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Financial Frictions, Firm Dynamics and the Aggregate Economy: Insights from Richer Productivity Processes

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1 Introduction

Output per capita differs vastly across countries. An extensive literature, e.g. Klenow and Rodríguez-Clare (1997) and Hall and Jones (1999), shows that differences in aggregate productivity mainly drive differences in output per worker. Firm heterogeneity is crucial to understand the differences in aggregate productivity. Firms are heterogeneous in their efficiency to transform inputs, mainly capital and labour, into output. As a result, the aggregate productivity of a country depends on the productivity distribution of firms that operate.

Furthermore, the allocation of resources across firms also matters for aggregate productivity. A growing literature in macroeconomics, starting with Guner, Ventura, and Xu (2008), Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), analyses how the allocation of resource affects aggregate productivity. The basic idea in this literature is that if the most efficient firms are not using a larger amount of inputs than less efficient firms, the total amount of output produced by the country is smaller than in a first-best where that does not happen.

What are the factors behind misallocation? Financial frictions are an obvious culprit. Financial frictions affect the capital allocation, as they prevent firms with low internal resources from installing their optimal capital level. Nevertheless, generating significant aggregate productivity and output losses from financial frictions in quantitative models of firm dynamics has been challenging, e.g. Buera, Kaboski, and Shin (2011) and Midrigan and Xu (2014).

The effects of financial frictions on firm dynamics and aggregate productivity depend crucially on the productivity shocks that firms face. On the one hand, as highlighted by Moll (2014), the persistence of productivity shocks determines the speed at which firms can accumulate internal funds and surpass financial frictions. If shocks are very persistent, firms that receive a sequence of favourable shocks will grow, retain profits, and finance their investment without borrowing. On the other hand, dispersion (variance), asymmetry (skewness) and tailedness (kurtosis) of shocks also matter. They determine the probability of an initially low productivity firm to have a good productivity realisation in the next period. After a good productivity realisation, this firm would like to invest a copious amount to benefit from the favourable shock, which may not be feasible given its level of internal funds. Therefore, if initially, low productivity firms have a significant probability of becoming highly productive tomorrow, they are likely to become a financially constrained firm as well. Finally, the variability of shocks also determines the firm's level of uncertainty in its investment decisions. Due to the time-to-build nature of investment decisions, firms decide how much capital to have for the next period based on their expected productivity. High uncertainty implies that many firms would end up with too little or too much capital for their realised productivity levels, as emphasised by Asker, Collard-Wexler, and De Loecker (2014).

Almost all existing papers on firm dynamics model firm-level productivity as a mere AR(1) process, despite the linkages among the productivity and financial frictions. Hence, all firms, independently of their current level of productivity, face the same persistence and shock variability. Furthermore, productivity shocks come from a well-behaved, symmetric Gaussian distribution.

In this paper, I nonparametrically estimate a non-linear and non-Gaussian firm-level productivity process. I use a comprehensive dataset, with more than 6.5 million firm-year observations, containing balance sheet information for Spanish firms from 1999 to 2014. I use recently developed techniques in the income dynamics literature by Guvenen, Karahan, Ozkan, and Song (2015), Arellano, Blundell, and Bonhomme (2017) and De Nardi, Fella, and Paz-Pardo (2019) to show that productivity dynamics are non-linear with non-Gaussian innovations.[1] The estimation allows persistence, variance, skewness and kurtosis of productivity shocks to depend on where the firms currently are in the productivity distribution.

I find that productivity persistence is hump-shaped, while shock variability is U-shaped with past productivity. Furthermore, skewness is decreasing, and shock kurtosis is hump-shaped with past productivity. These features contrast with the AR(1) productivity process usually used in the literature, implying very different productivity dynamics. Considering a low productivity firm, I find it has low persistence, so its past low productivity history barely matters. It also has more volatile and positively skewed shocks; therefore, there is a significant probability of receiving a good productivity realisation in the next period. The probability of a firm that is initially in the first decile of the productivity distribution to be above the median in the next period is 6.7% in the estimated process. This probability is only 1.3% if productivity dynamics are assumed to follow an AR(1). On top of that, the lower persistence and negative skewness of high productivity firms point out that these high productivity episodes are not long-lasting for some firms, slowing down the speed at which firms can surpass financial frictions through internal profit accumulation, faster in the high productivity states, as shown in Moll (2014).

I next build a model of firm dynamics to study how financial frictions affect aggregate productivity by distorting capital allocation across firms. The model economy builds on earlier papers on the role of financial frictions and firm dynamics, e.g. Cooley and Quadrini (2001), Gomes (2001),

[1] Arellano et al. (2017) use quantile regressions, while Guvenen et al. (2015) and De Nardi et al. (2019) study the earnings distribution conditional on previous earnings. All of them recover an earnings process that looks very different from the canonical AR(1) process.

Buera et al. (2011), Khan and Thomas (2013) and Midrigan and Xu (2014). Although, it has three main differences that set it apart from the existing literature. First and most importantly, the productivity process is non-linear and non-Gaussian instead of the AR(1) broadly used. Second, I model the firm life cycle and tie it to the data. Firms enter the market, and, as they age, they grow and decline depending on how their productivity evolves and their financial conditions. Finally, they eventually exit the market. Third, financial frictions are modelled through a size-dependent borrowing constraint, which nests the standard borrowing constraint with constant pledge-ability common in the literature, as in Gopinath, Kalemli-Ozcan, Karabarbounis, and Villegas-Sanchez (2017). Hence, what fraction of its capital a firm can pledge depends on its size.

In order to discipline the quantitative model, I use firm-level data to document several novel facts on misallocation and the financial behaviour of the firms. Misallocation of capital across firms in the data appears as a high average revenue product of capital (ARPK) for the constrained firms, which contrasts with the predictions in a perfectly competitive world without financial frictions, where the ARPK should be equalised across firms. In that sense, the standard deviation of ARPK has become the standard statistic used to assess capital allocation efficiency in the economy. Financial frictions mostly affect the capital level of young, small and high productivity firms. Those firms are less likely to have enough internal funds to sustain their optimal level of capital. Along those lines, the data confirm that the mean of ARPK and its standard deviation is larger for young, small and highly productive firms.

I also show how the leverage ratio, measured as debt over total assets, varies by firm characteristics. A significant fraction of firms, 29%, do not use costly debt. Furthermore, average leverage is decreasing with firm age and firm productivity but increasing with firm size. Furthermore, these patterns arise both in the extensive margin, probability of using costly debt, and the intensive margin, average leverage conditional on using debt.

The simulated economy is consistent with the empirical evidence on financial frictions. The model matches how the average level and dispersion of the ARPK changes with age, size and productivity. The model without financial frictions fails in accounting for those patterns. The model also matches the firm's financial behaviour. It generates a leverage distribution very similar to the one in the data. Second, it accounts for the negative relation of firm leverage with firm's age and productivity; and the positive relation with firm's size. As in the data, these regularities are present in both the extensive and intensive margin.

I then use the model to study how financial frictions affect firms' initial size and the growth over their life cycle. Moreover, I quantify the aggregate consequences of financial frictions. I obtain two main results. First, financial frictions affect the firm's life cycle. I compare the results from the benchmark model with the solution of a benevolent social planner that maximises total output taking the economy's structure as given. The social planner abstracts from financial friction by reallocating capital across firms taking into account only firm productivity. Compared to a world without financial frictions, an average entrant is three times smaller in the benchmark economy with financial frictions. Although the size gap between entrants and incumbents reduces over the firm's life cycle, it is not fully closed, pointing out that the process to overcome financial frictions through internal profit accumulation is slow. Indeed, it is prolonged for young (less than 5 years old) firms and only speeds up when firms mature (more than 5 years old).

Second, the aggregate effects of financial frictions are significant. Around 1/3 of the firms are constrained in their capital decision. The inefficient allocation of capital translates into productivity losses of 16%. These effects are much smaller in an economy with an AR(1) productivity process: only 1/4 of the firms are constrained, and the productivity losses from financial frictions are only 8%.

Finally, I do a decomposition exercise to analyse why the effects of financial frictions are more prominent in the model with non-linear and non-Gaussian productivity dynamics than in a standard AR(1). In order to do so, I run several parallel economies modifying the characteristics of the non-linear and non-Gaussian productivity process so that it inherits the characteristics of the AR(1) dynamics. Then, I compare the aggregate effects of financial frictions in these parallel economies. I find that around half of the more considerable productivity losses are due to the non-linearities (non-constant persistence and shock variability) while the non-Gaussian shocks (non-constant skewness and kurtosis) contribute another half.

The rest of the paper is organised as follows. Section 2 reviews the related literature and states the contribution of the paper. Section 3 describes the main dataset and variables used in the remaining of the paper. Section 4 covers the empirical part of the paper. It has three subsections that analyse the characteristics of the productivity dynamics, evidence on the presence of financial frictions and firms' financial behaviour. Section 5 sets up the model. Section 6 shows the benchmark economy. Section 7 quantifies the effects of financial frictions over the firm life cycle and their aggregate consequences. Finally, Section 8 concludes.

2 Related Literature

This paper relates to four strands of the literature: firm dynamics and financial frictions, misallocation, empirical finance and non-linear processes. In the firm dynamics and financial frictions literature, an early contribution by Cooley and Quadrini (2001) highlights that persistence in the productivity process and financial frictions are two key elements to obtain realistic firm dynamics.[2] In contrast to them, I introduce a richer productivity process directly estimated from the data. One feature is that persistence is non-linear, and it depends on past productivity. The paper shows that the productivity process is important for firm dynamics, but it also interacts with financial frictions. The negative effects of financial frictions over the firm life cycle are amplified under the non-linear productivity dynamics.

A recent paper, Chatterjee and Eyigungor (2019) show that if firms are subject to financial frictions, low-interest rate episodes can rationalise the rise in firm concentration, as recently noticed for the US. This paper shows that a non-linear and non-Gaussian productivity process, as estimated in the data, is crucial to generate the firm concentration levels seen for Spanish firms. I show that firm concentration is much smaller if productivity dynamics follow a standard AR(1).

Within the misallocation literature, Buera et al. (2011) and Midrigan and Xu (2014) study the effects of financial frictions in developing economies. They model a dual economy with formal and informal sectors. Both papers find that financial frictions prevent firms from entering into the formal economy, producing large losses in aggregate productivity. Nevertheless, they disagree on the effects of financial frictions once firms enter the formal economy. Buera et al. (2011) point out that they can be large; while, Midrigan and Xu (2014) find that they are small. The latter argues that firms can accumulate internal funds pretty fast in the most productive sector and overcome the effects of financial frictions. This paper differs from these two papers along several dimensions. First, it focuses on a developed economy, modelling only the formal sector. Second, it ties carefully firm entry and exit to the data to match the firm life cycle. Finally, the introduction of the non-linear productivity process affects the assessment of financial frictions. I find that financial frictions have important consequences in the formal sector.

The non-linear productivity process is key, as the aggregate productivity losses are twice as large as the ones implied under AR(1) productivity dynamics. The larger effects of financial frictions under a non-linear productivity process goes in line with the work of Asker et al. (2014),

[2] They document and rationalise through the lens of the model two empirical regularities. First, size dependence; conditional on firm age, firm growth and exit rates decreases with firm size. Second, age dependence; conditional on firm size, firm growth and exit rates are decreasing with firm age.

and especially Moll (2014). Asker et al. (2014) points out that firm uncertainty affects the investment decision of the firm, and it has consequences on aggregate productivity as the ex-ante optimal investment level may not be optimal ex-post; once the productivity shock realises. Moll (2014) highlights the importance of productivity persistence to financial frictions have an effect on aggregate productivity. The non-linear productivity process proposed in this paper has these two features, as persistence and shock variability depend on past productivity. The non-linear persistence and shock variability contribute to half of the larger effects of financial frictions. The other half is due to the non-Gaussian nature of productivity shocks.

David and Venkateswaran (2019) do a taxonomy of the frictions that affect the allocation of capital and quantify their importance. They find that correlated distortions are an essential source of capital misallocation in China and US.[3] This paper finds that average ARPK and its dispersion are higher for young, small, and high productivity firms. These are the firms most likely to be affected by financial frictions. Although they do not model financial frictions, they point out that financial frictions can generate those correlated distortions from a simple model. Therefore, financial frictions could account for a sizeable fraction of total misallocation. In this paper, I confirm that financial frictions generate correlated distortions that look like in the data. Finally, I find that financial frictions have a notable impact on aggregate productivity as they suggest.

Finally, Jo and Senga (2019) propose a set-up to evaluate policies aimed to ease financial frictions faced by firms and evaluate their aggregate effects. This paper differs from Jo and Senga (2019) in two main points. First, the focus is very different. They focus on a policy exercise, while this paper pursues a quantification of financial frictions. Second, they introduce a productivity process with non-Gaussian shocks. In this paper, the estimated productivity process is carefully tied to the data, featuring non-Gaussian productivity shocks and non-linear persistence and shock variability.

Regarding the empirical finance literature, the financial behaviour and capital structure of firms have been extensively studied, both empirically and theoretically, see, e.g. Lemmon, Roberts, and Zender (2008) and Graham and Leary (2011). However, most of the papers have focused on publicly-listed firms. The main reason is the lack of comprehensive datasets on privately-held companies. Although publicly-listed firms represent a significant fraction of total value-added, they are a small fraction of all the firms in the economy. As a consequence, their behaviour is not representative of the whole economy. This paper and the contemporary work of Dinlersoz,

[3] The term of correlated distortions has been used in the literature to refer to the situation when ARPs are positively correlated with firm characteristics, specially productivity.

Kalemli-Ozcan, Hyatt, and Penciakova (2018) fill this gap by studying the financial behaviour of privately-held firms. Although the focus of the papers is different, we find similar patterns with some differences discussed in Section 4.3. In particular, both papers find that large firms are more leveraged than small ones. The model economy presented in this paper can accommodate this fact. Chatterjee and Eyigungor (2019) build a firm dynamics model with default to account for the positive correlation of leverage and firm size, as well.

Finally, this paper relates to the recent literature on non-linear processes, see, e.g. Guvenen et al. (2015), Arellano et al. (2017) and De Nardi et al. (2019). The focus has been on estimating the income process of households and individuals. The main result is that the income process differs from a standard AR(1). Arellano et al. (2017) shows that the non-linear income process has a consequence on the saving and consumption behaviour of individuals, fitting better the empirical consumption dynamics than the canonical AR(1). I bring those techniques to the firm dynamics literature and estimate a rich process for productivity dynamics. The results show that it differs significantly from a standard AR(1) process. The non-linear and non-Gaussian productivity process is crucial to evaluate the effect of financial frictions. This paper shows that the effect of financial frictions over the firm's life cycle and their aggregate consequences are much larger under the estimated process than under AR(1) productivity dynamics.

3 Data

The main dataset is called *Central de Balances Integrada* (CBI) and it is compiled by Banco de España (BdE). Firms have the legal requirement to deposit their annual accounts at the Commercial Registry.[4] At the end of the financial year, the managers of Spanish firms collect all the information and elaborate the annual accounts. Then, they deposit them at the Commercial Registry during the first half of the year. The BdE has an agreement with the Commercial Registry, which gives access to that information. The annual accounts consist of three documents: balance sheet, income statement and annual report. The balance sheet reflects all the firm's assets and liabilities at the end of the financial year. The income statement shows all the sources of income and expenses. Finally, the annual report states all the relevant information not considered in the two previous documents, such as dividend payments and employment structure. In the paper, I use the data from 1999 to 2014 covering all economic sectors, which results in more than 12 million

[4] The Spanish law imposes penalties if a firm does not deposit their annual accounts in form and time. These penalties are from economic, imposed on the firm, to the legal inability of the managers to run other firms or make them respond against the firm liabilities with their assets in case of bankruptcy.

firm-year observations.

I focus on privately-held companies that are legally established as limited liability firms. There are several reasons for this selection. First, publicly-held companies are a minority in Spain.[5] Moreover, those companies have access to other funding sources, such as equity, which I do not consider in the proposed framework. Second, I do not include firms in the public sector since they have access to other funding sources. Finally, the sample does not include self-employed since they often are not limited liability firms, and hence, do not need to present their accounts at the Commercial Registry. The final sample represents 98.6% of all the firms in the database, and they account for 74% of total value-added and 91% of total employment.

In order to evaluate the representativeness of the sample for the Spanish economy, I compare it with *el Directorio Central de Empresas* (DIRCE). DIRCE provides aggregate information on the census of Spanish firms. Several points arise. First, the selected sample covers around 50% of all the firms in Spain, and more importantly, the coverage is stable over the studied period. In terms of employment, the coverage is more petite, around 30% of the total, mainly due to the focus on firms from the private sector.[6] Regarding the firm size distribution, the coverage is constant across different size groups. Finally, the coverage is similar if we restrict our attention to the manufacturing sector.

I next construct the main variables used in the analysis. From the balance sheet information, I recover capital, debt and net worth. Capital is measured as the book value of long-term assets.[7] The measure of capital is deflated at the 2-digits sector level using investment deflators from the Spanish National Accounts. Debt is defined as costly debt, which is the sum of long-term liabilities and costly short-term liabilities. These are the funds for which the firm has to pay interest and

[5] According to the Spanish Commission of Stock Exchange (CNMV), there are around 210 listed firms in Spain, which represent a negligible fraction of the total number of firms, more than 800 thousand firms.

[6] There are several reasons why the CBI does not cover all the firms in the economy. First, the team in charge of data management could not compile all the information arriving from the Commercial Registry, especially relevant when most of the information was not digital. For this reason, I disregard all the data before 1999. Second, some firms deposit their accounts after the deadline. Although the BdE receives several updates from the Commercial Registry during the year, if the firm commits their accounts very late, the information does not arrive at the BdE. Third, some firms do not deposit their annual accounts, a minority due to the legal consequences. Finally, the quality of the information presented by some firms is inferior; and therefore, BdE does not incorporate them in the CBI.

[7] Some papers, e.g. Hsieh and Klenow (2009), use perpetual inventory methods to compute capital. Both methods, perpetual inventory and book value, have drawbacks. The perpetual inventory method relies on a standard depreciation rate for all the capital. Not taking into account heterogeneity in the capital, buildings, computers, machines ..., introduces measurement error in the capital measure, see, e.g. Collard-Wexler and De Loecker (2016). The book value method does not suffer from this problem, as capital is computed after accounting depreciation, which is firm and capital specific. The main drawback of the measure of capital as book value is that it is reported at historical cost. This cost may differ from the actual one.

does not include other short-term funding sources, such as working capital. Finally, net worth is computed as the difference between total assets and total liabilities. I deflect all the variables using CPI at the province level.

From the income statement information, I recover value-added, wage bill, and profits. I compute value-added as revenue minus intermediate goods. The resulting variable is deflated at the 2-digits sector level using value-added deflators from the Spanish National Accounts. The wage bill corresponds to the total cost of employment, including wages, bonuses and social security payments. Finally, profits are measured after taking into account depreciation, fund provisions and taxes. Therefore, it is the available income that the firm can keep as internal funds or pay to the shareholders as dividends. The wage bill and profits are deflated using CPI at the province level.

From the annual report information, I recover employment and dividends. Employment is measured in full-time equivalent units. Therefore, it captures hiring heterogeneity across firms, full-time vs part-time, and the timing when the firm hires or fires a worker, making the employment measure comparable across firms. Finally, I recover the dividend payment from the approval of the profit distribution proposal that the managers make to shareholders. I deflate the dividends using CPI at the province level.

The key variable of interest is firm productivity. In order to estimate it, I first assume a functional form that links output (value-added) and inputs (capital and labour). As it has become standard in the literature, I use wage bill instead of employment to measure labour. The main advantage of wages is that they consider workers heterogeneity, such as education and experience, reflected in higher wages. I choose a Cobb-Douglas specification under decreasing returns to scale, governed by a span of control parameter (η).[8] The production function reads as

$$py_{si} = A_{si}[k_{si}^{\alpha_s} l_{si}^{1-\alpha_s}]^{\eta} \qquad \alpha_s \in (0,\ 1) \textit{ and } \eta \in (0,\ 1), \tag{1}$$

where py_{si} is value-added, A_{si} is total factor productivity (TFPQ), k_{si} is capital and l_{si} is labor of a firm i operating in sector s. The model economy in Section 5 displays exactly the same firm-level production function.

I allow for differential output to input elasticity at the 2-digits sector level, which is governed

[8] This is analogous to constant returns to scale production function and constant elasticity of substitution demand system with elasticity parameter, σ. The two models yield to the same decreasing returns to scale in the value-added production function when $\eta = \dfrac{\sigma-1}{\sigma}$.

by α_s. I do not allow, however, for a differential degree of decreasing returns to scale across sectors. After parameterisation, I invert the production function to infer the firm-level productivity.

Regarding the parameterisation, I rely on the static nature of the labour decision to recover the values of α_s at the sector level. In order to do so, I first solve for the labour decision at the firm level and then aggregate them at the sector level. The values of α_s are given by the following expression:

$$\alpha_s = 1 - \frac{1}{\eta}\frac{wL_s}{Y_s} = 1 - \frac{1}{\eta}\frac{\sum_{i=1}^{Ns} wl_{si}}{\sum_{i=1}^{Ns} py_{si}}, \tag{2}$$

where wL_s is the aggregate wage bill, Y_s is the aggregate value-added and N_s is the total number of firms operating in sector s. In order to reduce the scope of measurement error, I rely on aggregate information on value-added and wage bill from the Spanish National Accounts to recover the α_s.

Second, I assign a value to the decreasing returns to scale parameter, η. In order to do so, I follow an iterative process. I consider different values of η, and for each value, I estimate the firm productivity, A_{si}, and the underlying productivity process. Then, I solve the model economy with the estimated productivity process. In the model, the value of η has a direct influence on the standard deviation of the capital distribution, $SD(k_{si})$. As a result, I choose the value of η for which the model economy gives the best match to this moment. This procedure results in a value of η equal to 0.83. I also construct sector weights (ω_s); so that I can aggregate the sector-specific measures. In Appendix A, I provide further details on the estimation and the distribution of the recovered parameters.

Finally, I do a last sample restriction and cleaning of the resulting dataset. First, I drop tiny firms.[9] I only consider firms with more than 1,000 € in value-added, and 500 € in the capital in real €2010. Furthermore, I disregard all the firms with less than 0.5 employees in full-time equivalent units. Second, I clean the dataset from outliers and inconsistent observations. Regarding outliers, I do a 1% winsorisation of the lower and upper tail of the productivity distribution at the sector level. Regarding inconsistent observations, I drop firms that seem to report the variables with the wrong units. In order to do so, I compute average wage and drop firm-year observations with unrealistic figures. Finally, I disregard observations that appear to have huge rank reversals in the output, inputs and productivity distribution. For instance, firms are at the top decile of the sector productivity distribution but the bottom first percentile of the sector employment distribution. In Appendix A, I provide further information of this process. The final dataset consists of 6,500,945

[9] Firms with zero employment or minimal economic activity are particularly likely to be used as instrumental firms in order to avoid taxes or hide heritage to the fiscal authorities.

firm-year observations corresponding to 1,024,144 different firms covering from 1999 to 2014.

4 Empirics

The productivity process is key in the firm dynamics literature to yield realistic firm behaviour. Cooley and Quadrini (2001) document its importance for generating age and size dependence, e.g. young and small firms growing faster than their old and large counterparts. Moreover, it affects the ability of firms to accumulate enough internal funding to overcome financial frictions, as shown in Moll (2014). Despite its importance, modelling productivity dynamics as a standard AR(1) is common. The standard AR(1) process imposes several restrictions. First, the productivity persistence and shock variability are assumed to be the same for all the firms. Second, productivity shocks are assumed to come from a Gaussian distribution. I propose a flexible estimation procedure that overcomes these two drawbacks. And I show that the empirical productivity process differs substantially from the AR(1) assumption, prevalent in the firm dynamics literature.

I estimate the bivariate relation of today's and tomorrow's productivity non-parametrically, capturing the non-linearities and non-Gaussian nature of the shocks in the productivity process. There are two important concerns regarding this procedure. The first one is that the non-parametric estimation is data-intensive, particularly if you aim to capture the productivity dynamics at the distribution's tails. Second, the estimated firm-level productivity has a sector and aggregate component that evolves over the business cycle. As I am interested in the productivity dynamics in the stationary economy, I clean the estimated productivity from the sector-year variation. In other words, I standardize the estimated productivity at the sector-year level; and then I pool the data across sectors and years. It is important to note that I allow the production function to differ across sectors, as α_s is sector-specific.

I first discretize the standardized productivity in 16 non-equally spaced intervals, as shown in Figure 1, paying special attention to the tails of the distribution.[10] This is particularly important as it is well known that the size distribution is skewed to the right, [11] Furthermore, the output is very concentrated at the top of the distribution.[12] Therefore, it is essential to capture the produc-

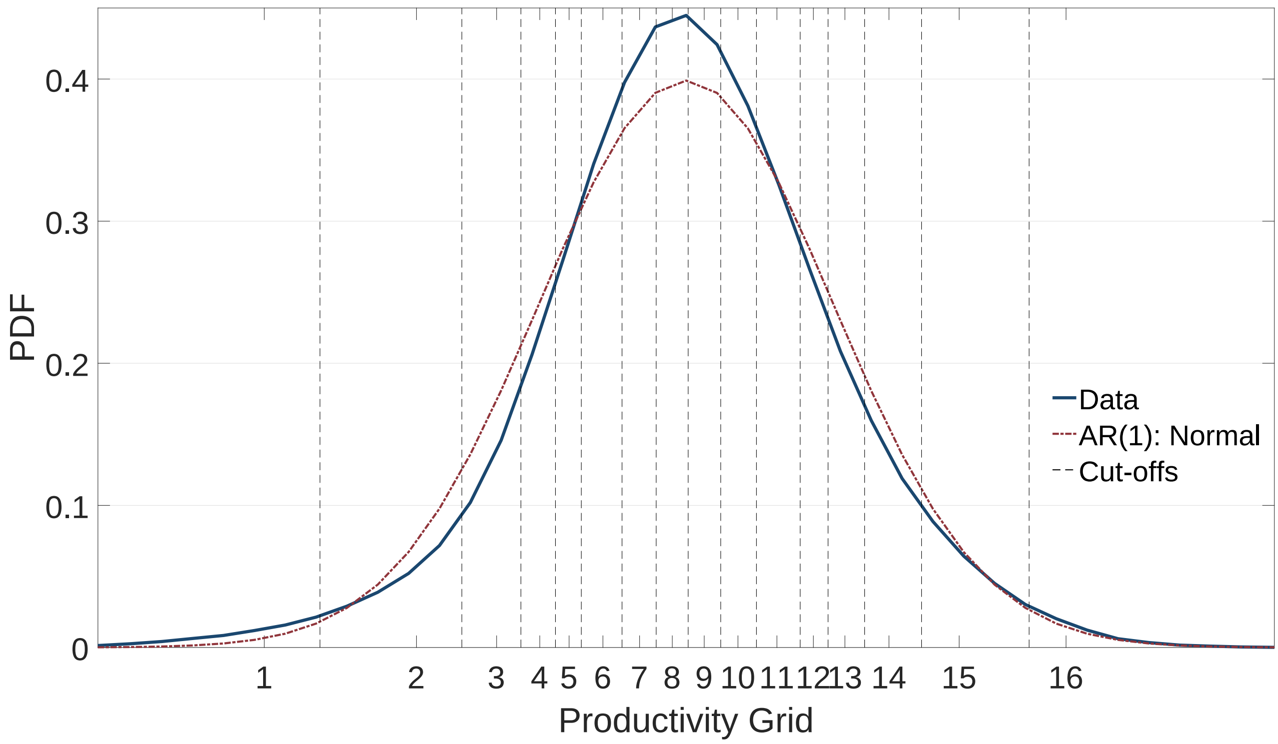
[10] I use the following quantiles as cut-offs: $Q_{0.01}$, $Q_{0.05}$, $Q_{0.10}$, $Q_{0.15}$, $Q_{0.20}$, $Q_{0.30}$, $Q_{0.40}$, $Q_{0.50}$, $Q_{0.60}$, $Q_{0.70}$, $Q_{0.80}$, $Q_{0.85}$, $Q_{0.90}$, $Q_{0.95}$, $Q_{0.99}$.

[11] see, e.g. Decker, Haltiwanger, Jarmin, and Miranda (2015) for an analysis of the skewness in the U.S. over time and its consequences for the economy.

[12] See, e.g. Autor, Dorn, Katz, Patterson, and Van Reenen (2019) and the note Philippon (2018) for the evolution of concentration in the U.S. and its consequences for investment and growth.

tivity behaviour of the low and middle productivity firms and the high productivity ones, which are responsible for a significant fraction of total output. Figure 1 contrasts the empirical productivity distribution with the one implied by a standard AR(1) process. The observed productivity distribution has a slightly longer tail at the left, i.e. it is negatively skewed. Therefore, there are a more significant fraction of very low productivity firms than very high productivity ones. Furthermore, the empirical distribution is more concentrated in its centre, i.e. high kurtosis, and therefore, having fatter tails. This translates into a more significant fraction of very low and high productivity firms than in the standard AR(1).

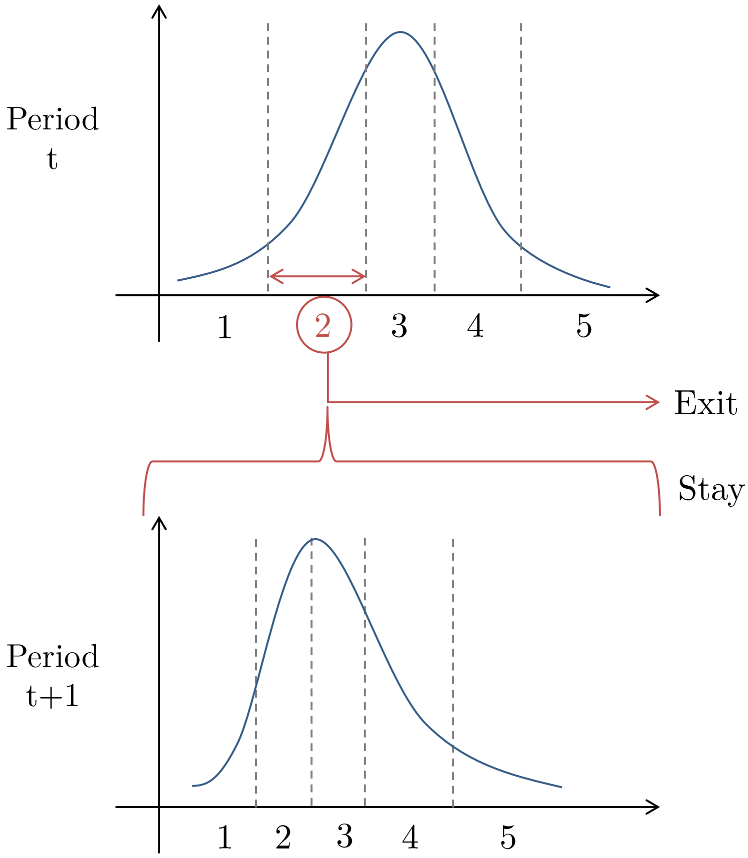
Figure 1: Productivity Distribution



4.1 Productivity Dynamics

The empirical firm's productivity distribution similarity with the one implied by a standard AR(1) hides richer productivity dynamics in the data than the ones implied by an AR(1) process. To estimate and characterize the productivity dynamics, I use firms that observed for at least two consecutive years. The estimation procedure is illustrated in Figure 2. Conditional on firms initially in one region of the productivity distribution, I estimate the exact quantiles as the discretization procedure for the next productivity distribution period. These conditional quantiles allow me to use the definitions of productivity persistence, shock variability, skewness and kurtosis used in Arellano et al. (2017).

Figure 2: Estimation of Productivity Dynamics

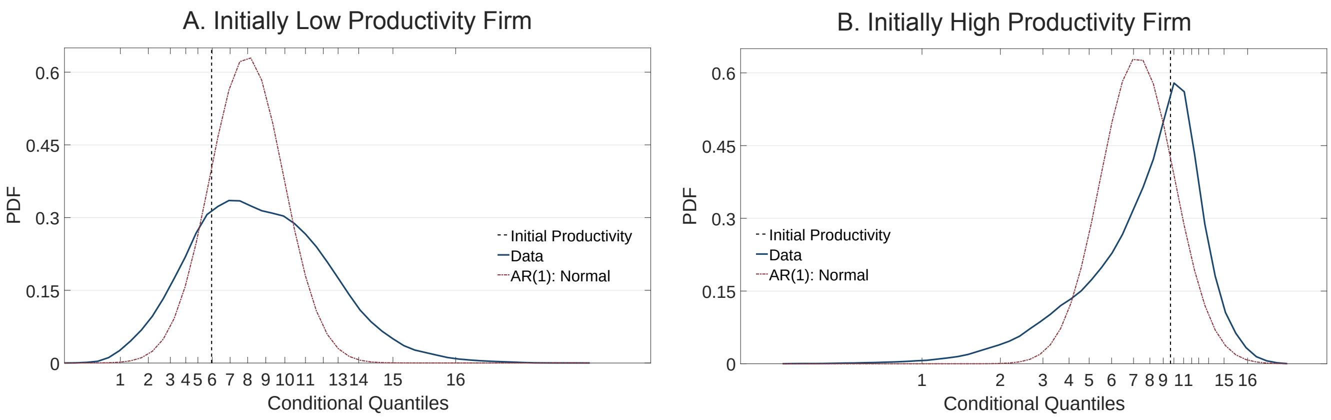


In Figure 3, I show the conditional productivity distribution for an initially low productivity firm (left) and an initially high productivity one (right) and compare them with the distributions implied if AR(1) dynamics are assumed. The conditional productivity distributions are far from being Gaussian. The empirical distribution is more dispersed for an initially low productivity firm and has a longer tail at the right, i.e. positive skewness. These features translate into a large probability of having a good productivity realization. Regarding an initially high productivity firm, there is a long tail at the left of the distribution, i.e. negative skewness, which contrasts with the symmetric distribution when AR(1) dynamics are assumed. And, it implies that high productivity firms have a large probability of having a large negative productivity shock. Therefore, good productivity realizations are not long-lasting for a large fraction of firms.

I also compute transitions to exit, i.e. the fraction of firms that leave the market, conditional on their initial productivity.[13] This is analogous to having an absorbing productivity state with zero productivity. Finally, I compute the entry rates of firms for each level of productivity. In Figure 4, I show the recovered entry and exit rates. Entry rates are the same for all the productivity levels, which means that entrants draw their productivity from the stationary distribution, a standard

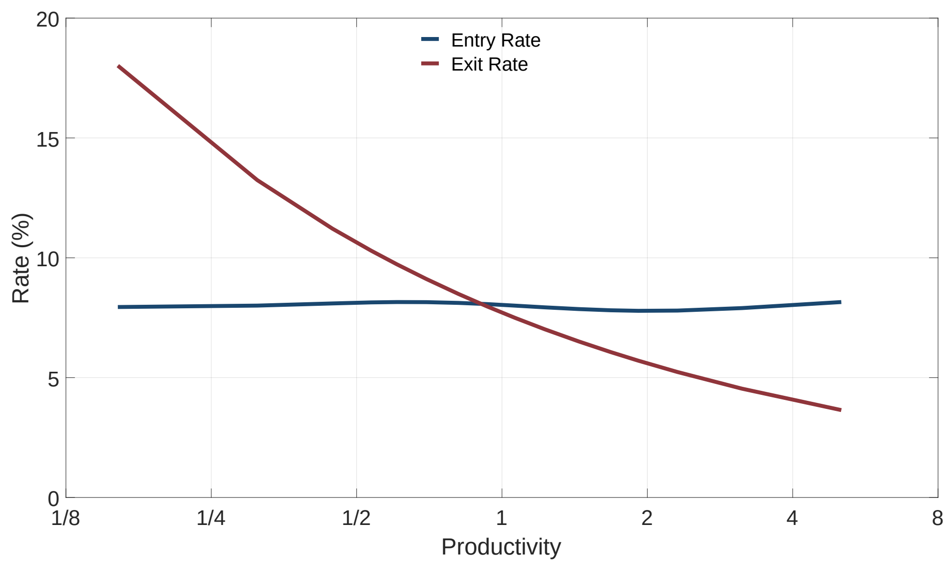
[13] As there is not an exit variable in the dataset, I infer firm exit from continuing firms. I use the panel dimension of the dataset to assess that a firm exits if it does not appear anymore in the following periods.

Figure 3: Conditional Productivity Distribution



assumption in models of firms dynamics with entry. On the other hand, exit rates decrease with firm productivity, so low productivity firms are more likely to exit. The entry and exit rates, together with the estimated transition probabilities, are the main ingredients that discipline the productivity dynamics in the model.

Figure 4: Entry and Exit Rates



Characteristics of the productivity process To compare the estimated productivity process with a standard AR(1) used in the literature, I estimate four objects. First, productivity persistence is defined as the fraction of productivity inherited in the next period conditional on facing the same productivity shock. The expression reads as follows:

$$\rho(log(A_{i,\ t-1}),\ \tau) = \frac{\partial Q(log(A_{i,\ t-1});\ \tau)}{\partial log(A_{i,\ t-1})}, \tag{3}$$

where $Q(log(A_{i,\ t-1});\ \tau)$ represents the quantile function of the productivity distribution in period t conditional on initial productivity, $log(A_{i,\ t-1})$, and τ is the quantile at which the function $Q(log(A_{i,\ t-1});\ \tau)$ is evaluated.

This gives a persistence estimate for each level of initial productivity and productivity shock. As I am interested in persistence conditional on initial productivity regardless of the productivity shock, I integrate over the shock distribution.[14] Therefore, the reported productivity persistence follows from this expression:

$$\rho(log(A_{i,\ t-1})) = E\left[\frac{\partial Q(log(A_{i,\ t-1});\ \tau)}{\partial log(A_{i,\ t-1})}\right]. \tag{4}$$

It is important to note that a standard AR(1) process features constant productivity persistence equals to the autoregressive parameter independently of the initial level of productivity and productivity shock.

Second, I define shock variability as the difference of two equally spaced quantiles from the median. It measures how wide is the subsequent period productivity distribution; and how much uncertainty the firm faces. The expression reads as follows:

$$\sigma(log(A_{i,\ t-1})) = Q(log(A_{i,\ t-1});\ \tau) - Q(log(A_{i,\ t-1});\ 1-\tau).^{15} \tag{5}$$

Third, shock skewness describes the asymmetry of the distribution. If the quantiles of the right tail are further away from the median than the left ones, the distribution exhibits positive or right skewness. If the contrary happens, it indicates negative or left skewness. The expression reads as follows:

$$sk(log(A_{i,\ t-1})) = \frac{Q(log(A_{i,\ t-1});\ \tau) + Q(log(A_{i,\ t-1});\ 1-\tau) - 2Q(log(A_{i,\ t-1});\ 0.5)}{Q(log(A_{i,\ t-1});\ \tau) - Q(log(A_{i,\ t-1});\ 1-\tau)}.^{16} \tag{6}$$

Finally, shock kurtosis or tailedness captures the concentration of probability in the central part of the distribution; and, therefore, the likelihood of having a small or very large productivity

[14] The main reason is that the investment decision is made before the productivity shock realizes, i.e. conditional on initial productivity.

[15] The previous expression is only valid for any $\tau \in (1/2,\ 1)$. In this case, I use $\tau = 0.75$, which corresponds to the interquartile range.

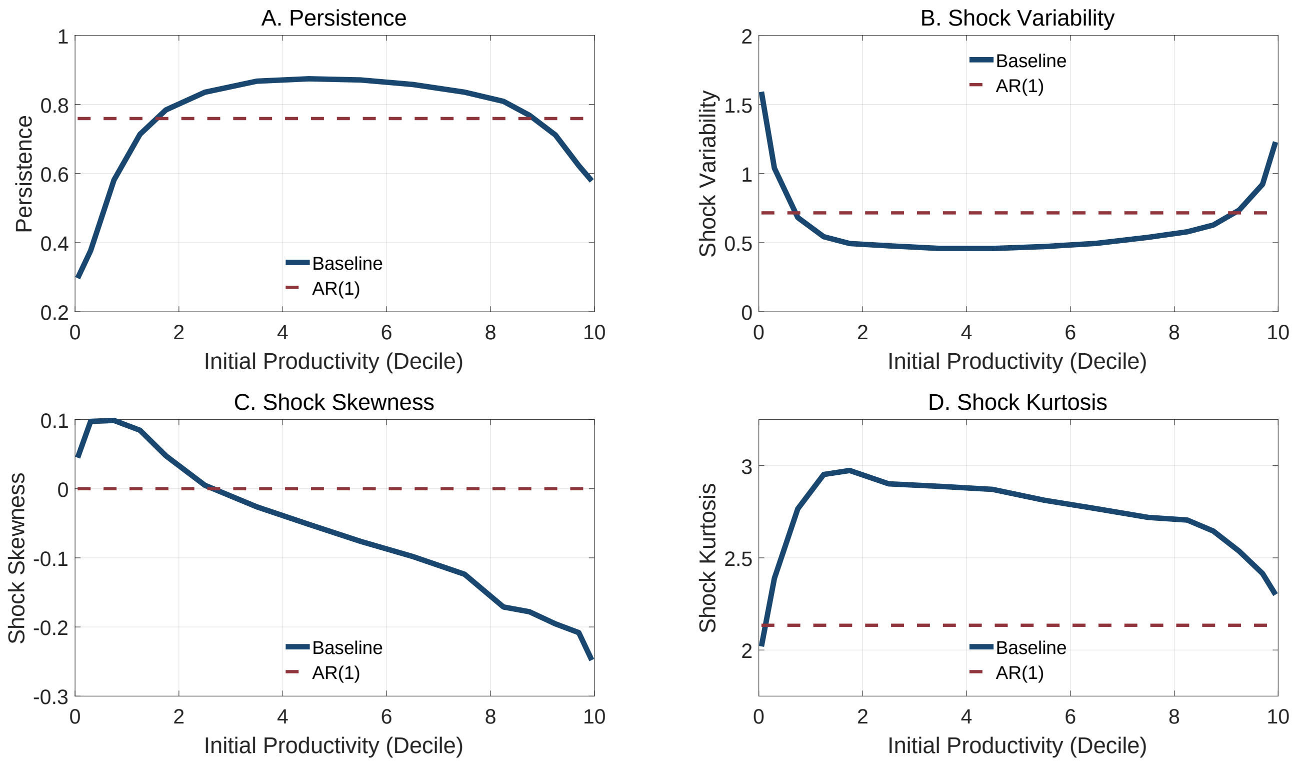
[16] The previous expression is only valid for any $\tau \in (1/2,\ 1)$. As in the previous case, I use $\tau = 0.75$.

shock. It has the following expression:

$$kur(log(A_{i,\ t-1})) = \frac{Q(log(A_{i,\ t-1});\ 1-\alpha) - Q(log(A_{i,\ t-1});\ \alpha)}{Q(log(A_{i,\ t-1});\ \tau) - Q(log(A_{i,\ t-1});\ 1-\tau)}.^{17} \tag{7}$$

In Figure 5, I plot the four main characteristics of the productivity process estimated for Spanish firms. First, the estimated productivity process is highly non-linear. Productivity persistence is hump-shaped, while shock variability is U-shaped with initial productivity. Second, productivity shocks are non-Gaussian. Shock skewness is decreasing, while shock kurtosis is hump-shaped with initial productivity. Importantly, the standard AR(1) productivity process features constant productivity persistence and shock variability, zero shock skewness and shock kurtosis close to 2.2, as defined here.

Figure 5: Characteristics of the Productivity Process



The estimated productivity process differs from a standard AR(1). What are the implications of these findings for firm behaviour and financial frictions? Under the estimated productivity process, low productivity firms are more likely to have a large positive productivity shock than in a standard AR(1) process. The transition probability from the first decile to the top decile is 0.8% in the estimated productivity process, while it is 0.0% in the AR(1) case. Similarly, the transition

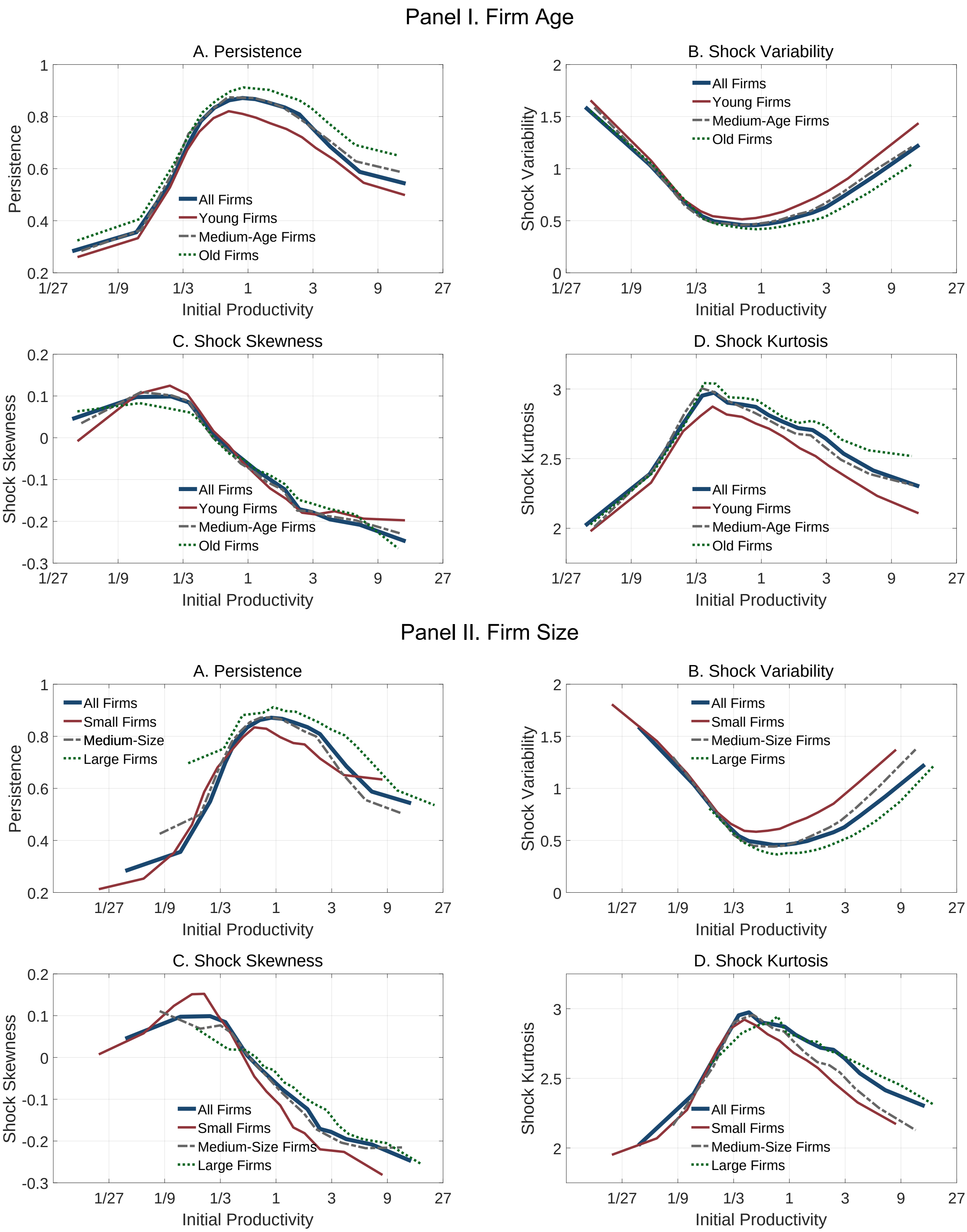
[17] The previous expression is only valid for any $\tau \in (1/2,\ 1)$ and $\alpha < 1-\tau$. I use $\tau = 0.75$ and $\alpha = 0.075$.

probability from the first decile to above the median contrasts from the 6.7% in the estimated process to the 1.2% implied by a standard AR(1) process. Some of these initially low productivity firms will not have enough internal funds to finance their optimal capital level, and therefore, they will be financially constrained. Furthermore, those good productivity realizations may not be long-lasting. I find that the transition probability from the top decile to below the median is 7.0% in the estimated productivity process versus a 1.2% if AR(1) productivity dynamics are assumed. Those bad realizations of productivity will slow down the internal profit accumulation of financially constrained firms.

Heterogeneity Does the estimated productivity process differ by firm age or size? I estimate the productivity process of young (1 to 5 years old), medium-age (6 to 10 years old), and old firms (more than 10 years old). I show the results in Panel I of Figure 6. The characteristics of the productivity process are remarkably similar for young, medium-age and old firms. Similarly, I also estimate the productivity process of small (first quartile of the size distribution), medium-size (second and third quartile), and large firms (fourth quartile). I show the results in Panel II of Figure 6. Again, the characteristics of the productivity process are similar for the three groups of firms. There is only a subtle difference in shock variability. Small highly-productive firms have a larger variation of the productivity shock. Therefore, they face slightly more uncertainty than large highly-productive firms. The results rule out the existence of compositional effects on the estimated productivity process. This is to say, the small persistence and large shock variability of the low productivity firms are not because those firms are young or small. It also rules out a component in my estimated productivity measure that varies with firm size or age, e.g. more measurement error in the data for young or small firms.

Robustness A natural question is whether the proposed approach can characterize the productivity dynamics properly. In order to tackle it, I do a Monte-Carlo simulation from an AR(1) productivity process with $\rho_a = 0.8$ (persistence parameter) and $\sigma_\varepsilon = 0.3$ (shock variability parameter). I simulate 1 million observations from the stationary distribution for two periods. Then, I implement the previous methodology to recover the parameters imposed in the simulation. The persistence parameter from the simulation is accurately estimated to 0.8 in all the range of the productivity distribution, except for the tails. Both at the very top and bottom of the productivity distribution, 1 percentile, the estimate of the persistence parameter jumps to 0.85. Regarding shock variability, a similar pattern arises. The estimation is very accurate in all the range of the productivity distribution, except for the tails, where it is slightly overestimated. Regarding shock skewness and shock kurtosis, the estimated parameters are close to their theoretical counterparts, even at the distribution's tails. These results can be found in Appendix B.1.2.

Figure 6: Heterogeneity of the Productivity Process



Another potential concern is that I treat the whole economy as one sector economy, standardizing the productivity data at the sector-year level. The main reason is that the proposed procedure is very data demanding, as I want to capture the dynamics at the tails of the productivity distribution. Therefore, pulling the data of all the sectors gives more power to the estimation strategy. As an alternative, I estimate the non-linear and non-Gaussian productivity process at the sector level and then aggregate it using 2-digits sector weights. I show the results in Appendix B.1.3. The main conclusion is that the sector by sector estimation yields very similar estimates.

In the estimation, I set $\eta = 0.83$, so that the model economy is able to match $SD(k_{si})$. A potential concern is the robustness of the characteristics of the productivity process to different values of the η parameter. I estimate the productivity dynamics by setting a wide range of η, from 0.75 to 0.90, which fall in the range usually used in the firm dynamics literature. Results are in Appendix B.1.4. I conclude that the main characteristics of the productivity process are robust to different levels of the decreasing returns to scale parameter, η.

Finally, the studied period from 1999 to 2014 covers a long period, including the Great Recession of 2007 in the middle. To check the robustness of the results over time, I split the studied period into two sub-periods. Before the Great Recession, the first one goes from 1999 to 2007, while the second period goes 2007-2014. Results are in Appendix B.1.5. The characteristics of the productivity process are very similar in the two periods showing the stability of the results.

4.2 Misallocation

Financial frictions affect firms by restricting their capital level below their optimal one. The standard approach to assess the existence of financial frictions in the literature has been through the following specification:

$$inv_{i,s,t} = \alpha \;+\; \beta cf_{i,s,t-1} \;+\; \tilde{\beta}' X_{i,s,t} \;+\; \varepsilon_{i,s,t}, \tag{8}$$

where $inv_{i,s,t}$ is the investment of firm i, in sector s and period t, $cf_{i,s,t-1}$ is the cash flow of firm i, in sector s and period $t-1$, and $X_{i,s,t}$ are controls. A positive estimated β coefficient has been pointed out as evidence of the existence of financial frictions. The reason is simple, if the firm is financially constrained and have a high cash flow in the past period, it can use those funds to self-finance itself. This will show up as high investment in the current period.

The usage of Equation 9 to show that firms experience financial constraints can be problematic. Gomes (2001) shows that a model with persistent productivity dynamics and time-to-build in the capital decision is enough to generate a positive coefficient. He proposes a model with productivity persistence, time-to-build and financial frictions. After simulating it, he estimates a positive coefficient as expected in a model with financial frictions. The puzzle is that the positive coefficient appears even in the specification without financial frictions. The main idea is as follows. If the firm has had a high cash flow in the past, it is likely to have experienced a high productivity shock. If productivity is persistent, then the firm expects to have higher productivity in the future. As capital takes time to build, it starts to invest today to take advantage of the expected higher productivity in the next period.

In this section, I propose a different methodology to show indirect evidence on financial frictions based on the misallocation literature. Hsieh and Klenow (2009) shows that with a Cobb-Douglas production function, the Average Revenue Product (ARP) should be equalized across firms in a perfectly competitive economy. Under frictionless input markets, firms would invest in inputs until the return of the last unit offsets its cost. As the cost of the inputs is the same across firms operating in the same sector due to perfect competition, the ratio of output over input, proportional to the marginal product under the Cobb-Douglas assumption, should be equalized. Regarding capital, we define ARPK as

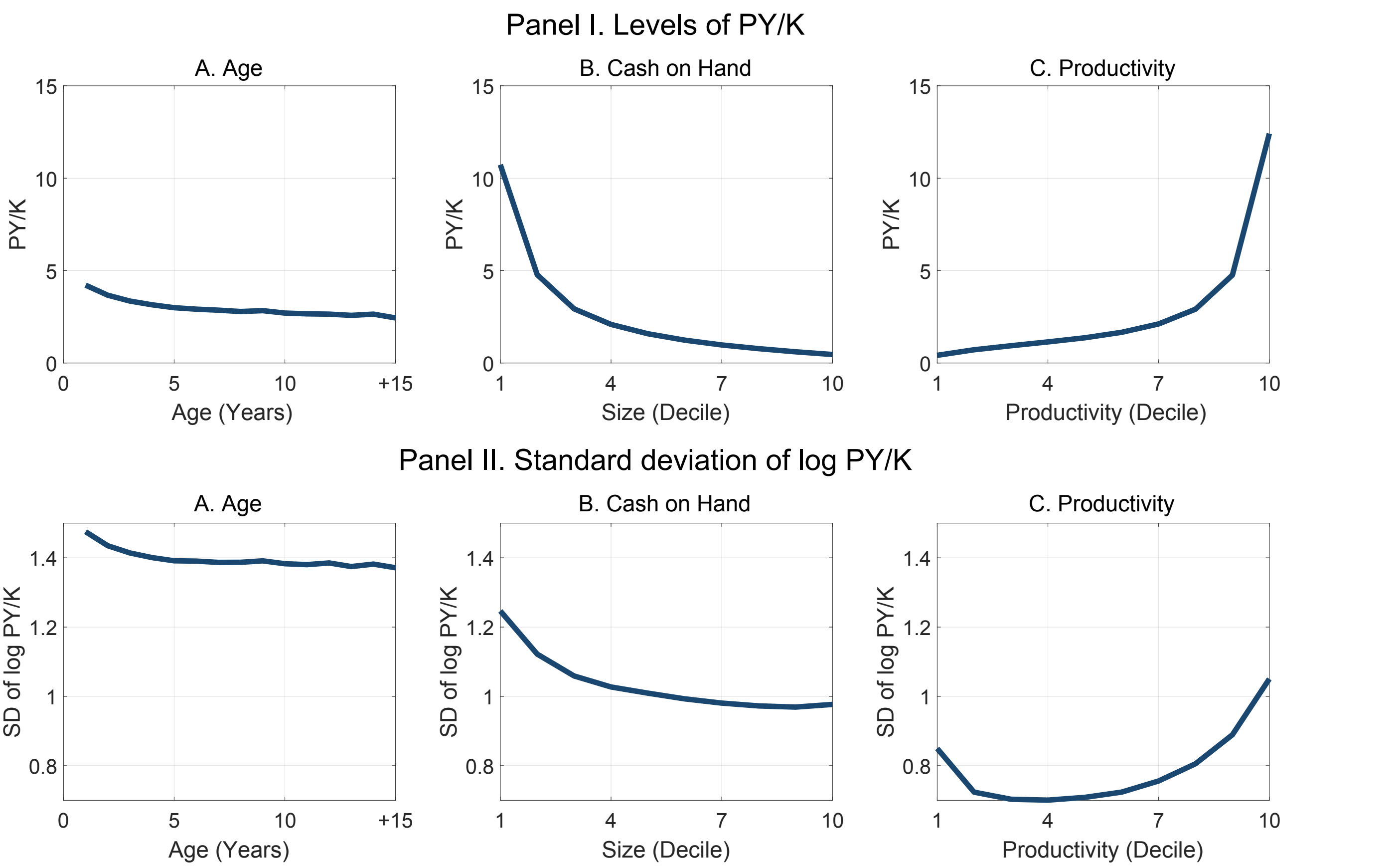
$$ARPK_{i,t} = \frac{py_{i,t}}{k_{i,t}}. \tag{9}$$

The difference of $ARPK$ among firms operating in the same sector can be due to capital misallocation. Therefore, the variance of $log(ARPK)$ has become the standard to measure allocation efficiency of capital at the sector level, see, e.g. David and Venkateswaran (2019).

I compute the mean of the $ARPK$ and standard deviation of $log(ARPK)$ at the sector level and then aggregate it, conditional on firm characteristics. The mean of the $ARPK$ conditional on the firm's characteristics captures distortions correlated with the firm's characteristics. On the other hand, the standard deviation of $log(ARPK)$ conditional on firm characteristics captures the variation within each group.

The results are shown in Figure 7. Panel I presents the mean $ARPK$ across firm characteristics, while panel II presents the profiles of the standard deviation of $log(ARPK)$. The results indicate the presence of financial frictions. Financial frictions should affect disproportionally young, small and high productivity firms. Young firms are unlikely to have enough internal funds to surpass financial frictions. In line with this prediction, I find that young firms have a larger $ARPK$, which gets slowly lower as firms age, as profit accumulation occurs. Furthermore, the standard deviation of $log(ARPK)$ is larger for young firms, as well. The reason is that not all the young firms are financially constrained, generating dispersion in $ARPK$ among them. The standard deviation of $log(ARPK)$ reduces as firms age as they accumulate internal funds to overcome financial frictions. Small firms are also limited by their current net worth to invest in capital. Finally, regarding firm productivity, high productivity firms have a high optimal level of capital, which they may not finance. Accordingly, I find an upwards sloping profile of mean of $ARPK$ and standard deviation of $log(ARPK)$ with firm productivity.

Figure 7: Profiles of PY/K



Robustness Two concerns might affect the previous analysis. First, during the studied period, the allocation of capital has been gradually deteriorating in Spain, as shown in Gopinath et al. (2017). To consider the increase in capital misallocation over time, I standardize the data on ARPK and log ARPK at the sector-year level. After the standardization, there is no trend in the allocation of capital during the studied period. The results are shown in Appendix B.2.1. The profiles look very similar under the two specifications. The only difference is smaller correlated distortions in the standardized specification, i.e. the relation of $ARPK$ with firm age, size, and productivity is flatter. Second and related to the previous concern, the studied period from 1999 to 2014 covers the Great Recession of 2007 in the middle. To check the robustness of the results across time, I split the studied period into two subperiods. The first one goes from 1999 to the Great Recession in 2007, while the second period goes from 2007 to 2014. Results are summarized in Appendix B.2.2. I conclude that the results are very similar in the two periods.

4.3 Financial Behavior

Empirical finance literature has focused on studying the financial behaviour of publicly-listed firms, see, e.g. Lemmon et al. (2008), and Graham and Leary (2011). The main reason is the lack of comprehensive datasets of privately-held companies. There are several reasons to believe that financial frictions affect differently these two groups, publicly-listed vs privately-held firms. First, publicly-listed firms have access to a wide range of fundraising instruments. They have access to the traditional bank-lending channel and can raise equity in stock markets and issue debt in bond markets. Second, publicly-listed firms are usually larger than privately-held firms, which may facilitate their access to credit. Indeed, Dinlersoz et al. (2018) shows that these two groups were affected differently by the recent financial crisis in 2007. On the other hand, without a consistent set of facts on the financial behaviour of privately-held companies, it is tough to evaluate models of firm dynamics with financial frictions. In this section, I fill this gap by providing evidence on how the debt structure of privately-held firms differs with firm characteristics.

In Table 1, I show the fraction of firms that do not use any costly debt and the leverage distribution, measured as costly debt over total assets. The usage of debt varies widely across firms. As we can see, the fraction of firms that do not use any debt is large, 29%. Nonetheless, there is a 5% of firms with leverage lower than 0.01, while another 5% of firms with leverage larger than 0.71 among firms with positive debt.

The considerable variation of firm leverage across firms raises two questions. First, how does leverage vary with firm characteristics? Second, are the patterns similar for the extensive and intensive margin? To answer the first question, I propose a non-parametric model to capture the correlation of leverage on firm characteristics (age, size and productivity). The specification reads as follows

$$Leverage_{i,s,t} = f(age_i) + g(size_i) + h(A_i) + \beta' X_{i,s,t} + \varepsilon_{i,s,t}, \tag{10}$$

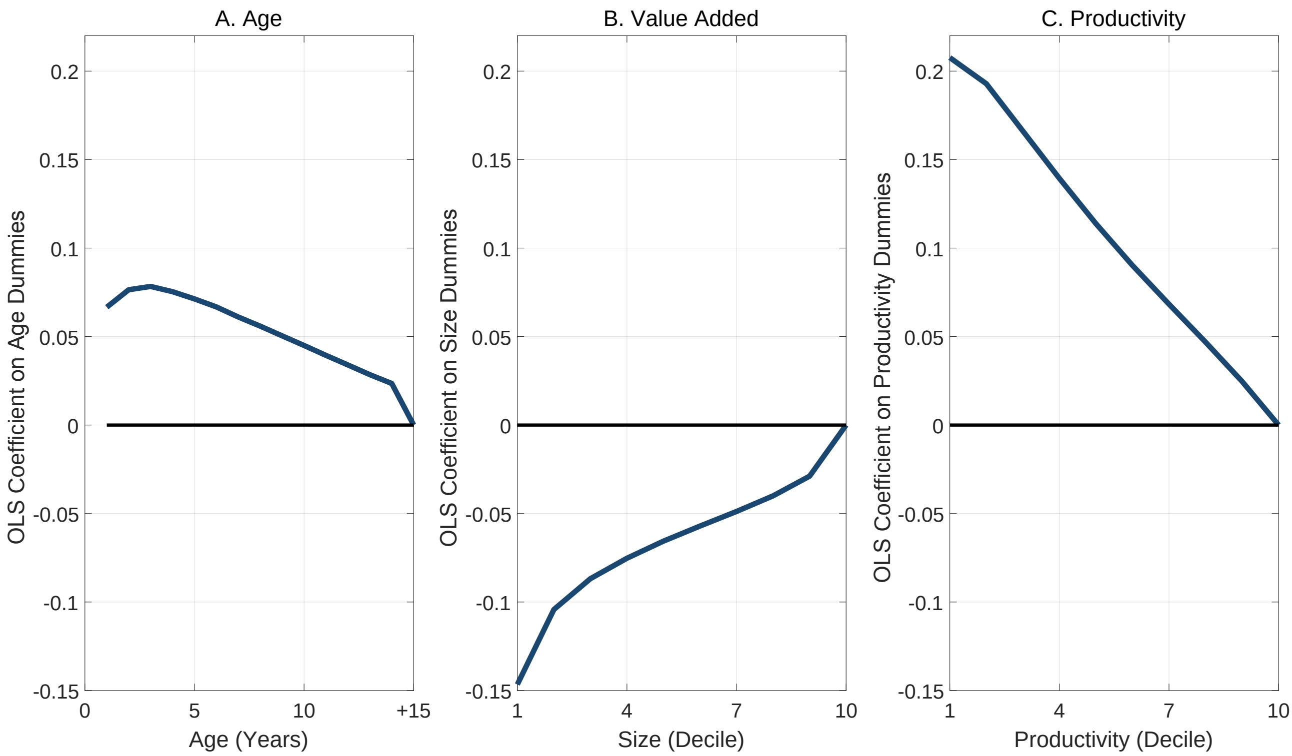
where $f(age_i)$ is a fully flexible function on firm age, which will be approximate by estimating the

Table 1: Leverage Distribution

	Data
Fraction with $Debt = 0$	0.29
$Percentile \mid Debt > 0$	
5	0.01
10	0.02
25	0.08
50	0.22
75	0.42
90	0.61
95	0.71

coefficients on age dummies. The $g(size_i)$ function is approximated with 10 dummies, corresponding to the deciles of the value-added distribution. The $h(A_i)$ function is also approximated with 10 dummies, corresponding to the deciles of the productivity distribution. Finally, $X_{i,s,t}$ are the controls, i.e. a full set of sector-year fixed effects. They aim to capture differential trends on the average financial behaviour across sectors over time.

Figure 8: Financial Behavior



Note: The omitted categories are +15 years old, top size decile and top productivity decile.

I show the estimated coefficients in Figure 8. Leverage is decreasing with firm age and firm productivity. On the contrary, it is increasing with firm size. As we have seen in Table 1, there are a non-negligible fraction of firms that do not use debt. Therefore, the relation shown in Figure 6 can come either from the extensive margin, probability of using costly debt, or intensive margin, average leverage conditional on being positive. To explore the extensive margin, I propose a probit model where the relation with firm age, size and productivity is estimated non-parametrically analogous to Equation 11. The estimated model is given by

$$P(Debt_{i,s,t} = 1 \mid age_i,\ size_i,\ A_i,\ X_{i,s,t}) = \Phi\left(f(age_i) + g(size_i) + h(A_i) + \beta^{'} X_{i,s,t}\right). \tag{11}$$

Regarding the intensive margin, I estimate Equation 11 conditional on firms having positive debt. Formally

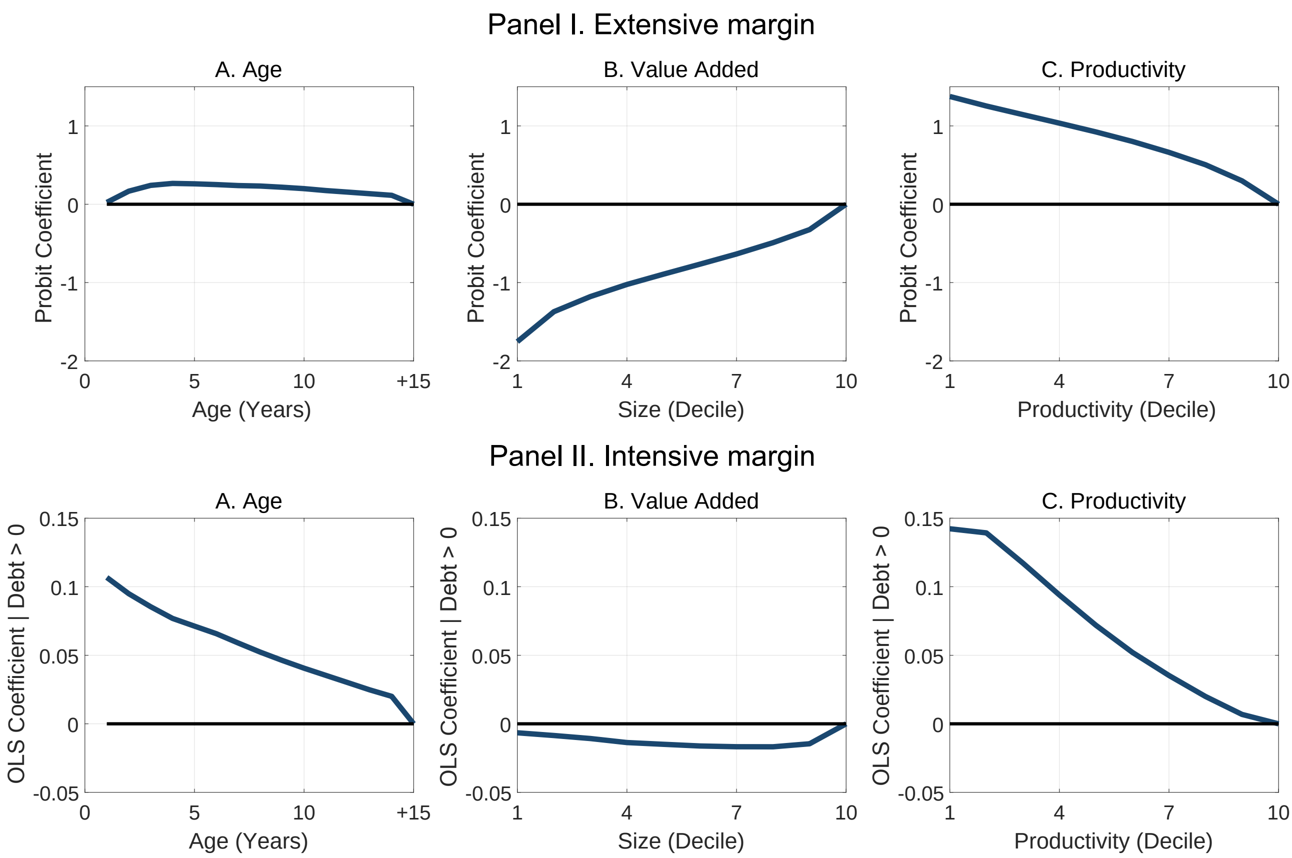
$$Leverage_{i,s,t} \mid Debt_{i,s,t} \geq 0 = f(age_i) + g(size_i) + h(A_i) + \beta^{'} X_{i,s,t} + \varepsilon_{i,s,t}. \tag{12}$$

The estimated functions are shown in Figure 9. Panel I shows the estimates for the extensive margin, while panel II shows the intensive margin. The results reveal that the negative correlation of leverage with firm age is mostly due to the intensive margin, as the probability of using debt is almost flat with firm age. The results differ markedly for firm size. Conditional to using debt, there is not much difference in the average leverage of firms of different sizes. But, smaller firms are much less likely than larger ones to use debt to finance their investment. Finally, the negative relation of leverage with firm productivity appears in both the intensive and extensive margin. Low productivity firms are more likely to use debt to finance their investment, and when they use it, they finance a larger fraction of their total assets.

Robustness Three concerns might affect the previous analysis. First, the finance literature has focused on profitability, measured as profits over total assets, instead of productivity. Indeed, the negative relation of firm leverage and profitability has been a puzzle in the literature, see, e.g. Frank and Goyal (2009) . To see the robustness of the results, I extend the previous models controlling for firm profitability. The results and further details are exposed in Appendix B.2.1. The main conclusion is that the relations presented here are very similar even when I control by firm profitability.

Second, in a very similar framework to the one proposed here, Dinlersoz et al. (2018) find a positive relation between leverage and productivity. The main difference between the two frameworks is the definition of firm productivity. In Dinlersoz et al. (2018), they rely on labour productivity,

Figure 9: Financial Behavior - Extensive and Intensive Margin



Note: The omitted categories are +15 years old, top size decile and top productivity decile.

defined as value-added over labour, while I rely on total factor productivity. In Appendix B.2.2, I show that if I estimate their specification with labour productivity, I find a positive coefficient on firm productivity as well. This paper measures firm productivity as TFP, which is more appropriate for two reasons. First, labour is treated as a static decision in a perfectly competitive framework. Therefore, as shown in Section 4.2, firms will hire labor until the $ARPL$ (labor productivity) is equalized across firms. In that sense, labour productivity is capturing distortions in the labour market that prevents firms from hiring the optimal level of employment. Second, even if the $ARPL$ is positively correlated with firm productivity, as more productive firms may face larger frictions that prevent them from hiring the optimal amount of labour, the measure of productivity used here is more comprehensive. It uses the two main production factors in its calculation, labour and capital.

Finally, I check whether the results change over time. As I did in previous sections, I divide the studied period into two, before the Great Recession, 1999 to 2007, and during and after the Great Recession, 2007-2014. Results are summarized in Appendix B.2.2. The results are very similar in the two periods. The main difference appears in the leverage-size relationship, which gets steeper in the Great Recession period, suggesting that the financial crisis of 2007 has affected disproportionally to small firms, which are the ones most likely to be constrained.

4.4 A Recap

In this section, I have provided three new sets of facts. First, I show that Spanish firms face a highly non-linear productivity process with non-Gaussian shocks. I show that productivity persistence is hump-shaped with past productivity, while shock variability is U-shaped. I also show that shock skewness is decreasing with past productivity, while shock kurtosis is hump-shaped. The productivity process uncovered in the estimation procedure is very different from a standard AR(1) process, the workhorse in the firm dynamics literature. Under the estimated process, a low productivity firm has a larger probability of becoming highly productive in the next periods. On top of that, those high productivity episodes are not long-lasting. These features of the estimated process are crucial to understand the effects of financial frictions on the firm life cycle and the aggregate economy.

Second, I show that the ARPK and the standard deviation of log ARPK decrease both with firm age and size, while they are increasing with firm productivity. This is suggestive evidence on the presence of financially constrained firms, especially among the young, small and highly productive ones.

Finally, I have studied the financial behaviour of Spanish firms exploiting variation on the leverage ratio. I first show that a large fraction of firms that do not use costly debt, 29%, and the leverage distribution is very dispersed. I also show that the average leverage correlates with firm characteristics. It decreases with firm age and productivity, while it increases with firm size. These patterns are present in both the extensive margin, probability of using costly debt, and the intensive margin, average leverage conditional on using costly debt.

5 Model

This section presents a model of firm dynamics with financial frictions and the non-linear and non-Gaussian productivity dynamics as estimated in the previous section. Firms are heterogeneous in their productivity levels, which evolves stochastically according to the Markov process presented in Section 4.1. They produce a homogeneous good combining capital and labour in a Cobb-Douglas production technology under decreasing returns to scale,

$$py = F(k,\ l,\ A) = A_{shift}A[k^{\alpha}l^{1-\alpha}]^{\eta} \qquad \alpha \in (0,\ 1) \ and \ \eta \in (0,\ 1), \tag{13}$$

where py is value-added, A_{shift} is the aggregate component of total factor productivity, A is the idiosyncratic component of productivity, k is capital and l is labor. This is the same expression as Equation 1, which was used to compute firm-level productivity in the data.

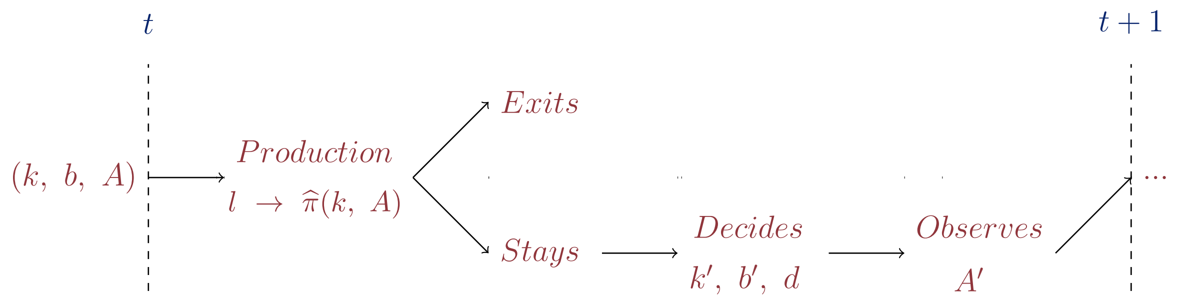
The objective of a firm is to maximize its current value plus continuation value. The firm chooses how much to invest, how much to borrow to finance its investment, how much labour to hire and how much it pays in dividends to the households. The choice of capital takes place before the productivity of the current period realizes, common information friction used in the firm dynamics literature, capturing the time-to-build nature of capital. Furthermore, investment is limited by a borrowing capacity that depends on internal funds and firm productivity. The choice of labour is static, and it is not subject to any friction. Dividends are the residual amount left after production occurs, and the firm adjusts its capital and borrowing levels. Finally, all the markets are perfectly competitive, and firms take prices as given.

In Figure 10, I summarize the decision tree of an incumbent firm. A firm enters in the period with a level of capital (k), borrowing (b), which can be positive or negative depending on if the firm is a borrower or saver, and productivity (A). At this stage, the firm decides how much labour to hire to maximize its per-period profits:

$$\widehat{\pi}(k,\ A) = \max_{\{l\}} \left\{ F(k,\ l,\ A) - wl \right\}, \tag{14}$$

where l is the amount of labour and w is the wage rate.

Figure 10: Timing - Incumbent Firm



After the production occurs, firms receive an exit shock $\vartheta(A)$, which depends on the firm productivity. If the firm exits, it is liquidated, and the surplus returns to the household. Formally, the following expression gives the firm value:

$$V^{exit}(k,\ b,\ A) = \widehat{\pi}(k,\ A) + (1-\delta)k - b. \tag{15}$$

If the firm stays, it decides how much to invest in capital and how to finance it, depending on its internal funds and productivity level. Borrowing is limited by the installed capital and a size-dependent pledge-ability parameter. Formally,

$$b' \leq \theta\Big(\frac{k'}{k'_u(A)}\Big)^{\Psi} k'. \tag{16}$$

This borrowing constraint follows Gopinath et al. (2017) and has two components. First, the firm's level of capital will install for the next period, k'. And the pledge-ability component, $\theta\Big(\frac{k'}{k'_u(A)}\Big)^{\Psi}$, which captures the fraction of the installed capital subject to collateralization. I assume it is a non-linear function of the installed capital, k' and the optimal level of capital the firm would like to install, $k'_u(A)$. If the firm has enough internal funds, such that $k' = k'_u(A)$, the borrowing constrained turns the standard one used in the firm dynamics literature, $b' \leq \theta k'$. Therefore, the parameter θ governs the maximum amount of capital a firm can pledge. The parameter Ψ governs the difference in pledge-ability among firms that differ in their level of internal funds. Therefore, it is the penalty that the financial markets impose to firms with low internal resources. Importantly, this specification nests the usual borrowing constraint with constant pledge-ability parameter, $b' \leq \theta k'$, if $\Psi = 0$.

Finally, dividends are the remaining funds after the investment and borrowing decisions are made. They are constrained to be non-negative, as firms are not allowed to raise equity. Formally,

$$d \equiv (1-\tau)\widehat{\pi}(k,\ A) + (1-\delta)k - b - k' + qb', \tag{17}$$

where δ is the depreciation rate of capital and q is the price of the firm's debt to obtain funding. The value of q is a general equilibrium object that determines the funding cost of firms. The parameter τ disciplines the wedge between the value-added and the after taxes profits. It captures any friction or conditions in the environment not considered in the model, such as taxes. This wedge is returned to the household as a lump sum not to distort firm decisions. Finally, the firm observes the next period productivity and the process restarts.

The problem of an incumbent firm that stays, in recursive formulation, reads as follows:

$$
\begin{aligned}
V(k,\ b,\ A) = \max_{\{k',\ b',\ d\}} \quad & d + \\
& \beta(1-\vartheta(A))E\left[V(k',\ b',\ A')|A\right]\ + \\
& \beta(1-\vartheta(A))E\left[\tau\widehat{\pi}(k',\ A')|A\right]\ + \\
& \beta\vartheta(A)E\left[\widehat{\pi}(k',\ A') + (1-\delta)k' - b'|A\right],
\end{aligned}
\tag{18}
$$

subject to

$$d = (1-\tau)\widehat{\pi}(k,\ A) + (1-\delta)k - b - k' + qb' \geq 0,\ and \tag{19}$$

$$b' \leq \theta\Big(\frac{k'}{k'_u(A)}\Big)^{\Psi} k', \tag{20}$$

where β is the subjective discount factor, $E[\ .\ |A]$ is the expectation conditional on today's productivity (A). It contains the dynamics of the productivity process and it is the main source of uncertainty firms face. Finally, Equation 19 is the non-equity issuance constraint, and Equation 20 reflects the borrowing constraint.

Exiting firms are replaced by new entrant firms. The timing of the entry problem is summarized in Figure 11. First, entrants observe the distribution of equity (initial internal funds) and firm productivity $\Omega(e_0,\ A_0)$.[18] The initial level of internal funds conditional on firm productivity is assumed to follow a log-normal distribution. Formally,

$$\Omega(e|a)\ \sim\ N\Big(\mu_e + \frac{\sigma_e}{\sigma_a}\rho_{a,e}(a-\mu_a);\ (1-\rho_{a,e}^2)\sigma_e^2\Big), \tag{21}$$

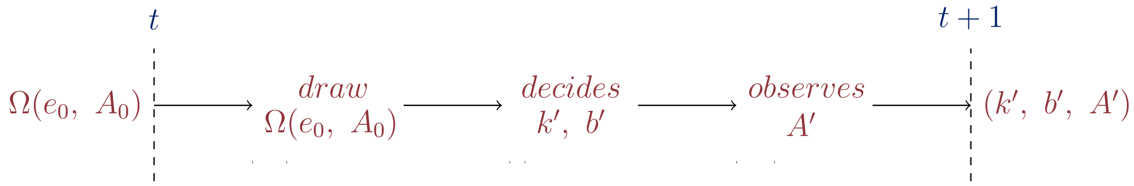
where a stands for $log(A)$. While the marginal distribution with respect to productivity $\Omega(a)$ is directly estimated from the data, as shown in Figure 4.

The entrant firm gets a draw $(e_0,\ A_0)$ from the distribution. Given their equity e_0 and initial productivity A_0, the firm decides the capital investment (k') and how much to finance (b'). Finally, the firm observes the next period productivity and starts production according to the state $(k',\ b',\ A')$. At this stage, the firm becomes incumbent and the sequence of events is described according to Figure 10.

Households There is a representative household that owns the firms. The household maximizes the discounted flow of per-period utility. The household provides 1 unit of labour inelastically, and

[18] This is equivalent to $k_0 = 0$ and $b_0 = -e_0$.

Figure 11: Timing - Entrant Firm



it decides how much to consume of the homogeneous good produced by the firms. It also owns the firms, and the bonds firms use to finance investment. Finally, she receives the dividends flow paid by the firms. The household problem, in recursive formulation, reads as follows:

$$V^h(\Lambda,\ \Phi) = \max_{\{C^h,\ \Lambda',\ \Phi'\}} \left\{U\left(C^h\right) + \beta V^h(\Lambda',\ \Phi')\right\}, \tag{22}$$

subject to

$$\begin{aligned} C^h + q\Phi' + \int_{k'xb'xA'} \rho_1(k',\ b',\ A')\Lambda'(k',\ b',\ A')d\left[k'xb'xA'\right] \leq \\ w + \Phi + \int_{kxbxA} \rho_0(k,\ b,\ A)\Lambda(k,\ b,\ A)d\left[kxbxA\right] + \tau \int_{kxbxA} \widehat{\pi}(k,\ A)\Lambda(k,\ b,\ A)d\left[kxbxA\right], \end{aligned} \tag{23}$$

where Λ is the measure of firms and Φ is the amount of bonds the household holds. $\rho_1(k',\ b',\ A')$ is the price (ex-dividend) of firm's shares with state $(k',\ b',\ A')$, while $\rho_0(k,\ b,\ A)$ is the price (dividend inclusive) of firm's shares with state $(k,\ b,\ A)$.

Equilibrium A stationary recursive competitive equilibrium consists of prices $(w,\ q,\ \rho_0,\ \rho_1)$, quantities $(l,\ k',\ b',\ d,\ C^h,\ \Lambda,\ \Phi)$, a distribution $\mu(k,\ b,\ A)$, a mass of firms (M) and values $(V^{exit},\ V,\ V^h)$ such that: First, V^{exit} and V solve the firm's problem and $(l,\ k',\ b',\ d)$ are the associated policy functions. Second, V^h solves the household's problem and $(C^h,\ \Lambda,\ \Phi)$ are the associated policy functions. Third, all the markets clear: labor market, bond market, stock market and good market, which does due to Walras' law. Finally, the distribution of firms $\mu(k,\ b,\ A)$ is a fixed point consistent with the policy functions $(k',\ b')$, the exogenous exit rate $(\vartheta(A))$, the entry distribution $\left(\Omega(e_0,\ A_0)\right)$ and the law of motion for productivity (A).

5.1 Aggregation

From the firm-level behaviour and using the distribution of firms ($\mu(k,\ b,\ A)$), we can aggregate the economy to obtain the main economic variables. The total output is given by

$$Y = \int_{kxbxA} F(k,\ l,\ A)\ \mu(k,\ b,\ A)\ d\left[kxbxA\right]. \tag{24}$$

Similarly, total capital and labour are given by

$$K = \int_{kxbxA} k\ \mu(k,\ b,\ A)\ d\left[kxbxA\right] \quad \textit{and} \quad L = \int_{kxbxA} l\ \mu(k,\ b,\ A)\ d\left[kxbxA\right] = 1. \tag{25}$$

I define aggregate productivity as

$$A_g = \frac{Y}{K^{\alpha} L^{1-\alpha}}, \tag{26}$$

where $\alpha \in (0,\ 1)$ is a parameter governing the K/L ratio in the economy. It can be shown that aggregate productivity is an expression with three main elements: the average firm-level productivity, the allocation of resources across firms, and the number of firms. This last component arises from the decreasing returns to scale of the production function at the firm level.

Following the same procedure, other variables can be aggregated, like total debt, profits and dividends.

5.2 Solution of the Model

The model set-up is similar to the one developed in Khan and Thomas (2013). Therefore, I follow their strategy in order to solve the model. In this section, I describe the main points of the solution method, and I provide further details in the Appendix C.1.

First, let me define the cash-on-hand variable. Cash-on-hand is the total amount of available resources the firm has after undertaking production, selling the undepreciated capital and paying its debts. From the accounting point of view, the closest counterpart is the firm net worth. Formally, it is defined as

$$e(k,\ b,\ A) = (1-\tau)\widehat{\pi}(k,\ A) + (1-\delta)k - b. \tag{27}$$

Depending on their level of cash-on-hand, we can classify the firms into three categories. The first group of firms are the unconstrained ones. A firm that currently can implement the optimal level of capital as well as in the future, regardless of its productivity path. They invest up to the optimal unconstrained capital level ($k'_u(A)$), and have debt, or savings, such that they will be unconstrained in the future ($b'_u(A)$). In Appendix C.1, I provide the derivation for $k'_u(A)$, which has a closed-form solution in my set-up, and the algorithm to find $b'_u(A)$. These firms are the only ones that pay positive dividends, as they have accumulated enough internal funds that prevent them from being constrained in the future. Dividends are determined as the residual of the available cash-on-hand after the capital and borrowing decision is made, as shown in Equation 17.

The second group of firms are labelled as constrained type I. A firm that currently can implement the optimal unconstrained level of capital ($k'_u(A)$), but not the borrowing ($b'_u(A)$). These firms are currently unconstrained, but they can be constrained in the future depending on their productivity shocks. The non-equity issuance constraint, (Equation 19) is binding for them, determining the threshold that divides constrained from unconstrained firms. Formally,

$$e(k,\ b,\ A) - k'_u(A) + qb'_u(A) = 0 \qquad \rightarrow \qquad \hat{e}(A) = k'_u(A) - qb'_u(A). \tag{28}$$

These firms do not pay dividends. They find it optimal to retain all the profits, as internal funding, up to the point they become unconstrained, i.e. ensure the borrowing constraint will not be binding in the future.

Finally, there is a third group of firms labelled as constrained type II. A firm that currently cannot implement the optimal unconstrained level of capital ($k'_u(A)$). Therefore, the capital allocation of this group of firms is distorted by financial frictions. For this type of firms, both the non-equity issuance (Equation 19) and borrowing constraint (Equation 20) are binding. Formally,

$$\left.\begin{aligned} & e(k,\ b,\ A) - k'_u(A) + qb' = 0 \\ & b' = \theta k'_u(A) \end{aligned}\right\} \qquad \rightarrow \qquad \hat{\hat{e}}(A) = (1 - q\theta)k'_u(A). \tag{29}$$

These firms do not pay dividends, as they accumulate all the profits up to a point they become unconstrained.

6 Benchmark Economy

This section calibrates the model and evaluates its performance along several dimensions: firm life cycle, capital misallocation, and firm's financial behaviour.

6.1 Calibration

There are 11 parameters in the model that I calibrate to match 11 moments in the data. Table 2 shows the estimated parameters and their values. It also shows the targeted moments, their value in the data and the model. In the calibration of the decreasing returns to scale parameter (η), I apply a discrete search grid method to match the standard deviation in the capital distribution ($SD(k)$). For each value of η, I estimate the productivity process and calibrate the remaining 10 parameters of the model using a simulated method of moments. I minimize the sum of the squared residuals between a set of moments computed in the model and the data. Although all the moments are jointly determined through the internal mechanisms of the model, some parameters are particularly relevant for matching certain moments.

Table 2: Calibration

Parameter	Value	Moment	Data	Model
η	0.83	$SD(k)$	1.79	1.76
β	0.97	K/Y	2.0	2.2
α	0.35	K/L	4.0	4.1
δ	0.05	Inv/Y	0.12	0.13
A_{shift}	1.22	L	15.5	15.5
θ	0.81	$avg(Lev)$	0.19	0.19
Ψ	0.48	$P_{95}^{Lev} \| Debt > 0$	0.71	0.71
τ	0.43	$Profits/Y$	0.15	0.15
μ_e	1.95	k_{ent}	0.36	0.36
σ_e	1.92	$SD(k_{ent})$	0.95	0.95
$\rho_{a,e}$	0.02	$\rho(a_{ent}; e_{ent})$	0.05	0.05

The η parameter is estimated to be 0.83 matching very well the $SD(k)$. The estimated values of the subjective discount factor (β), the output to capital elasticity (α) and the depreciation rate (δ) fall in the usual range consider in the firm dynamics literature. The average productivity of firms (A_{shift}) sets the average firm size in the model as in the data, 15.5 employees. The two parameters governing the borrowing constrained, θ and Ψ, are set to match two moments of the leverage distribution: average leverage ($avg(Lev)$) and the percentile 95 (P_{95}^{Lev}). They imply that the maximum fraction of capital that a firm can use as collateral is 0.81. The value of Ψ differs from 0, rejecting a specification of the borrowing constraint with a constant pledgeability parameter. The wedge, τ, is set to match the after-tax profits over total output in the economy. Finally, the parameters governing the initial level of equity of entering firms are set to match moments of the firm entry distribution: average size of entrants with respect to incumbents ($avg(k_{ent})$), the standard deviation of capital distribution ($SD(k_{ent})$) and the correlation between initial productivity and equity ($\rho(a_{ent}; e_{ent})$).

I as well calibrate a version of the model where productivity dynamics evolve according to a standard AR(1) process. Formally,

$$a_t = \rho a_{t-1} + X_t + \sigma\varepsilon_t \quad \varepsilon_t \sim N(0,\ 1), \tag{30}$$

where a_t stands for $log(A_t)$ and X_t is a full collection of sector-year fixed effects. The autoregressive parameter, ρ is estimated to 0.81, and the shock variability, σ, to 0.34. The values of the remaining parameters and the value of their moment counterparts are shown in Appendix D.1.

6.2 Model Validation

The model performs well in the dimensions targeted in the calibration strategy. However, How does the model behave among other dimensions? And, more importantly, are the mechanisms of the model consistent with firm behaviour?

Non-Targeted Moments I check the consistency of the model, opposing a set of non-targeted moments with the data. The results are summarized in Table 3. First, regarding the firm size distribution, the model captures very well firm concentration. The top 1% of the firms accumulate around 1/3 of total resources both in the model and in the data.

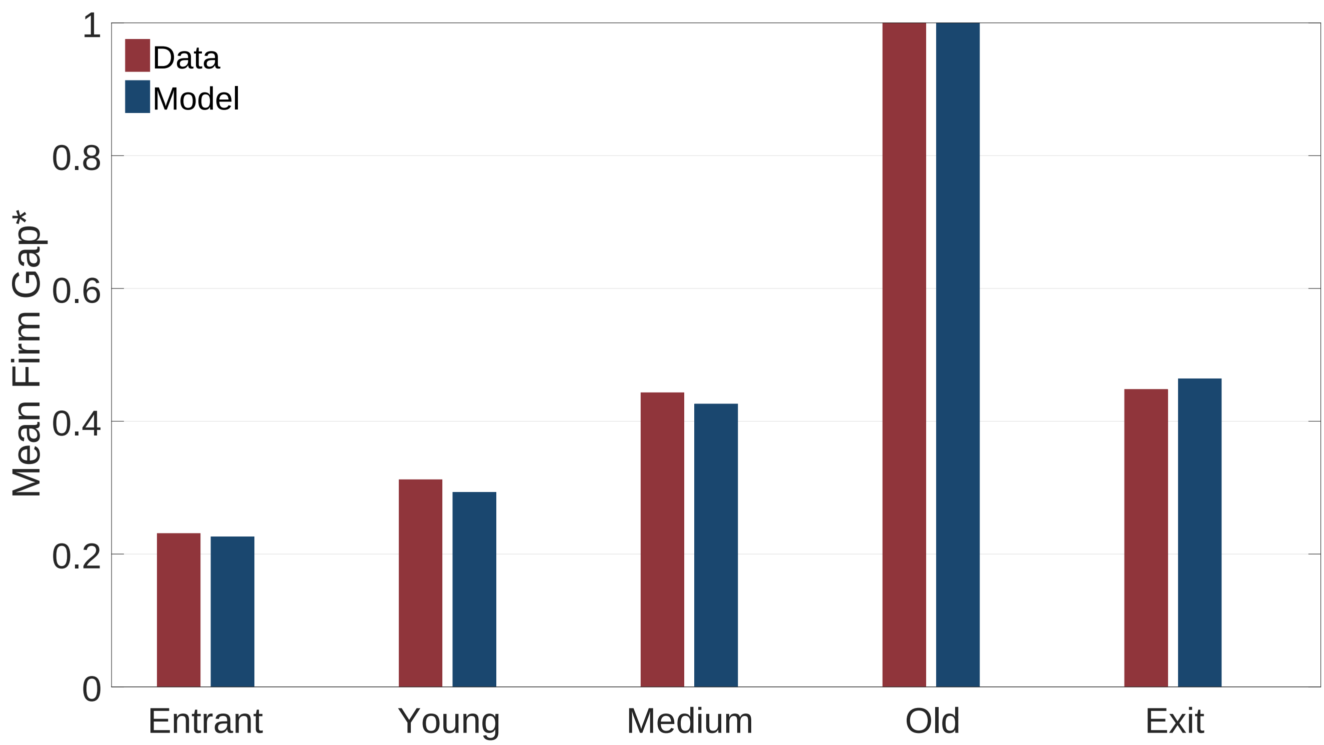
Table 3: Non-Targeted Moments

Moment	Data	Baseline
$Concentration_{99}(K)$	0.34	0.33
$P_{10}^{Lev}\|Debt > 0$	0.03	0.08
$P_{25}^{Lev}\|Debt > 0$	0.09	0.15
$P_{50}^{Lev}\|Debt > 0$	0.22	0.29
$P_{75}^{Lev}\|Debt > 0$	0.42	0.51
$P_{90}^{Lev}\|Debt > 0$	0.61	0.67
$Debt/Y$	0.81	0.82
$Debt > 0$	0.71	0.57
$Div > 0$	0.01	0.00
Div/Y	0.14	0.00
$Med(K_{ent})$	0.08	0.08

On the financial side, the model matches the debt to output and the leverage distribution quite well. However, the fraction of firms with positive debt is smaller in the model than in the data. This is consistent with a more precautionary dividend-paying behaviour in the model economy than in reality. The fraction of firms paying dividends and the dividend to output ratio is smaller in the model than in the data. The main reason is that the firms are too precautionary in the model. They save retaining all the profits up to the point they ensure to be unconstrained regardless of any productivity path, even if this is very unlikely. The model also matches other moments of the firm entry distribution, such as the median size of entrants, $Med(K_{ent})$.

Firm Life Cycle In Figure 12, I show the firm life cycle in the data and the model. The model can match very well the firm life cycle. As in the data, firms enter very small in the economy, 25% the size of an old firm, more than 10 years old. They gradually grow over the firm life cycle. Although the process is prolonged, a medium-age firm, 6 to 10 years old, is half the size of an old firm. Finally, firms eventually exit the market. The average size of an exiting firm is half the one of an old firm.

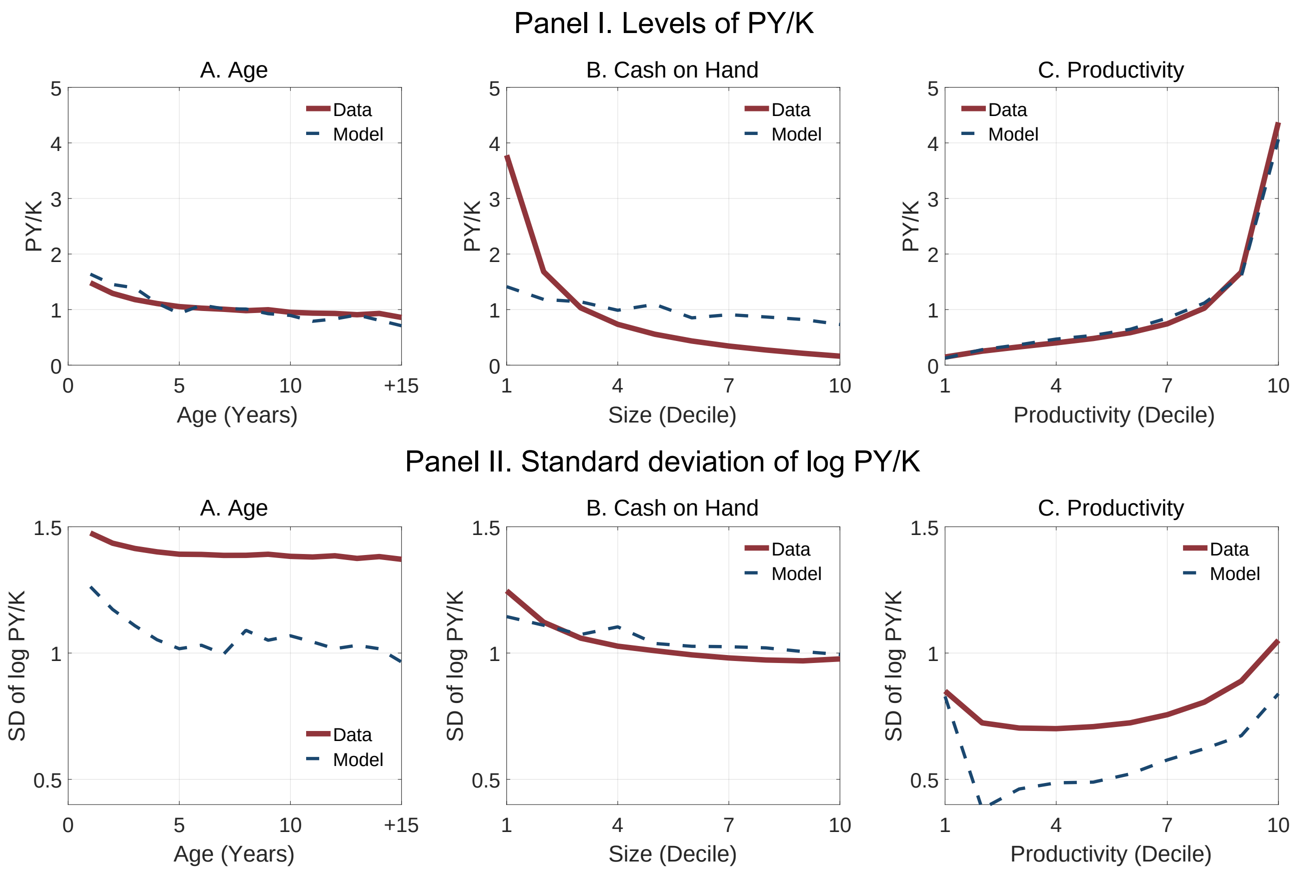
Figure 12: Firm Life Cycle



Notes: Young: 1-5 years old, Medium: 6-10 years old, Old: more than 10 years old. *Mean Firm Gap with respect to an old firm.

Misallocation Figure 13 shows how the level and dispersion of ARPK behaves in the model and the data. Panel I shows the level of ARPK. The model does an excellent job generating the patterns by age and productivity. Both in the data and in the model, young and high productivity firms have higher levels of ARPK. In the model economy, these are exactly the firms that are more likely to be financially constrained. The average ARPK is also larger for smaller firms. While the model can generate the same pattern, this is more muted. Nevertheless, a model without financial frictions could not generate a negative relation between firm size and the level of ARPK. The flatter profile of ARPK with firm size suggests other distortions, apart from financial frictions, affecting small firms in the Spanish economy.

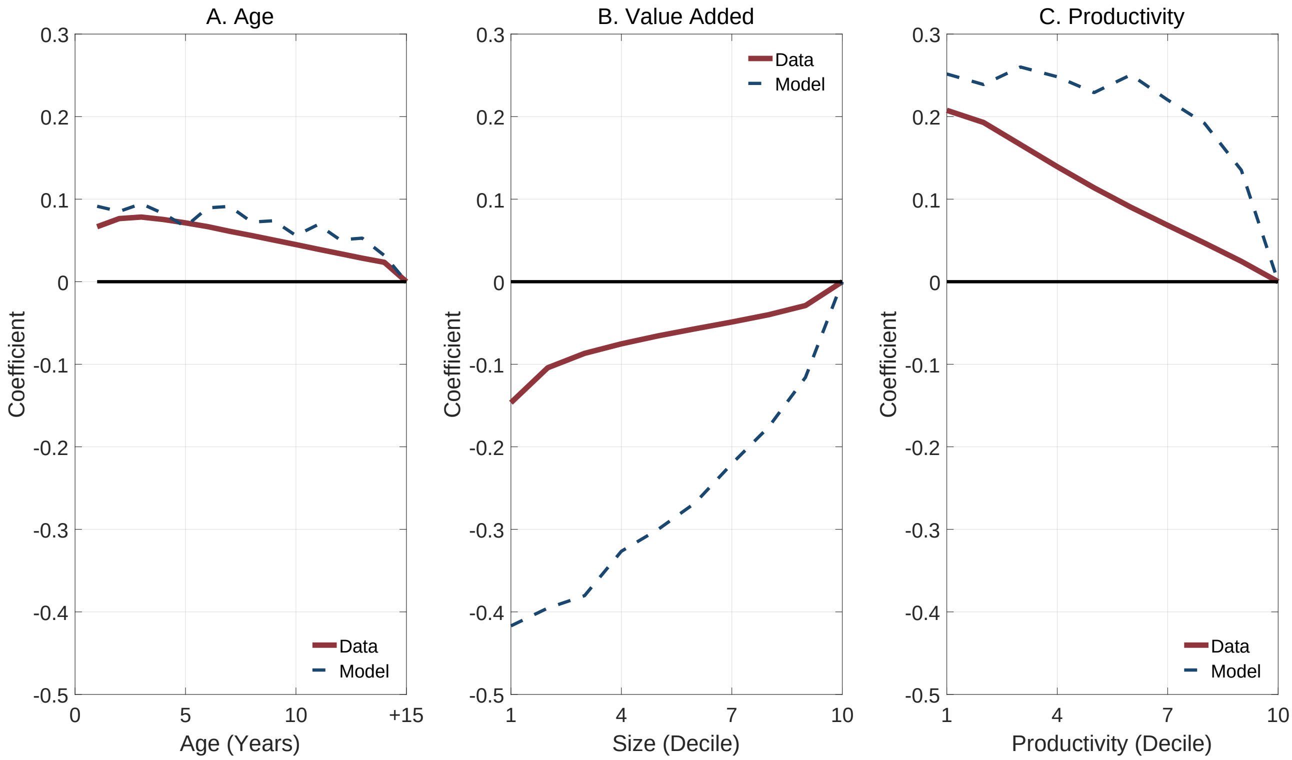
Figure 13: Profiles of PY/K



Panel II of Figure 13 shows how the dispersion of ARPK varies with firm age, size and productivity. Both in the model and the data dispersion of ARPK is declining with age and size, while it is U-shaped with productivity. The level of dispersion in ARPK in the model, on the other hand, is lower than in the data. Overall, the variation of ARPK is 1.33 in the data, while 1.07 in the model. The level of the dispersion in ARPK, however, can be made arbitrarily large if I allow for idiosyncratic distortions in firms capital decisions, as in David and Venkateswaran (2019).

Firm Financial Behavior I also evaluate how the model captures the financial behaviour of firms by running the same regressions in Section 4.3 with the simulated data from the model. In Figure 14, I compare the data and model counterparts of Equation 11. The model can capture the relation of average leverage with firm age, size and productivity. The main discrepancy is with firm size, as the model overstates the estimated elasticity.

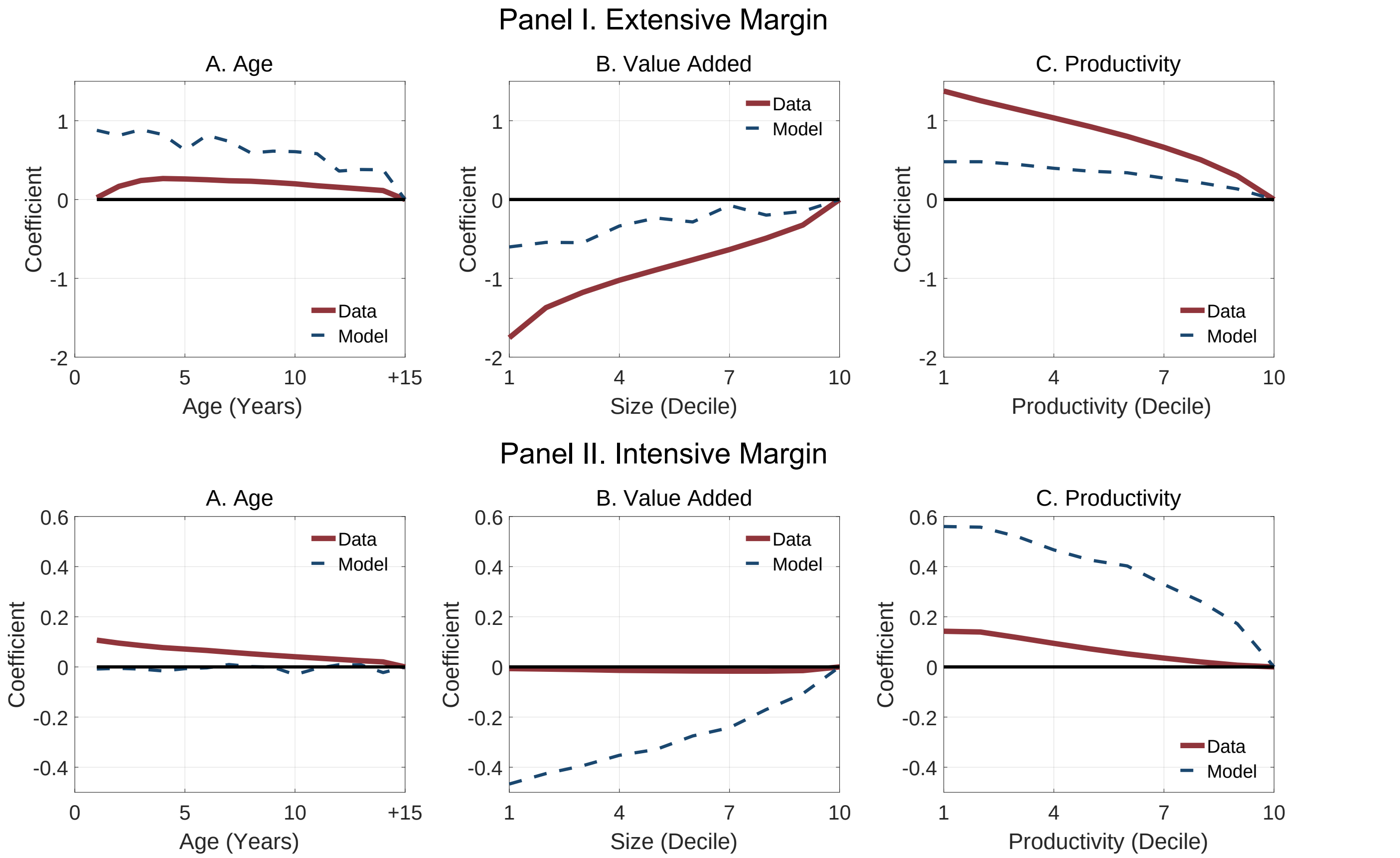
Figure 14: Financial Behavior



Panel I and II of Figure 15 show the results for the extensive margin (Equation 12) and intensive margin (Equation 13), respectively. Regarding the intensive margin, the model slightly overstates the elasticity with firm age; while, it understates it with firm size and productivity in the extensive margin. Regarding the intensive margin, the opposite pattern arises. The model slightly understates the elasticity with firm age; while, it overstates it with firm size and productivity.

Overall, the model captures the firm's financial behaviour pretty well, despite not explicitly targeting the calibration. It matches reasonably well the variation in firm leverage and its relation with firm characteristics. Furthermore, the model can distinguish the variation in firm leverage between the extensive and intensive margin, as it is in the data.

Figure 15: Financial Behavior - Extensive and Intensive Margin



7 The Effects of Financial Frictions

The model has two main mechanisms that affect capital allocation: financial frictions and uncertainty in the capital decision in the form of time-to-build. In order to disentangle these two mechanisms, I solve the problem of a benevolent social planner.[19] The planner can allocate available capital in the economy optimally without any financial frictions. However, the planner faces the same informational friction as in the benchmark economy, i.e. she has to decide on investment before she observes the productivity shocks of the firms due to the time-to-build nature of capital. The social planner takes the total amount of capital and labor from the benchmark economy as given and allocate it to maximize aggregate output.[20] Furthermore, the social planner takes the total number of firms and their productivity level as given. The problem is

$$\max_{\{k^{SP}(A_i)\}_{i=1}^{N}} \sum_{i=1}^{N} E\big(\hat{F}(k^{SP}(A_i),\ A')|A_i\big), \tag{31}$$

[19] Solving for the social planner problem to quantify the effects of financial frictions has been used in the misallocation literature, e.g. Buera et al. (2011).

[20] Labor is not subject to any friction. Therefore, the labor policy function is the same in both problems, decentralized and social planner.

subject to

$$K = \int_{kxbxA} k \ \mu(k, \ b, \ A) \ d\left[kxbxA\right] = \sum_{i=1}^{N} k^{SP}(A_i), \tag{32}$$

where

$$\left\{ \hat{F}(k^{SP}(A_i), \ A_i') = \max_{\{l\}} \left\{ F(k^{SP}(A_i), \ l, \ A_i') \right\} \right\}_{i=1}^{N}, \tag{33}$$

and

$$L = \int_{kxbxA} l \ \mu(k, \ b, \ A) \ d\left[kxbxA\right] = \sum_{i=1}^{N} l^{SP}(k^{SP}(A_i), \ A_i'), \tag{34}$$

and N is the total number of firms.

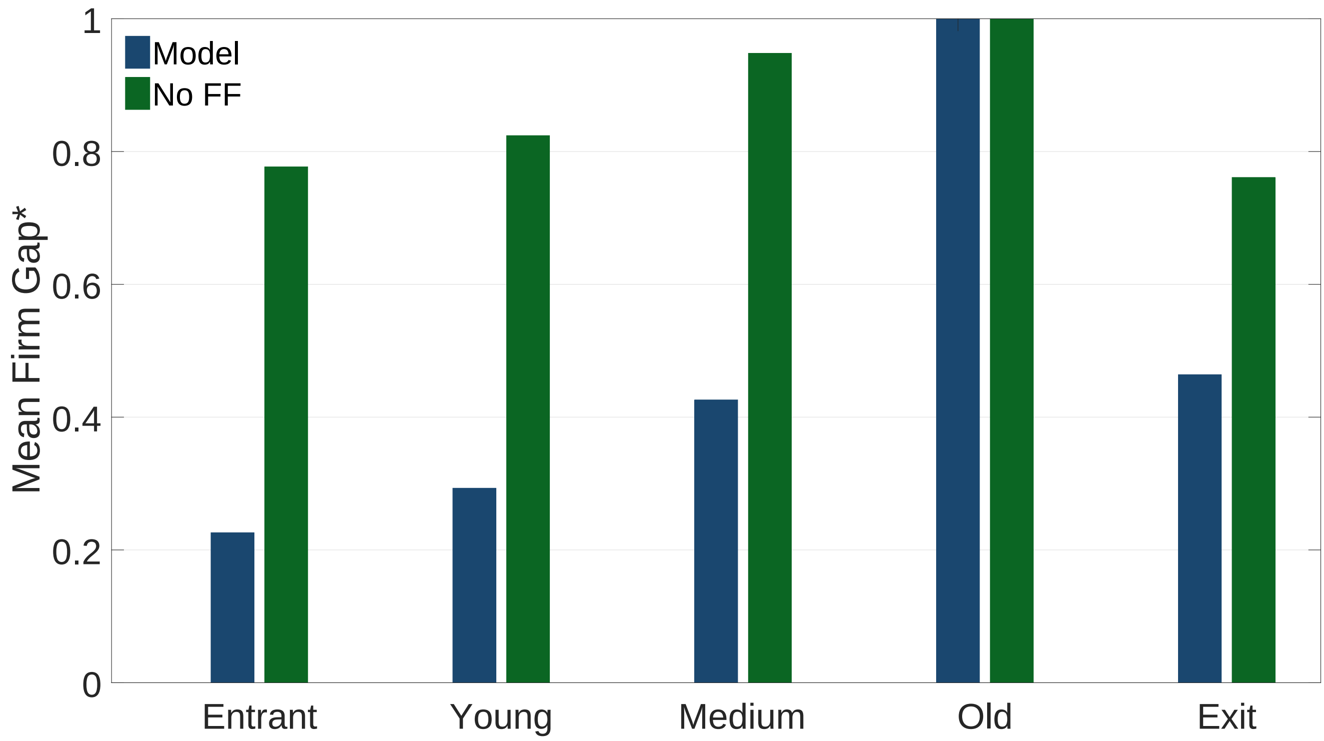
The solution to the social planner problems yields an allocation of capital that satisfies:

$$k^{SP}(A) \propto E\left(A'^{\frac{1}{1-\eta(1-\alpha)}} | A\right)^{\frac{1-\eta(1-\alpha)}{1-\eta}}. \tag{35}$$

Firm Life Cycle I evaluate how financial frictions affect the firm life cycle by comparing the results from the benchmark model and the one from the social planner problem. Figure 16 shows the results.

Financial frictions have a very significant effect on the firm life-cycle. In a world without financial frictions, entrants are much larger. They are only 20-25% smaller than an average old firm, more than 10 years old. Entrant firms are much smaller in the data, implying a large effect of financial frictions on entering firms. Overall, the average size of an entrant, compared to an old firm, will be three times larger in the absence of financial frictions. The existing firms also look very different without financial frictions. They are as large as entrants; their size is about 80% of old firms. In the benchmark economy, on the other hand, they were much smaller. The gap between firm sizes in the benchmark economy and social planner problem gets smaller over the firm life-cycle. Although, the process is prolonged and incomplete in most cases. An exiting firm will be 60% larger in the absence of financial frictions. The results when the productivity dynamics follow an AR(1) process are in Appendix D.3.1. The main takeaway is that the effects of financial frictions are much smaller under the standard AR(1) productivity dynamics, as the gap between the model and the social planner problem are closer in this case.

Figure 16: Effects of Financial Frictions: Firm Life Cycle



Notes: Young: 1-5 years old, Medium: 6-10 years old, Old: more than 10 years old. *Mean Firm Gap with respect to an old firm.

7.1 Aggregate Effects of Financial Frictions

In this section, I quantify the aggregate consequences of financial frictions. In Table 4, I summarize the main results. I evaluate the aggregate effects of financial frictions by looking at three statistics. First, the fraction of firms which capital decision is constrained due to financial frictions is 1/3 in the benchmark economy. Second, financial frictions prevent firms from investing their optimal level of capital, which translates into variation in the $ARPK$. The model generates a $SD(logARPK)$ of 1.07 versus the 1.33 present in the data. The remaining variation is due to other frictions that affect the allocation of capital not modelled in this paper, e.g. idiosyncratic distortions. Nevertheless, not all the variation in $ARPK$ is due to financial frictions, as firms face uncertainty in the capital decision. Using the social planner problem solution, I conclude that 20% of the variation in $ARPK$ is due to negative effects of financial frictions, i.e. (1.07-0.84)/1.07. Finally, I compute the productivity losses from the inefficient allocation of capital. Productivity losses are large, 32%, and half of them, 16%, result from the misallocation generated by financial frictions.

The aggregate effects of financial frictions are more muted if productivity dynamics follow a standard AR(1) process. The fraction of firms that are financially constrained is 1/3, the variation in $ARPK$ is smaller, and the aggregate productivity losses from financial frictions are only half:

Table 4: Aggregate Consequences of Financial Frictions

	Baseline	AR(1)
No Constrained (% of firms)	65.6%	74.6%
Constrained (% of firms)	34.4%	25.4%
SD(log ARPK)	1.065	0.847
SD(log ARPK) No FF	0.843	0.684
Productivity Loss (%)	31.5%	18.6%
Productivity Loss FF (%)	16.4%	8.1%

8%.[21]

An interesting question is why the benchmark economy produces larger effects of financial frictions in the aggregate economy compared to the standard AR(1) case. In order to answer this question, I do a decomposition exercise where I shut down one by one the differential characteristics of the estimated productivity dynamics with the AR(1) process. The results of the exercise are shown in Table 5.

Table 5: Decomposition of the Aggregate Effects

	(1)	(2)	(3)	(4)	(5)
No Constrained (% of firms)	65.6%	64.2%	57.8%	73.3%	74.6%
Constrained (% of firms)	34.4%	35.9%	42.2%	26.7%	25.4%
SD(log ARPK)	1.065	1.150	1.125	0.999	0.847
SD(log ARPK) No FF	0.843	1.023	0.933	0.823	0.684
Productivity Loss (%)	31.5%	32.6%	30.0%	25.1%	18.6%
Productivity Loss FF (%)	16.4%	11.5%	9.6%	11.2%	8.1%

Notes: (1) Benchmark, (2) Benchmark + Gaussian Shocks, (3) Benchmark + Gaussian Shocks + Constant Shock Variability, (4) Benchmark + Gaussian Shocks + Constant Productivity Persistence and (5) AR(1).

Column 1 contains the results of the benchmark economy. Column 2 solves the model with non-Gaussian productivity shocks. The difference in aggregate productivity losses is 4.9 p.p.. This is slightly more than 50% the gap between the benchmark economy and the AR(1) case, column 5. Therefore, half of the more considerable aggregate productivity losses are due to the non-Gaussian nature of productivity shocks. The other half is due to the non-linear productivity persistence

[21] In Appendix D, I explore the robustness of the results under different specifications of the borrowing constraint proposed in the literature. First, I solve the model using the standard borrowing constraint ($b' \leq \theta k'$). Second, I also solve the model using an earnings-based borrowing constraint ($b' \leq \theta E\left[\widehat{\pi}(k', A')|A\right]$), as recently used in Drechsel (2019). In all the cases, the aggregate productivity losses are at least twice as large when productivity dynamics follow the estimated non-linear and non-Gaussian dynamics instead of a standard AR(1) process.

and shock variability. To set these two elements apart, column 3 adds constant shock variability to column 2, while column 4 adds constant productivity persistence. I find that differential shock variability accounts for around 30% (3.1 p.p.) of the difference between the benchmark economy and the AR(1) case, while differential persistence is responsible for slightly less than 20% (1.5 p.p.).

8 Conclusion

In this paper, using a comprehensive dataset of Spanish firms, I first show that the productivity process that firms face is highly non-linear with non-Gaussian shocks. Low productivity firms have low productivity persistence, high shock variability and positive skewness, implying a larger probability of having a good productivity realization than in a standard AR(1) process. These firms may not have enough internal funds to finance their investment needs. Therefore, they will be financially constrained. Furthermore, these periods of high productivity are not long-lasting since high productivity firms have lower productivity persistence than the implied under an AR(1) process. This reduces the speed of financially constraint firms to accumulate internal funds through profit accumulation and surpass financial frictions. These two features that tell apart the estimated productivity process from a standard AR(1) process are fundamental to quantify the effects of financial frictions on the economy.

I then build a firm dynamics model with financial frictions where firm productivity evolves as estimated in the data. I discipline the model with a host of evidence on firm dynamics, misallocation, and firms' financial behaviour. Under the lens of the model, the effects of financial frictions are large. It affects the firm's life cycle, as firms enter the economy three times larger in a world without financial frictions than in the data. Furthermore, the process of profit accumulation to overcome financial frictions is slow and incomplete for many firms. I find that exiting firms are on average 60% larger in an economy without financial frictions.

The effects of financial frictions over the firm life cycle translate into substantial aggregate productivity losses through resource misallocation. About 1/3 of all firms are financially constrained in the benchmark economy, and financial frictions lower the aggregate productivity by 16%. These figures are much smaller if productivity dynamics evolve according to the standard AR(1) process common in the literature, 1/4 and 8%, respectively.

In the framework presented in this paper, productivity dynamics are exogenous and financial frictions do not affect their evolution. However, financial frictions may distort the firms' incentives to undertake investment opportunities to increase their productivity. Therefore, the effects of financial frictions may be even larger if this channel is important. In Petit and Ruiz-García (2019), we extend the standard firm dynamics model to incorporate endogenous productivity dynamics.

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Appendices

A Data

In this section, I provide further details about the dataset. I first show the sample selection and the cleaning procedure. Then, I compare the resulting dataset with the census of Spanish firms to check the sample representativeness. Finally, I show the parameters of the production function.

A.1 Sample Selection

In Table A1, I show the sample selection step by step. First, I select non-publicly-listed firms (column 1). Publicly-listed firms represent 0.1% of the total, around 5% of economic activity in terms of value-added and employment. Second, I select non-public firms (column 2). Public firms represent 0.5% of the total, around 15% of economic activity in terms of value-added, and 3% in terms of employment. Third, I select limited liability firms (column 3). Non-limited liability firms represent 0.8% of total and around 3% of total activity in terms of value-added and employment. The final sample represents 98.6% of the firms, accounting for 74% of value-added and 91% of employment.

A.2 Cleaning

Nest, I summarize the steps I take to arrive at the final dataset used in the paper.

1.- I drop all the observations with a real wage (nominal wage bill over CPI over the number of employees) lower than the 1st percentile and larger than the 99th percentile (no applied to missing wage, public and public sector firms). I drop 111,992 observations in this step.

2.- I drop observations with more than 100,000 workers, as the largest Spanish firm has a bit more than 80,000 employees (no applied to public and public sector firms). I drop 46,320 observations in this step.

3.- I apply filters to detect errors on key variables: on the sector of activity (if the firm is classified in an economic sector of the CNAE classification), age (if the firm has a reliable age), province (if the firm's headquarters are classified in one of Spanish provinces), value-added (if the firm has positive value-added), capital (if the firm has positive value for capital), wage bill (if the firm has positive value in the wage bill). I drop 7,289,899 observations in this step.

4.- I restrict the analysis to the years from 1999 to 2014, both included. I drop 2,426,715 observations in this step.

5.- The number of observations left after the previous cleaning are 7,767,289.

6.- I drop economic sectors with capital share lower than 0; those are 5 economic sectors out of 59. I drop 378,191 observations in this step.

7.- I keep firms with a value-added in real terms of more than 1,000 euros in 2010, capital of more than 500 euros in 2010, wage bill of more than 3,000 euros in 2010. I drop 214,815 observations in this step.

8.- I drop weird observations:

8.1.- Firms that are at the top 90th percentile of the total factor productivity, value-added, capital, wage bill, labour, revenue productivity, average revenue product of capital, and average revenue product of labour distribution and at the bottom 1st percentile of any of the other distributions.

8.2.- Firms that are at the bottom 10th percentile of the total factor productivity, value-added, capital, wage bill, labour, revenue productivity, average revenue product of capital, and average revenue product of labor distribution and at the top 99th percentile of the other distributions.

8.3.- 287,928 observations are detected as weird observations.

9.- I drop outliers: Firms that are at the bottom 1st percentile or the top 99th percentile of the total factor productivity, revenue productivity, average revenue product of capital, and average revenue product of labour. 504,038 observations are detected as outliers.

10.- From the combination of the two previous steps, I drop 602,597 observations.

11.- I drop sectors with less than 5,000 firms (6 out of 54 economic sectors). 17,535 observations are dropped; as a result, all sectors have at least 100 firms in a given year.

12.- There are 1,505,436 out of 6,500,945 firm-year observations that cannot be followed in two consecutive years.

13.- The final sample includes 6,500,945 firm-year observations from 1999 to 2014 from 1,024,144 different firms.

13.1.- In the before crisis period (1999-2007), there are 3,371,530 firm-year observations from 745,296 different firms.

13.2.- In the after crisis period (2007-2014), there are 3,553,697 firm-year observations from 822,242 different firms.

A.3 Sample Representativeness

Comparing the final database with the Spanish directory (Table A2 and Table A3). The selected sample covers around 50% of all the firms, and the coverage is constant over the studied period. In terms of employment, the coverage is smaller, around 30% of the total, due to the focus on private firms. Regarding the firm size distribution, the coverage is consistent across all size groups. It is only slightly lower for tiny and very large firms. The coverage is very similar in the manufacturing sector (Table A4 and Table A5).

A.4 Parameterization

I recover the parameters governing the elasticity of output with respect to capital at the sector level. The estimated parameters are shown in Figure A1, Panel A, the unweighted average and median are 0.32 and 0.29, respectively. The weighted average and median are 0.38 and 0.35, respectively. I compute sector-specific weights ω_s to aggregate the economy. There are 50 sectors at the 2-digits level. In Figure A1, Panel B, I plot the sector-weight distribution. The average and median sector weights are 2.0% and 1.1%, respectively.

Table A1: Sample Selection

	(1)	(2)	(3)	Total	Sample Selection
Firms	0.1	0.5	0.8	1.4	98.6
Value Added	5.1	19.9	3.0	26.0	74.0
Capital	11.5	21.8	4.3	34.3	65.7
Wage Bill	4.6	14.5	2.4	20.3	79.7
Employment	4.2	3.3	2.3	8.7	91.3
Total Assets	10.1	18.0	3.6	28.6	71.4
Equity	9.3	20.0	4.0	30.1	69.9

Notes: (1) Public listed firms, (2) No public firms and (3) No limited liability firms.

Table A2: Sample Representativeness. Aggregate

Year	Employment	Wage Bill	Firms
1999	22.2	31.9	43.1
2000	23.0	27.3	44.0
2001	24.5	44.4	45.7
2002	26.4	29.8	46.8
2003	28.8	31.0	49.2
2004	31.0	30.7	50.3
2005	32.8	32.1	51.5
2006	33.7	33.1	50.6
2007	32.3	31.8	46.2
2008	35.4	32.8	47.4
2009	34.0	30.4	46.5
2010	34.2	31.0	48.6
2011	34.7	31.7	49.0
2012	34.9	32.1	48.3
2013	35.6	32.8	47.5
2014	36.9	34.0	51.1
Average	31.3	32.3	47.9

Table A3: Sample Representativeness. Firm Size Distribution

Year	1-5	5-20	20-50	50-200	+200
1999	25.8	46.1	46.2	34.8	32.0
2000	28.2	49.0	47.9	34.5	30.5
2001	31.4	50.4	55.0	35.8	30.6
2002	33.2	52.0	57.3	40.1	31.8
2003	36.1	57.0	64.5	44.6	35.7
2004	38.2	60.2	68.2	48.7	37.5
2005	39.8	64.0	70.4	50.5	40.4
2006	39.6	62.3	70.2	53.7	43.0
2007	36.3	57.8	64.5	47.2	40.4
2008	39.7	63.0	68.1	48.7	41.8
2009	40.0	59.9	64.5	46.9	51.2
2010	41.6	62.8	71.2	52.1	56.5
2011	42.3	62.6	72.1	54.0	58.6
2012	42.1	61.2	70.7	53.8	58.5
2013	40.0	63.0	75.2	57.8	56.0
2014	41.5	71.2	82.8	68.1	60.3
Average	37.2	58.9	65.5	48.2	44.1

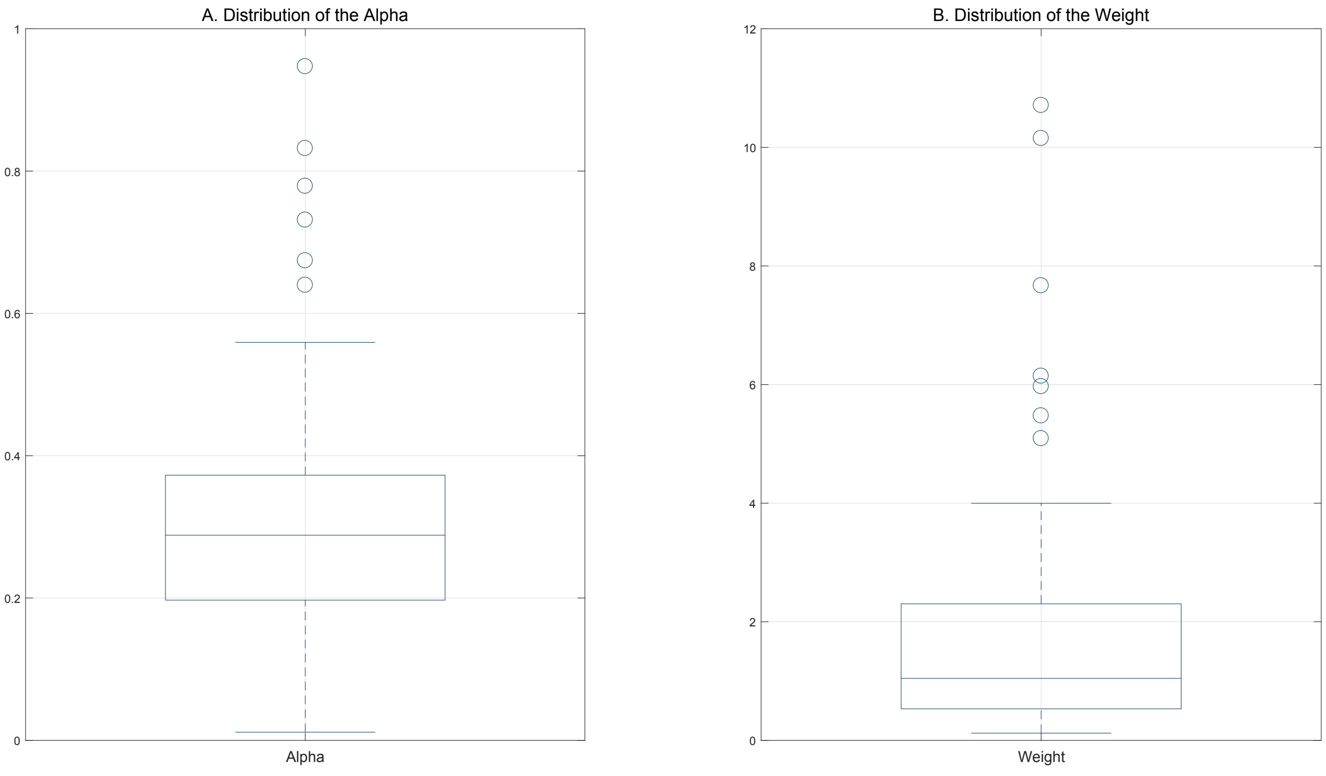
Table A4: Sample Representativeness. Aggregate (Manufacturing)

Year	Employment	Wage Bill	Firms
1999	29.6	51.1	43.8
2000	29.8	39.7	44.8
2001	31.9	82.8	46.9
2002	34.7	38.5	48.1
2003	37.3	42.7	51.6
2004	40.1	42.3	53.2
2005	42.0	43.8	56.3
2006	43.2	44.6	56.4
2007	42.9	44.2	53.3
2008	46.3	44.8	46.4
2009	46.3	43.4	45.2
2010	47.6	44.7	47.0
2011	47.9	45.9	47.1
2012	49.6	47.2	47.6
2013	50.9	48.6	48.9
2014	54.8	52.0	54.9
Average	42.2	47.3	49.5

Table A5: Sample Representativeness. Firm Size Distribution (Manufacturing)

Year	1-5	5-20	20-50	50-200	+200
1999	25.2	44.1	46.2	35.1	35.7
2000	27.5	47.0	48.0	34.7	32.9
2001	31.3	49.0	52.7	36.6	33.2
2002	33.1	50.9	54.8	39.9	33.3
2003	37.1	56.7	60.6	43.4	36.0
2004	39.6	60.9	61.1	47.4	39.4
2005	42.4	66.5	65.6	49.8	41.4
2006	43.7	65.2	64.4	50.6	43.7
2007	41.7	62.3	60.8	45.8	43.0
2008	36.2	62.0	73.2	52.7	44.9
2009	37.2	58.5	67.6	49.3	52.0
2010	38.2	62.9	74.3	54.0	58.7
2011	38.4	62.9	75.8	56.5	61.4
2012	39.8	62.8	72.8	56.8	63.7
2013	39.7	67.0	78.1	62.2	62.3
2014	43.7	74.6	87.2	76.4	69.3
Average	37.2	59.6	65.2	49.5	46.9

Figure A1: Alpha and Weight distribution



B Empirics

In this section, I provide additional details and robustness exercises on the empirical analysis of the main paper. There are three sections covering the three empirical sections of the paper.

B.1 Productivity Dynamics

In this section, I provide further details and robustness exercises on the estimated productivity process.

B.1.1 Persistence

Persistence depends on initial productivity and the productivity shock, as shown in equation xx. In the main paper, I integrate over the productivity shock, as shown in equation xx. The persistence of the shock process conditional on initial productivity and the productivity shock is shown in figure B1, B2 and B3.

B.1.2 Estimation

A concern is that the procedure describe in section 4.1 to characterize the productivity process is not able to capture its characteristics. In order to show the reliability of the estimation procedure, I do a Monte-Carlo simulation of 1 million firms from a AR(1) productivity process with parameters, $\rho = 0.8$ and $\sigma = 0.3$. In this case, we know that persistence should be flat on initial productivity and with a value of 0.8. Shock variability should be flat conditional on initial productivity and with a value close to 0.4. Shock skewness should be flat conditional on initial productivity and with a value of 0. Finally, shock kurtosis should be flat conditional on initial productivity and with a value close to 2.1. The results of this exercise are shown in figure B4 and B5. As we can see, the procedure used to characterize the productivity process captures well the dynamics of the AR(1) process. It suffers an upwards bias in the tails of the distribution in the estimation of persistence and shock variability. Importantly, the upper bias will go against; and therefore, dampens the results found in the empirical section of the paper.

B.1.3 Data as One Sector Economy

I estimate the productivity process sector by sector, instead of pooling the data of all the sectors. Then, I aggregate using the sector weights ω_s. The results are show in figure B6, where the sector by sector estimation is labelled version 2. The results look very similar to the baseline described in the main paper.

B.1.4 Decreasing Returns to Scale

The productivity estimation is sensitive to the decreasing returns to scale (DRS), governed by the parameter η. I repeat the estimation of the productivity process for different values of η. The results are shown in figure B7. The main takeaway is the robustness of the characteristics of the productivity process to the range of DRS used in the literature.

B.1.5 Studied Period Heterogeneity

The time period used, from 1999 to 2014, has the Great Recession of 2007 in the middle. In order to evaluate the consistency of the characteristics of the productivity process across time and specially in the period of recession and recovery, I split the sample in two sub-periods. The first one, before the Great Recession, from 1999 to 2007; and the second one during and after the Great Recession, from 2007 to 2014. The results are shown in figure B8. As we can see, the characteristics of the productivity process has been pretty stable during the whole period.

B.1.6 AR(1) Productivity Process

I estimate the standard AR(1) process for comparison. The specification is as follows

$$log(A_{it}) = \alpha \ + \ \rho_a log(A_{it}) \ + \ \sigma_\varepsilon \varepsilon_{it} \qquad \varepsilon_{it} \sim N(0,\ 1). \tag{36}$$

As in the non-linear productivity process, I choose η such that the model matches the variation of the firm size distribution. The results are summarized in Table B.1. The value of η that yields the best fit is 0.78. The estimation of the AR(1) productivity process results on a persistence parameter (ρ_a) of 0.813 and shock variability (σ_ε) of 0.336. These values fall in the standard range used in the firm dynamics literature. Another interesting point is the stability of the estimated ρ_a and σ_ε parameters to different values of the span of control parameter (η).

Figure B1: Persistence of the Productivity Process Conditional on Past Productivity and Productivity Shock

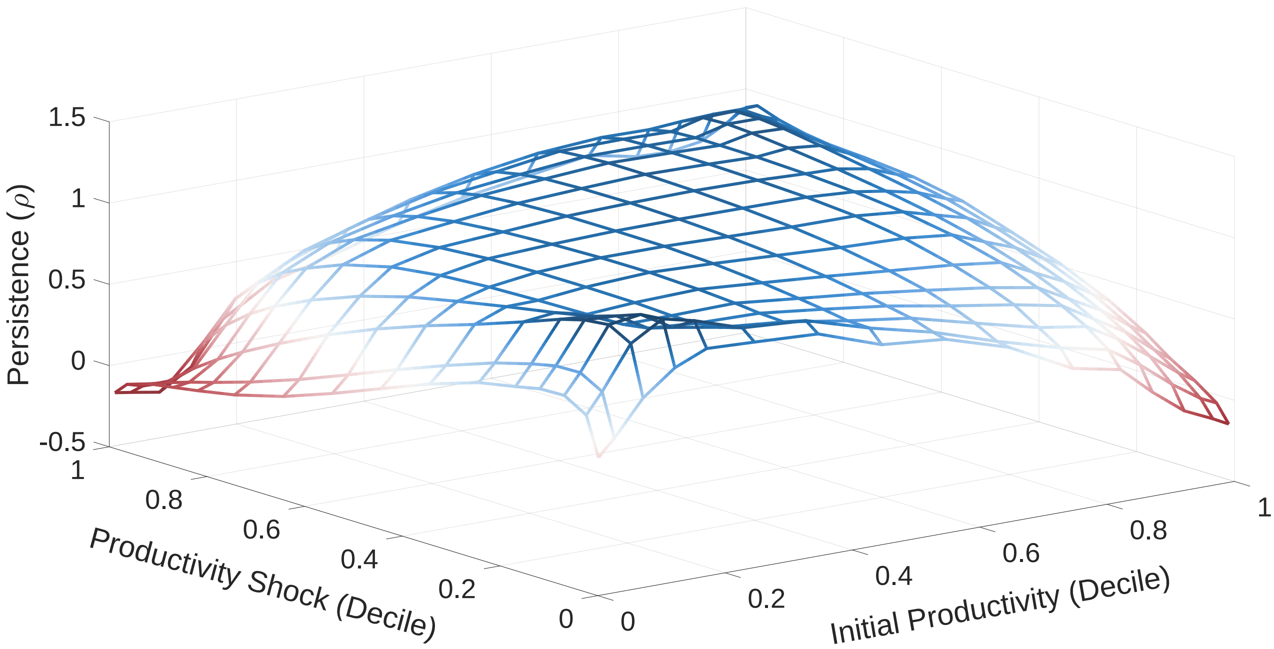


Table B1: Estimation of the coefficients of the AR(1) process

η	ρ_a	σ_ε
0.75	0.8130	0.3324
0.77	0.8127	0.3350
0.78	0.8128	0.3364
0.79	0.8133	0.3378
0.80	0.8137	0.3369
0.81	0.8126	0.3361
0.82	0.8133	0.3376
0.83	0.8135	0.3392
0.85	0.8146	0.3408

Figure B2: Persistence of the Productivity Process Conditional on Past Productivity and Productivity Shock

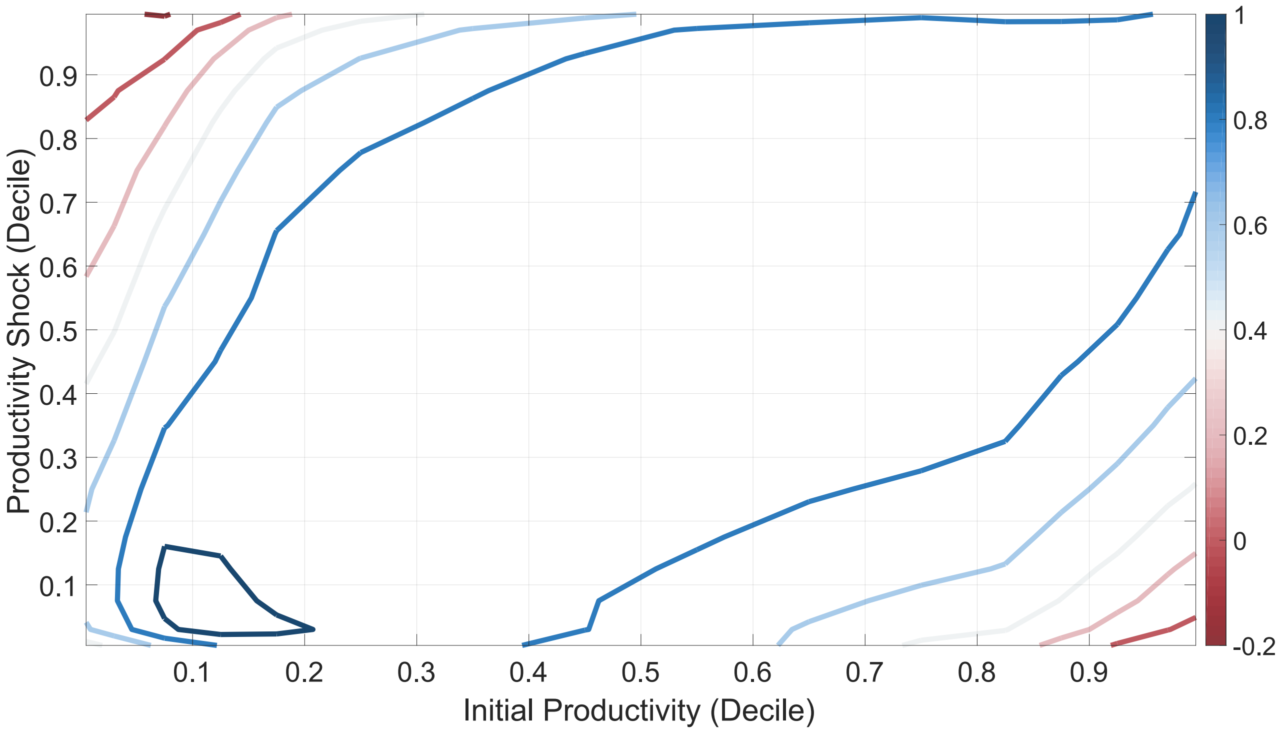


Figure B3: Conditional Persistence

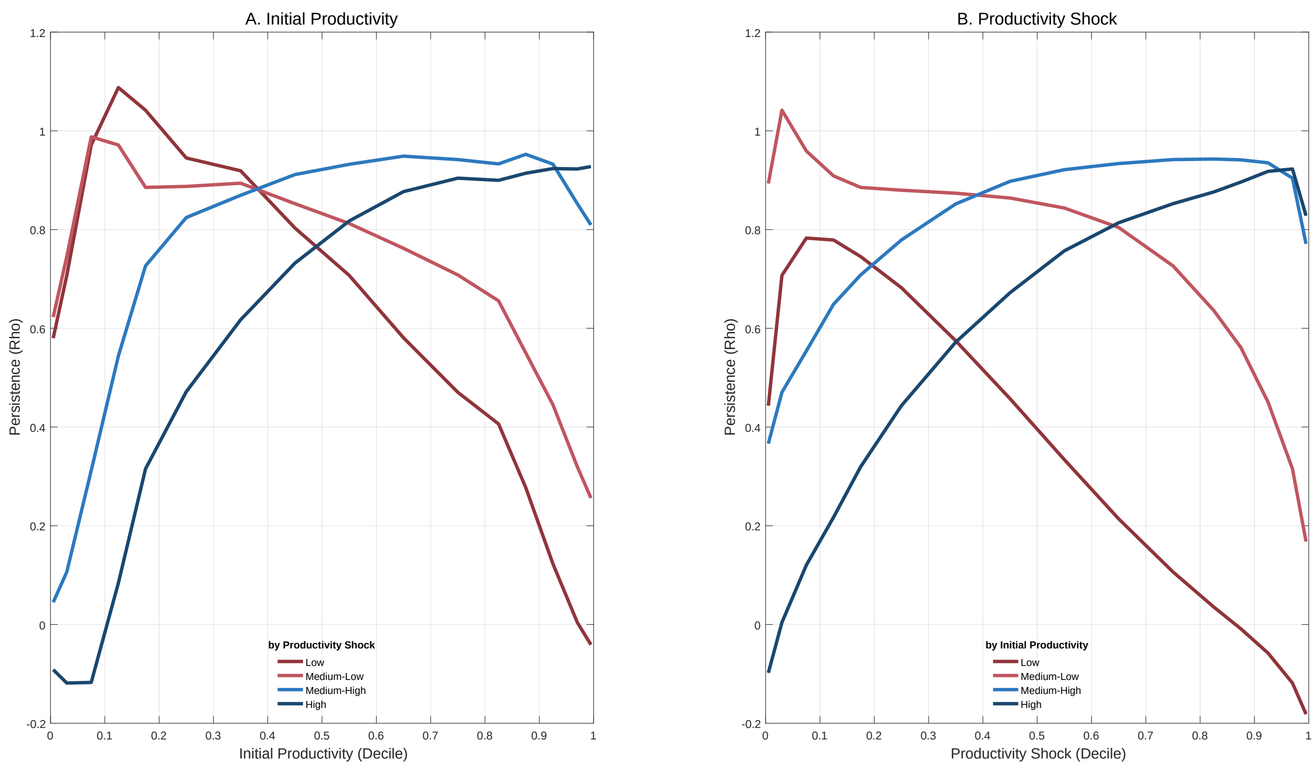


Figure B4: Characteristics of the Productivity Process - Simulation

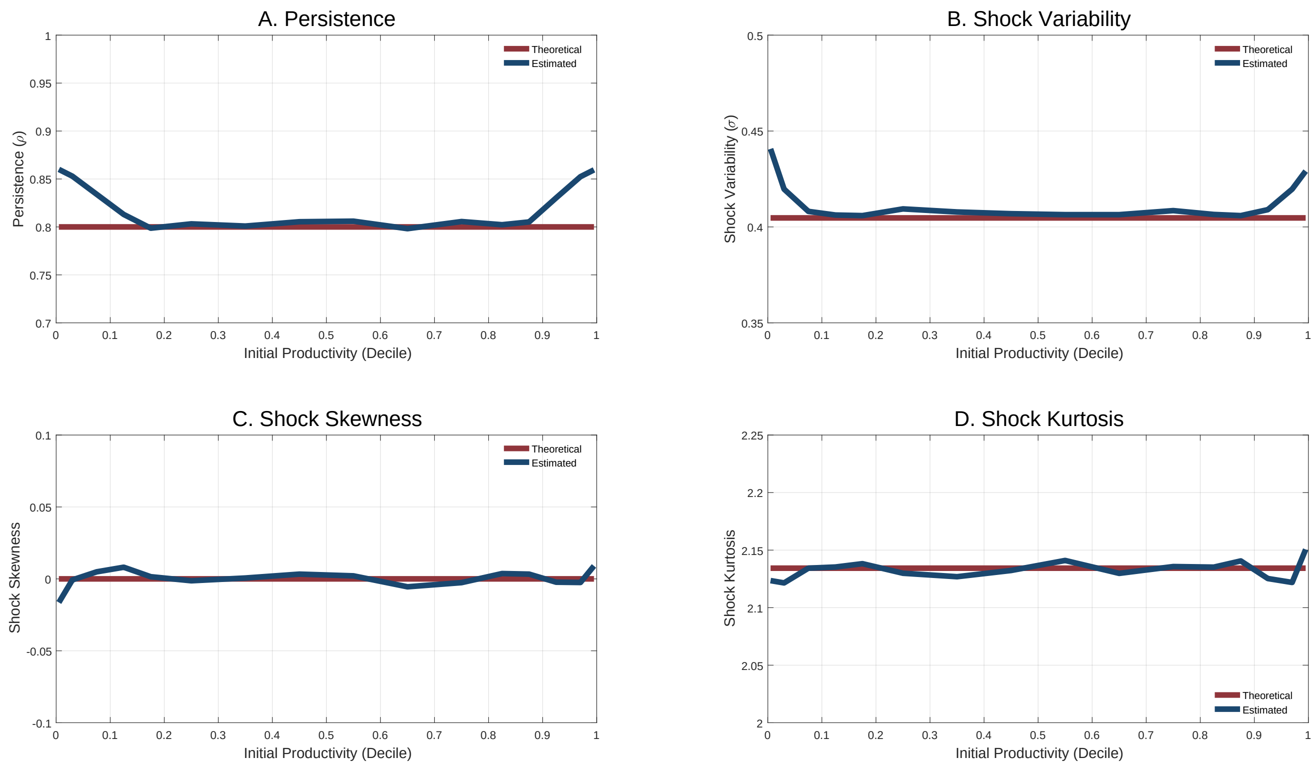


Figure B5: Persistence of the Productivity Process Conditional on Past Productivity and Productivity Shock

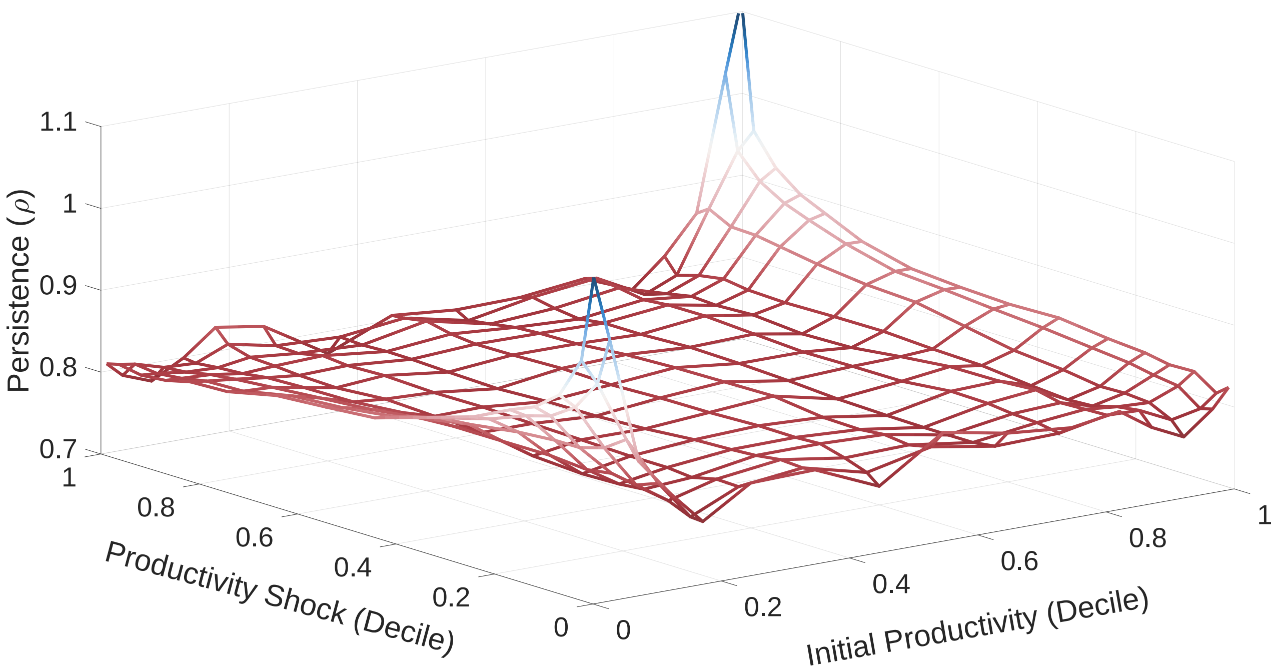


Figure B6: Characteristics of the Productivity Process - Different Specification

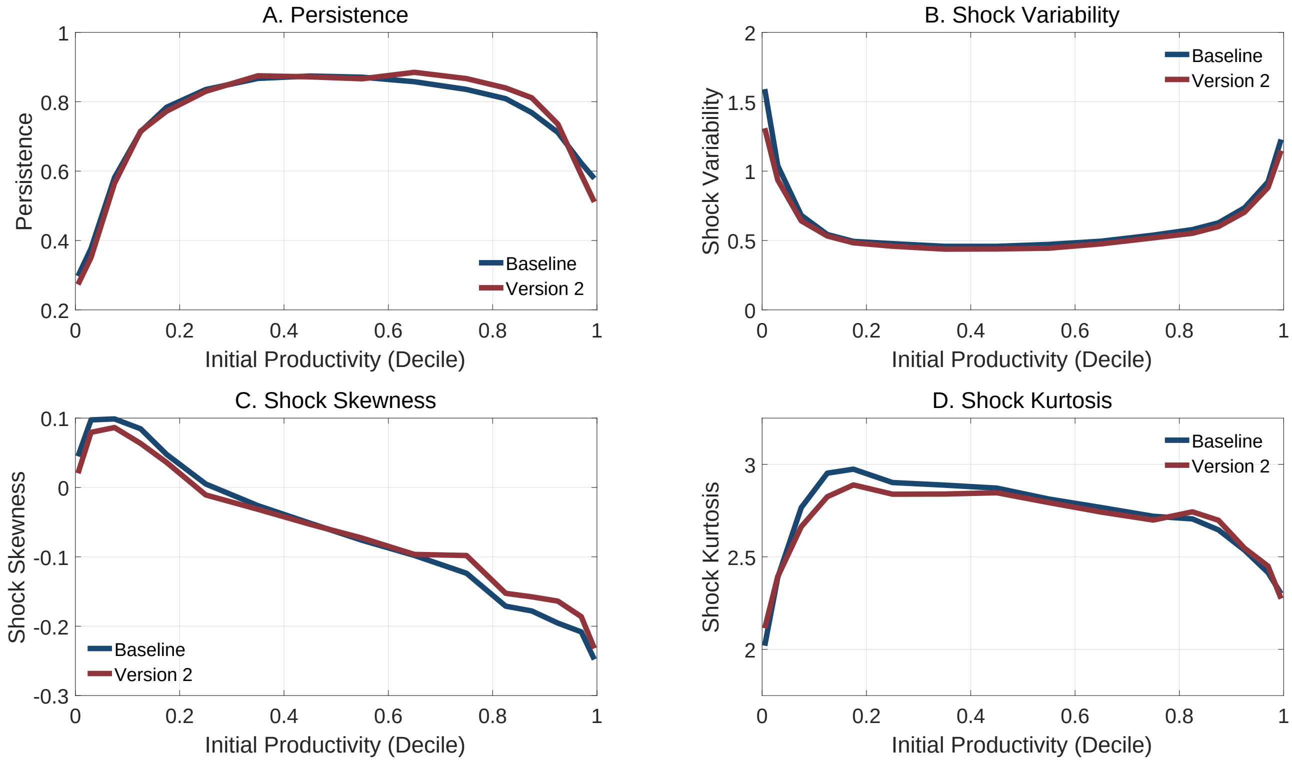


Figure B7: Characteristics of the Productivity Process - Different DRS

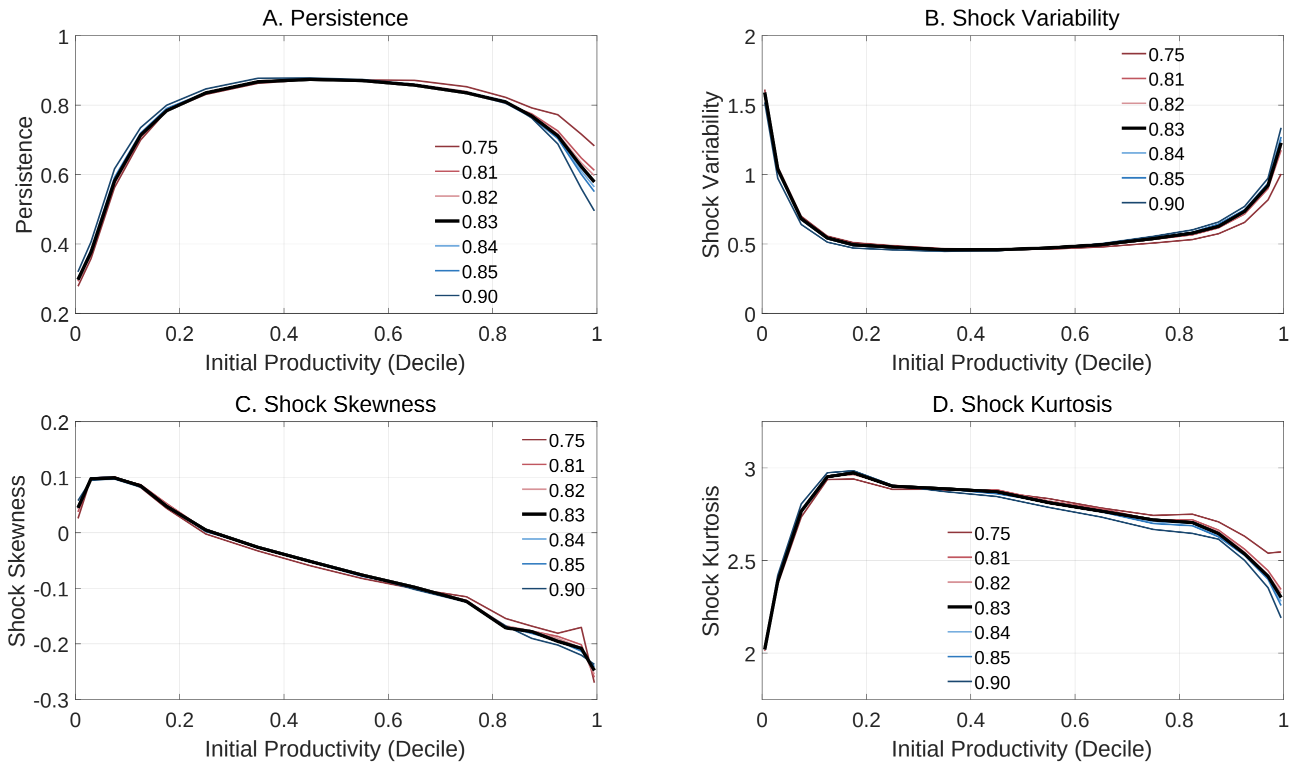
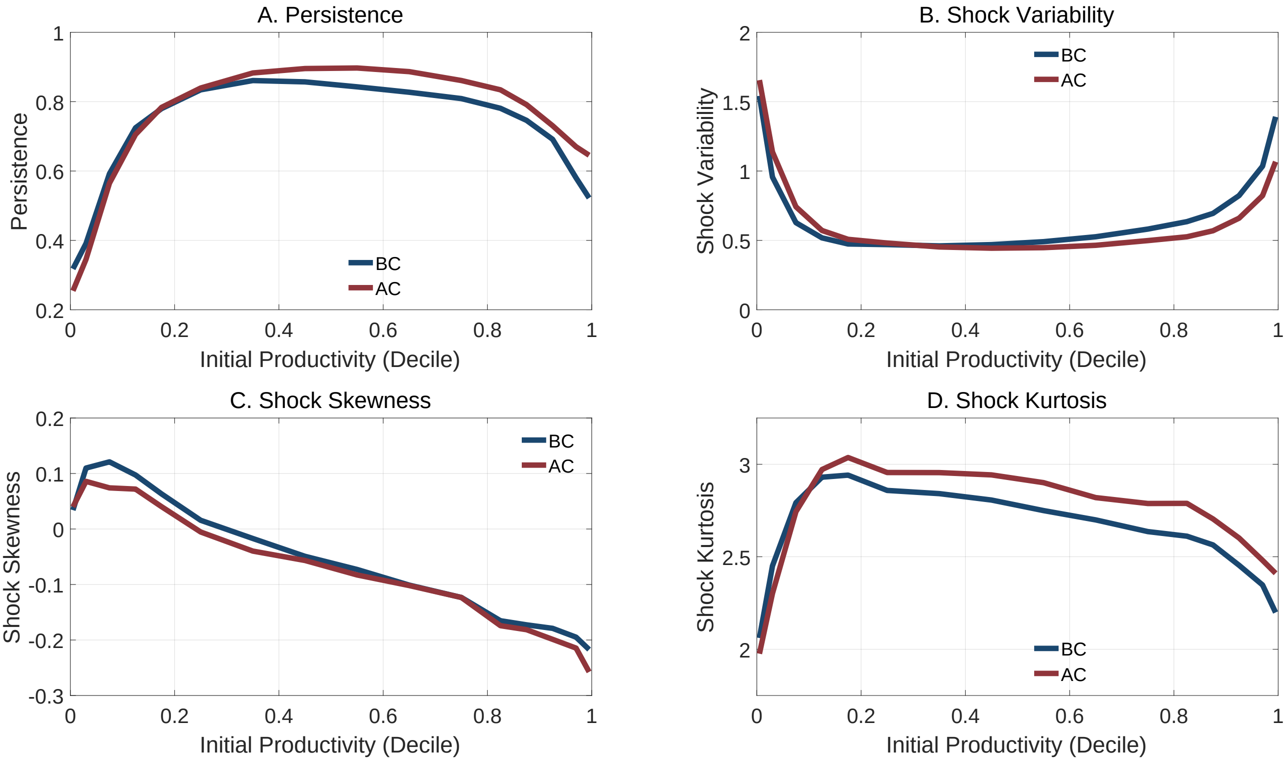


Figure B8: Characteristics of the Productivity Process - Different Periods



B.2 Misallocation

In this section, I provide additional details and robustness exercises on the empirical analysis of the main paper. There are three sections, each of them covering the three empirical sections of the paper.

B.3 Productivity Dynamics

B.3.1 Persistence

Persistence depends on initial productivity and the productivity shock, as shown in the main text. Then, I integrate over the productivity shock distribution. Figures B1, B2, and B3 show the persistence of the productivity conditional on the initial level and the shock.

B.3.2 Estimation

One potential concern is if the procedure described in section 4.1 to characterise the productivity process accurately describes its characteristics. In order to show the reliability of the estimation procedure, I do a Monte-Carlo simulation of 1 million firms from a AR(1) productivity process with parameters, $\rho = 0.8$ and $\sigma = 0.3$. In this case, we know that persistence should be a horizontal line on initial productivity at 0.8. Shock variability should be as well constant conditional on initial productivity and with a value close to 0.4. As the productivity shocks are from a Gaussian distribution, the skewness should be flat conditional on initial productivity and with a value of 0. Moreover, kurtosis should be flat conditional on initial productivity and with a value close to 2.1. The results of this exercise appear in figures B4 and B5. As we can see, the procedure captures well the dynamics of the AR(1) process. Nonetheless, it suffers an upwards bias at the distribution's tails in estimating persistence and shock variability. Importantly, the upper bias goes against the financial frictions mechanism, dampening the results found in the empirical section of the paper.

B.3.3 Data as One Sector Economy

Next, I estimate the productivity process sector by sector instead of pooling the data of all the sectors. Then, I aggregate using the sector weights ω_s. The results are in figure B6, where the sector by sector estimation is labelled as version 2. The results look very similar to the baseline described in the main paper.

B.3.4 Decreasing Returns to Scale

The productivity estimation is sensitive to the decreasing returns to scale (DRS), governed by the parameter η. I repeat the estimation of the productivity process for different values of η. The results are in figure B7. Overall, the estimated productivity process characteristics prevail in the range of DRS used in the literature.

B.3.5 Studied Period Heterogeneity

The Great Recession of 2007 is in the middle of the studied period. I split the sample into two sub-periods to evaluate the consistency of the characteristics of the productivity process across time. The first sub-period goes from 1999 to 2007, and the second one goes from 2007 to 2014. The results are in figure B8. As we can see, the characteristics of the productivity process are alike during the whole period.

B.3.6 AR(1) Productivity Process

For the sake of comparison with the non-parametric estimation, I estimate the standard AR(1) process. The specification is as follows:

$$log(A_{it}) = \alpha \ + \ \rho_a log(A_{it}) \ + \ \sigma_\varepsilon \varepsilon_{it} \qquad \varepsilon_{it} \sim N(0,\ 1). \tag{37}$$

As in the non-linear productivity process, I choose η such that the model matches the variation of the firm size distribution. The results are summarized in Table B.1. The value of η that yields the best fit is 0.78. The estimation of the AR(1) productivity process results on a persistence parameter (ρ_a) of 0.813 and shock variability (σ_ε) of 0.336. These values fall in the standard range commonly used in the firm dynamics literature. Another interesting point is the stability of the estimated ρ_a and σ_ε parameters to different values of the span of control parameter (η) as in the non-parametric estimation.

Figure B9: Persistence of the Productivity Process Conditional on Past Productivity and Productivity Shock

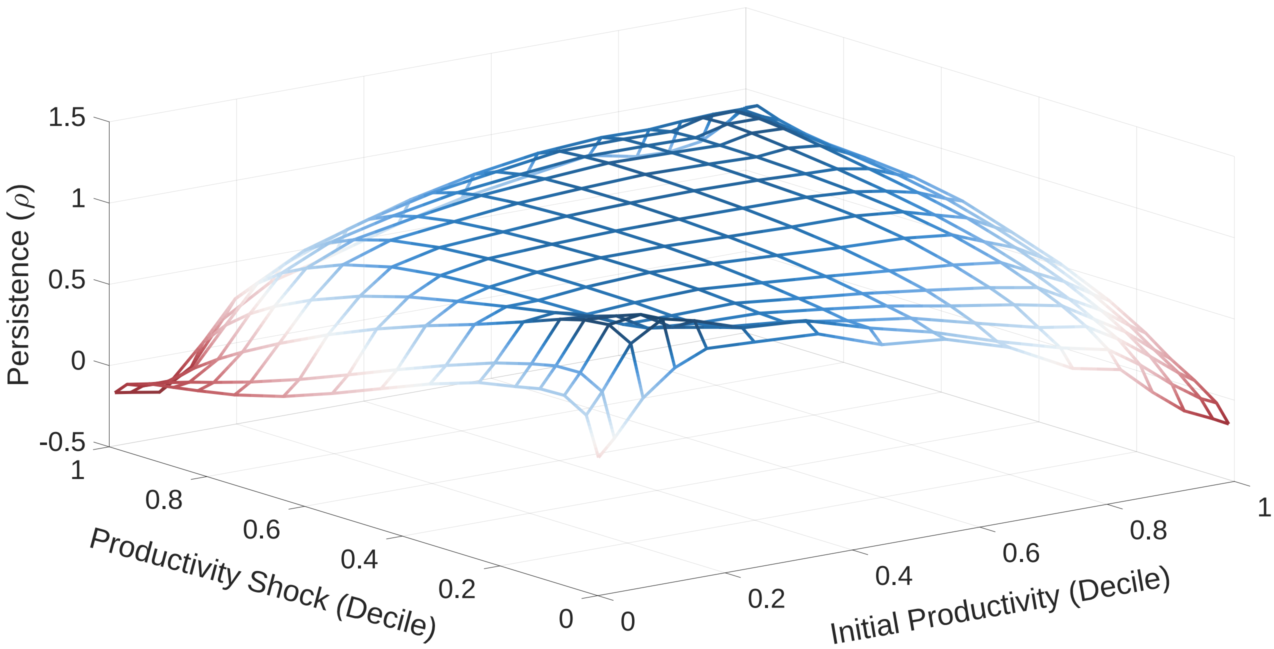


Table B2: Estimation of the coefficients of the AR(1) process

η	ρ_a	σ_ε
0.75	0.8130	0.3324
0.77	0.8127	0.3350
0.78	0.8128	0.3364
0.79	0.8133	0.3378
0.80	0.8137	0.3369
0.81	0.8126	0.3361
0.82	0.8133	0.3376
0.83	0.8135	0.3392
0.85	0.8146	0.3408

Figure B10: Persistence of the Productivity Process Conditional on Past Productivity and Productivity Shock

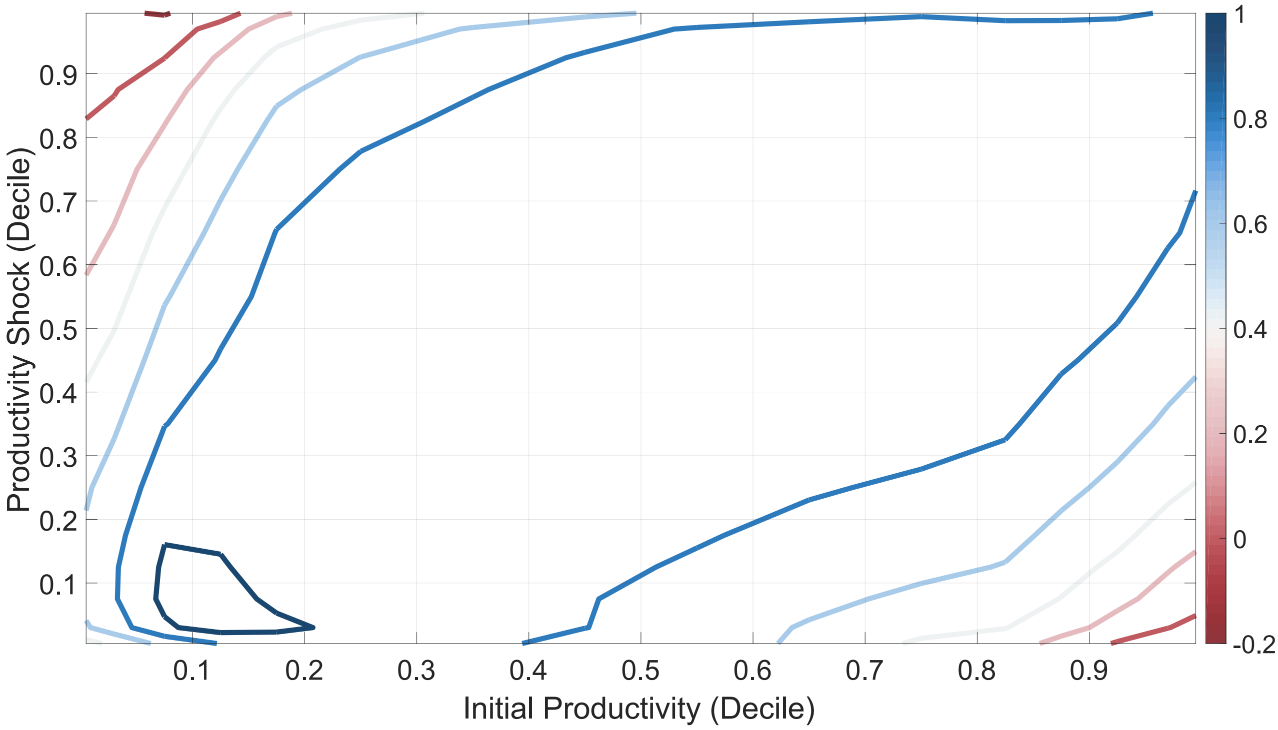


Figure B11: Conditional Persistence

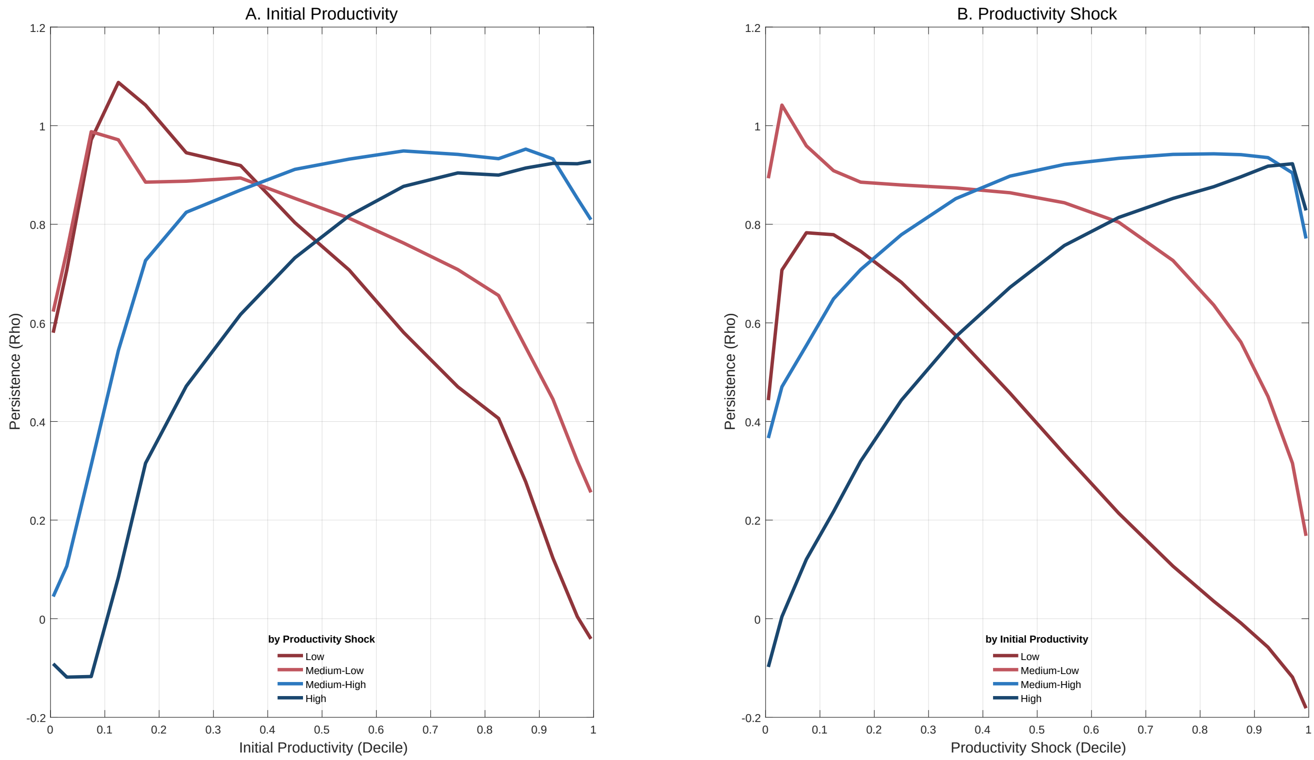


Figure B12: Characteristics of the Productivity Process - Simulation

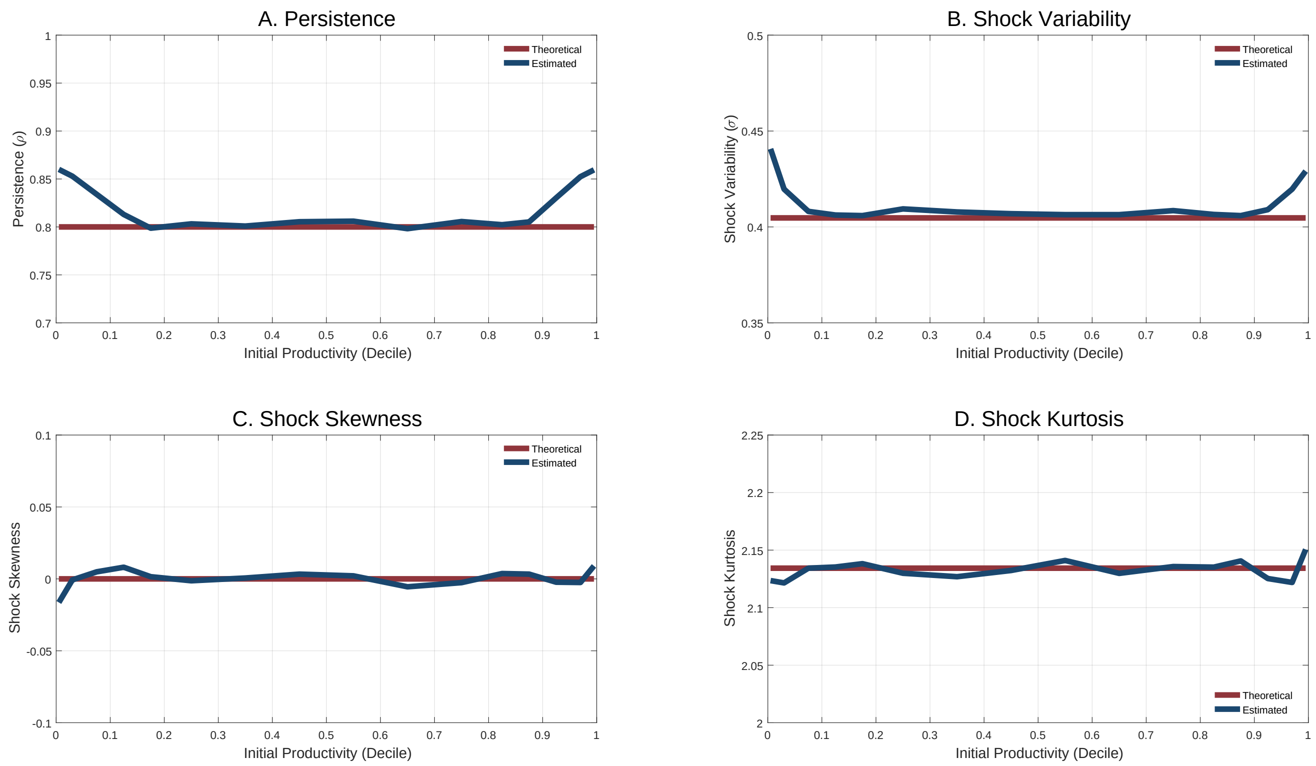


Figure B13: Persistence of the Productivity Process Conditional on Past Productivity and Productivity Shock

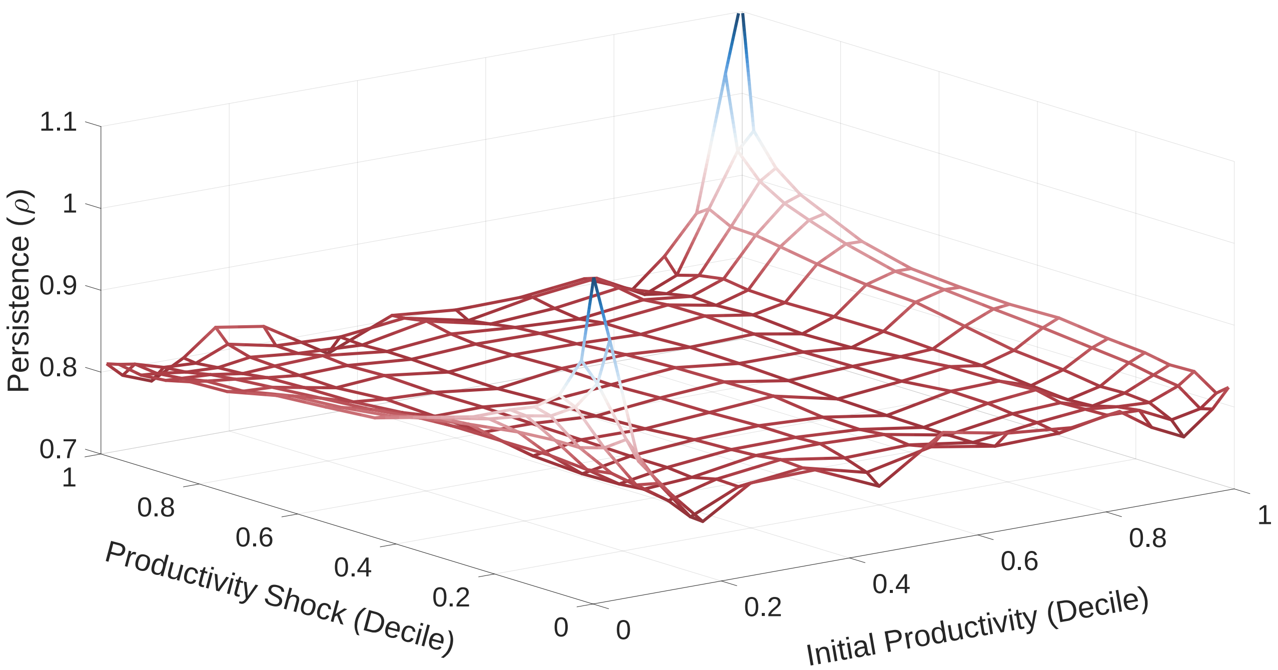


Figure B14: Characteristics of the Productivity Process - Different Specification

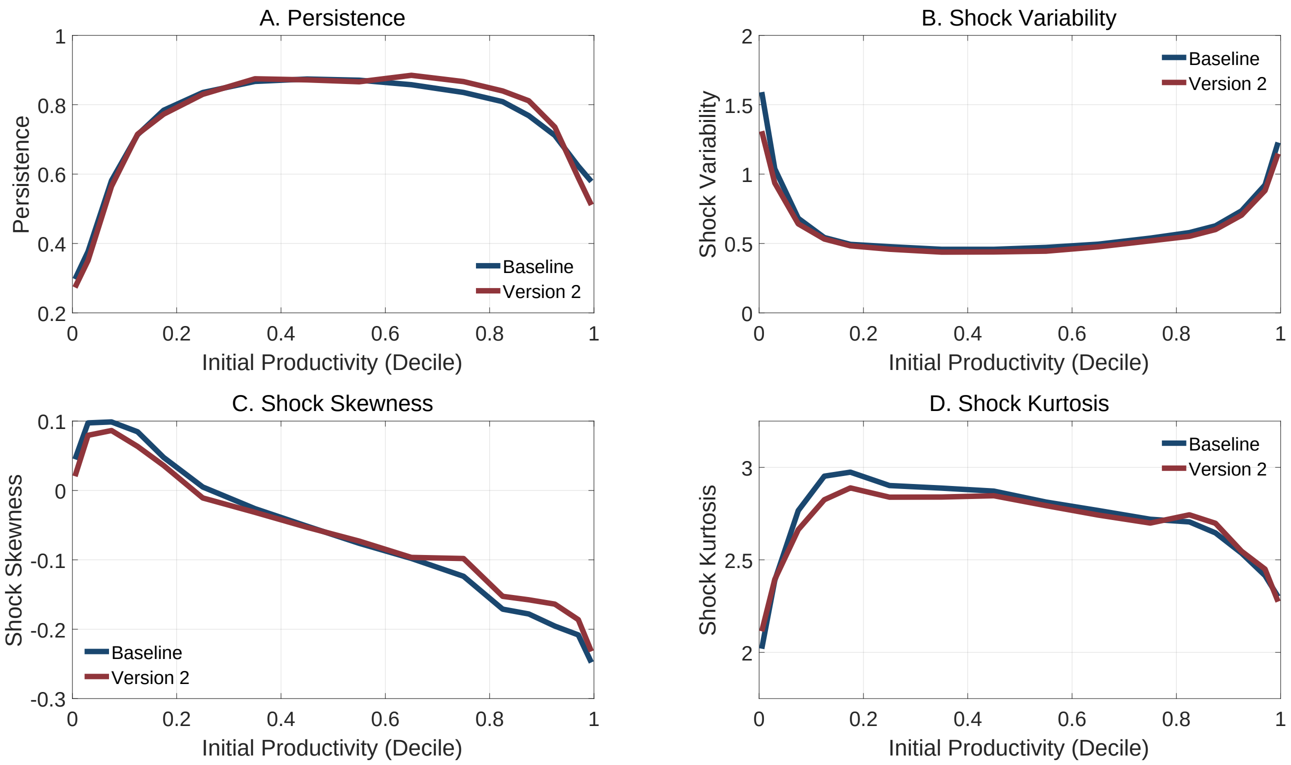


Figure B15: Characteristics of the Productivity Process - Different DRS

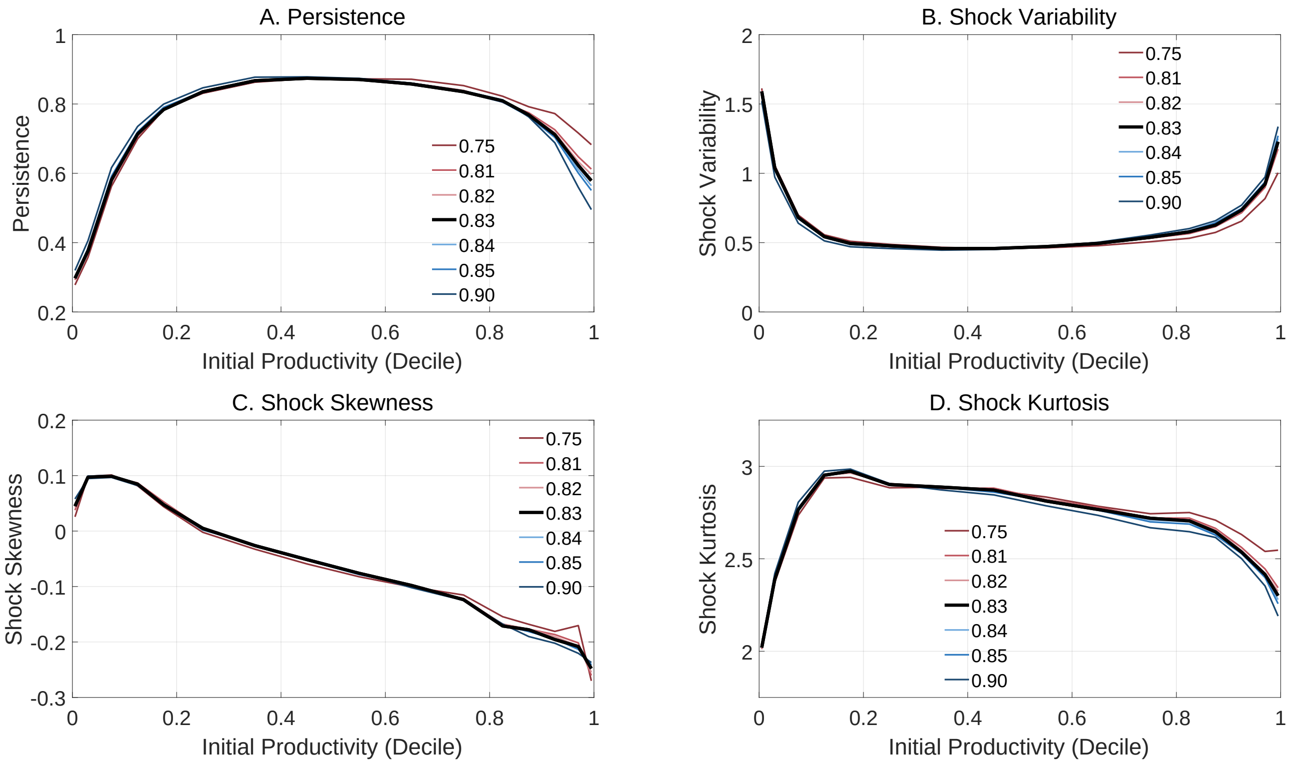
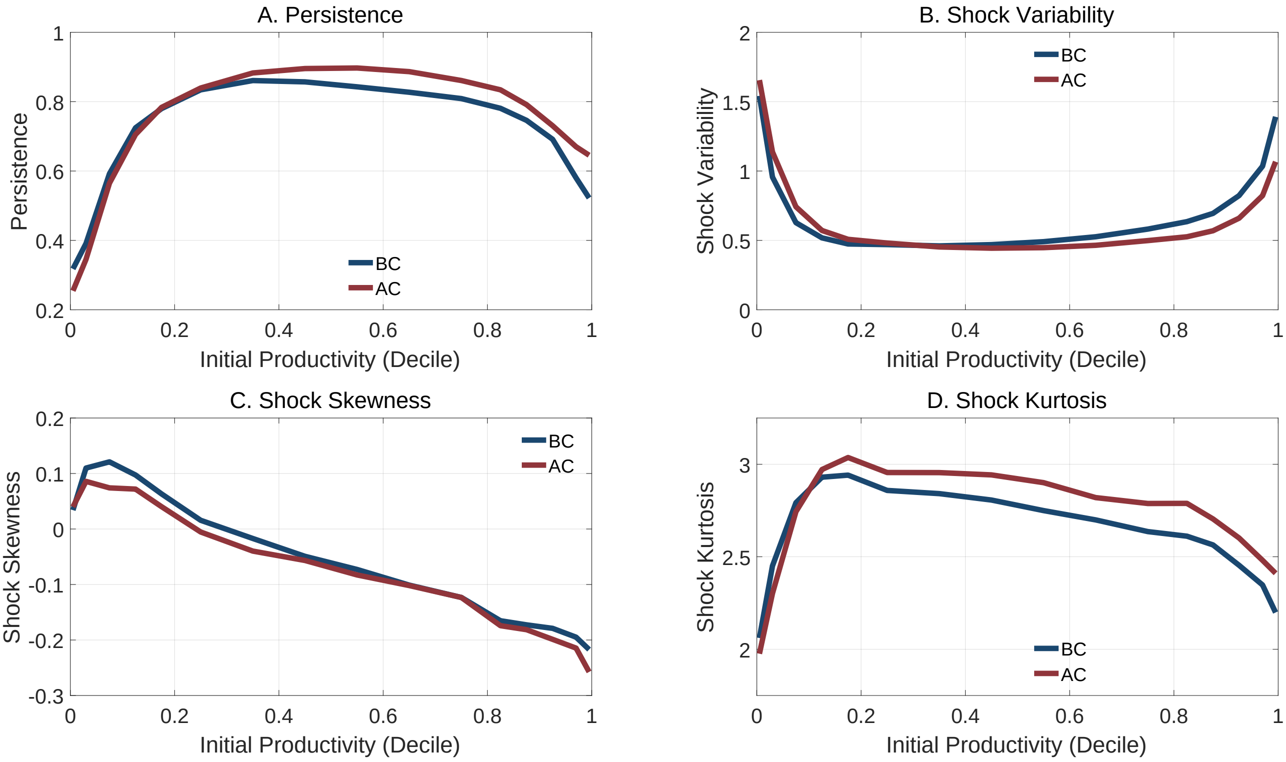


Figure B16: Characteristics of the Productivity Process - Different Periods



B.4 Misallocation

In this section, I provide the robustness checks on the results on misallocation conditional on firm characteristics.

B.4.1 Trend in Misallocation

During the studied period, there is an increasing trend in misallocation in Spain, see, e.g. Gopinath et al. (2017). In order to show that the results on misallocation shown in the main paper are not due to the increase in misallocation, I standardise the ARPK at the sector-year level. Therefore, the time series of variance of log ARPK does not have any trend on time. The results are shown in figure B9. The standardised profiles are labelled version 2. As we can see, the results are similar in the two versions. Of course, the standard deviation of log ARPK is lower in version 2 due to the standardisation procedure.

B.4.2 Studied Period Heterogeneity

The period from 1999 to 2014 holds the Great Recession of 2007 in the middle. In order to evaluate the consistency of the misallocation facts across time and especially in the period of recession and recovery, I split the sample into two sub-periods. The first period, before the Great Recession, goes from 1999 to 2007, while the second period goes from 2007 to 2014. The results are in figure B10. As we can see, the misallocation profiles have been pretty stable during the whole period.

Figure B17: Profiles of PY/K - Specification

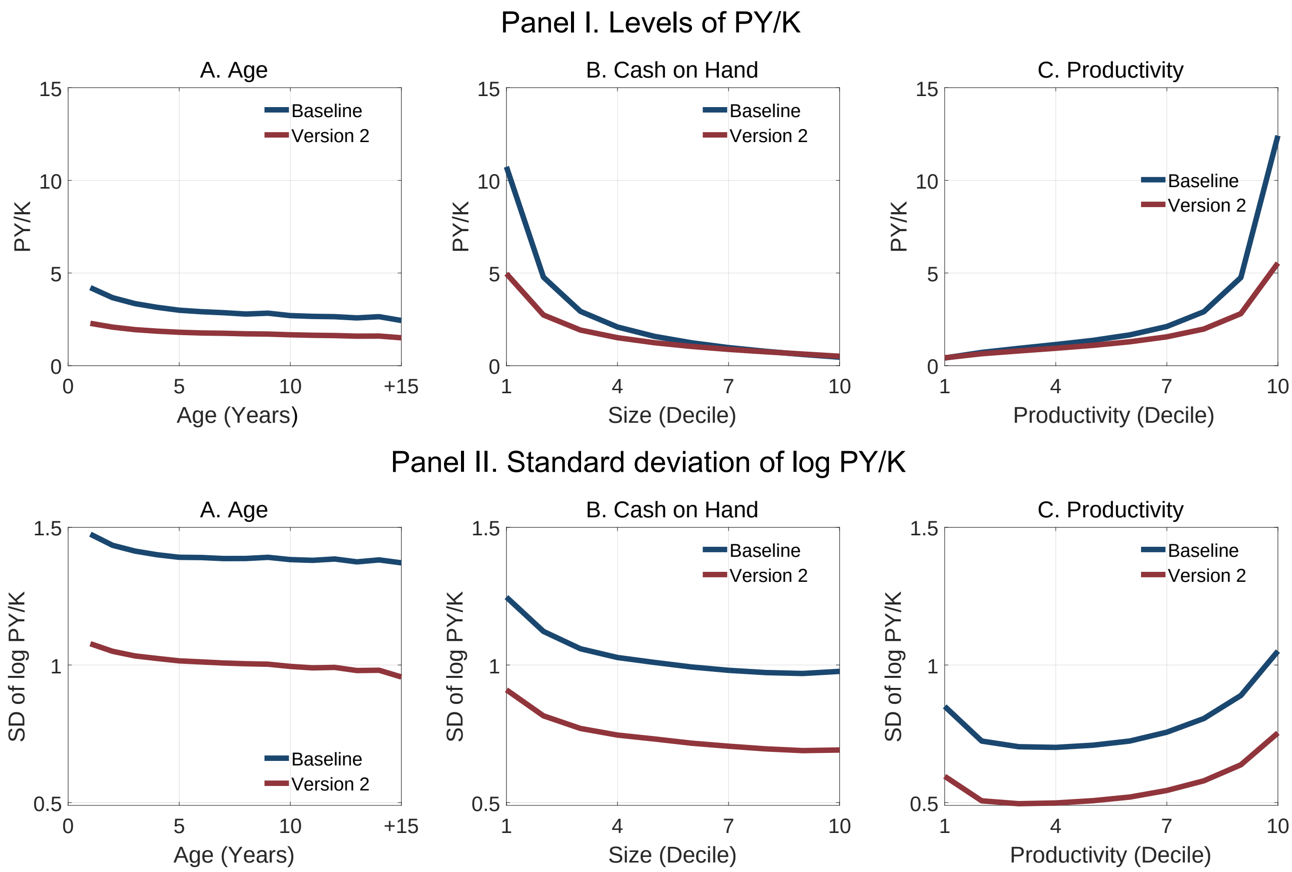
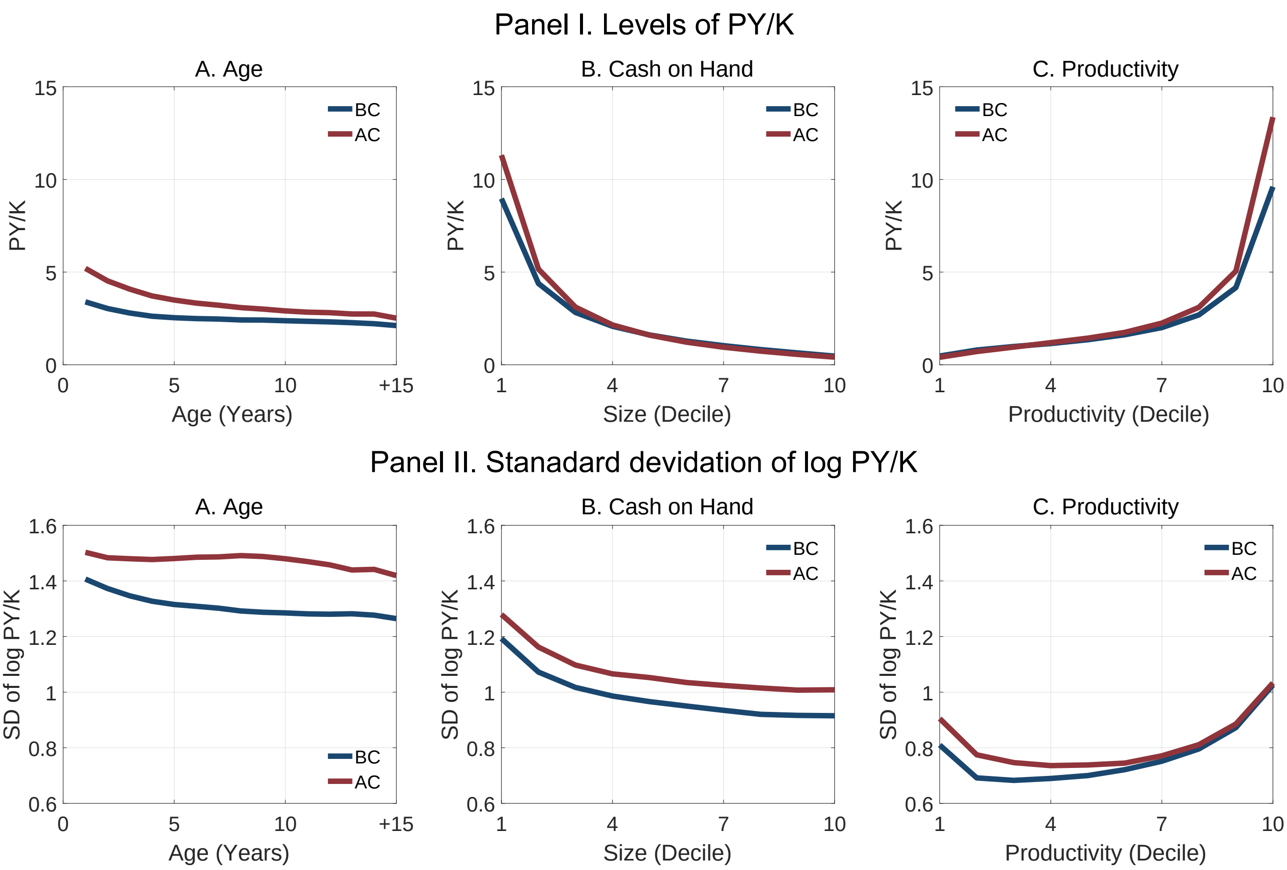


Figure B18: Profiles of PY/K - Periods



B.5 Financial Behavior

In this section, I provide the robustness checks on the financial behaviour of Spanish firms.

B.5.1 Controlling by Firm Profitability

The corporate finance literature has looked at the financial behaviour of firms with particular attention to the relation between leverage and profitability, measured as profits over total assets. This literature usually finds a negative relation, known as a leverage-profitability puzzle, see, e.g. Graham and Leary (2011). The focus of this paper is on firm productivity, finding the negative relation with size as well. Next, I show if the negative relation of leverage and productivity survives once I control with firm profitability. Figures B11 and B12 show the results of this specification, version 2. As we can see, the profiles are very similar in the baseline and version 2.

B.5.2 labor Productivity Dinlersoz et al. (2018)

To be completed.

B.5.3 Studied Period Heterogeneity

The Great Recession of 2007 is in the middle of the studied period. In order to evaluate the consistency of financial behaviour across time and particularly in the period of recession and recovery, I split the sample into two sub-periods. This is important as the main characteristic of the Great Recession is that it affected disproportionally the financial sector; therefore, the level of credit in the economy. The period before the Great Recession goes from 1999 to 2007, while the second period goes from 2007 to 2014. The results are in figures B13 and B14. As we can see, the financial behaviour has been pretty stable during the whole period. If anything, it seems that the Great Recession affected the credit of small and medium-sized firms, which are less leverage.

Figure B19: Financial Behavior - Specification

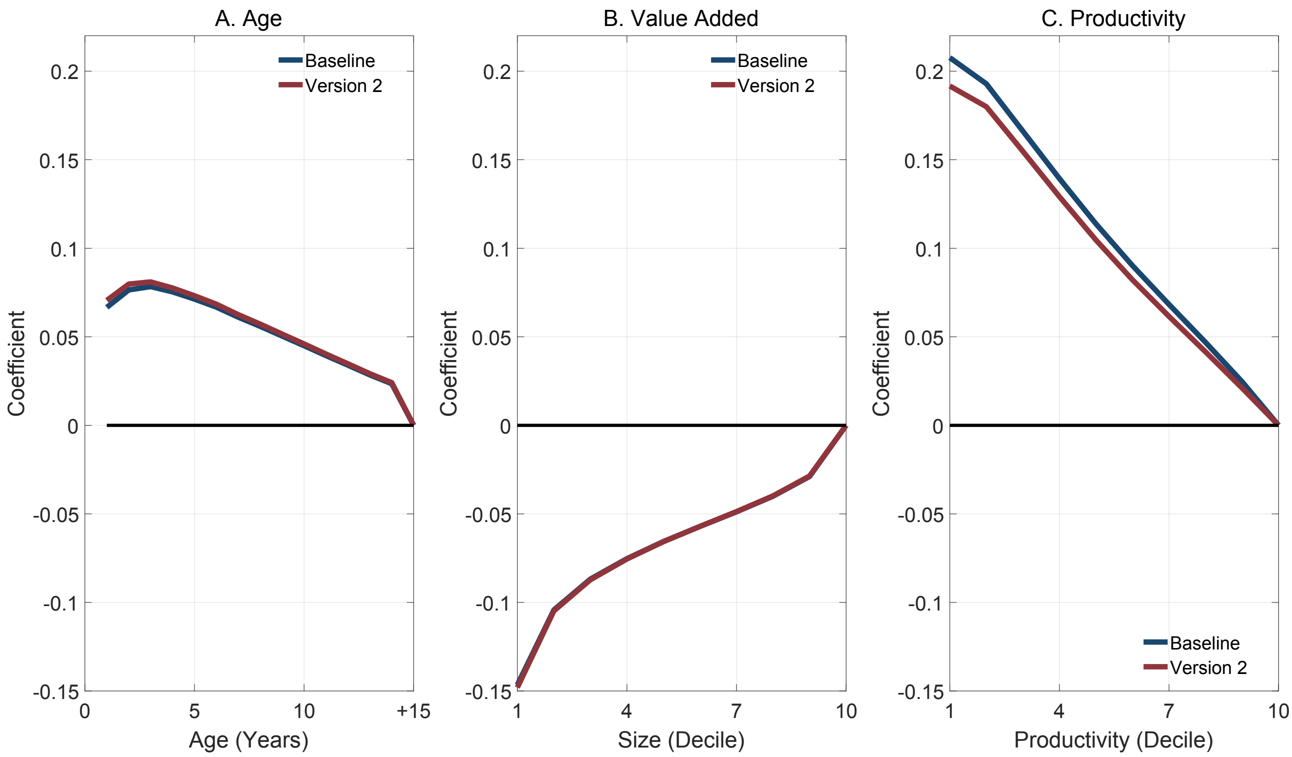


Figure B20: Financial Behavior - Extensive and Intensive Margin - Specification

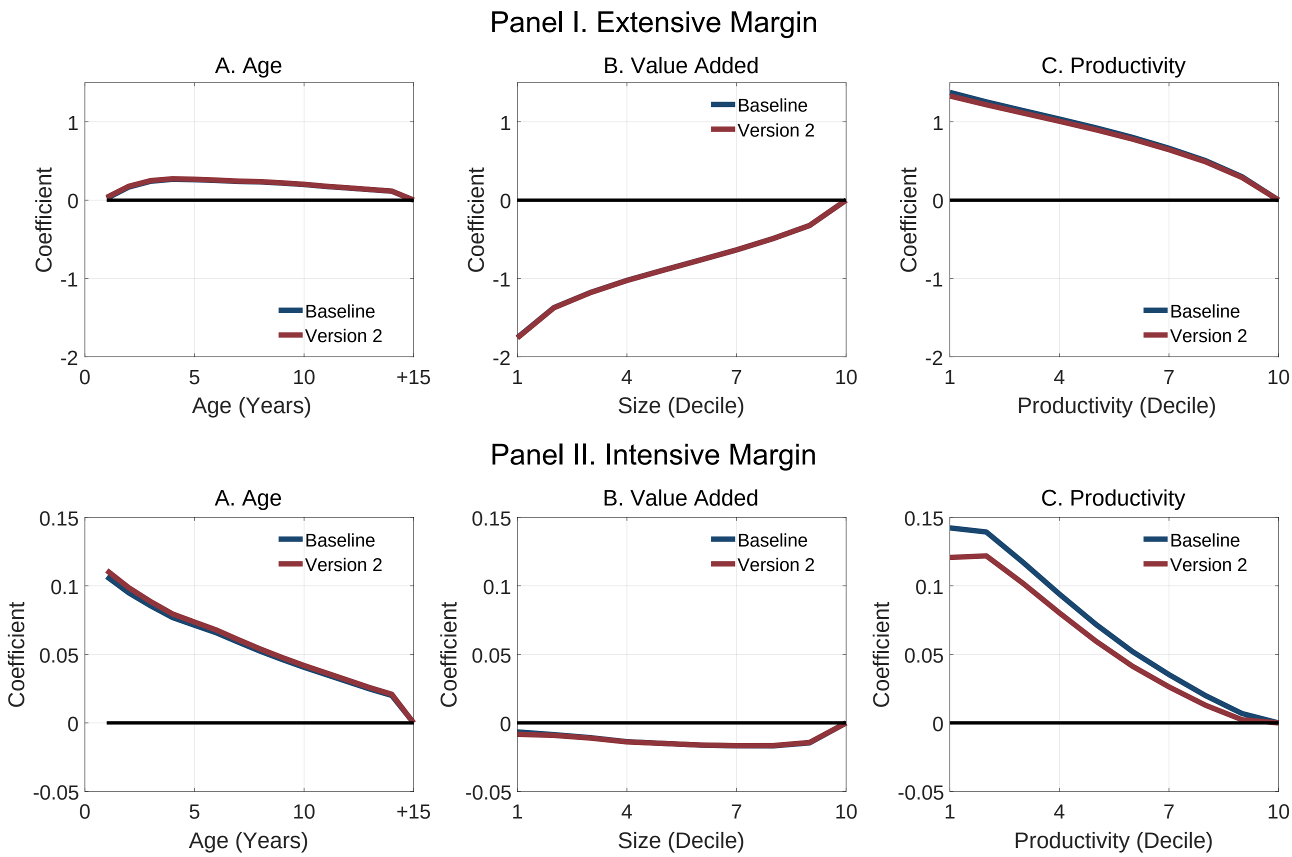


Figure B21: Financial Behavior - Periods

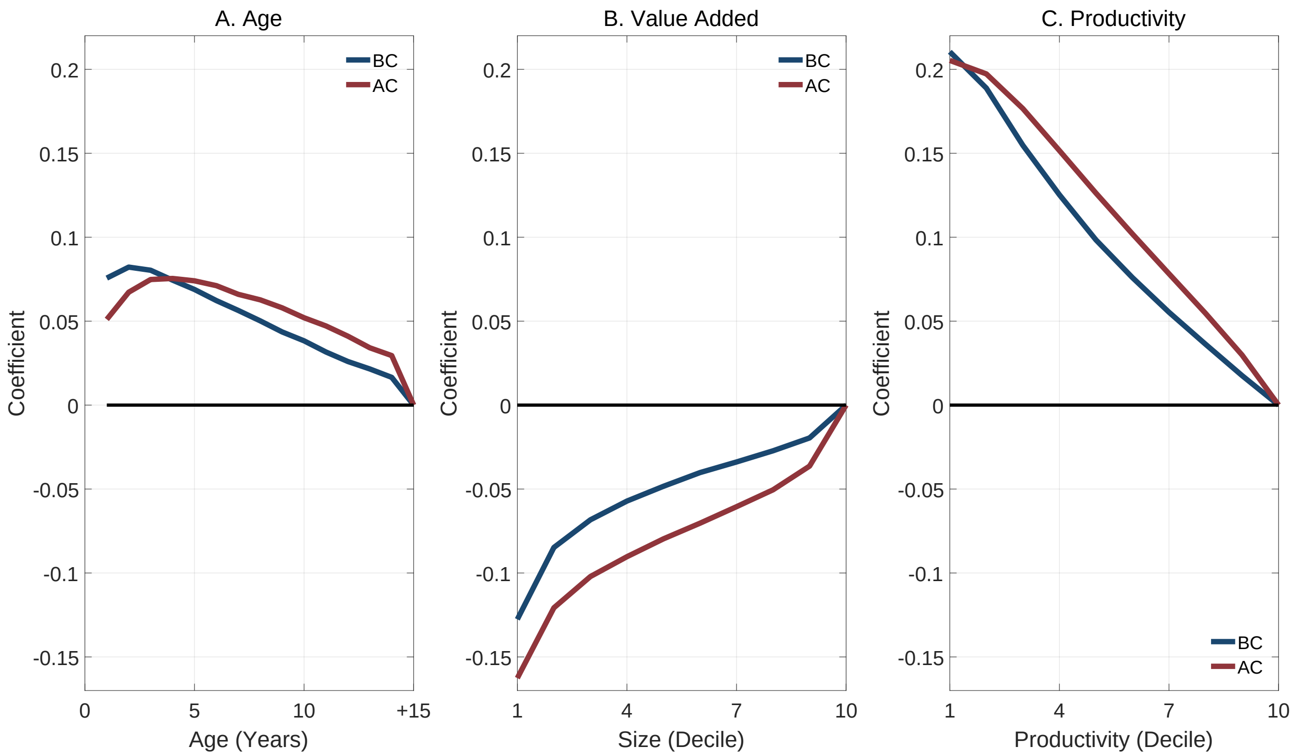
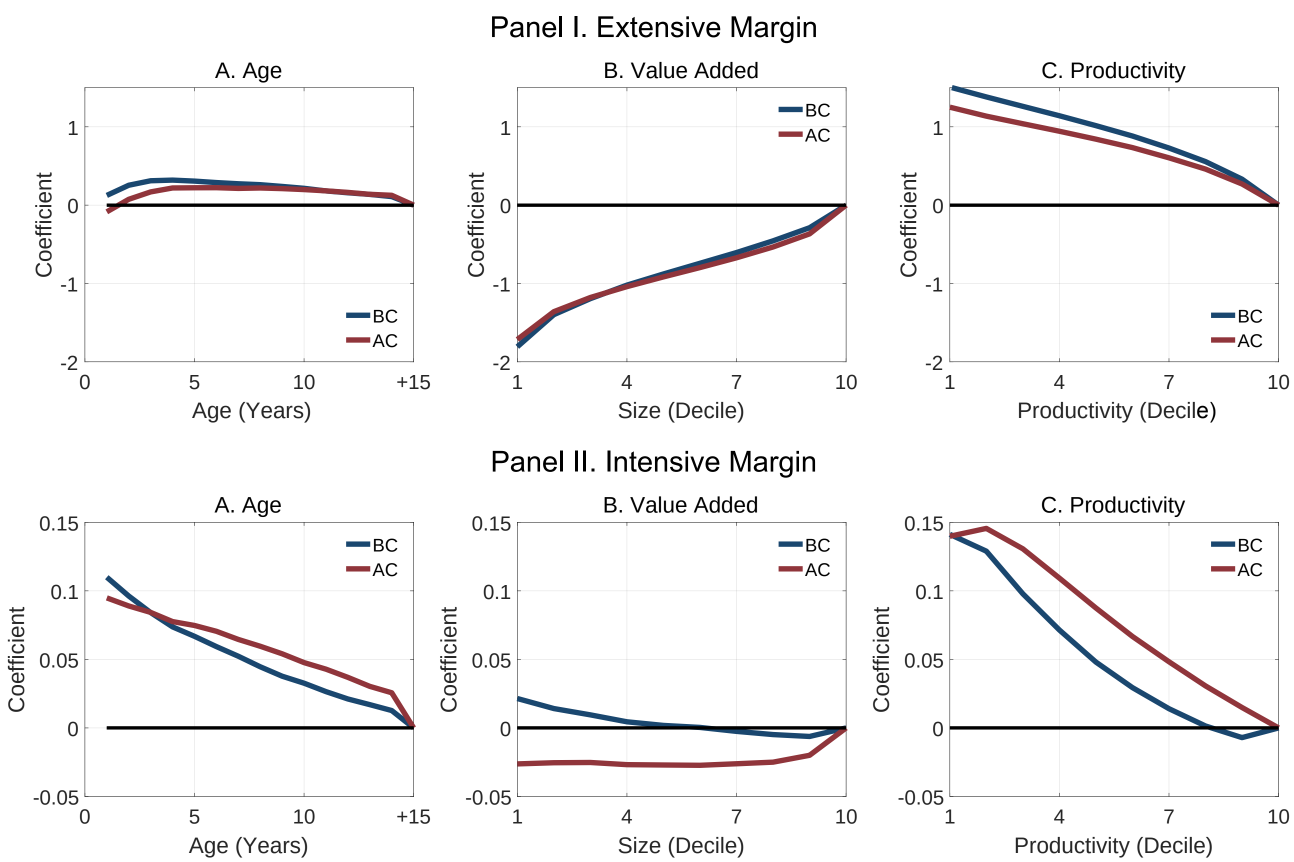


Figure B22: Financial Behavior - Extensive and Intensive Margin - Periods



C Model

In this section, I provide further details on the model, the solution method and results from the model.

C.1 Solution Algorithm

I follow the algorithm developed in Khan and Thomas (2013). First, I solve the model without financial frictions to obtain the optimal unconstrained policy function of capital $k'_u(A)$. Next, I solve the optimal policy function for borrowing, defined as the maximum borrowing (or minimum saving if it is positive) that allows the firm to implement the optimal policy function for borrowing and capital regardless of the productivity shock. This borrowing (or saving) level guarantees that the firm will not be constrained in the future, i.e. current and future multipliers of the borrowing constraint are zero. Next, I characterize the type of firms depending on their state. First, I find the states that allow the firm to achieve the optimal policy functions (capital and borrowing). These are unconstrained firms. Second, I find the states that allow the firm to achieve the optimal capital function but not the borrowing function (the non-equity issuance constraint is binding). These are constrained type-I firms. Then, I find the capital policy function of capital and borrowing of firms that cannot implement the optimal capital (borrowing and non-equity issuance constraints are binding). These are constrained type-II firms. Finally, I find the optimal dividend policy function. Note that it will be only positive for the unconstrained firms.

I simulate a sample of 10,000 firms over 100 periods and take the last two periods to evaluate the performance. The simulation converges in the main variables after the 100 periods, as appears in figure C1.

C.2 Figures from the Model

In this section, I provide the figures that summarize the solution of the model. Figure C2 shows the optimal unconstrained policy function for capital. As we can see, the NL productivity process produces a non-linear policy function. The higher level of capital for each level of productivity in the NL productivity process arises from the higher value of η in the calibration.

Figure C3 shows the 3 types of firms depending on their financial health with respect to firm productivity and cash-on-hand.

Finally, figure C4 shows how financial frictions translates into lower firm value compared to the unconstrained case.

Figure C1: Convergence of the Model

Figure C2: Optimal Unconstrained Policy Functions for Capital

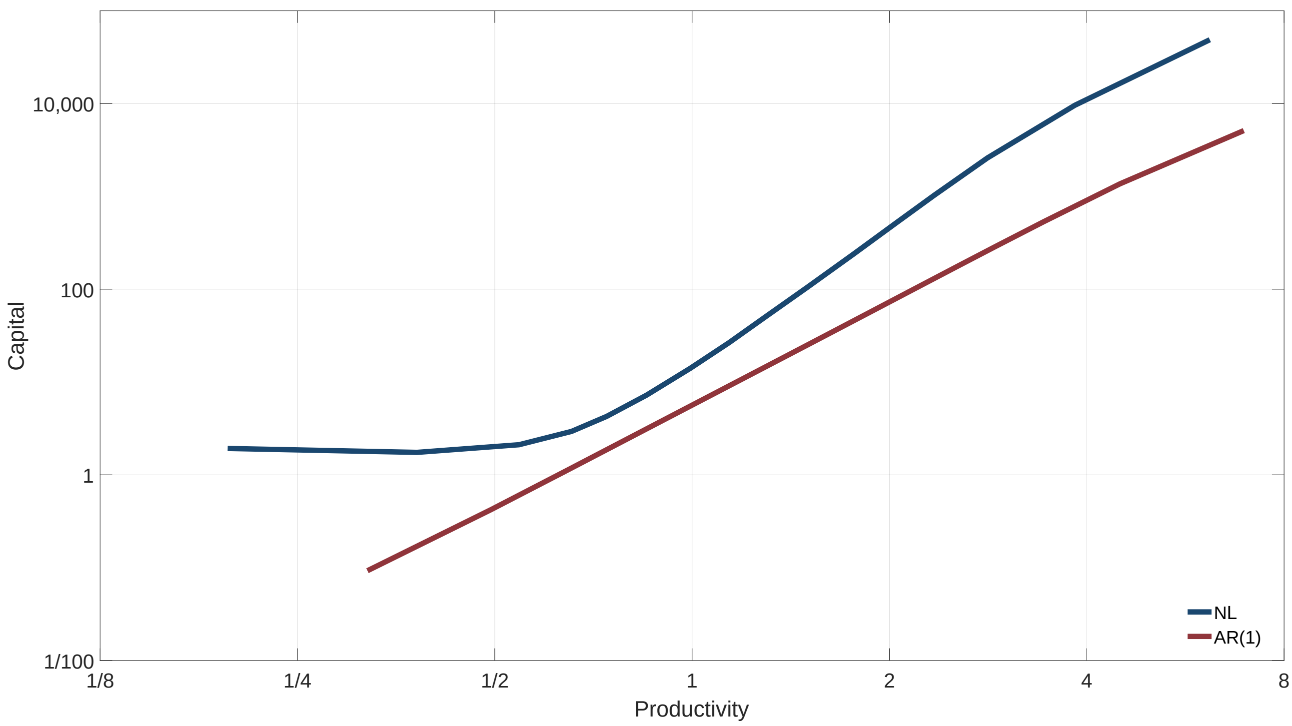


Figure C3: Firm Type by Cash-on-Hand and Productivity Levels

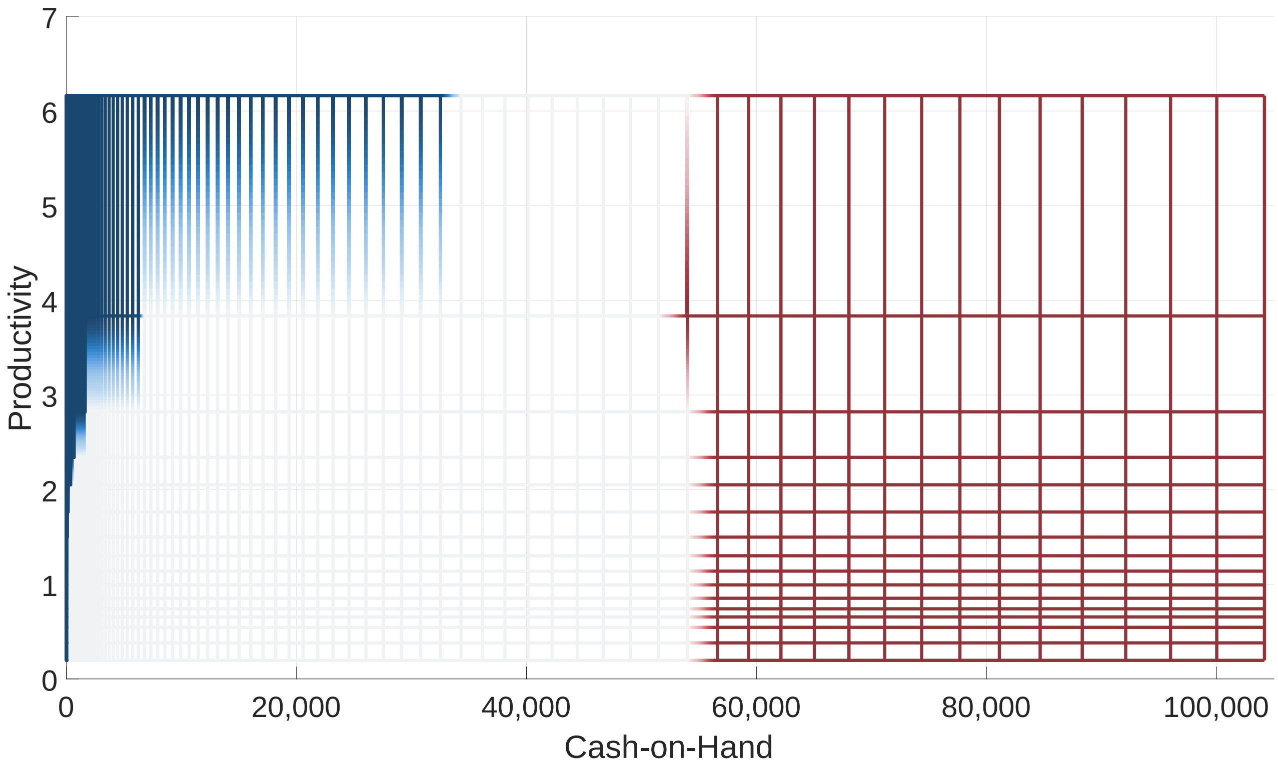
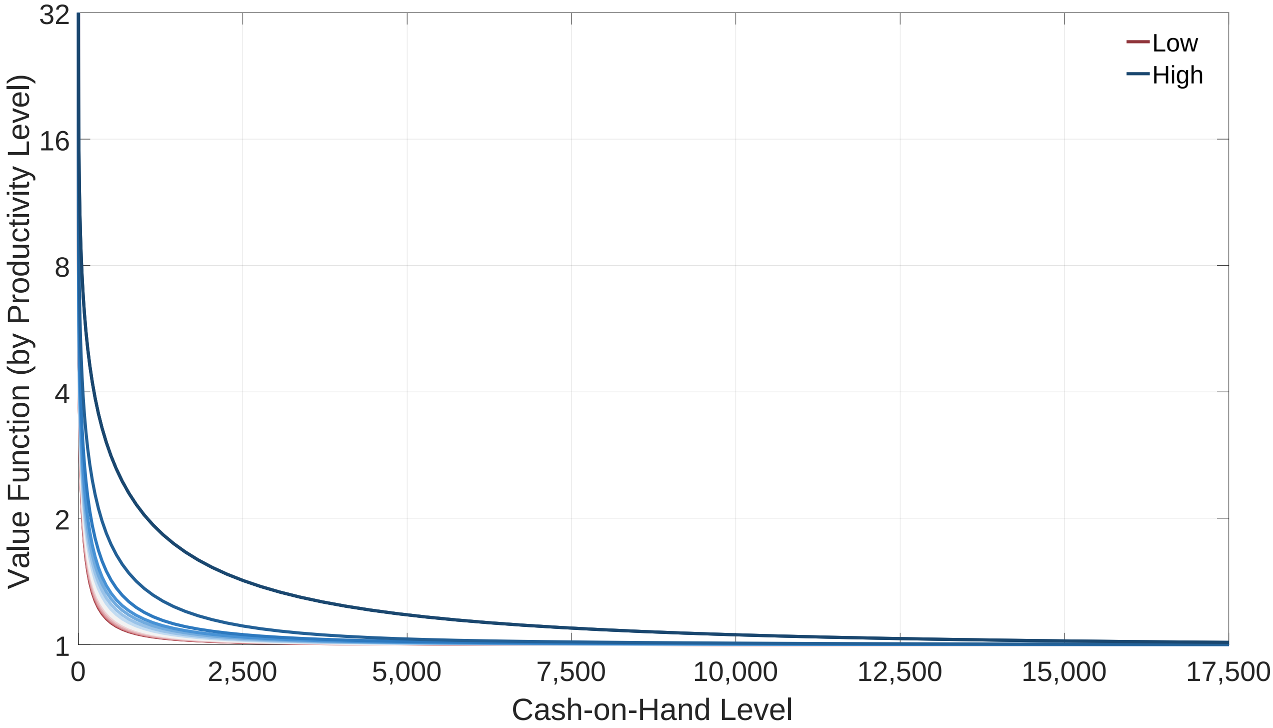


Figure C4: Value Function (NFF/FF) by Cash-on-Hand and Productivity Levels



D Results

This section provides the results when the productivity process follows the standard AR(1) used in the literature. Finally, I review other forms of borrowing constraints used in the literature.

D.1 AR(1)

This section shows the results of the model when productivity dynamics follow an AR(1) process. Table D1 shows the calibration of the parameters.

D.1.1 Firm Life Cycle

Figures D1 and D2 show the firm life cycle in terms of entry and exit and firm ageing.

D.1.2 Misallocation

Figure D3 shows the results on misallocation across firm characteristics.

D.1.3 Financial Behavior

Figures D4 and D5 show the results on financial behaviour.

Table D1: Moments of the calibration - Size Dependent Borrowing Constraint

Parameter	Value	Moment	Data	Model
η	0.83	$SD(k)$	1.79	1.76
β	0.97	K/Y	2.0	2.2
α	0.35	K/L	4.0	4.1
δ	0.05	Inv/Y	0.12	0.13
A_{shift}	1.22	L	15.5	15.5
θ	0.81	$Leverage$	0.19	0.19
Ψ	0.50	$P_{95}^{Leverage}$	0.71	0.71
τ	0.43	$Profits/Y$	0.15	0.15
μ_e	1.95	k_{ent}	0.36	0.36
σ_e	1.92	$SD(k_{ent})$	0.95	0.95
$\rho_{a,e}$	0.02	$\rho(a_{ent};e_{ent})$	0.05	0.05

Figure D1: Firm Life Cycle - Entry and Exit

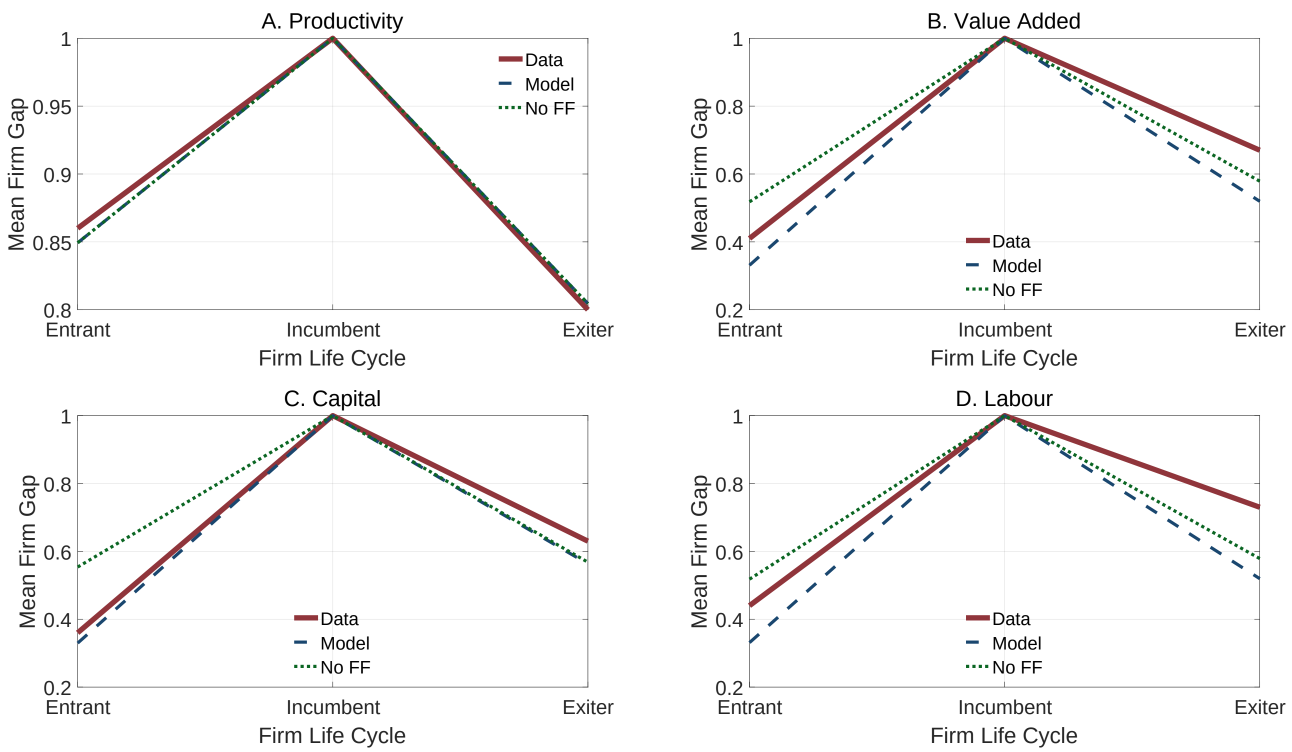


Figure D2: Firm Life Cycle - Firm Ageing

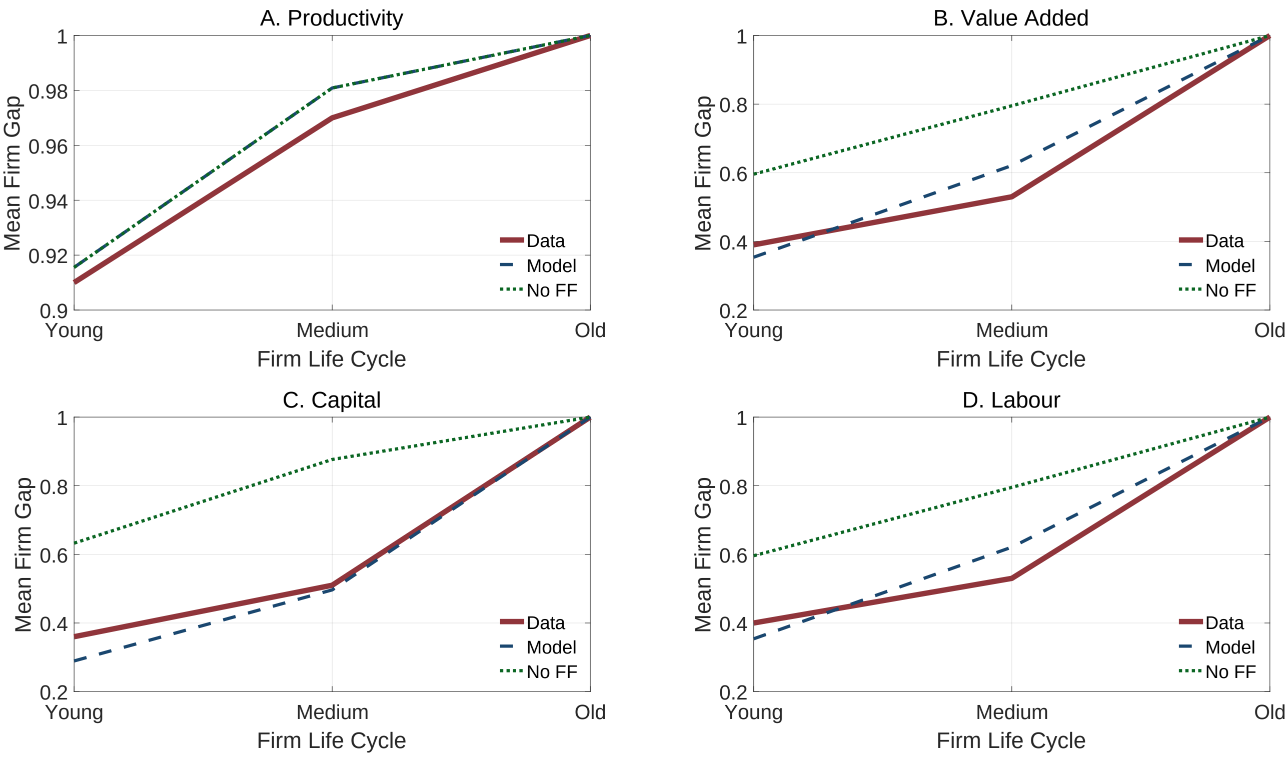


Figure D3: Profiles of PY/K

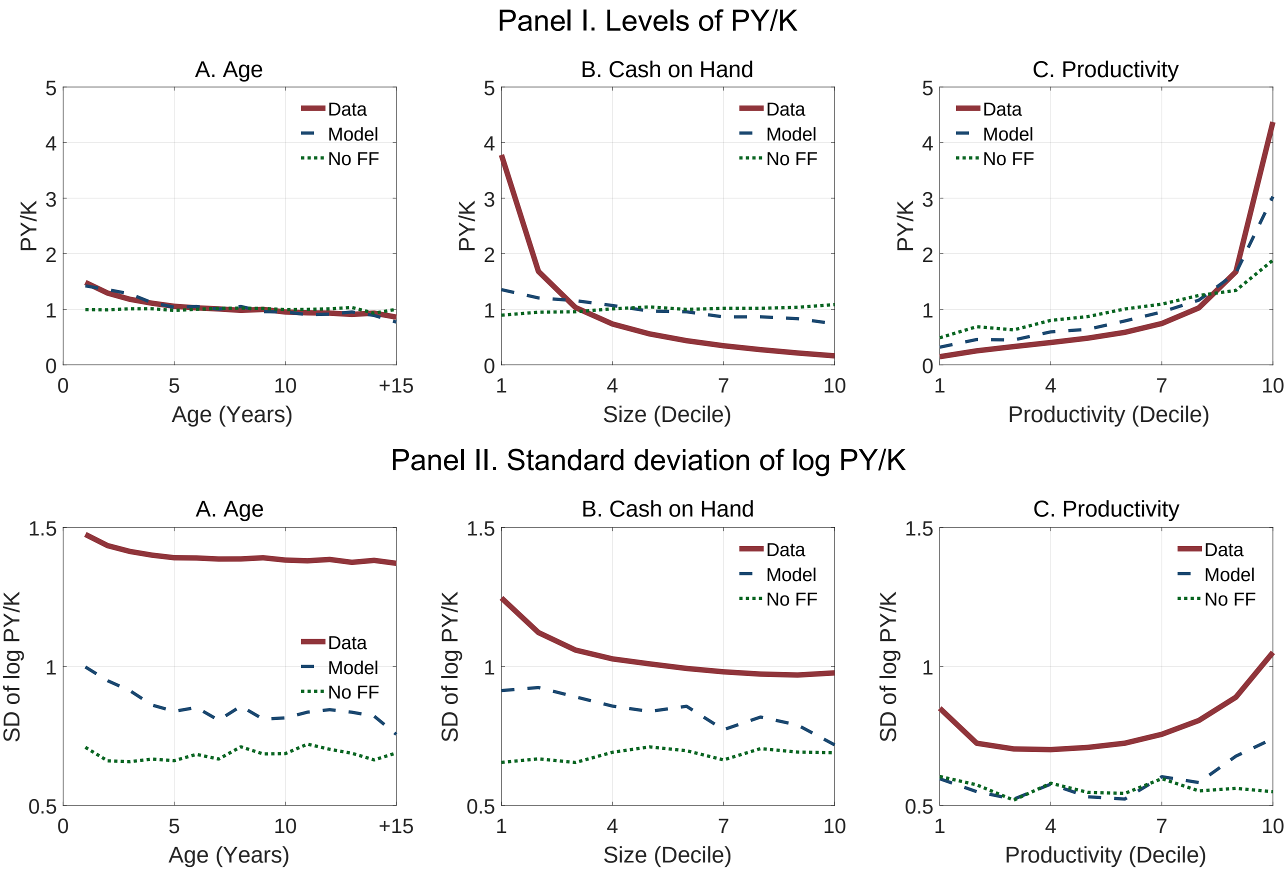


Figure D4: Financial Behavior

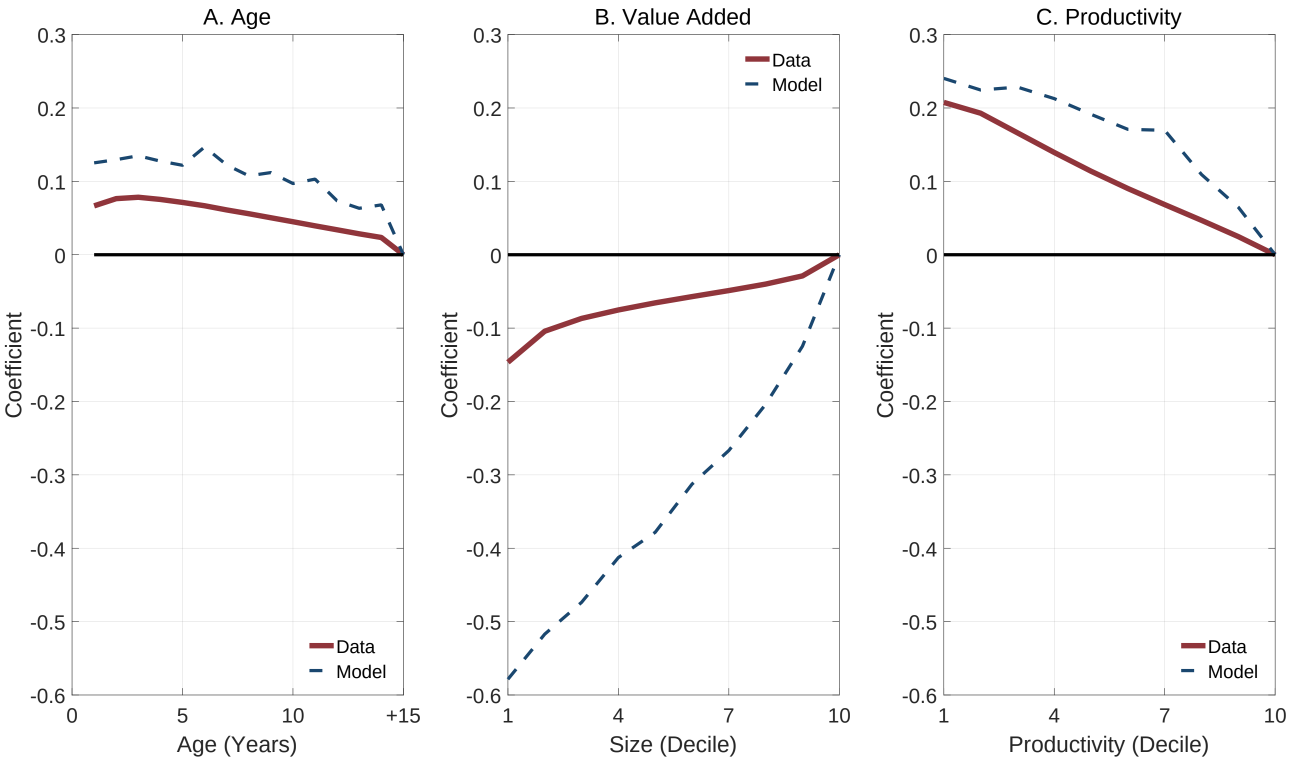
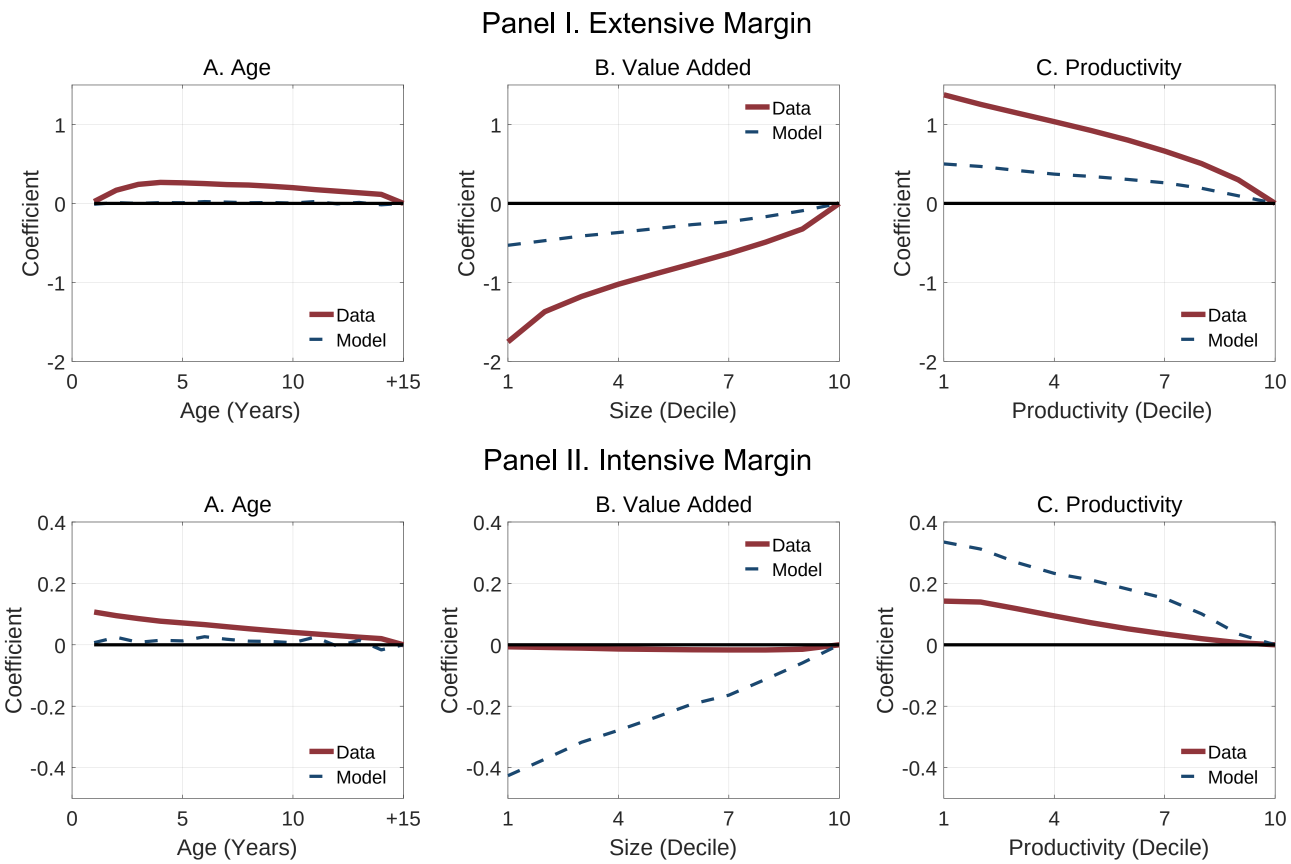


Figure D5: Financial Behavior - Extensive and Intensive Margin



D.2 Other Functional Forms of Borrowing Constraint

In this section, I explore other forms of borrowing constraint broadly used in the literature.

D.2.1 Standard Borrowing Constraint: Target Average Leverage

I start by analysing the most commonly used in the literature, where the pledgeability parameter, θ, is calibrated to match the average leverage.

Table D2: Moments of the calibration - Standard Borrowing Constraint Target Leverage

Moment	Data	N-L	AR(1)	Target
l	15.5	15.48	15.49	A
$SD(k)$	1.79	1.774	1.770	η
K/Y	2.0	2.04	2.06	β
K/L	4.0	3.79	4.06	α
Inv/Y	0.12	0.119	0.124	δ
$Leverage$	0.19	0.190	0.190	θ
$Profits/Y$	0.15	0.150	0.150	ϕ
k_{ent}	0.36	0.360	0.360	μ_e
$SD(k_{ent})$	0.95	0.950	0.950	σ_e
$\rho(a_{ent}; e_{ent})$	0.05	0.050	0.050	$\rho_{a,e}$

Table D3: Calibration Standard Borrowing Constraint Target Leverage

Parameter	N-L	AR(1)
A_{shift}	1.222	1.49
η	0.83	0.78
β	0.97	0.95
α	0.35	0.35
δ	0.04	0.04
θ	0.319	0.443
ϕ	0.503	0.471
μ_e	1.82	2.44
σ_e	1.89	1.77
$\rho_{a,e}$	0.023	0.031

Figure D6: Firm Life Cycle - Entry and Exit

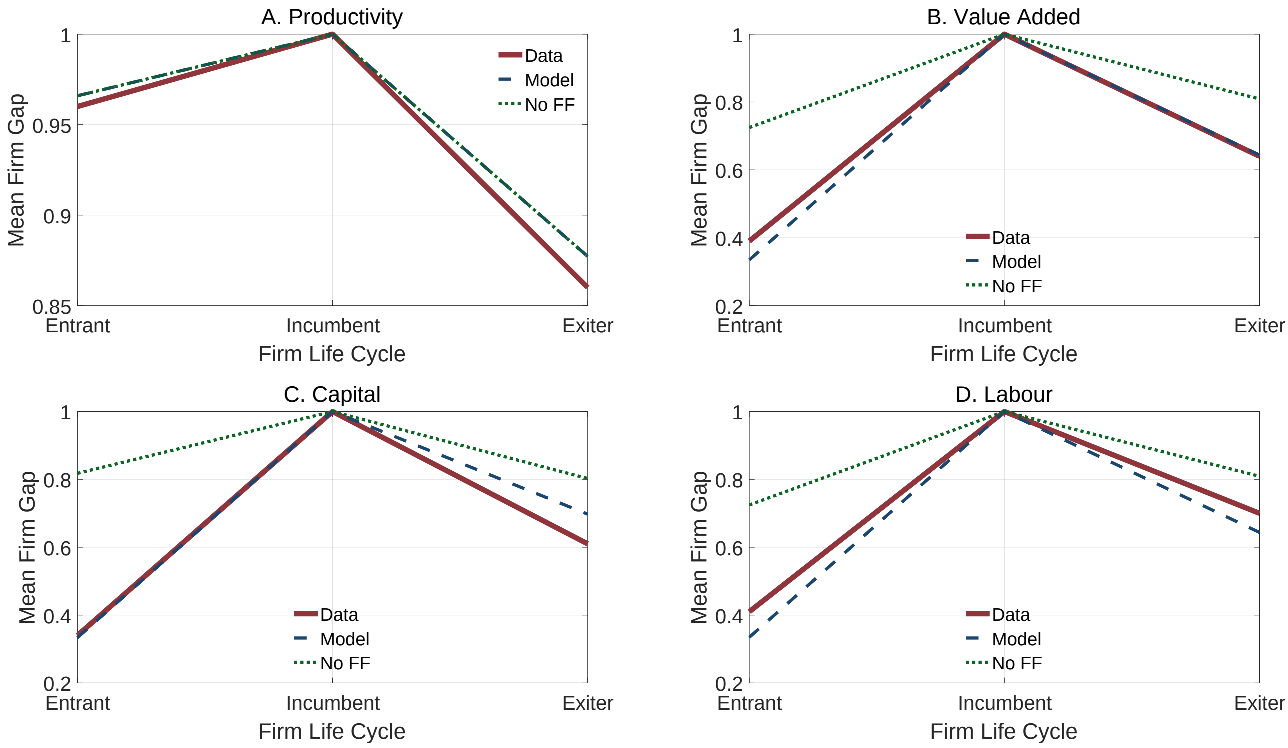


Table D4: Aggregate Consequences - Standard Borrowing Constraint Target Leverage

	N-L	AR(1)
No Constrained Firms	0.0001	0.0064
Constrained Type I Firms	0.4351	0.6298
Constrained Type II Firms	0.5648	0.3638
SD(log MRPK)	1.0809	0.8165
SD(log MRPK) No FF	0.8474	0.6838
Productivity Loss	0.3059	0.1700
Productivity Loss FF	0.1515	0.0626

Figure D7: Firm Life Cycle - Firm Ageing

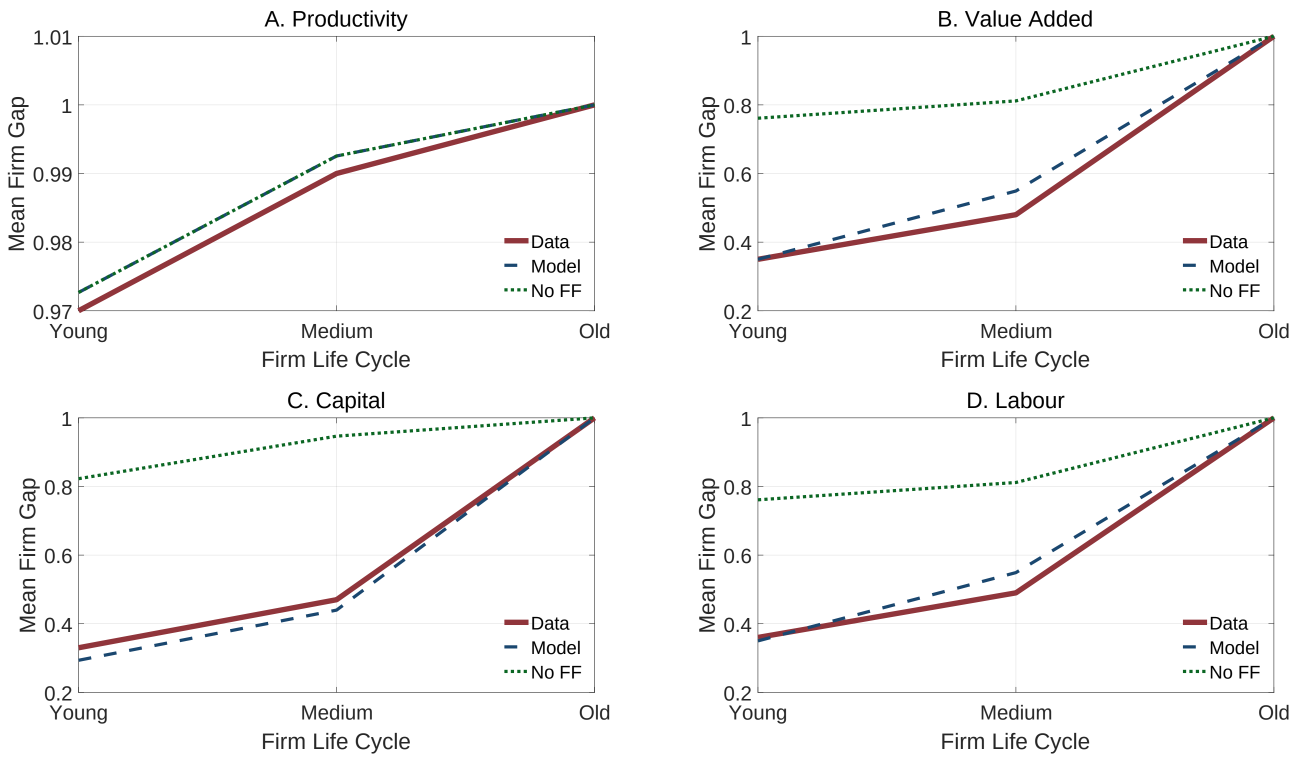


Figure D8: Profiles of PY/K

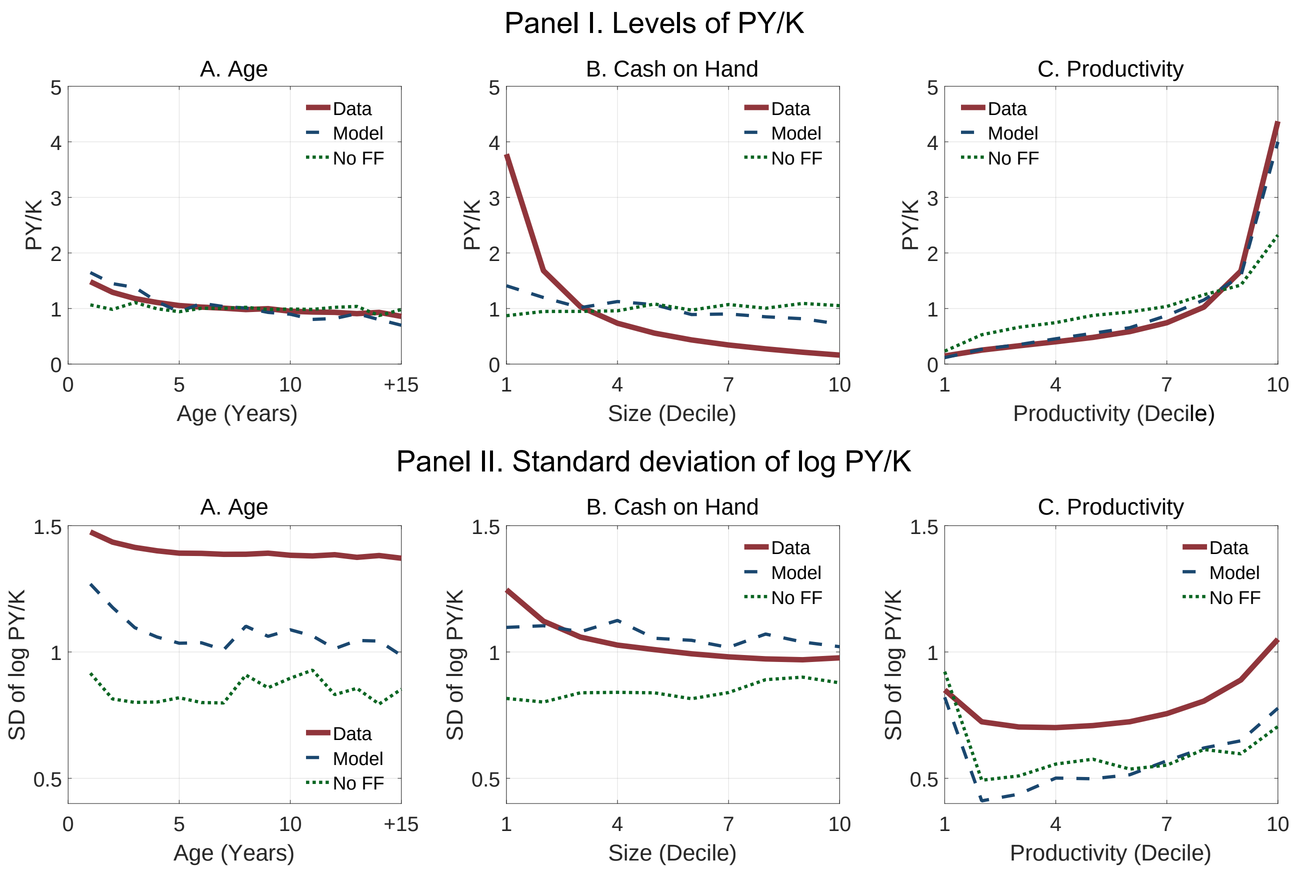


Figure D9: Financial Behavior

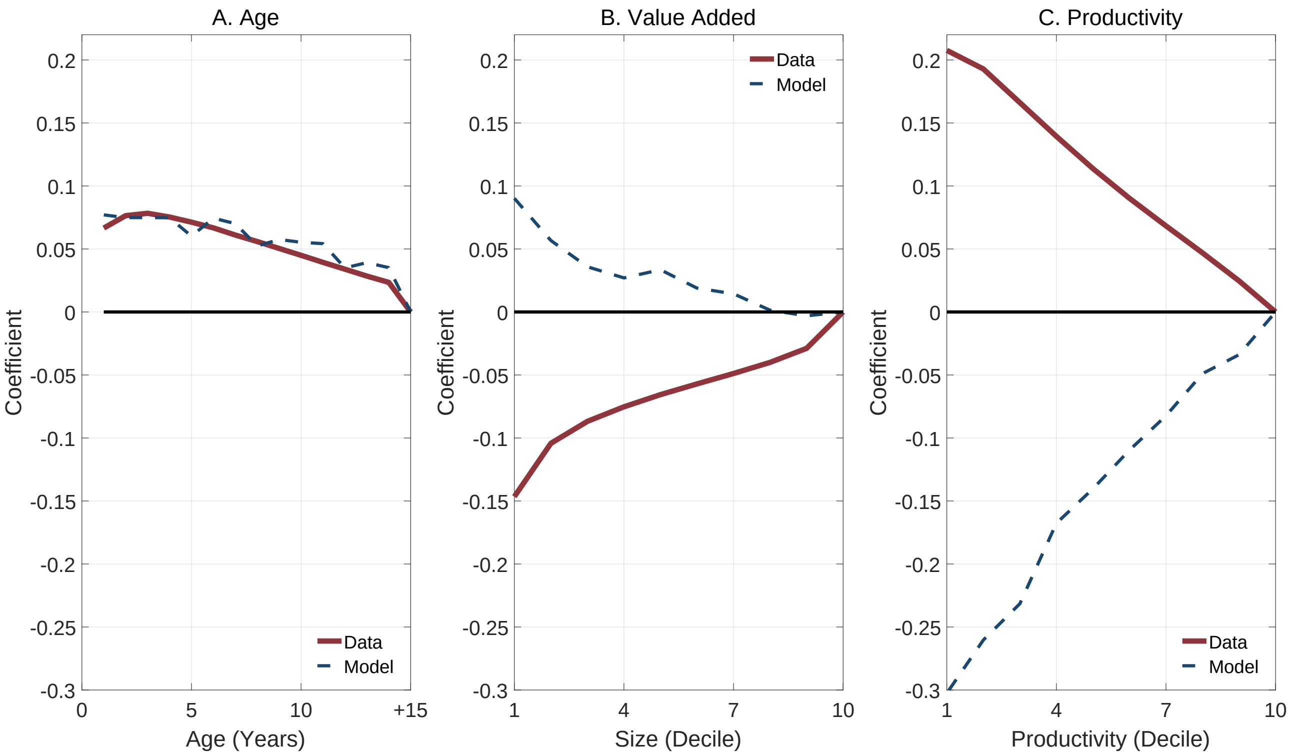
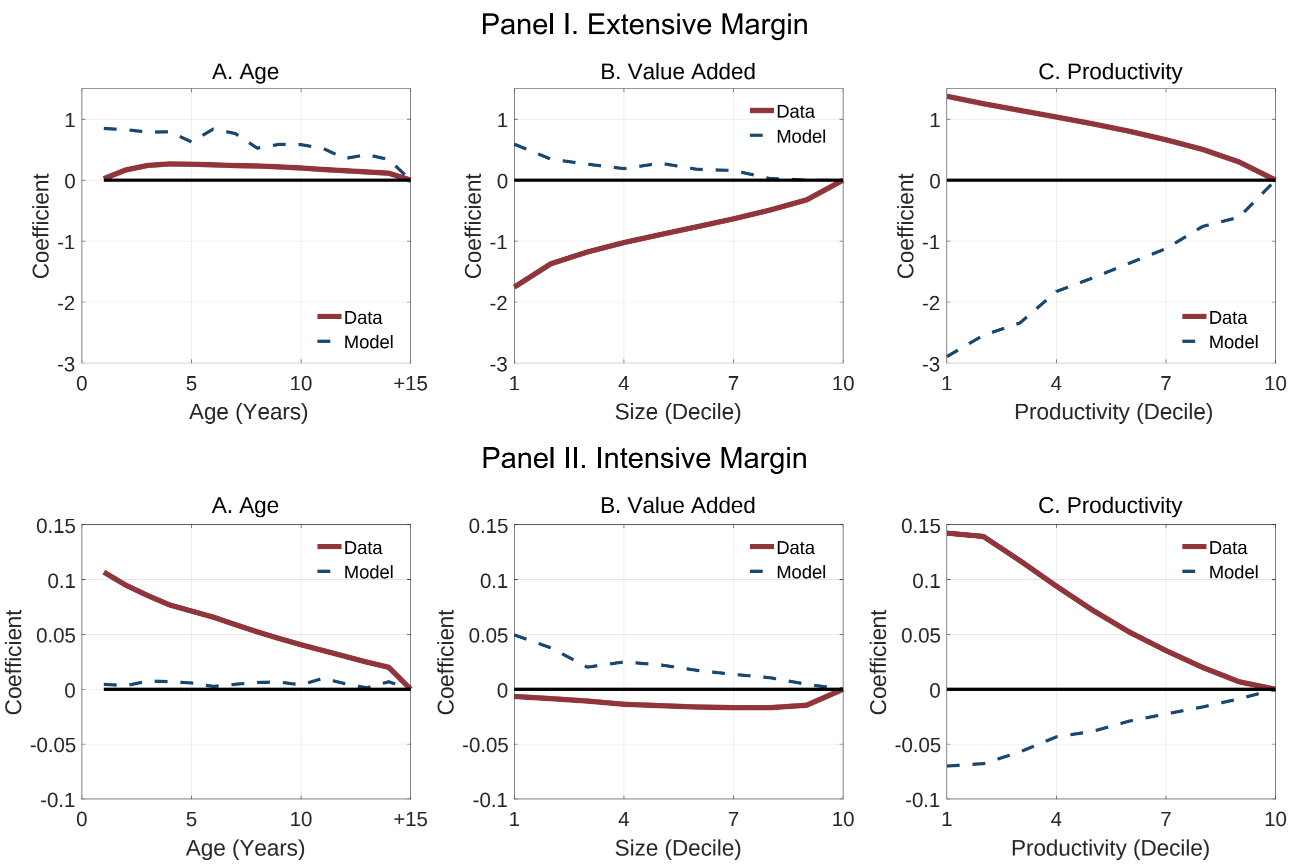


Figure D10: Financial Behavior - Extensive and Intensive Margin



D.2.2 Standard Borrowing Constraint: Target Debt to Output Ratio

Next, I analyse the most commonly used borrowing constraint in the literature, where the pledgeability parameter, θ, is calibrated to match the debt to output ratio in the economy.

Table D5: Moments of the calibration - Standard Borrowing Constraint Target Debt to Output Ratio

Moment	Data	N-L	AR(1)	Target
l	15.5	15.47	15.47	A
$SD(k)$	1.79	1.930	1.811	η
K/Y	2.0	2.08	2.16	β
K/L	4.0	3.85	4.25	α
Inv/Y	0.12	0.109	0.116	δ
$Debt/Y$	0.19	0.811	0.810	θ
$Profits/Y$	0.15	0.150	0.150	ϕ
k_{ent}	0.36	0.361	0.360	μ_e
$SD(k_{ent})$	0.95	0.949	0.950	σ_e
$\rho(a_{ent}; e_{ent})$	0.05	0.050	0.049	$\rho_{a,e}$

Table D6: Calibration Standard Borrowing Constraint Target Debt to Output Ratio

Parameter	N-L	AR(1)
A_{shift}	1.165	1.455
η	0.83	0.78
β	0.97	0.95
α	0.35	0.35
δ	0.05	0.04
θ	0.513	0.572
ϕ	0.551	0.479
μ_e	1.52	2.317
σ_e	2.149	1.823
$\rho_{a,e}$	0.025	0.034

Figure D11: Firm Life Cycle - Entry and Exit

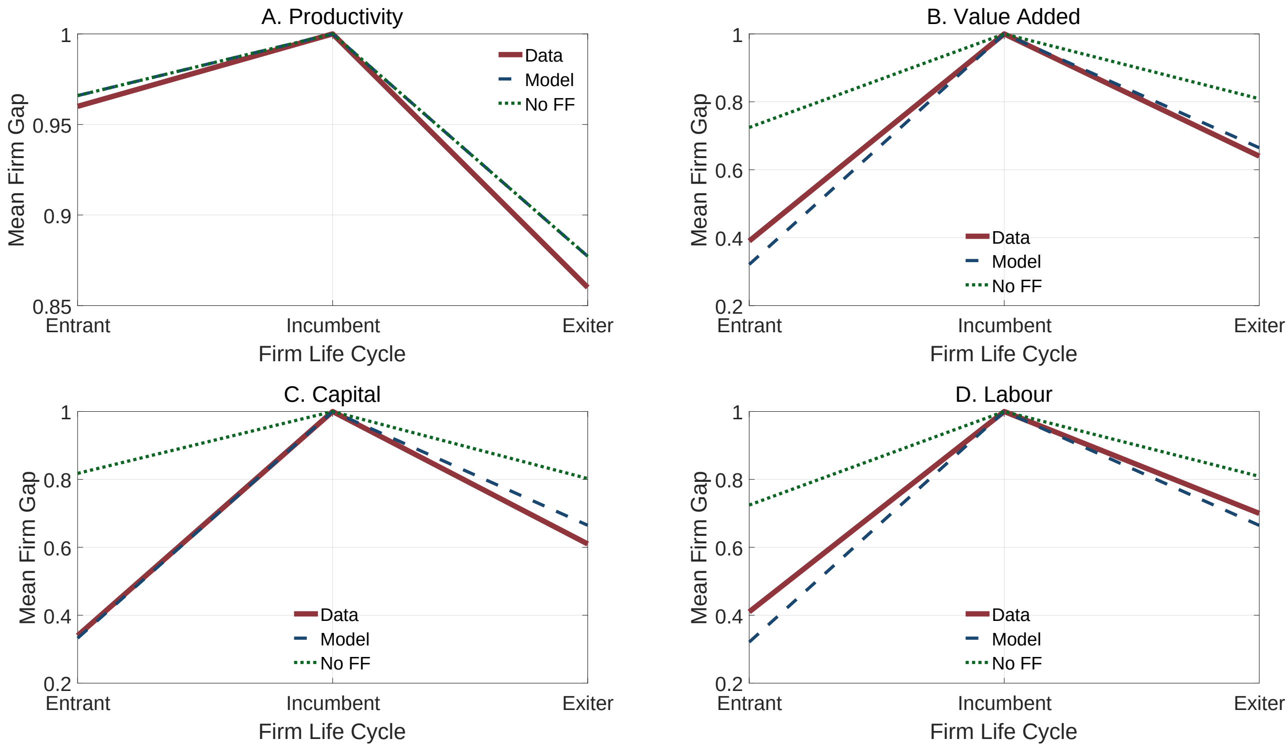


Table D7: Aggregate Consequences - Standard Borrowing Constraint Target Debt to Output Ratio

	N-L	AR(1)
No Constrained	0.0001	0.0070
Constrained Type I	0.5384	0.6566
Constrained Type II	0.4615	0.3364
SD(log MRPK)	1.0254	0.7959
SD(log MRPK) No FF	0.8474	0.6879
Productivity Loss	0.2756	0.1607
Productivity Loss FF	0.1144	0.0522

Figure D12: Firm Life Cycle - Firm Ageing

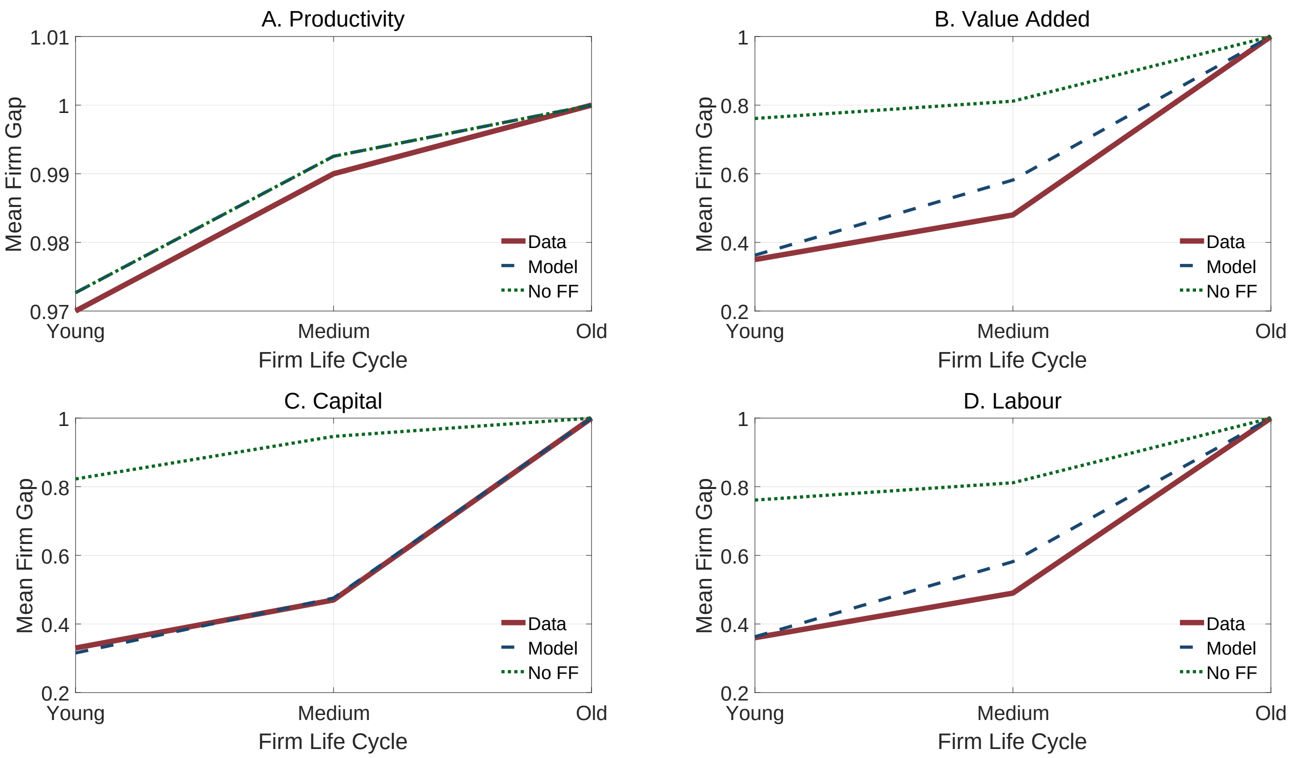


Figure D13: Profiles of PY/K

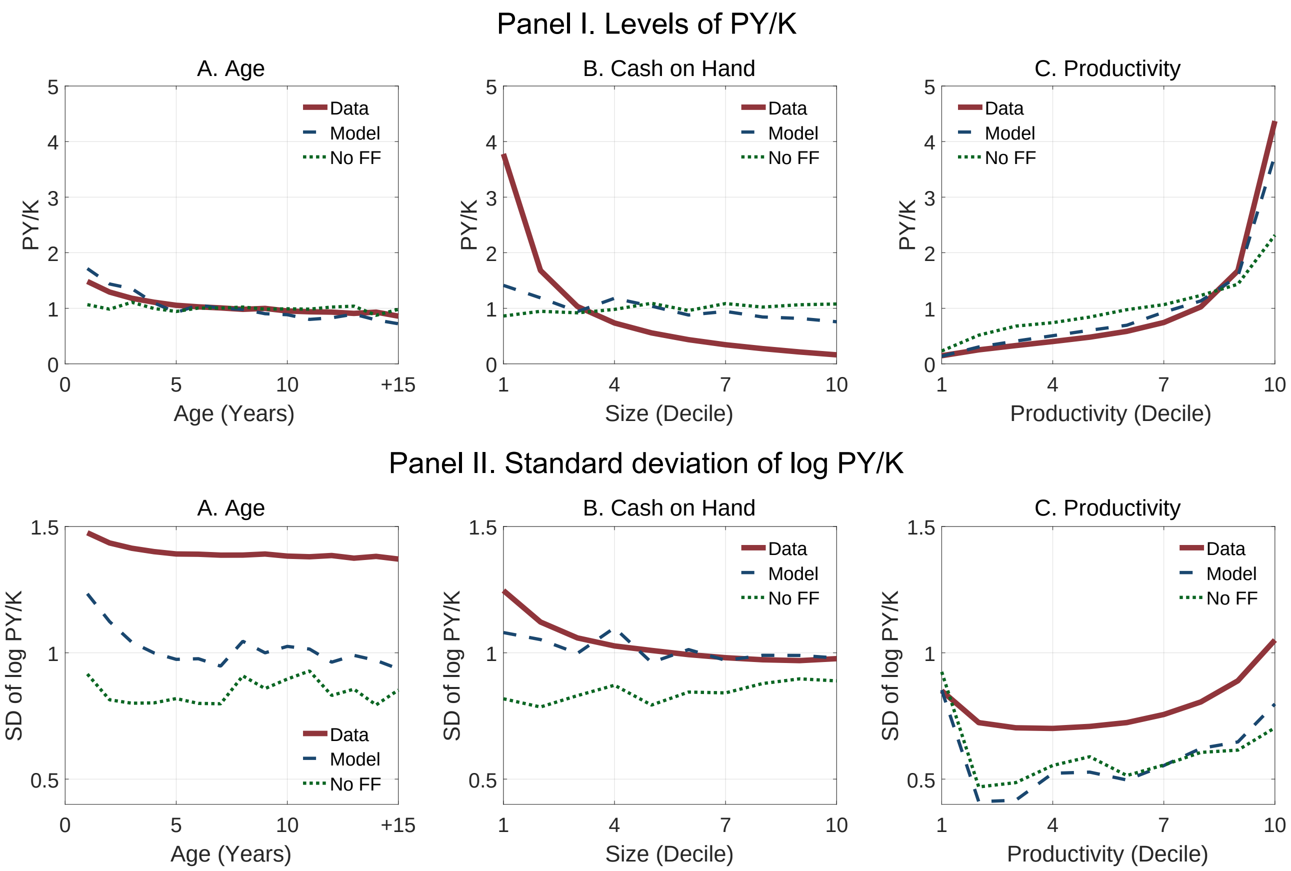


Figure D14: Financial Behavior

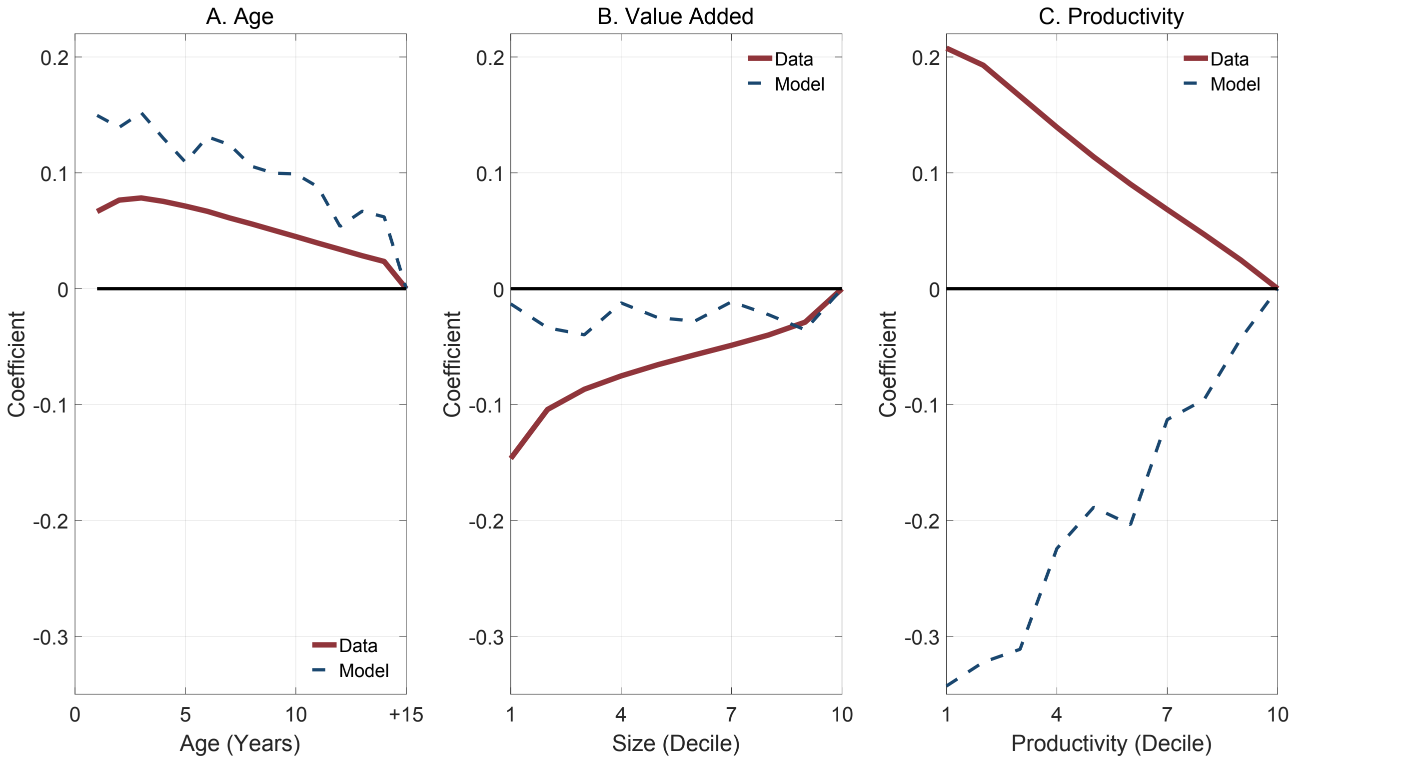
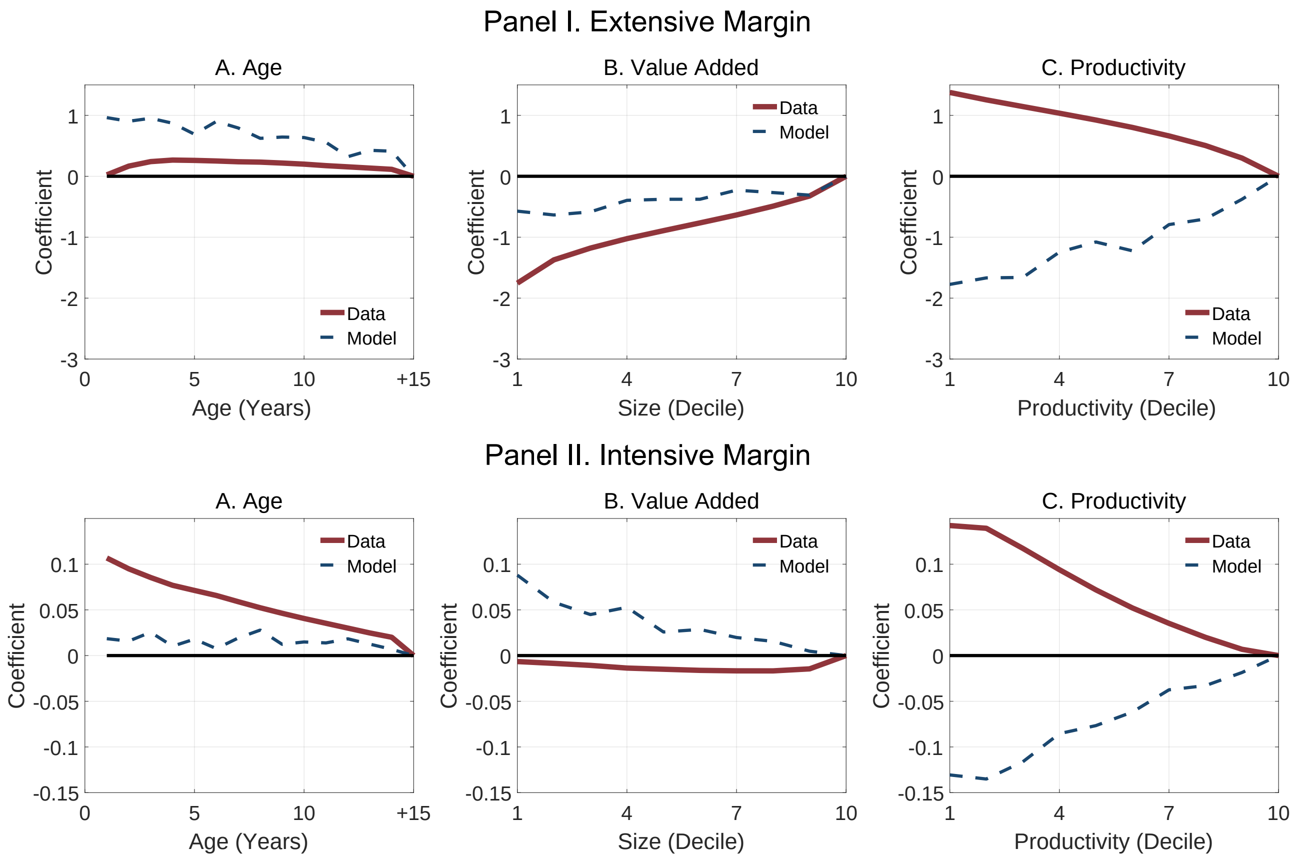


Figure D15: Financial Behavior - Extensive and Intensive Margin



D.2.3 Borrowing Constraint with Profits: Target Average Leverage

Finally, I analyse the case where the borrowing constraint is earnings based instead of collateral-based. This formulation of borrowing constraint is growing up, and it is motivated by the existence of earnings covenants in the debt contracts, as shown in Drechsel (2019).

Table D8: Moments of the calibration - Borrowing Constraint with Expected Profits

Moment	Data	NP	AR(1)	Target
l	15.5	15.47	15.47	A
$SD(k)$	1.79	1.820	1.770	η
K/Y	2.0	1.78	1.93	β
K/L	4.0	3.75	3.80	α
Inv/Y	0.12	0.113	0.119	δ
$Leverage$	0.19	0.190	0.190	θ
$Profits/Y$	0.15	0.150	0.150	ϕ
k_{ent}	0.36	0.360	0.360	μ_e
$SD(k_{ent})$	0.95	0.951	0.939	σ_e
$\rho(a_{ent}; e_{ent})$	0.05	0.050	0.050	$\rho_{a,e}$

Table D9: Calibration Borrowing Constraint with Expected Profits

Parameter	N-L	AR(1)
A_{shift}	1.207	1.501
η	0.83	0.78
β	0.97	0.95
α	0.35	0.35
δ	0.05	0.04
θ	0.953	1.140
ϕ	0.518	0.461
μ_e	1.217	2.034
σ_e	2.300	2.017
$\rho_{a,e}$	0.043	0.059

Figure D16: Firm Life Cycle - Entry and Exit

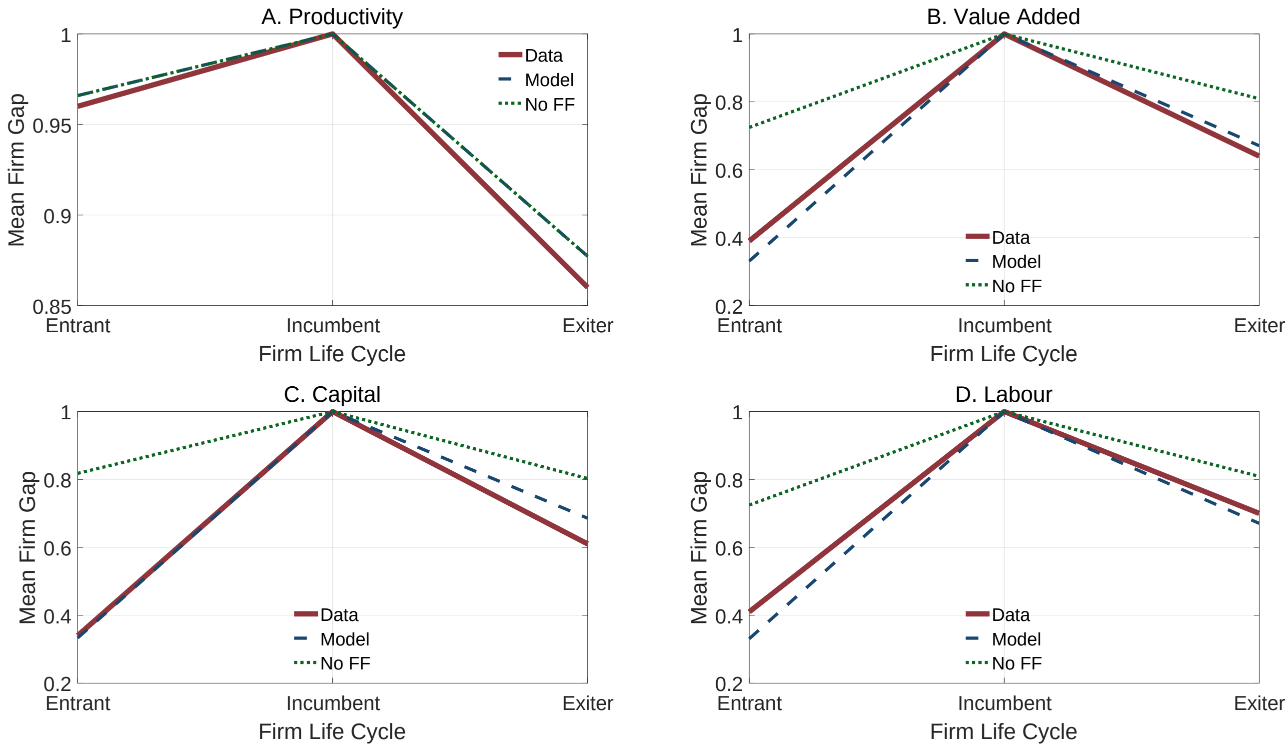


Table D10: Aggregate Consequences - Borrowing Constraint with Expected Profits

	N-L	AR(1)
No Constrained	0.0001	0.0075
Constrained Type I	0.4186	0.5336
Constrained Type II	0.5813	0.4589
SD(log MRPK)	1.0326	0.8024
SD(log MRPK) No FF	0.8474	0.6838
Productivity Loss	0.2684	0.1611
Productivity Loss FF	0.1056	0.0526

Figure D17: Firm Life Cycle - Firm Ageing

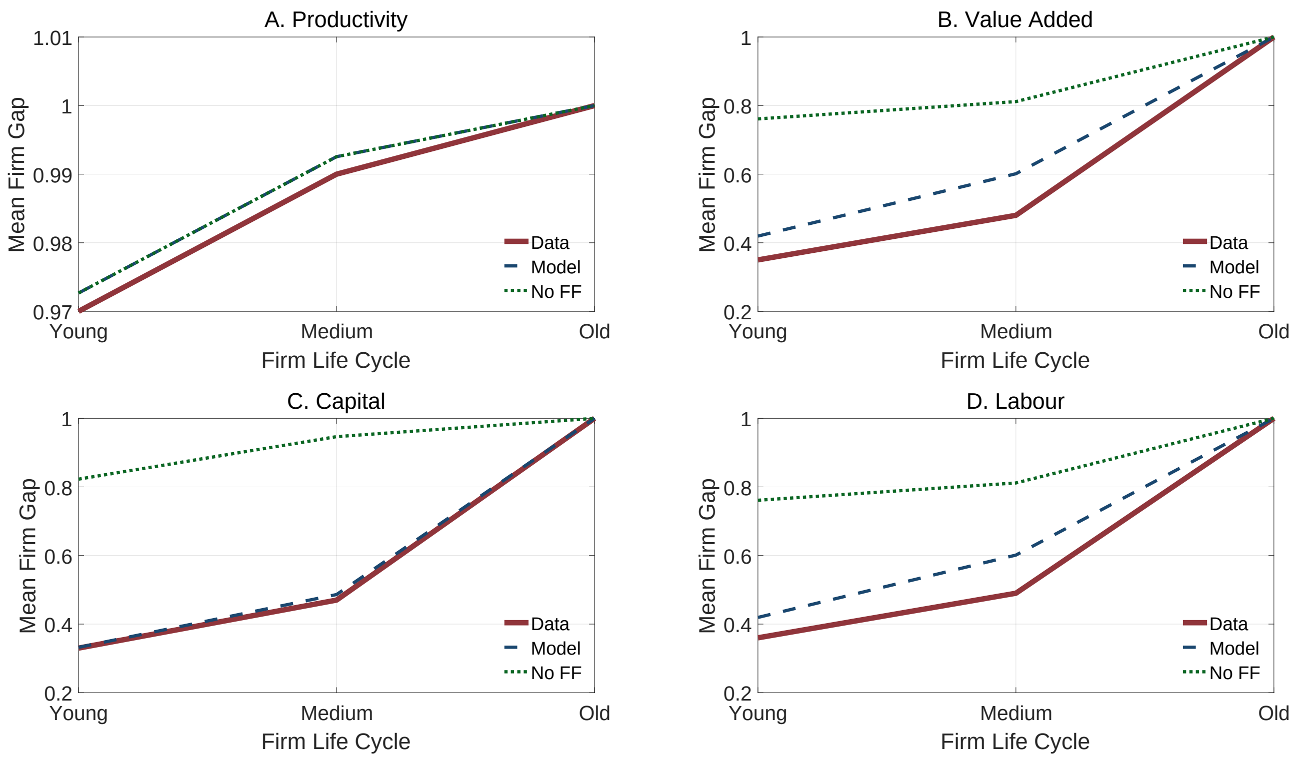


Figure D18: Profiles of PY/K

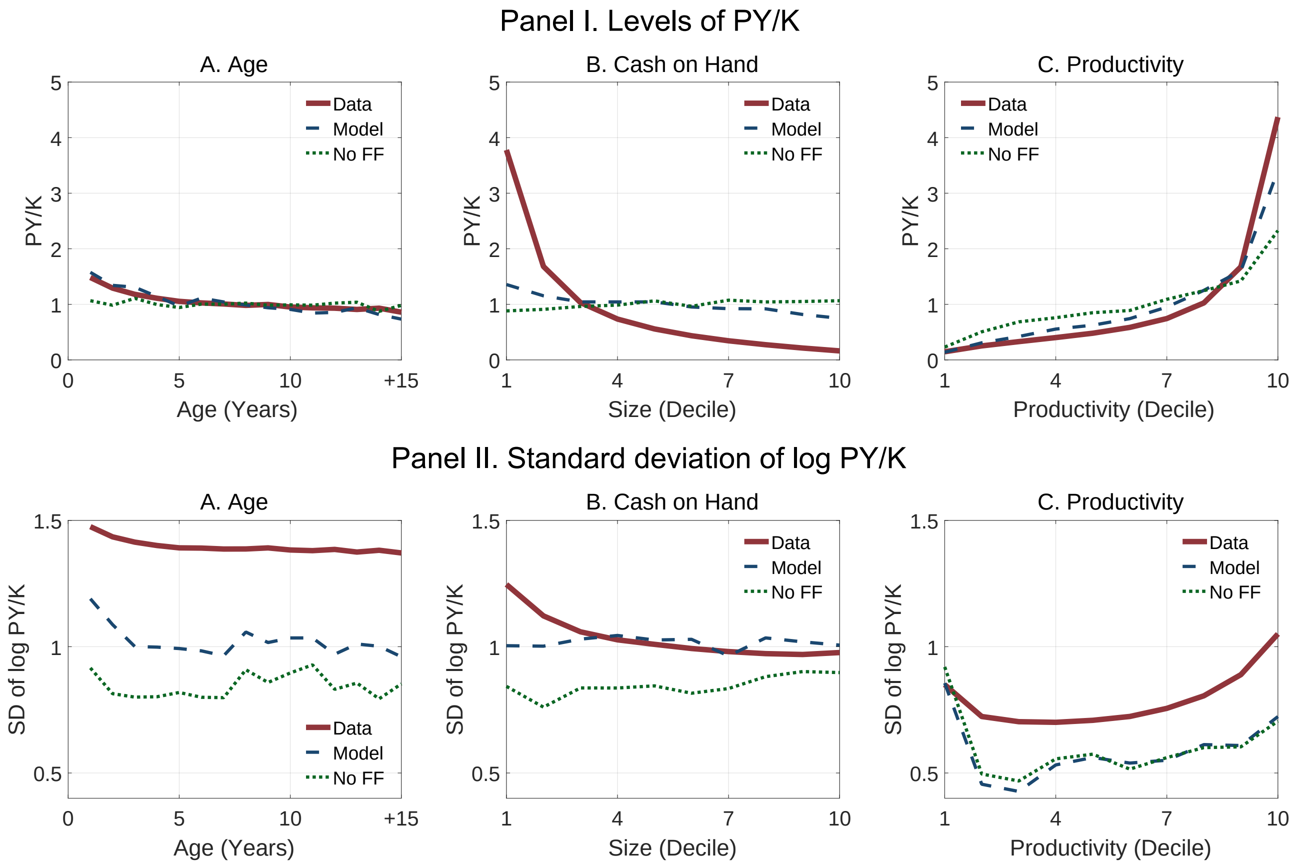


Figure D19: Financial Behavior

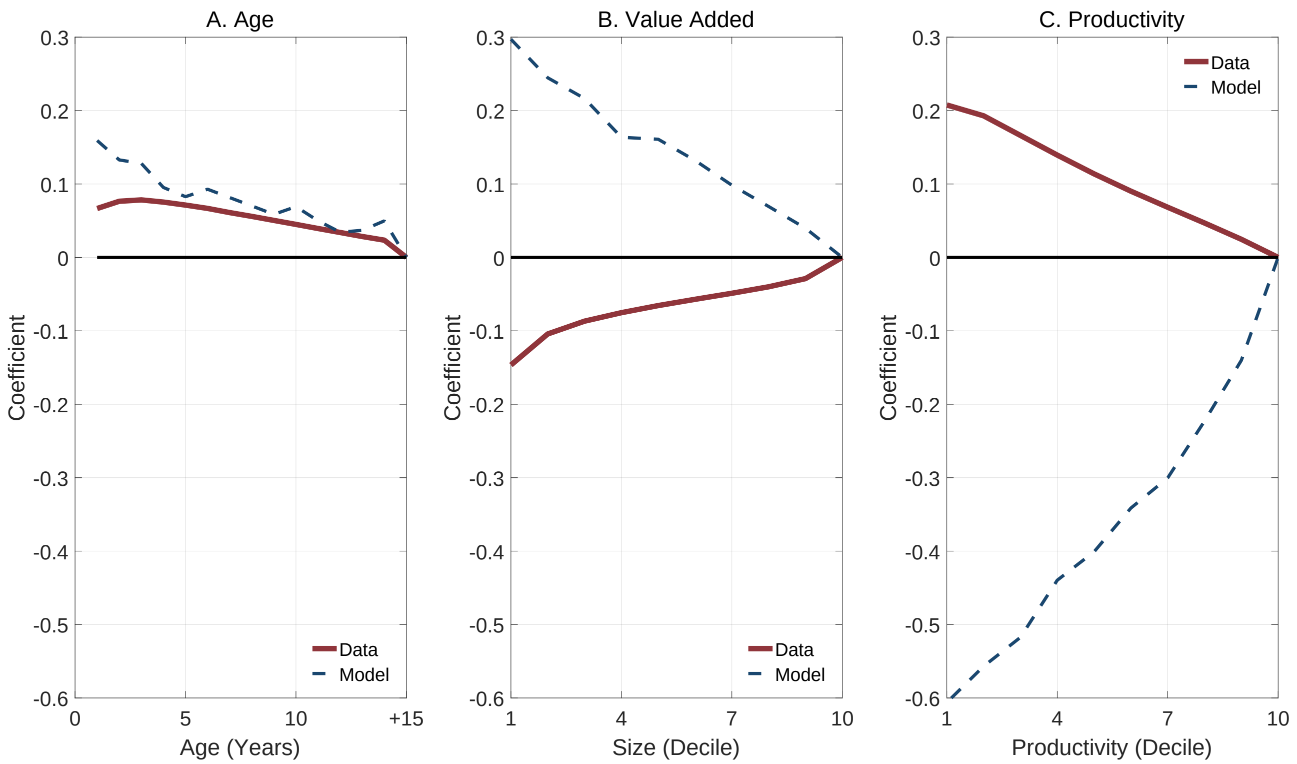


Figure D20: Financial Behavior - Extensive and Intensive Margin