the relative price of goods.

With a homogeneous good, replacing the net return on domestic capital with domestic output, the asset price equation (5.1) simplifies as follows:

$$P_{K,t} = E_t \sum_{i=1}^{\infty} \Lambda_{t+i|t} r_{K,t+i} = E_t \sum_{i=1}^{\infty} \delta^i \left(\frac{nY_{t+i} + (1-n)Y^*_{t+i}}{nY_t + (1-n)Y^*_t} \right)^{-\rho} Y_{t+i} \tag{6.5}$$

where the expression in parenthesis in the discount factor is the growth rate of world consumption. An analogous expression holds for the foreign economy.

[23] With home bias in consumption, the response of the terms of trade to a given output shock is not necessarily monotonic, see Corsetti et al. (2008) for a detailed analysis. This observation is relevant in numerical analysis for elasticity of substitution below 1/2.

Now, in the limiting case of $\rho = 1$ (utility from consumption is logarithmic) the log-expansion of the utility function has no higher order terms, that is, $U(C_t) = \log(C_t)$ to any order of accuracy. Hence, under our assumptions (equal-size countries producing a homogeneous good normalized such that $\bar{Y} = \bar{Y}^* = 1$), expected utility in autarky is identical for the two countries, $EU(C_t^{au}) = EU(C_t^{*,au})$. By proposition 2, then, the difference in utility under complete markets coincides with $\mathcal{R}^R$. In other words, it only depends on changes in asset prices. It follows that, moving from autarky to perfect insurance has no smoothing effect. Relative gains from risk sharing are driven exclusively by asset valuation.

Under log preference, (6.5) simplifies to:

$$P_{K,t} = 2E_t \sum_{i=1}^{\infty} \delta^i \left(1 + x_{t+i}\right)^{-1} \tag{6.6}$$

where $x_t = \frac{Y_t^*}{Y_t}$. Combining this with the symmetric expression for the other country

$$P_{K,t}^* = 2E_t \sum_{i=1}^{\infty} \delta^i \left(1 + x_{t+i}^{-1}\right)^{-1} \tag{6.7}$$

we obtain

$$P_{K,t} - P_{K,t}^* = 2E_t \sum_{i=1}^{\infty} \delta^i \frac{(1 - x_{t+i})}{(1 + x_{t+i})} \tag{6.8}$$

Our key result for the limiting case of log preferences in our structurally symmetric economy follows from taking a log expansion of order m of the ratio on the right-hand side of this expression

$$\frac{(1 - x_{t+i})}{(1 + x_{t+i})} \approx -\frac{x_{t+i}}{2} + \frac{x_{t+i}^3}{24} - \frac{x_{t+i}^5}{240} - \ldots + O\left(x_{t+i}^{m+1}\right), \tag{6.9}$$

Relative asset prices are affected only by odd moments of the distribution of relative income x_t: only differences in the degree of asymmetry of the distributions matter for asset price revaluation and hence welfare under log preferences. Gains from risk sharing would be missed by focusing only on second moments.

This main conclusion is strengthened when we move away from the log-preference case. Specifically, it can be shown that the *even* derivatives of (6.5) (evaluated at the symmetric deterministic steady state) are multiplied by the term $(1 - \rho)$, while the *odd* derivatives – those capturing the asymmetries in the distribution – are not. Even moments (e.g., variance or kurtosis) have an extra, direct effect on relative asset prices and welfare as long as $\rho \neq 1$—and will thus affect the RGRS.

Comparing two countries with asymmetric kurtosis in the distribution of output, higher values of ρ amplify the range of variation in the discount factor, making the country with relatively fatter tail relatively riskier. Thus, the difference in asset prices will be increasing in ρ. It is worth reiterating that all these results would go unnoticed if we restricted our attention to Gaussian stochastic processes, widely used in the literature, as the odd moments of Gaussian distributions are always zero. That said, we also note that the above results are derived under the Assumption 1, which implies independence of the two income processes.

7 Quantitative Analysis

In this section, we reconsider and generalize our main results using quantitative analysis. In particular we allow for endogenous labor and cross-country differences in preferences over consumption goods (home bias). In a richer specification of the model, we can assess the (SE and LE) components of relative welfare gains and the macroeconomic effects of risk sharing, as function of key structural parameters.

In addition to expanding on the consumption smoothing and relative wealth channels highlighted in the previous section, our quantitative analysis will highlight a novel one. Gains from risk sharing may also materialize via a terms of trade channel—reflecting any adjustment in relative labor supply and demand for home and foreign goods across borders. A country that experiences an improvement in its terms of trade can enjoy a higher consumption-to-labor ratio for any given asset price revaluation. To bring forward the implications of risk sharing for the terms of trade, we assess the model for different degrees of home bias, as this is the key parameter determining the terms-of-trade effect, and contrast standard CRRA preferences with GHH preferences, since the latter rule out wealth effect on labor supply.

In our exercises we will restrict attention to specific values for the trade elasticity ($\theta = 1.5$) and risk aversion ($\rho = 4$), bearing in mind their role in determining our results as explained above.[24]

[24] We chose a relatively high value for risk aversion following Coeurdacier and Gourinchas (2016) in a related analysis.

7.1 Parametrization

We set the frequency of the model to annual (reflecting the frequency of our main PWT database). The share of labor in production is $1-\alpha = 0.7$, the risk aversion parameter is $\rho = 4$, the inverse Frisch elasticity of labor supply is $\varphi = 1.75$ (see Attanasio et al., 2018), the weight of disutility from working is normalized to $\chi = 20$,[25] and the trade elasticity of substitution is $\theta = 1.5$,[26] and the discount factor is $\delta = (1 + 0.02)$ implying a real rate of 2% per year. We assume that the persistence of the TFP process is $\varphi_D = 0.7$, amounting to the average TFP across OECD countries. We assume that $\iota = \iota^* = 1$.

The baseline moments are set at the median value shown in Table 1. In particular, denoting the 5-th percentile by x and the 95-th by y, for each moment, we set the total variance ($\Gamma := \gamma + \gamma^*$), total skewness ($\Phi := \phi + \phi^*$) and total kurtosis ($N := \eta + \eta^*$) at the value corresponding to $x + y$ (see Table 1 for the specific values). Then we set the range of values for γ, ϕ and η at $\{50\%, 60\%, 70\%, \frac{y}{j}\}$ where $j = \{\Gamma, \Phi, N\}$ respectively. We present our analysis at first focusing on GHH preferences, which allow us to purge out the terms-of-trade effect stemming from income effects on labor supply. We then discuss our baseline with CRRA preferences, using the Appendix to elaborate on the comparison of the two cases.

7.2 Relative Gains from Risk Sharing by Channels and Moments

The results from our quantitative exercises are shown in Tables 2 through 5 in the text and in Appendix C. To enhance comparison across tables, we report the RGRS as share of the global gains from risk sharing, i.e. the sum of Home and Foreign gains: we denote this share by $RGRS_s$. Correspondingly, we scale the LE and SE by total welfare gains, with notation LE_s and SE_s respectively.

All our tables include five panels. The first three panels report, respectively, $RGRS_s$, and its decomposition into the two additive terms SE_s and LE_s. The last two panels show the relative adjustment in asset prices and terms of trade. In each

[25] This value yields approximately hours worked equal to 20% of total time, in line with US data. That said, the value of χ does not affect the results.

[26] Consistently with our discussion of Proposition 3, the sign of RGRS (and the direction of the channels) change around $\theta = 1$. For reasons of space, we show results only for $\theta > 1$, as this is the typical range considered in the literature.

panel, rows describe the results for four degrees of home bias $(1 - \lambda)$, in decreasing order (1=no home bias). Columns show results for different values of each moment, in the ranges defined above.

Results assuming GHH preferences are shown in Tables 2 through 4. Focusing on asymmetric variances first, the results shown in table 2 confirms and generalizes our analytical results. The country with the most volatile output gains more. In relative terms, however, the gains from improved smoothing(the level effect shown in the second panel of the table) are to a large extent offset by a negative level effect (LE, in the third panel). The opposite is true for the country with a lower volatility of output. This country gains mainly in terms of average utility from consumption and leisure. In the limiting case of no-home-bias ($\lambda = 1$, corresponding to PPP), the SE and LE offset each other exactly. This generalizes Proposition 3, stated for endowment economies, to a production economy under GHH preferences.

The drivers of the level effect are shown in the bottom part of the Table 2. Observe that both the relative price of assets and the relative price of goods (terms of trade) deteriorate for the country with the more volatile output—improve for the other country. Indeed, the country with a low output volatility enjoys higher relative wealth and purchasing power. Note that, with GHH preferences, the terms of trade adjust only with home bias in consumption. This is because, with home bias, changes in relative income and wealth modify the composition of global demand in favor of the output produced by the safer (and thus richer) country—driving up its relative price. With GHH preferences, when λ approaches 1–the case of PPP–the terms of trade do not move at all.

Comparing the results in three tables Tables 2 through 4, it is apparent that tail risk magnifies the RGRS among heterogeneous countries. To appreciate this point, in each table, consider a "risky" (Home) country in the upper decile of the distribution of the corresponding moment, for the intermediate degree of home bias (.75). Relative to the median (Foreign) country, the risky Home country only gains 0.4 percentage points (of total gains) more than the Foreign country, if it falls in the upper decile of the variance distribution (Table 2, first panel). However, it gains 8.7 percentage points more if it falls in the upper decile of the (negative) skewness distribution (Table 3, first panel); and 8.8 percentage points more if it falls in the upper decile of the kurtosis distribution (Table 4, first panel). With tail risk, the relative gains for the riskier country are 20 times larger (8.7 or 8.8 versus 0.4).

In the three tables, the SE and the LE have opposite sign, but the relative weight of these two components is different. In the Tables for skewness and kurtosis

(Tables 3 and 4), the SE for the risky country (i.e., a country whose output distribution has the fatter tail, or the larger mass on negative realization), is much larger relative to the LE. Insuring against extreme realizations of output yields large gains in terms of consumption smoothing. The level effect (the macroeconomic "price" of the insurance), while non negligible, plays a smaller role compared to the case of asymmetric variances in Table 2.

A notable result is that, everything else equal, the riskier country gains more (the RGRS are higher) when home bias is high. This result is only in part explained by the SE component of the RGRS. As shown in Tables 3 and 4, what matters is that, when home bias is high, the risky country actually suffers smaller losses in terms of LE (LE is less negative in the third panel of the tables). This is so, despite the fact that the fall in asset prices is more pronounced (fourth panel). But while a large fall in asset prices makes the risky country relatively worse-off in financial terms, the loss in real purchasing power of their residents remains contained because of a moderate deterioration in their terms of trade (fifth panel).

As is well known, GHH preferences insulate labor supply from wealth effects– for more general preferences, these wealth effects activate an additional mechanism by which risk sharing may cause terms of trade adjustment. If only for this reason, it is particularly instructive to consider the case of CRRA preferences. To save space, we only discuss results for the variance in the main text (Table 5) – we present the results for the other moments in the Appendix C.

With CRRA preferences, when moving from autarky to perfect risk sharing (κ), the safer country consumes more and works less on average. This implies that, through the effects of market integration, the terms of trade tend to improve for the safer country. The gains in purchasing power may be strong enough to tilt the $RGRS_s$ in its favor under PPP and for an intermediate degree of home bias. Yet home bias interacts with risk. A strong degree of home bias tilts the $RGRS_s$ back in favor of the riskier country, in line with the results using GHH preferences. Note that, in Table 5, the $RGRS_s$ changes sign between the first and the second row. The nonlinear effects of labor supply and terms of trade adjustment under CRRA preferences mitigate and can even reverse the $RGRS_s$ for riskier countries.

Table 2: Welfare and Relative Price Effects of Risk Sharing by Volatility of TFP, GHH preferences

Home-Bias	$\gamma^{\dagger}$ 50%	60%	70%	95%
		$RGRS_s{}^{\dagger\dagger}$		
0.2	0.0000	0.4311	0.8665	2.1008
0.75	0.0000	0.0824	0.1655	0.4005
1.	0.0000	0.0000	0.0000	0.0000
		$SE_s{}^{\dagger\dagger}$		
0.2	0.0000	1.1634	2.3387	5.6798
0.75	0.0000	1.5242	3.0626	7.4155
1.	0.0000	2.3436	4.7085	11.3920
		$LE_s{}^{\dagger\dagger}$		
0.2	0.0000	−0.7322	−1.4722	−3.5790
0.75	0.0000	−1.4418	−2.8970	−7.0150
1.	0.0000	−2.3436	−4.7085	−11.3920
		Relative Gains in Asset Prices		
0.2	0.0000	−1.1350	−2.2700	−5.3422
0.75	0.0000	−0.3353	−0.6706	−1.5781
1.	0.0000	−0.1983	−0.3965	−0.9332
		Home Average Terms of Trade		
0.2	0.0000	0.8172	1.6345	3.8465
0.75	0.0000	0.0631	0.1262	0.2970
1.	0.0000	0.0000	0.0000	0.0000

All measures are in percentages. $n = \frac{1}{2}$, $\theta = 1.5, \rho = 4$, $\phi = \phi^* = 50\%$, $\eta = \eta^* = 50\%$. Fourth-order approximation.

† Columns refer to the distribution displayed in Table 1

†† Permanent consumption equivalent (pce) units relative to total welfare in pce units.

Table 3: Home bias in consumption and home relative skewness of TFP - GHH preferences

Home-Bias	$\phi^\dagger$ 50%	60%	70%	95%
	$RGRS_s{}^{\dagger\dagger}$			
0.2	0.0000	0.8312	1.6624	10.1457
0.75	0.0000	0.7160	1.4320	8.7421
1.	0.0000	0.5380	1.0761	6.5698
	$SE_s{}^{\dagger\dagger}$			
0.2	0.0000	0.9693	1.9386	11.8312
0.75	0.0000	0.9424	1.8848	11.5060
1.	0.0000	0.9040	1.8080	11.0381
	$LE_s{}^{\dagger\dagger}$			
0.2	0.0000	−0.1381	−0.2762	−1.6855
0.75	0.0000	−0.2264	−0.4527	−2.7638
1.	0.0000	−0.3659	−0.7319	−4.4682
	Relative Gains in Asset Prices			
0.2	0.0000	−0.2697	−0.5394	−3.2931
0.75	0.0000	−0.0734	−0.1468	−0.8965
1.	0.0000	−0.0424	−0.0848	−0.5175
	Home Average Terms of Trade			
0.2	0.0000	0.1720	0.3441	2.1007
0.75	0.0000	0.0101	0.0203	0.1237
1.	0.0000	0.0000	0.0000	0.0000

All measures are in percentages. $n = \frac{1}{2}$, $\theta = 1.5$, $\rho = 4$, $\gamma = 50\%$, $\phi^* = 1 - \phi$, $\eta = \eta^* = 50\%$. Fourth-order approximation.

† Columns refer to the distribution displayed in Table 1

†† Permanent consumption equivalent (pce) units relative to total welfare in pce units.

Table 4: Home bias in consumption and home relative kurtosis of TFP - GHH preferences

	$\eta^{\dagger}$			
Home-Bias	50%	60%	70%	95%
		$RGRS_s{}^{\dagger\dagger}$		
0.2	0.0000	3.0267	6.0527	10.2219
0.75	0.0000	2.6073	5.2144	8.8078
1.	0.0000	1.9592	3.9184	6.6192
		$SE_s{}^{\dagger\dagger}$		
0.2	0.0000	3.2188	6.4368	10.8706
0.75	0.0000	2.8238	5.6474	9.5392
1.	0.0000	2.3098	4.6195	7.8035
		$LE_s{}^{\dagger\dagger}$		
0.2	0.0000	−0.1921	−0.3841	−0.6487
0.75	0.0000	−0.2165	−0.4330	−0.7314
1.	0.0000	−0.3505	−0.7011	−1.1843
		Relative Gains in Asset Prices		
0.2	0.0000	−0.4376	−0.8751	−1.4784
0.75	0.0000	−0.1071	−0.2141	−0.3618
1.	0.0000	−0.0570	−0.1139	−0.1925
		Home Average Terms of Trade		
0.2	0.0000	0.2185	0.4369	0.7381
0.75	0.0000	0.0095	0.0190	0.0321
1.	0.0000	0.0000	0.0000	0.0000

All measures are in percentages. $n = \frac{1}{2}$, $\theta = 1.5$, $\rho = 4$, $\gamma = 50\%$, $\phi = \phi^* = 50\%$, $\eta^* = 1 - \eta$. Fourth-order approximation.

† Columns refer to the distribution displayed in Table 1

†† Permanent consumption equivalent (pce) units relative to total welfare in pce units.

Table 5: Home bias in consumption and home relative volatility of TFP - CRRA preferences

Home-Bias	$\gamma^{\dagger}$ 50%	60%	70%	95%
		$RGRS_s{}^{\dagger\dagger}$		
0.2	0.0000	0.0959	0.1922	0.4566
0.75	0.0000	-0.0289	-0.0580	-0.1374
1.	0.0000	-0.0545	-0.1090	-0.2584
		$SE_s{}^{\dagger\dagger}$		
0.2	0.0000	1.2578	2.5197	5.9872
0.75	0.0000	1.5613	3.1265	7.4146
1.	0.0000	2.0900	4.1849	9.9189
		$LE_s{}^{\dagger\dagger}$		
0.2	0.0000	-1.1619	-2.3275	-5.5306
0.75	0.0000	-1.5902	-3.1844	-7.5520
1.	0.0000	-2.1444	-4.2939	-10.1773
		Relative Gains in Asset Prices		
0.2	0.0000	-0.4028	-0.8056	-1.8959
0.75	0.0000	-0.1370	-0.2740	-0.6449
1.	0.0000	-0.0822	-0.1645	-0.3871
		Home Average Terms of Trade		
0.2	0.0000	0.3983	0.7965	1.8745
0.75	0.0000	0.1347	0.2694	0.6339
1.	0.0000	0.0818	0.1636	0.3851

All measures are in percentages. $n = \frac{1}{2}$, $\theta = 1.5$, $\rho = 4$, $\phi = \phi^* = 50\%$, $\eta = \eta^* = 50\%$. Fourth-order approximation.

† Columns refer to the distribution displayed in Table 1

†† Permanent consumption equivalent (pce) units relative to total welfare in pce units.

Table 6: Risk-sharing effects: 90% interval

	5%	95%
$RGRS_s$	-9.504	9.774
SE_s	-14.892	15.387
LE_s	-9.310	9.010
Asset Prices	-1.843	1.916
Terms of Trade	-0.291	0.317

Note: For the methodology, see the note to Figure 3.

7.3 Model Simulations

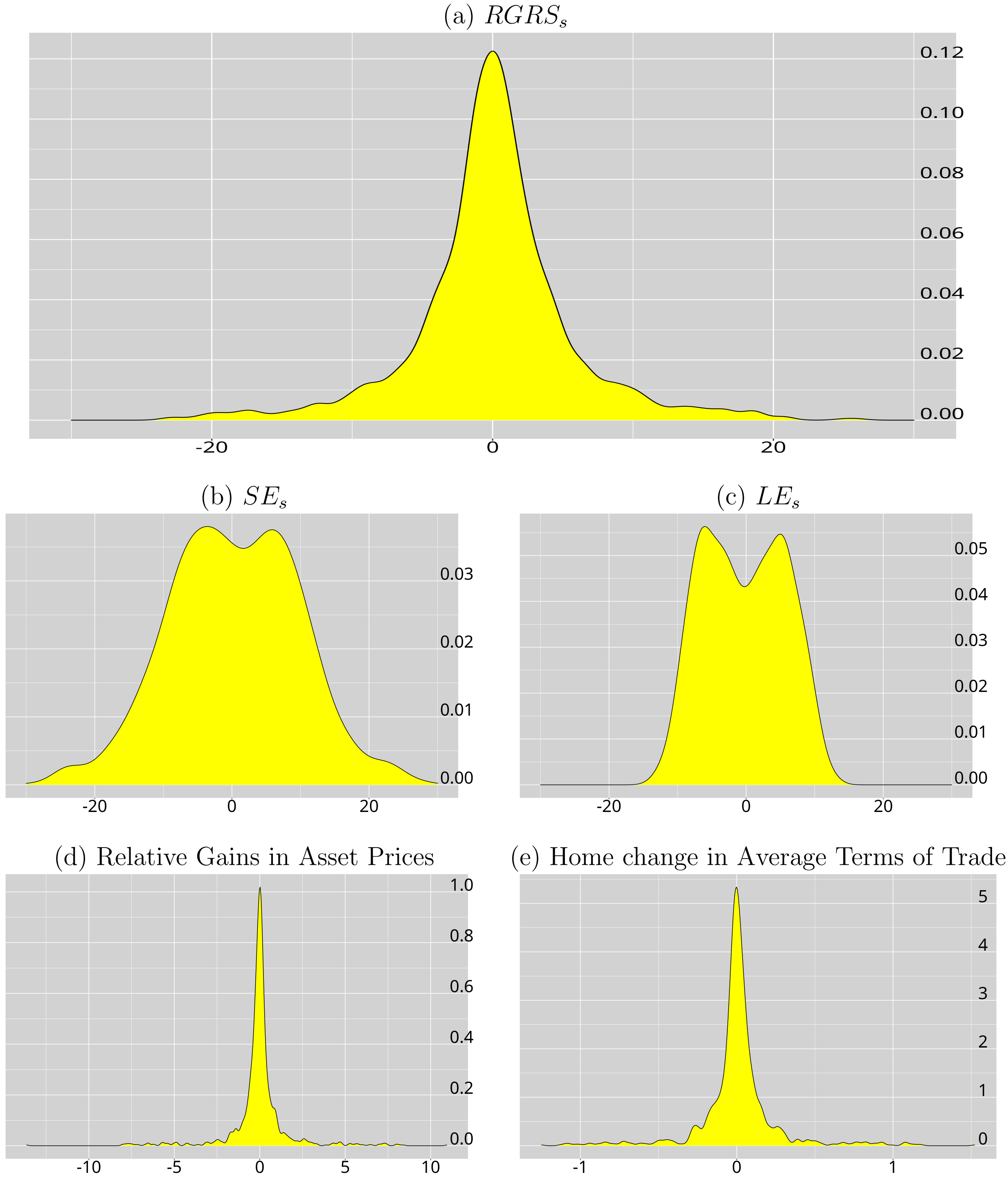
So far we have carried out our analysis focusing on each higher moment in the distribution of GDP by country separately. In this section, we will use the empirical evidence in Section 2 (see also Table 9), to assess the potential gains from risk sharing treating the different moments jointly. In doing so, we will be in a good position to capture the full extent of asymmetries in the distribution of GDP across borders.

We conduct our exercise by randomly drawing 1449 non-repeated pairs of countries from Table 9, each draw generating a pair of vectors of moments for Home and Foreign. The results are shown in Figure 3 and Table 6. Starting from Figure 3, in panel (a) we plot the distribution of the Relative Gains from Risk Sharing $RGRS_s$ predicted by our model. The following panels refer the decomposition of $RGRS_s$ into smoothing effect SE_s (panel (b)), and level effect LE_s (panel (c)), as well as to the distribution of the relative change in asset prices (panel (d)) and in the Home terms of trade (panel (e)), all relative to autarky. The summary of these results in Table 6 reports also the 90% interval of each distribution.[27]

Our new figure highlights at least three remarkable results. To start with, as shown in panel (a), many country pairs gain about the same from risk sharing (corresponding to the mass around zero). Yet, there is a dense tail of countries that draw significantly larger or lower relative gains (right and left tail of distribution). Considering the 90% interval of the distribution, the relative gains/losses from risk

[27] A comment is in order about taking country pairs as our unit of observation, as opposed to, say, pairing a country with regions obtained from aggregating countries. Aggregating countries into regions would be more directly representative of the gains that one individual country could obtain from joining a larger integrated capital market (or aggregate risk sharing institutional arrangement). However, aggregation would also implicitly internalize some of the potential gains from risk sharing, as these would already be achieved through within-region diversification. We find our approach closer in line with the goal of bringing our evidence to bear on the potential gains from, and effects of, aggregate GDP risk sharing.

Figure 3: Distribution of Risk Sharing Effects



Note: The frequency distributions shown in each panel are derived by randomly drawing 1449 non-repeated pairs of countries using the estimated moments displayed in Table 9 for our sample. Specifically, for each pair of country, we assign moments to a Home and a Foreign country (eliminating repetition), and compute the measures on the x-axis of each panel. Total moments (i.e. Γ, Φ and N) change through draws (country pairs). In the simulations we assume that $\lambda = 0.75$, i.e. a considerable degree of trade openness.

sharing (as a percentage of the total gains) are of the order of 10 percentage points on either side.

In light of our analysis, relatively riskier countries tend be the winner in terms of improved smoothing, reflecting heterogeneity in exposure to tail risk. The decomposition of $RGRS_s$ into smoothing and level effects, in panels (b) and (c), corroborates this analytical insight. A significant group of riskier countries mostly gains in terms of improved smoothing, in excess of the loss from a lower average level of wealth and consumption. A significant group of safer countries mostly gains in terms of relative wealth and consumption–these gains more than offsetting the relative loss in smoothing. The range of relative gains from smoothing corresponding to the 90% interval of the distribution is comprised between -14.8% and 15.5% of total gains from risk sharing; the corresponding range for the level effect is between -9.3% and 9.0%.

The level effect in panel (c), in turn, maps into the distribution of changes in asset prices and the terms of trade, shown in the following two panels. In our analysis, safer countries experience a relative re-valuation of their assets and an appreciation of their terms of trade (right tail of asset gains' distribution in panel (d) and left tail of terms of trade distribution in panel (e)). Riskier countries experience capital losses in their assets and a depreciation of their terms of trade (left tail of asset gains' distribution and right tail of terms of trade distributions). The 90% interval for the changes in asset prices ranges approximately from -1.8% to 1.9%; for terms of trade, from -0.3% to 0.3%.

The main takeaway is straightforward. Tail risk enhances the relative gains from capital market integration of countries that are more exposed to it–our empirically-motivated exercise suggests that riskier countries have a potential relative gains advantage for riskier up to 10 percentage points of the total gains from risk sharing. This relative advantage however corresponds to significant differences in the sign, magnitude and combinations of the SE and LE components of the gains across risky and safe countries. In relative terms, the consumption smoothing component can contribute up to a 15 percentage point advantage for riskier countries. The level effect component produces relative gains and losses up to 10% of the total gains from risk sharing. Safer countries may benefit from an average revaluation of their total assets just short of 2% in excess to riskier countries.

8 Conclusions

In this paper, we have reconsidered the welfare and macroeconomic effects of insuring fundamental output risk across borders, offering a decomposition of welfare gains into a smoothing and a level effect, as well as a discussion of these effects by moments of the distribution of output and by transmission channels. After mapping output risk into income risk, highlighting the role of key structural features of the economy, we have brought volatility, kurtosis and skewness in the distribution of output to bear on our decomposition of the welfare gains from risk sharing. We articulate the level component of these gains distinguishing between two channels: a relative wealth channel, reflecting the revaluation of a country assets at the new equilibrium prices, and a terms of trade channel, reflecting changes in the relative price of domestically produced goods. These adjustments are the general equilibrium analog of the price of insurance that riskier countries pay, safer countries obtain, when joining a capital market union. Either type of countries gains from insuring aggregate risk, but the composition of these gains differs. While riskier countries tend to gain mostly in terms of consumption smoothing, they may lose out in terms of average consumption and labor effort.

Based on the empirical evidence for a large sample of countries, we offer an assessment of the potential gains from macroeconomic risk sharing, decomposing them by smoothing and level effects and by channels. In this exercise, tail risk features prominently in shaping these gains. The distribution of relative gains is bi-modal. A significant group of riskier countries benefit mostly from consumption smoothing, and a significant group of safer countries benefit mostly from the level effects of capital market integration.

In the majority of applied and policy assessments of the gains from capital market integration, risk sharing is measured in terms of volatility and correlation of consumption, sometimes in relation to the volatility of output, most often ignoring the real exchange rate in breach of the theoretical condition for smoothing. Our analysis clarifies that these indicators can at best provide an incomplete assessment of capital market integration. By focusing on what we dub the smoothing effect of risk sharing, they miss the main channels through which relatively safer countries gain. Providing macroeconomic insurance is rewarded with a larger share of world output to the residents in the safer regions—corresponding to a rise in relative wealth and an appreciation of their terms of trade. Looking forward, the challenge for the literature is to devise indicators of these level effects, which we show are likely to play a non secondary role in defining why many countries have a clear interest in achieving international risk sharing, especially at times of heightened tail risk.

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Appendix

A Higher-order accurate solution of the risk-sharing constant

In the simple Lucas' endowment model discussed in the main text, the risk-sharing constant κ,[28] defined as

$$\kappa = \frac{C_{2,t}}{C_{1,t}} \tag{A.1}$$

depends recursively on exogenous variables, i.e. assuming $\iota = \iota^* = 1$,

$$\frac{1}{n + (1-n)\kappa} = \frac{E_0 \sum_{t=0}^{\infty} \delta^t \left(D_{w,t}\right)^{-\rho} D_{1,t}}{E_0 \sum_{t=0}^{\infty} \delta^t \left(D_{w,t}\right)^{-\rho} D_{w,t}} \tag{A.2}$$

In general, and specifically when there is production, the right-hand-side of equation (A.2) is endogenous (e.g. the discount factor depends on consumption, and income depends on production). Therefore, to find the risk sharing constant in this more general setting we typically need to use some fixed-point algorithm.

We propose a closed-form solution for κ based on perturbation methods. We present a version that is accurate up to second order but that is straightforward to extend to higher orders. In describing our solution we follow a procedure that allows for easy numerical implementation, e.g. using Dynare.

Our solution is based on the observation that κ depends on second order terms, and in particular on the variance of the exogenous processes. We can thus represent the risk-sharing constant as

$$\log(\kappa) := \bar{\kappa} E_t \varepsilon_{\kappa,t+1}^2 \tag{A.3}$$

so that

$$\bar{\kappa} E_t \varepsilon_{\kappa,t+1}^2 = \widetilde{C}_{2,t} - \widetilde{C}_{1,t} \tag{A.4}$$

[28] The term "constant" refers to time invariance. This coefficient is not invariant to risk.

where $\bar{\kappa}$ is the unknown parameter we want to solve for, and $\varepsilon_{\kappa,t}$, is a mean-zero iid auxiliary shock with variance denoted by σ_κ^2. This implies that $E_t\varepsilon_{\kappa,t+1}^2 = \sigma_\kappa^2$. So far we have thus re-scaled the original kappa by σ_κ^2.

The second-order solution of a DSGE model can be written in a second-order VAR form as (e.g. following Dynare notation)

$$y_t = Ay_{t-1} + Bu_t + \frac{1}{2}\left[C\left(y_{t-1} \otimes y_{t-1}\right) + D\left(u_t \otimes u_t\right) + 2F\left(y_{t-1} \otimes u_t\right)\right] + \frac{1}{2}G\vec{\Sigma^2} \quad \text{(A.5)}$$

where $y_t \in \mathbb{R}^{n_y}$ is the vector of all the n_y variables (endogenous and exogenous excluding innovations), $u_t \in \mathbb{R}^{n_i}$ is the vector of all the n_i (iid) innovations, A, B, C, D, F, G are conformable matrices, and for any column vectors x and z, $(x \otimes z)$ is the vectorized outer product of these vectors. $\Sigma^2 := E_t\left(u_{t+1}u_{t+1}'\right)$, and $\vec{\cdot}$ is the vectorization operator.[29]

The key term in equation (A.5) is the last one, which shifts the mean of variables in proportion to the exogenous risk, captured by the variance matrix Σ^2 (also referred to as the stochastic steady state in the literature).

Using regular perturbations (see e.g. Lombardo and Uhlig, 2018), none of the matrices in (A.5) depends on exogenous risk. This means that the only place where σ_κ appears is in Σ^2.

The vector y_t contains the variable measuring Arrow-Debreu securities. Assume the latter are in position i_{AD}, and that σ_κ occupies position j_{σ_κ} in the vector $\vec{\Sigma^2}$. Then we have that

$$\begin{aligned} y_t[i_{AD}] = A[i_{AD},:]y_{t-1} + B[i_{AD},:]u_t + \frac{1}{2}\left[C[i_{AD},:]\left(y_{t-1} \otimes y_{t-1}\right)\right. \\ \left. + D[i_{AD},:]\left(u_t \otimes u_t\right) + 2F[i_{AD},:]\left(y_{t-1} \otimes u_t\right)\right] + \frac{1}{2}G[i_{AD},:]\vec{\Sigma^2} \end{aligned} \quad \text{(A.6)}$$

where for a matrix X, $X[i,j]$ denotes the element in row i and column j, and where $X[i,:]$ denotes the row i of matrix X; for a vector z, $z[j_{\sigma_\kappa}]$ is the $j_{\sigma_\kappa} - th$ element in z. In particular, $\vec{\Sigma^2}[j_{\sigma_\kappa}] = \sigma_\kappa^2$.

Note that if we set $\bar{\kappa} = 1$, we can solve for σ_κ^2 that satisfies some restriction on $y_t[i_{AD}]$. In particular we know that under complete markets it must be that $y_0[i_{AD}] = 0$

[29] To date, Dynare returns only the product $G\vec{\Sigma^2}$ in the variable "oo_.dr.ghs2". In order to implement our algorithm this product must be factorized in the two components. This can be easily done by modifying Dynare function `dyn_second_order_solver.m` at about line 173, by adding a new variable e.g. `dr.G=LHS\(-RHS);`, where LHS and RHS are variables defined in the function.

(Ljungqvist and Sargent, 2012). One way to implement this condition is to assume that at time 0 and -1 the economy was at the stochastic steady state, i.e. all elements of equation (A.6) are zero except the last one, i.e.[30]

$$y_0[i_{AD}] = 0 = \frac{1}{2} G[i_{AD}, :]\vec{\Sigma^2} \tag{A.7}$$

Then we can solve for σ_κ^2 as

$$\sigma_\kappa^2 = -\frac{G[i_{AD}, j_{\sigma_\kappa}^\perp]\vec{\Sigma^2}[j_{\sigma_\kappa}^\perp]}{G[i_{AD}, j_{\sigma_\kappa}]} \tag{A.8}$$

where $j_{\sigma_\kappa}^\perp$ denotes all the elements excluding j_{σ_κ}.

Now we simply need to swap values, i.e.

$$\begin{aligned} \bar{\kappa} &\leftarrow \sigma_\kappa^2 \\ \sigma_\kappa^2 &\leftarrow 1. \end{aligned} \tag{A.9}$$

With this assignment of values, κ is the second-order accurate risk-sharing constant that implements complete markets.

Our proposed algorithm, correctly implements complete markets up to second order accuracy. It should be noted also that our approach does not affect the first-order solution. This solution correctly describes growth rates of variables, since the risk-sharing constant is invariant to time (Ljungqvist and Sargent, 2012).

Our approach is reminiscent of the solution algorithm proposed by Devereux and Sutherland (2011) (DS) to solve for portfolio shares up to second order. DS introduce an auxiliary iid shock in the budget constraint of investors as a placeholder for portfolio shares. By knowing the position of this auxiliary shock DS can then use simple linear algebra to derive the shares. Although we solve a different problem, our algorithm shares with DS the idea of using auxiliary iid shocks as placeholders for parameters that would otherwise drop out of the perturbed solution.

[30] Equally easily implementable is any other condition, e.g. $Ey_0[i_{AD}] = 0$.

B GHH preferences: Analytics

A large number of papers assumes that households' preferences are such that there is no wealth effect on labor supply, following the seminal work of **?**, GHH. GHH preferences are non-separable in consumption and labor, i.e. (for the Home country)

$$U\left(C_t, L_t\right) := \frac{\left(C_t - \chi \frac{L_t^{1+\phi}}{1+\phi}\right)^{1-\rho}}{1-\rho}, \tag{B.1}$$

and an identical expression for the foreign country.

Under these preferences the first-order conditions (3.15a) and (3.15b) can be written as

$$C_t : U_C(C_t, L_t) := \left(C_t - \chi \frac{L_t^{1+\phi}}{1+\phi}\right)^{-\rho} = \lambda_t \tag{B.2}$$

$$L_t : \; - U_L(C_t, L_t) := \chi L_t^{\phi} U_C(C_t, L_t) = w_t \lambda_t, \tag{B.3}$$

so that labor supply does not depend directly on the marginal utility of consumption.

Importantly, these preferences imply that the risk-sharing condition (**??**) does not simply depend on relative consumption and the exchange rate, but on labor too, i.e.

$$\log \zeta_t^* - \log \zeta_t - \log Q_t = \log \zeta_0^* - \log \zeta_0 - \log Q_0 := \widetilde{\kappa}_{GHH}. \tag{B.4}$$

The same technique discussed above can be used to solve for κ_{GHH} at any order of approximation.

C Further Quantitative Results (CRRA preferences)

This appendix shows the effect of asymmetries in skewness and kurtosis under CRRA preferences.

Table 7: Home bias in consumption and home relative skewness of TFP - CRRA preferences

Home-Bias	$\phi^{\dagger}$ 50%	60%	70%	95%
	$RGRS_s{}^{\dagger\dagger}$			
0.2	0.0000	0.6266	1.2531	7.6504
0.75	0.0000	0.8052	1.6103	9.8310
1.	0.0000	0.7160	1.4321	8.7432
	$SE_s{}^{\dagger\dagger}$			
0.2	0.0000	0.7147	1.4294	8.7264
0.75	0.0000	0.8932	1.7865	10.9063
1.	0.0000	0.8299	1.6599	10.1340
	$LE_s{}^{\dagger\dagger}$			
0.2	0.0000	−0.0881	−0.1762	−1.0760
0.75	0.0000	−0.0881	−0.1761	−1.0753
1.	0.0000	−0.1139	−0.2278	−1.3908
	Relative Gains in Asset Prices			
0.2	0.0000	−0.0425	−0.0850	−0.5190
0.75	0.0000	−0.0096	−0.0193	−0.1177
1.	0.0000	−0.0054	−0.0107	−0.0654
	Home Average Terms of Trade			
0.2	0.0000	0.0405	0.0811	0.4949
0.75	0.0000	0.0076	0.0151	0.0923
1.	0.0000	0.0043	0.0087	0.0531

All measures are in percentages. $n = \frac{1}{2}$, $\theta = 1.5$, $\rho = 4$, $\gamma = 50\%$, $\phi^* = 1 - \phi$, $\eta = \eta^* = 50\%$. Fourth-order approximation.

† Columns refer to the distribution displayed in Table 1

†† Permanent consumption equivalent (pce) units relative to total welfare in pce units.

Table 8: Home bias in consumption and home relative kurtosis of TFP - CRRA preferences

Home-Bias	$\eta^\dagger$ 50%	60%	70%	95%
	$RGRS_s{}^{\dagger\dagger}$			
0.2	0.0000	0.9644	1.9287	3.2581
0.75	0.0000	1.2393	2.4785	4.1869
1.	0.0000	1.1021	2.2042	3.7235
	$SE_s{}^{\dagger\dagger}$			
0.2	0.0000	1.0375	2.0749	3.5051
0.75	0.0000	1.2601	2.5201	4.2572
1.	0.0000	1.1227	2.2453	3.7930
	$LE_s{}^{\dagger\dagger}$			
0.2	0.0000	−0.0731	−0.1462	−0.2470
0.75	0.0000	−0.0208	−0.0416	−0.0703
1.	0.0000	−0.0206	−0.0411	−0.0695
	Relative Gains in Asset Prices			
0.2	0.0000	−0.0281	−0.0562	−0.0949
0.75	0.0000	−0.0047	−0.0095	−0.0160
1.	0.0000	−0.0023	−0.0047	−0.0079
	Home Average Terms of Trade			
0.2	0.0000	0.0253	0.0506	0.0855
0.75	0.0000	0.0018	0.0035	0.0060
1.	0.0000	0.0008	0.0016	0.0027

All measures are in percentages. $n = \frac{1}{2}$, $\theta = 1.5$, $\rho = 4$, $\gamma = 50\%$, $\phi = \phi^* = 50\%$, $\eta^* = 1 - \eta$. Fourth-order approximation.

† Columns refer to the distribution displayed in Table 1

†† Permanent consumption equivalent (pce) units relative to total welfare in pce units.

D Distribution of moments across countries

Table 9 shows the three moments of interest for the PWT9.1 list of countries. Country iso-code to English name conversion is shown in Table 10.

Table 9: Sample moments

Country	Stdev	Skewness	Kurtosis	Country	Stdev	Skewness	Kurtosis	Country	Stdev	Skewness	Kurtosis
ARG	6.775	0.654	0.850	FIN	3.794	-0.174	0.421	MKD	3.426	-1.371	3.464
AUS	2.333	-0.892	1.269	FRA	2.535	-0.621	-0.364	MLT	5.172	2.263	10.812
AUT	1.972	-0.678	-0.043	GBR	2.755	-0.693	3.072	NLD	2.915	-0.594	0.041
BEL	2.788	-0.922	1.488	GRC	4.746	-0.530	0.735	NOR	3.986	-0.322	1.204
BGR	5.852	-1.711	3.701	HRV	7.398	-2.056	5.286	NZL	3.437	-0.733	0.961
BRA	5.266	0.404	-0.426	HUN	3.586	-0.999	4.587	POL	4.647	-1.833	3.815
CAN	2.704	-1.308	2.372	IDN	5.781	-0.058	-0.014	PRT	4.214	-0.244	0.781
CHE	2.566	-0.644	3.554	IND	4.377	-0.427	0.175	ROU	5.047	-0.341	0.701
CHL	6.762	-1.063	4.346	IRL	5.461	1.876	11.222	RUS	9.937	-0.413	-0.644
CHN	4.417	-1.261	3.371	ISL	6.571	-0.516	1.189	SAU	12.873	0.465	0.915
COL	3.419	0.037	-0.082	ISR	3.664	-0.048	-0.186	SVK	6.453	-2.532	9.426
CRI	3.597	-0.705	1.919	ITA	3.110	0.289	0.411	SVN	4.726	-1.839	4.098
CYP	7.781	0.026	2.036	JPN	3.866	0.065	-0.832	SWE	2.940	-0.527	0.233
CZE	4.854	-2.601	9.711	KOR	5.496	-0.814	1.673	TUR	4.951	-0.274	-0.741
DEU	2.483	-0.552	0.019	LTU	8.289	-1.620	1.549	USA	2.205	-0.616	0.344
DNK	2.918	-0.450	-0.039	LUX	5.390	-1.174	3.912	ZAF	3.124	0.724	1.687
ESP	3.655	-0.589	0.030	LVA	9.815	-2.046	5.330	ZMB	11.232	-0.517	0.455
EST	8.143	-2.430	8.429	MEX	4.763	-2.289	9.673				

Table 10: List of countries

ABW = Aruba	FRA = France	MYS = Malaysia
AGO = Angola	GAB = Gabon	NAM = Namibia
AIA = Anguilla	GBR = United Kingdom	NER = Niger
ALB = Albania	GHA = Ghana	NGA = Nigeria
ARE = United Arab Emirates	GIN = Guinea	NIC = Nicaragua
ARG = Argentina	GMB = Gambia	NLD = Netherlands
ATG = Antigua and Barbuda	GNB = Guinea-Bissau	NOR = Norway
AUS = Australia	GNQ = Equatorial Guinea	NPL = Nepal
AUT = Austria	GRC = Greece	NZL = New Zealand
BDI = Burundi	GRD = Grenada	OMN = Oman
BEL = Belgium	GTM = Guatemala	PAK = Pakistan
BEN = Benin	HKG = China, Hong Kong SAR	PAN = Panama
BFA = Burkina Faso	HND = Honduras	PER = Peru
BGD = Bangladesh	HTI = Haiti	PHL = Philippines
BGR = Bulgaria	HUN = Hungary	POL = Poland
BHR = Bahrain	IDN = Indonesia	PRT = Portugal
BHS = Bahamas	IND = India	PRY = Paraguay
BLZ = Belize	IRL = Ireland	PSE = State of Palestine
BMU = Bermuda	IRN = Iran (Islamic Republic of)	QAT = Qatar
BOL = Bolivia (Plurinational State of)	IRQ = Iraq	ROU = Romania
BRA = Brazil	ISL = Iceland	RWA = Rwanda
BRB = Barbados	ISR = Israel	SAU = Saudi Arabia
BRN = Brunei Darussalam	ITA = Italy	SDN = Sudan
BTN = Bhutan	JAM = Jamaica	SEN = Senegal
BWA = Botswana	JOR = Jordan	SGP = Singapore
CAF = Central African Republic	JPN = Japan	SLE = Sierra Leone
CAN = Canada	KEN = Kenya	SLV = El Salvador
CHE = Switzerland	KHM = Cambodia	STP = Sao Tome and Principe
CHL = Chile	KNA = Saint Kitts and Nevis	SUR = Suriname
CHN = China	KOR = Republic of Korea	SWE = Sweden
CIV = Cote d'Ivoire	KWT = Kuwait	SWZ = Eswatini
CMR = Cameroon	LAO = Lao People's DR	SYC = Seychelles
COD = Congo, Democratic Republic	LBN = Lebanon	SYR = Syrian Arab Republic
COG = Congo	LBR = Liberia	TCA = Turks and Caicos Islands
COL = Colombia	LCA = Saint Lucia	TCD = Chad
COM = Comoros	LKA = Sri Lanka	TGO = Togo
CPV = Cabo Verde	LSO = Lesotho	THA = Thailand
CRI = Costa Rica	LUX = Luxembourg	TTO = Trinidad and Tobago
CYM = Cayman Islands	MAC = China, Macao SAR	TUN = Tunisia
CYP = Cyprus	MAR = Morocco	TUR = Turkey
DEU = Germany	MDG = Madagascar	TWN = Taiwan
DJI = Djibouti	MDV = Maldives	TZA = U.R. of Tanzania: Mainland
DMA = Dominica	MEX = Mexico	UGA = Uganda
DNK = Denmark	MLI = Mali	URY = Uruguay
DOM = Dominican Republic	MLT = Malta	USA = United States of America
DZA = Algeria	MMR = Myanmar	VCT = St. Vincent & Grenadines
ECU = Ecuador	MNG = Mongolia	VEN = Venezuela (Bolivarian Republic of)
EGY = Egypt	MOZ = Mozambique	VGB = British Virgin Islands
ESP = Spain	MRT = Mauritania	VNM = Viet Nam
ETH = Ethiopia	MSR = Montserrat	ZAF = South Africa
FIN = Finland	MUS = Mauritius	ZMB = Zambia
FJI = Fiji	MWI = Malawi	ZWE = Zimbabwe