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Over the past two decades, validation of choice models has focused on predictive validity rather than parameter bias. In real-world validation of choice models, true parameter values are unknown, so examination of parameter bias is not possible. In contrast, the main focus of this study is parameter bias in simulated scanner-panel choice data with known parameter values. Study of parameter bias enables the assessment of a fundamental issue not addressed in the choice modeling literature—the extent to which the logit choice model is capable of distinguishing unobserved effects that give rise to persistence in observed choices (e.g., heterogeneity and state dependence). Although econometric theory provides some information about the causes of bias, the extent of such bias in typical scanner data applications remains unclear. The authors present an extensive simulation study that provides information on the extent of bias resulting from the misspecification of four unobserved effects that receive frequent attention in the literature—choice set effects, heterogeneity in preferences and market response, state dependence, and serial correlation. The authors outline implications for model builders and managers. In general, the potential for parameter bias in choice model applications appears to be high. Overall, a logit model with choice set effects and the Guadagni–Little loyalty variable produces the most valid parameter estimates.

Parameter Bias from Unobserved Effects in the Multinomial Logit Model of Consumer Choice

Beginning with the pioneering application by Guadagni and Little (1983), many methodological advancements have been made on fitting choice models for frequently purchased products to supermarket scanner data. For example, researchers have given an increasing amount of attention to habit persistence (e.g., Roy, Chintagunta, and Haldar 1996), state dependence (e.g., Erdem 1996; Keane 1997; Roy, Chintagunta, and Haldar 1996), unobserved heterogeneity in consumer preferences (e.g., Ailawadi, Gedenk, and Neslin 1997; Chintagunta, Jain, and Vilcassim 1991; Currim 1981; Fader and Lattin 1993; Gupta and Chintagunta 1994; Jones and Landwehr 1988; Kamakura and Russell 1989), response to marketing mix (e.g., Currim, Meyer, and Le 1988; Gönül and Srinivasan 1993; Jain, Vilcassim, and Chintagunta 1994), and choice sets (e.g., Andrews and Manrai 1998a, b; Andrews and Srinivasan 1995; Bronnenberg and Vanhonacker 1996; Chiang, Chib, and Narasimhan 1999; Siddarth, Bucklin, and Morrison 1995). Typically, only information on consumers' purchase histories and demographic characteristics, the purchase environment facing consumers, and current choices are available to the analyst modeling choice behavior. These effects are largely unobservable with scanner-panel data.

One fundamental issue is the extent to which choice models are capable of distinguishing such unobservable effects. For example, consider heterogeneity and state dependence (Heckman 1981a, b). Consumers may make repeat purchases of brands either because previous choice outcomes affect current choices (state dependence) or simply because

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consumers have strong preferences for certain brands (heterogeneity). Likewise, choice set effects (see Andrews and Manrai 1998a, b) could be another explanation for persistence in choices. If consumers consider their most preferred brands, as suggested by Horowitz and Louviere (1995), then heterogeneous choice sets and heterogeneous preferences may be confounded. If consumers consider only previously purchased brands, as suggested by Siddarth, Bucklin, and Morrison (1995), then choice set heterogeneity and state dependence may be confounded. Furthermore, habit persistence (Roy, Chintagunta, and Haldar 1996) and heterogeneity in preferences (Heckman 1981a, b) are believed to result in serial correlation in the residuals and therefore may be empirically indistinguishable. In summary, there may be several unobserved explanations for observed patterns of persistent choices. Intuition suggests that it may not be possible for choice models to identify correctly the underlying cause of persistence in choices, which may result in spurious habit, state dependence, preference heterogeneity, or choice set effects.

Although the potential for spurious identification of unobserved effects has been recognized for some time, no one has attempted to study these identification issues carefully in simulated settings resembling typical scanner-panel data sets. Because newer, more sophisticated choice models are typically validated using scanner-panel data, true values of parameters that underlie the choice process are unknown. Consequently, researchers have relied on how well models fit the data, validation sample hit rates, and significance of parameter estimates to validate their models. Parameter bias could not be examined with scanner-panel data sets.

In contrast, we focus on parameter bias. Why is the effect of misspecified unobserved components on parameter bias important? Imagine an applied analyst, in a company setting, who is faced with a household-level scanner-panel data set on brand choice in a frequently purchased product category. Given the proliferation of models in the literature, how should the analyst estimate a model that will permit an accurate representation of the effects of intrinsic brand preferences, price, promotion, and loyalty on brand choice? Do all potential unobservable effects need to be modeled? Can the models be simplified without risk? In other words, are some effects more important than others? Is this contingent on the objective of the data analyst (prediction versus explanation of consumer behavior based on parameter estimates)? If certain effects are not modeled, will other parameter estimates be biased? Are these biases expected to be small or large in magnitude? To what extent could such biases affect price and promotion decisions? Although econometric theory provides guidance on the effect of misspecification on parameter bias, the actual extent of bias in typical scanner data applications remains unclear.

In addition to parameter bias, incorrect identification of unobserved effects could have other important consequences for marketing managers. First, spurious effects produce misleading conclusions about the nature of consumers' decision processes and therefore lead to inappropriate marketing strategies. For example, if spurious choice set effects are identified, managers may incorrectly focus on methods of gaining consideration of their brand (e.g., point-of-purchase displays) when, in reality, consideration of their brand may already be sufficient. Second, understanding the effectiveness of price reductions and promotions may depend on the correct identification of unobserved components that affect choices. Promotions may be much more effective if there is true state dependence or choice set effects than if there is habit or heterogeneity in preferences (Erdem 1996; Keane 1997; Siddarth, Bucklin, and Morrison 1995). Third, misspecification of unobserved components could very well produce misleading information on other unobserved components. This is because key parameters may adjust to compensate for the misspecification. For example, Andrews and Manrai (1998b) and Abramson, Buchmueller, and Currim (1998) report that the brand loyalty coefficient is larger and therefore state dependence is exaggerated when choice set or brand preference heterogeneity is not modeled, even though predictive validity is not seriously affected by the misspecification.

In this study, we use an extensive simulation to assess parameter bias resulting from misspecified unobserved components. The experimental factors manipulated include state dependence, serial correlation, heterogeneity in preferences and responses to marketing mix, and choice set heterogeneity. Variation in these four factors produces a full-factorial design with 81 data conditions. As discussed previously, these effects have received much attention in the literature in the past decade. However, no previous study has performed an extensive, simultaneous analysis of these misspecified unobserved components.

As choice modelers move farther away from Guadagni and Little's (1983) original model, the work is becoming more and more complex through incorporation of various unobserved effects. Because of the potential for confounding such effects, it is becoming progressively more difficult to appreciate many new models: Each one seems to offer a new methodological twist that is hard to compare with all other approaches. It is unclear whether, in a collective sense, modelers are making genuine progress. In this article, we offer a step back to regain some perspective on today's choice models. In a sense, our objective is to investigate which of today's "bells and whistles" really contribute to better parameter estimates and forecasting accuracy.

In the next section, we discuss the biases that we expect to find in the simulation on the basis of econometric theory and previous research. We then describe the simulation study, the unobserved components selected, levels chosen for each component, data sets simulated, and models estimated over each data set. Subsequently, we outline the main findings from the simulation study. Finally, we present implications for choice modelers, limitations, and suggestions for further research.

EXPECTED EFFECTS

In general, econometric theory suggests that, when models are overspecified, estimators of the coefficients are unbiased but inefficient (Kmenta 1986). Although we expect this result to be true for overspecified unobserved components in general, overspecification could lead to spurious effects if there are other underspecified unobserved components, possibly resulting in severe parameter bias. Thus, overspecification of some components may not cause parameter bias unless there is underspecification of others.

Econometric theory shows that underspecification typically results in biased and inconsistent estimators, producing what is known as omitted variables bias. Underspecification of unobserved effects, similar to omitted variables, should have serious consequences for parameter bias. According to the literature cited subsequently, underspecification of state dependence, choice set effects, or heterogeneity is likely to produce spurious effects in other unobserved components, which may exacerbate parameter bias. However, underspecified serial correlation in residuals should not affect bias in parameters (Kmenta 1986) if least squares regression results hold in the case of qualitative dependent variables.

Results from econometric theory might not apply to all misspecified unobserved components, such as those investigated in this study. If the results apply, it is unclear what the magnitude or seriousness of the bias will be. If so, simulation methods can supplement theoretical predictions on the existence of bias with information on the magnitude of bias that results in various misspecification conditions.

Empirical research suggests that it may be difficult for choice models to identify correctly the underlying cause of persistence in choices, which possibly results in spurious habit, state dependence, preference heterogeneity, or choice set effects.

Andrews and Manrai (1998b) focus on the consequences of misspecified choice set heterogeneity in a simulated environment. They show that spurious state dependence and parameter bias result when choice set effects are underspecified. Coefficients are biased toward zero unless the variable is used in the screening of brands, in which case it is difficult to predict the direction of bias. Chintagunta, Kyriazidou, and Perktold (1998) investigate the sensitivity of various estimation methods to (1) the extent of preference heterogeneity, (2) the correlation of individual heterogeneity and exogenous control variables, and (3) the assumption of exogeneity of the initial observations.

Most other studies use scanner-panel data to investigate specification issues. Keane (1997) studies heterogeneity and state dependence in the context of scanner-panel data. He shows that there is evidence of state dependence in purchases of ketchup even after a complex heterogeneity structure is controlled for. When heterogeneity is not controlled for, the effect of state dependence for a brand purchased on the previous occasion is roughly equivalent to a $.27 price reduction on the current occasion. When various sources of heterogeneity are controlled for, the effect of state dependence is comparable to only a $.05 price reduction. The study concludes that failing to control for heterogeneity exaggerates the degree of state dependence (see also Heckman 1981a, b).

Chiang, Chib, and Narasimhan (1999), using scanner-panel data, find that ignoring heterogeneity in choice sets understates the impact of marketing-mix variables but overstates the impact of preferences and past purchases. They also provide some evidence that heterogeneity in preferences and market response is exaggerated when heterogeneity in choice sets is not controlled for. Roy, Chintagunta, and Haldar (1996) find evidence of habit persistence (manifested as serial correlation in the residuals) in ketchup data unless state dependence and heterogeneity are controlled for. As in Keane's (1997) work, their study shows that ignoring heterogeneity exaggerates the extent of state dependence but that there is still evidence of state dependence even when heterogeneity is controlled for.

Our introduction section outlines how unobserved effects due to choice sets, heterogeneity in preferences and market response, and state dependence are conceptually related. The studies reviewed in this section indicate that these unobserved effects are empirically associated as well. Altogether, this overview suggests that logit models will not be able to distinguish the unobserved effects examined in this study.

SIMULATION STUDY

Data

This simulation experimentally varies four factors, each at three levels. The factors are state dependence (smoothing factor λ levels of 0, .75, and 1), serial correlation (correlation coefficient ρ levels of 0, .45, and .90), heterogeneity (none, discrete distributions for preference and market response parameters, and continuous distributions for preference and market response parameters), and choice set heterogeneity (none, consumers form choice sets using Siddarth, Bucklin, and Morrison's [1995] promotion expansion strategy [SBM] or Bronnenberg and Vanhonacker's [1996] strategy [BV]). All possible combinations of these four factors are investigated, which results in a full-factorial design with $3^4 = 81$ experimental conditions. Three data sets (replications) are generated for each condition, which results in 243 data sets. Each data set contains a total of 3000 purchases (1000 purchases to be used for initialization of loyalty variables and choice sets, 1000 to be used for model estimation, and 1000 to be used for model validation). Each data set is constructed such that it contains 15 purchases from each of 200 consumers: 5 purchases are used for initialization, 5 for estimation, and 5 for validation. Overall, the simulation generates $3000 \times 243 = 729{,}000$ choices from 48,600 "consumers."

The simulation assumes that consumers make choices from a universal set that contains five brands. Two binary variables intended to represent promotional activities (e.g., aisle display, store feature advertisement) and two normally distributed continuous variables (e.g., price, ad exposure) are generated to represent the attributes consumers use to make choices. The two continuous variables are generated such that the average correlations of the attributes across brands (e.g., the average correlation of price across brands) are a moderate .30, which reflects some degree of similarity among brands. The binary variables are generated such that they take values of 1 about 10% of the time, comparable to the promotion frequency observed in actual scanner-panel data applications.

Given the attribute data, the choices are generated. The brand-specific constants α_i are chosen such that, in the absence of marketing-mix effects, the market shares of Brands 1 through 5 are .10, .10, .15, .25, and .40, respectively. These values are chosen to ensure that all brands have enough choices in each data set to produce reliable estimates of brand-specific constants. The four β^k parameters are assigned values of –1 (price), 1 (ad exposure), 5 (0/1 store feature), and 4 (0/1 aisle display) for all data sets except those containing market response heterogeneity. Experimentation shows that the values chosen for the β^k parameters do not affect the conclusions drawn from the simulation, though they do affect the fit of the models.

The state dependence factor varies the value of the smoothing parameter λ across three levels: 0, .75, and 1. State dependence is operationalized as the Guadagni–Little (1983; GL) measure of brand loyalty:

$$\text{(1)} \qquad BL^h_{i,t} = \lambda \times BL^h_{i,t-1} + (1 - \lambda) \times \begin{cases} 1 & \text{if household h purchased brand i at } t-1 \\ 0 & \text{otherwise,} \end{cases}$$

where $BL^h_{i,t}$ is household h's brand loyalty for brand i at occasion t, initialized at t = 0 with equal values for all brands. The values of λ used in the analysis encompass the range of empirical estimates obtained in the literature. Studies using lagged dependent variables (e.g., Roy, Chintagunta, and Haldar 1996) assume first-order state dependence, which is equivalent to a λ value of zero. Empirical evidence suggests that the smoothing factor is usually closer to .75 (e.g., Guadagni and Little 1983). When $\lambda = 1$, the loyalty variable indicates that there is no state dependence, which is equivalent to assuming zero-order choice behavior (Bass 1993). When $\lambda = 0$ or $\lambda = .75$, the importance weight for $BL^h_{i,t}$ in the utility function is set to 4, which is again consistent with empirical estimates from prior research. When $\lambda = 1$, the value of $BL^h_{i,t}$ is the same for all brands (because it is initialized that way), and the importance weight for $BL^h_{i,t}$ does not affect choice probabilities. We assume the true value to be zero for purposes of calculating bias, because empirical models will recover a value of zero if they are operating properly.

The serial correlation factor has three levels: 0, .45, and .90. Because it is not yet feasible to model serial correlation in the residuals of logit models, the literature provides little empirical evidence as to what levels of serial correlation are realistic in scanner-panel data. Roy, Chintagunta, and Haldar (1996) find no evidence of serial correlation (suggesting a value of zero), but they note that this finding is dependent on the product category chosen. Keane's (1997) probit model produces estimates from .134 to .262, which are bracketed well by our 0 and .45 conditions. We would not expect to find negatively autocorrelated residuals in scanner-panel applications, so we investigate only positive serial correlation.

The heterogeneity factor has three levels: none, discrete distributions for preferences and market response, and continuous distributions for preferences and market response. When there is no heterogeneity, all consumers have the preference structure and response pattern described previously. When there are discrete heterogeneity distributions, we assume that each consumer belongs to one of two segments. Each segment k has its own set of preferences $\alpha^k{}_i$, which produces (in the absence of marketing-mix effects) market shares of .10, .10, .15, .25, and .40 for Segment 1 and .40, .25, .15, .10, and .10 for Segment 2. The response coefficients $\beta^k{}_i$ for Segment 1 are –1, 1, 5, and 4, and for Segment 2 are –5, 4, 1, and 2. Kamakura and Russell's (1989) study, on which this examination of latent-class heterogeneity is based, reports comparable empirical differences between segment parameters. Segment membership is determined randomly, and each consumer has a 50% chance of belonging to each segment.

When there are continuous heterogeneity distributions, the coefficients are drawn from normal distributions with means as described for Segment 1 previously. The standard deviations of the coefficients are 1 for the preferences so that households may prefer different brands. For the marketing-mix coefficients (–1, 1, 5, and 4), the standard deviations are .3, .3, 1, and 1. These were chosen so that the coefficients would rarely, if ever, change signs. For example, we want the price coefficient to be negative for all households.

The choice set heterogeneity factor has three levels: no choice set formation, formation of choice sets using the SBM promotion expansion strategy, and formation of choice sets using the BV strategy. The promotion expansion strategy assumes that consumers choose from one of three choice sets: the full set of available brands, the set of previously purchased brands, or the set of previously purchased brands augmented by currently promoted brands. Applications of the promotion expansion and similar models show usage of a choice set other than the full set by 62% (detergent; Siddarth, Bucklin, and Morrison 1995) and 73% (yogurt and detergent; Andrews and Manrai 1998a) of consumers. Thus, we assume that 70% of consumers choose from reduced choice sets, and the remaining 30% choose from the full set of available brands. Consumers are assumed to use their assigned choice set formation strategy consistently for the duration of their 15 purchases.

The third level of the choice set heterogeneity factor assumes that consumers form choice sets using the BV strategy, which calculates brand consideration probabilities π^h_{it} that are then used to compute choice probabilities as

$$\text{(2)} \qquad P^h_{it} = \frac{\pi^h_{it} \times \exp\left(v^h_{it}\right)}{\sum_j \pi^h_{it} \times \exp\left(v^h_{it}\right)},$$

where P^h_{it} is the choice probability and v^h_{it} is the utility of brand i for household h at t. To keep the information set comparable to that of the promotion expansion model, we generate brand consideration probabilities as a function of past purchases and promotion as follows:

$$\text{(3)} \qquad \pi^h_{it} = \frac{\exp\left(-2 + 2 \times \text{Prom}_{it} + 2 \times \text{Prev}^h_{it}\right)}{1 + \exp\left(-2 + 2 \times \text{Prom}_{it} + 2 \times \text{Prev}^h_{it}\right)},$$

where $\text{Prom}_{it} = 1$ if brand i is promoted at t, 0 otherwise, and $\text{Prev}^h_{it} = 1$ if household h has previously purchased brand i. The consideration probabilities range from .12 to .88, which would produce a choice set size distribution comparable to that of Bronnenberg and Vanhonacker's (1996) in-store sensitive segment.

Models

The simulation fits nine models to each of the 243 data sets. The nine models were chosen largely according to their prominence in the choice modeling literature, though some concession was made to practicality, given that the chosen models would need to be estimated 243 times each. Model 1 is the zero-order logit model, which contains only brand-specific constants and market response parameters. This model is included because it underspecifies all four major components examined in this study. With five brands used in the simulation, we have four brand-specific constants (the value of the fifth constant is fixed at zero, because only four are identified) and four marketing-mix coefficients, for a total of eight parameters per logit model.

Model 2 is the logit model with the GL specification of the brand loyalty variable (see Equation 1) added to the utility function. The brand loyalty variable is initialized with the initialization period market shares. This model underspecifies heterogeneity in preferences and market response, choice sets, and serial correlation but is specified to handle state dependence correctly. With the two additional loyalty parameters, ten parameters are required per logit model.

Model 3 is the logit model with Fader and Lattin's (1993) specification of the loyalty variable. Fader and Lattin's study presents an alternative measure of brand loyalty that is intended to handle state dependence, heterogeneity, and nonstationarity in preferences. However, Model 3 is not well suited to first-order state dependence ($\lambda = 0$), so it may be misspecified in this condition. In addition, Fader and Lattin's specification does not model heterogeneity in market response, so we do not expect Model 3 to explain heterogeneity well either. Fader and Lattin's loyalty variable is included in the logit specification, which contains four brand-specific constants and four marketing-mix coefficients. The total parameter count is 14.

The latent segment heterogeneity models Kamakura and Russell (1989) describe estimate segment-specific preference and market response parameters. Model 4 correctly specifies heterogeneity in preferences and market response but underspecifies state dependence, choice set effects, and serial correlation. For the sake of computational feasibility, we estimate models that have two latent segments. Each model requires 8 parameters for each latent segment (as detailed for the zero-order logit model previously) plus another parameter that determines segment sizes, for a total of 17 parameters for each model. Note that if two-segment models outperform zero-order logit in any condition in which there is no heterogeneity in preferences and market response, there is no real need to estimate models with more segments to demonstrate spurious effects.

Model 5 is the model with choice set heterogeneity, which is based on the SBM promotion expansion model. Model 5 reflects the possibility that the consumer could use any of three screening strategies to form choice sets on any given purchase occasion. Consumers are assumed to consider (1) only previously purchased brands, (2) previously purchased brands augmented by currently promoted brands, or (3) all available brands. Model 5 requires only two more parameters than the zero-order logit model, for a total of ten. The model correctly specifies heterogeneity in choice sets but underspecifies state dependence, heterogeneity in preferences and market response, and serial correlation.

Model 6 is Roy, Chintagunta, and Haldar's (1996) model, which is intended to explain heterogeneity in preferences and market response, habit persistence (serial correlation), and state dependence. Model 6 underspecifies choice set effects. The utility function contains a lagged dependent variable to explain state dependence, but the model will be misspecified when the true state dependence is not first order (i.e., when $\lambda = .75$). Brand-specific preference and response parameters are handled with the latent-class formulation, as in Kamakura and Russell's (1989) models, to account for heterogeneity. Habit persistence, which is assumed to induce serial correlation in the residuals, is handled by setting up a first-order reinforcement model for the choice probabilities, conditional on the last purchase. The serial correlation coefficient is not allowed to vary across latent segments (see Roy, Chintagunta, and Haldar 1996), and because there are two segments, Model 6 requires 20 parameters.

Model 7 is the choice set model described previously (Model 5) but with GL loyalty variables included in the utility specification. Model 7 correctly specifies state dependence and choice set effects but underspecifies heterogeneity in preferences and market response and serial correlation. The model requires 12 parameters.

Model 8 is the two-segment latent-class model described previously (Model 4) but with segment-specific GL brand loyalty variables. This adds 4 parameters (2 smoothing parameters and 2 coefficients) to each model, for a total of 21. Model 8 underspecifies choice set heterogeneity and serial correlation but correctly specifies state dependence and heterogeneity in preferences and market response.

Model 9 is specified to explain heterogeneity in preferences and market response, state dependence, and choice set effects, leaving only serial correlation underspecified. A two-segment version adds 2 choice set parameters per segment to Model 8, resulting in a total of 25 parameters. Model 9 is the most fully specified of the models, and we expect that it will be the best-performing model of the group.

This simulation does not estimate a model that correctly specifies all four unobserved components, because experimentation has shown that there is no acceptable technology available for modeling the serial correlation component in logit models. Also, this simulation does not investigate the effectiveness of recently developed Bayesian models (e.g., Chiang, Chib, and Narasimhan 1999) that require simulation-based estimation methods because of the high degree of computational effort required. The complete simulation requires 2187 optimization runs.

SIMULATION RESULTS

The Effects of Misspecification: Regression Results

To analyze the effects of misspecification on parameter bias, we use dummy-variable regression analysis. The bias measure is computed as $100(\hat{\beta}_i - \beta_i)/\beta_i$, where the hat indicates a parameter estimate. Because the biases in preferences, market responses, and brand loyalty are often very different, we use three bias measures—the average bias in preference coefficients, the average bias in market response coefficients, and the brand loyalty coefficient.[1]

When there is discrete heterogeneity in the data, there are two sets of true coefficients, and when the model is latent class (Models 4, 6, 8, and 9), there are two sets of estimated coefficients, so special procedures must be used to compute bias. First, we match estimated segments with true segments on the basis of the highest percentage of correct allocation of subjects to segments. For example, we compute the percentage of correct allocation when the first estimated segment (of which membership is determined by posterior probabilities) is matched with the first true segment and when the first estimated segment is matched with the second true segment. Second, we use the matching that produces the highest percentage of correct allocation to assign to each consumer a true and an estimated vector of coefficients. Third, using the assigned true and estimated vectors of coef-

[1]For the brand loyalty coefficient, we analyze the estimated values of the coefficient instead of bias, because it is not possible to compute the bias measure when $\lambda = 1$. The true value of the loyalty coefficient is zero in this condition, which necessitates division by zero.

ficients, we compute the bias for each coefficient for each consumer. Finally, we average the biases across consumers and across variables to obtain the average bias for that model–data set combination. When there is heterogeneity in the data but not in the model, the estimated coefficients will be the same for all consumers, whereas the true coefficients vary across consumers. Likewise, when there is no heterogeneity in the data but the model is latent class, the true coefficients will be the same for all consumers, whereas the estimated coefficients (may) vary across consumers.

In Table 1, we show the regression results for three dependent measures: percent bias in response coefficients, percent bias in preference constants, and the estimated loyalty coefficients. For each factor (state dependence, serial correlation, heterogeneity, and choice sets), the baseline level corresponds to no effect. For example, the baseline level of the state dependence factor is the one with $\lambda = 1$, which corresponds to no state dependence. Table 1 shows the main effects for each factor level as well as the interaction of each level with model type. Model 1 (zero-order logit) is the baseline model. Table 1 can be used to predict the performance of a model given assumptions about the effects likely to be found in the data (e.g., state dependence, choice set effects).

As an example to aid interpretation, consider Table 1, Panel A. To compute the predicted bias in response coefficients for Model 2 (Model 2, the logit model with GL loyalty) when there is $\lambda = .75$ state dependence, we add the main and interaction effects for both the constant (–7 + 6) and state dependence for $\lambda = .75$ (–18 + 19), which results in a predicted bias of zero. Using Table 1, Panel A, the computations showing the predicted biases in response coefficients for each of the correctly specified models is shown in Table 2. Models 3 and 6 have no closely corresponding data generation processes and therefore are excluded from the computations. All models recover response parameters reasonably well when correctly specified.

In the main effects column of Table 1, Panel A, we show that underspecified state dependence produces significant and serious bias in response coefficients. When state dependence is first order ($\lambda = 0$), the baseline level of bias in response coefficients is –40% (i.e., 40% underestimation), whereas –18% bias results if the underspecified state dependence is generated from $\lambda = .75$ (which is more likely to be the case). Underspecified serial correlation does not produce significant bias in response coefficients (–2% and –7% for the $\rho = .45$ and $\rho = .90$ conditions). Underspecified heterogeneity does not produce significant bias in response coefficients either. However, underspecified choice sets, if generated by the SBM promotion expansion strategy, generate significant bias in response coefficients (–27%). Underspecified choice sets do not produce significant bias if the sets are generated using the more compensatory BV strategy. Thus, of the effects examined in this study, only underspecified state dependence and choice sets produce significant bias in response coefficients.

In Table 1, Panel B, we show the regression analysis for percent bias in preference constants. Correctly specified models do well in some cases but not as well in others, as the predicted values show (see Table 3). On the whole, models do not recover preference constants as well as they recover market response parameters.

In the main effects column of Table 1, Panel B, we show that significant bias in preference constants results only from underspecified discrete heterogeneity—and this bias is an astounding –105%. Underspecified choice set effects of the SBM variety produce an estimated –24% bias, but the bias is not statistically significant (t = –1.3). Thus, state dependence and choice set effects produce bias in response coefficients, and heterogeneity produces bias in preference constants, whereas serial correlation produces bias in neither.

In Table 1, Panel C, we show the regression analysis for the loyalty coefficients. Only Models 2, 7, 8, and 9 have loyalty coefficients. The predicted loyalty coefficients for the models, when correctly specified, are given in Table 4.

Note that the true loyalty coefficient is 4.0 for $\lambda = .75$. Models 2 and 7 do a reasonable job of recovering the loyalty coefficient when correctly specified, but Models 8 and 9 do less well. Indeed, Model 8 recovers the loyalty coefficient poorly. Perhaps the explanation for the relatively poor recovery of the loyalty coefficient for Models 8 and 9 is that those models are designed to recover heterogeneity.

In the main effects column of Table 1, Panel C, we show that state dependence significantly increases the loyalty coefficient, as we expected. Serial correlation at $\rho = .90$ produces significant positive bias (1.39) in the size of the loyalty coefficient. Also, underspecified heterogeneity (whether continuous or discrete) produces negative (though insignificant) bias (–.71 and –.72) in the size of the loyalty coefficient. Conventional wisdom holds that underspecified heterogeneity would be explained as state dependence, which produces positive bias in the loyalty coefficient (Keane 1997; Roy, Chintagunta, and Haldar 1996). Our study does not support this finding. Underspecified choice sets (of the SBM variety) produce significant bias in the loyalty coefficient (1.36).

To summarize the regression analyses, underspecified state dependence produces significant bias (–40% for $\lambda = 0$, –18% for $\lambda = .75$) in response coefficients. Underspecified serial correlation increases the value of the loyalty coefficient (by 1.39). Underspecified discrete heterogeneity produces significant bias (–105%) in preference constants but does not increase the value of the loyalty coefficient, as is commonly held. Underspecified continuous heterogeneity produces comparatively minor problems. Whether the true coefficients have continuous or discrete distributions in real-world applications is unknown, though some researchers have suggested that the assumption of discrete heterogeneity distributions is not realistic (Allenby, Arora, and Ginter 1998). Underspecified choice sets, if generated by the SBM promotion expansion strategy, produce significant bias in response coefficients (–27%), large but insignificant bias in preference constants (–24%), and a significant increase in loyalty coefficients (1.36). In contrast, choice sets produced by the BV strategy produce no significant biases. Whether true choice sets are generated by promotion expansion screening strategies or compensatory analyses in real-world applications is unknown, though there is much more evidence that choice sets are formed through a noncompensatory screening such as that described by Siddarth, Bucklin, and Morrison (1995; see also Andrews and Manrai 1998a).

Table 1

REGRESSION RESULTS: PARAMETER ESTIMATES AND t–VALUES.

A: Percent Bias in Response Coefficients (R^2 = .30, n = 2187)

	Main Effect		Interactions by Model Type															
Effect			*Model 2*		*Model 3*		*Model 4*		*Model 5*		*Model 6*		*Model 7*		*Model 8*		*Model 9*	
Constant	–7	–.8	6	.5	8	.6	10	.7	8	.6	40	3.0	11	.8	34	2.5	58	4.3
State: λ = 0	–40	–5.1	29	2.6	24	2.2	–3	–.2	–6	–.5	101	9.2	26	2.3	43	3.9	66	6.0
State: λ = .75	–18	–2.3	19	1.7	12	1.0	1	.1	3	.3	7	.6	17	1.5	20	1.8	39	3.5
Serial: ρ = .45	–2	–.3	2	.2	1	.1	0	.0	1	.1	–11	–1.0	2	.2	13	1.2	–11	–1.0
Serial: ρ = .90	–7	–1.0	15	1.3	8	.7	2	.2	3	.3	1	.1	15	1.4	15	1.4	25	2.3
Heterogeneity: discrete	9	1.2	–10	–.9	–7	–.6	–8	–.7	0	.0	–43	–3.9	–11	–1.0	–34	–3.1	–70	–6.3
Heterogeneity: continuous	–8	–1.0	–11	–1.0	–8	–.7	–3	–.2	–5	–.4	–9	–.8	–14	–1.3	–13	–1.1	–46	–4.1
Choice set: SBM	–27	–3.5	8	.7	8	.7	–1	–.1	22	2.0	–23	–2.1	24	2.1	2	.2	29	2.6
Choice set: BV	–4	–.5	3	.3	3	.3	–1	–.1	4	.3	6	.6	5	.5	–12	–1.1	4	.4

B: Percent Bias in Preference Constants (R^2 = .39, n = 2187)

	Main Effect		Interactions by Model Type															
Effect			*Model 2*		*Model 3*		*Model 4*		*Model 5*		*Model 6*		*Model 7*		*Model 8*		*Model 9*	
Constant	0	.0	–18	–.6	–93	–2.9	4	.1	2	.0	41	1.3	–8	–.2	–22	–.7	66	2.1
State: λ = 0	–1	–.1	3	.1	–369	–14.1	2	.1	–9	–.4	43	1.6	–2	–.1	84	3.2	55	2.1
State: λ = .75	12	.6	–7	–.3	–3	–.1	8	.3	0	.0	–6	–.2	–10	–.4	–2	–.1	26	1.0
Serial: ρ = .45	0	.0	1	.0	–30	–1.1	–6	–.2	–1	.0	–12	–.5	–1	–.1	34	1.3	–55	–2.1
Serial: ρ = .90	–5	–.2	–1	.0	–40	–1.5	–1	.0	–4	–.1	–32	–1.2	–4	–.1	0	.0	–26	–1.0
Heterogeneity: discrete	–105	–5.7	23	.9	210	8.0	103	3.9	–2	–.1	67	2.6	11	.4	71	2.7	12	.5
Heterogeneity: continuous	–12	–.6	0	.0	67	2.6	2	.1	–3	–.1	9	.3	–4	–.1	38	1.5	–50	–1.9
Choice set: SBM	–24	–1.3	–2	–.1	47	1.8	–26	–1.0	16	.6	–46	–1.8	12	.5	–2	–.1	19	.7
Choice set: BV	–3	–.2	0	.0	6	.2	1	.0	2	.1	–2	–.1	2	.1	–17	–.7	19	.7

C: Estimated Loyalty Coefficient (R^2 = .25, n = 972)

	Main Effect		Interactions by Model Type						
Effect			*Model 2*	*Model 7*		*Model 8*		*Model 9*	
Constant	1.64	2.0	0	–.50	–.4	–1.47	–1.3	–.36	–.3
State: λ = 0	1.51	2.2	0	.60	.6	3.01	3.1	3.76	3.9
State: λ = .75	1.97	2.9	0	.57	.6	.50	.5	1.90	2.0
Serial: ρ = .45	.24	.4	0	.09	.1	1.78	1.9	.00	.0
Serial: ρ = .90	1.39	2.0	0	.27	.3	1.23	1.3	2.32	2.4
Heterogeneity: discrete	–.71	–1.0	0	–.21	–.2	–.62	–.6	–1.96	–2.0
Heterogeneity: continuous	–.72	–1.1	0	–.10	–.1	1.69	1.8	–1.51	–1.6
Choice set: SBM	1.36	2.0	0	–1.38	–1.4	2.11	2.2	–.81	–.8
Choice set: BV	.45	.7	0	–.24	–.3	–.48	–.5	.04	.0

Notes: Model 1: zero-order logit; Model 2: logit with GL (1983) loyalty; Model 3: logit with Fader–Lattin (1993) loyalty; Model 4: heterogeneity in preferences and market response (Kamakura and Russell 1989); Model 5: heterogeneity in choice sets (Siddarth, Bucklin, and Morrison 1995); Model 6: habit, state dependence, and heterogeneity (Roy, Chintagunta, and Haldar 1996); Model 7: choice set Model 5 with GL loyalty; Model 8: heterogeneity Model 4 with GL loyalty; Model 9: heterogeneity, state dependence, and choice sets.

Table 2
PREDICTED BIASES IN RESPONSE COEFFICIENTS

	Model 1	*Model 2*	*Model 4*	*Model 5*	*Model 7*	*Model 8*	*Model 9*
Constant	–7	–1	3	1	4	27	51
State: λ = .75		1			–1	2	21
Heterogeneity: discrete			1			–25	–61
Choice set: SBM				–5	–3		2

Table 3
PREDICTED BIASES IN PREFERENCE CONSTANTS

	Model 1	*Model 2*	*Model 4*	*Model 5*	*Model 7*	*Model 8*	*Model 9*
Constant	0	–18	4	2	–8	–22	66
State: λ = .75		5			2	10	38
Heterogeneity: discrete			–2			–34	–93
Choice set: SBM				–8	–12		–5
Predicted bias	0	–13	2	–6	–18	–46	6

Table 4
PREDICTED LOYALTY COEFFICIENTS

	Model 2	*Model 7*	*Model 8*	*Model 9*
Constant	1.64	1.14	.17	1.28
State: λ = .75	1.97	2.54	2.47	3.87
Heterogeneity: discrete			–1.33	–2.67
Choice set: SBM		–.02		.55
Predicted coefficient	3.61	3.66	1.31	3.03

Means and Standard Deviations by Model Type and Experimental Condition

In Table 5, we show the means and standard deviations of the experimental factors shown to be most important in Table 1, namely, models (Panel A), choice sets (Panel B), heterogeneity (Panel C), and state dependence (Panel D). We show the means not only for preference, market response, and loyalty but also for two key predictive criteria (Bayesian information criterion [BIC][2] and the prediction log-likelihood). Italics indicate our assessment of the best model for each data condition.

In Panel A of Table 5, we show that Model 9 (with state dependence, heterogeneity, and choice set effects) has the best predictive validity of all models when averaged across all 81 experimental conditions. Although Model 9 has the best predictive validity, Model 7 (with state dependence and choice set effects) has the most appealing coefficient bias (taking variance into account). Notice that the standard deviations of the biases in coefficients are at least four times as high for Model 9 as for Model 7. The true loyalty coefficient for Table 5, Panel A, should be 4(2/3) + 0(1/3) = 8/3 = 2.67, because two-thirds of the data conditions have a loyalty coefficient of four and the remaining one-third (corresponding to λ = 1) have a loyalty coefficient of zero. Model 7 also does the best job of recovering the loyalty coefficient, with an average estimate of 2.83 (a bias of only 6%). Model 4 (with heterogeneity only) has good bias numbers (especially preference), but because the model's predictive accuracy is far out of the range of the best models, we cannot recommend it is a best model. It is a bit shocking that the model with the best overall bias numbers has an average bias of –53% in preference coefficients. As we discuss subsequently, most of this bias is due to underspecified discrete heterogeneity.

Thus, Model 9 has the best overall predictive accuracy, and Model 7 has the best overall parameter estimates. Superior predictive validity does not necessarily imply superior parameter bias, as is commonly assumed in empirical applications in the literature.

In Table 5, Panel B, we show the interaction of model type and choice set usage. Model 8 has the best predictive validity when there are no choice set effects (Model 9 is overspecified in this condition). However, Model 9 has the best predictive validity when choice sets are generated according to the SBM model, as we would expect. The SBM model formulation apparently does not explain choice set effects generated according to Bronnenberg and Vanhonacker (1996), as Model 8 performs better than Model 9 in the BV choice set usage condition.

From Table 5, Panel B, it is apparent that Models 2 and 7 have nearly identical performance when there are no choice set effects. This is strong evidence that Model 7 has no spurious choice set effects when there are truly no choice set effects in the data. Model 7 also has the best parameter bias when the choice sets are generated according to the SBM model. As before, the standard deviations of the biases for latent-class Models 8 and 9 are high. However, Model 8 has surprisingly good parameter bias figures in the BV choice set usage condition.

Also note in Table 5, Panel B, the average loyalty coefficient for Model 2 when there are no choice set effects (2.86) and when there are SBM choice set effects (4.22). The loyalty coefficient is biased because of the underspecified choice set effects, as is suggested in Table 1, Panel C. The magnitude of bias (4.22 – 2.86 = 1.36) is exactly the same as is predicted in the "Choice set: SBM" row in Table 1, Panel C. Likewise, the estimated loyalty coefficient for Model 8 is 3.92 when there are no choice set effects and 7.39 when there are SBM choice set effects, a difference of

[2]BIC = –2log L + pln(n), where log L is the value of the maximized log-likelihood function from the estimation sample, p is the number of parameters required, and n is the sample size.

Table 5
MEANS AND STANDARD DEVIATIONS
(Standard Deviations in Parentheses)

A: By Model Type

	Predictive Validity				*Parameter Bias*					
Model	*BIC*		*Log-Likelihood*		*Preference Constants*		*Response Coefficients*		*Loyalty Coefficients*	
1	2338	(366)	–1140	(184)	–46	(51)	–40	(23)		—
2	1568	(299)	–719	(172)	–59	(41)	–15	(15)	3.47	(1.35)
3	1606	(268)	–753	(125)	–177	(242)	–18	(15)		—
4	2160	(328)	–1034	(218)	–15	(39)	–34	(24)		—
5	1856	(173)	–937	(112)	–46	(51)	–24	(22)		—
6	1592	(400)	–713	(208)	2	(125)	10	(89)		—
7	1539	(288)	–692	(165)	*–53*	*(43)*	*–8*	*(14)*	*2.83*	*(1.47)*
8	1521	(282)	–643	(177)	0	(224)	6	(66)	5.07	(7.96)
9	*1503*	*(265)*	*–615*	*(169)*	20	(189)	30	(109)	4.35	(5.12)

B: By Model Type and Choice Set Usage Condition

	Predictive Validity						*Parameter Bias*								
	None		*SBM Usage*		*BV Usage*		*None*			*SBM Usage*			*BV Usage*		
Model	*BIC*	*Log-Likelihood*	*BIC*	*Log-Likelihood*	*BIC*	*Log-Likelihood*	*Preference Constants*	*Response Coefficients*	*Loyalty Coefficients*	*Preference Constants*	*Response Coefficients*	*Loyalty Coefficients*	*Preference Constants*	*Response Coefficients*	*Loyalty Coefficients*
1	2147	–1052	2656	–1292	2212	–1074	–37	–29	—	–62	–56	—	–41	–33	—
	(329)	(169)	(184)	(99)	(330)	(169)	(56)	(22)	—	(38)	(12)	—	(54)	(22)	—
2	1585	–721	1570	–723	1548	–713	*–49*	*–8*	*2.86*	–75	–27	4.22	–52	–9	3.32
	(284)	(169)	(323)	(178)	(291)	(170)	*(46)*	*(14)*	*(1.50)*	(25)	(10)	(0.70)	(44)	(13)	(1.33)
3	1630	–764	1595	–738	1592	–756	–186	–11	—	–163	–31	—	–183	–12	—
	(266)	(128)	(273)	(119)	(265)	(129)	(274)	(13)	—	(213)	(9)	—	(237)	(13)	—
4	1982	–949	2461	–1186	2035	–968	3	–22	—	–48	–51	—	1	–27	—
	(301)	(220)	(116)	(112)	(284)	(217)	(22)	(25)	—	(45)	(11)	—	(21)	(23)	—
5	1941	–984	1739	–864	1887	–964	–42	–22	—	–51	–28	—	–44	–23	—
	(179)	(114)	(112)	(74)	(153)	(105)	(54)	(22)	—	(48)	(22)	—	(52)	(23)	—
6	1512	–667	1747	–799	1516	–674	27	27	—	–44	–24	—	22	29	—
	(337)	(186)	(459)	(220)	(351)	(191)	(119)	(86)	—	(37)	(26)	—	(170)	(119)	—
7	1597	–720	1469	–649	1551	–707	*–48*	*–7*	*2.77*	*–60*	*–11*	*2.75*	–50	–6	2.98
	(285)	(169)	(277)	(148)	(290)	(170)	*(46)*	*(13)*	*(1.51)*	*(37)*	*(14)*	*(1.46)*	(46)	(14)	(1.46)
8	*1510*	*–628*	1570	–680	*1481*	*–622*	15	19	3.92	–11	–6	7.39	*–5*	*3*	*3.89*
	(266)	*(182)*	(305)	(164)	*(268)*	*(180)*	(103)	(68)	(3.56)	(372)	(89)	(12.9)	*(38)*	*(23)*	*(1.86)*
9	1538	–629	*1471*	*–597*	1500	–621	17	30	4.01	11	31	4.55	32	30	4.50
	(268)	(180)	*(256)*	*(143)*	(269)	(180)	(143)	(88)	(3.90)	(208)	(118)	(5.96)	(211)	(121)	(5.33)

Table 5
CONTINUED

C: By Model Type and Heterogeneity Condition															
	Predictive Validity						Parameter Bias								
	None		Discrete		Continuous		None			Discrete			Continuous		
Model	BIC	Log-Likelihood	BIC	Log-Likelihood	BIC	Log-Likelihood	Preference Constants	Response Coefficients	Loyalty Coefficients	Preference Constants	Response Coefficients	Loyalty Coefficients	Preference Constants	Response Coefficients	Loyalty Coefficients
1	2259	–1098	2363	–1150	2393	–1171	–7	–40	—	–113	–31	—	–19	–48	—
	(430)	(219)	(318)	(161)	(332)	(161)	(26)	(26)	—	(5)	(21)	—	(22)	(18)	—
2	1346	–599	1729	–809	1628	–750	–27	–8	3.94	–109	–9	3.23	–39	–27	3.23
	(291)	(172)	(169)	(84)	(279)	(170)	(27)	(16)	(1.53)	(4)	(11)	(1.07)	(20)	(9)	(1.28)
3	1400	–650	1754	–824	1663	–784	–230	–14	—	–126	–11	—	–175	–29	—
	(248)	(110)	(157)	(63)	(250)	(121)	(333)	(16)	—	(33)	(12)	—	(245)	(9)	—
4	2113	–1044	2085	–928	2281	–1131	–11	–30	—	–13	–29	—	–21	–41	—
	(311)	(205)	(388)	(244)	(233)	(143)	(49)	(27)	—	(37)	(23)	—	(27)	(19)	—
5	1718	–877	1932	–958	1917	–976	–5	–23	—	–112	–14	—	–20	–36	—
	(153)	(120)	(139)	(96)	(137)	(93)	(27)	(26)	—	(5)	(17)	—	(19)	(17)	—
6	1468	–682	1566	–629	1741	–829	15	27	—	–22	–7	—	12	11	—
	(409)	(195)	(347)	(182)	(397)	(196)	(101)	(105)	—	(38)	(20)	—	(187)	(110)	—
7	*1320*	*–572*	1692	–777	*1604*	*–727*	*–16*	*0*	*3.42*	–110	–2	2.49	*–32*	*–22*	*2.59*
	(264)	*(155)*	(177)	(94)	*(269)*	*(163)*	*(19)*	*(13)*	*(1.76)*	(4)	(7)	(1.04)	*(15)*	*(7)*	*(1.37)*
8	1398	–608	1480	–555	1684	–766	3	21	5.19	–32	–4	3.86	29	0	6.16
	(286)	(165)	(191)	(124)	(278)	(167)	(104)	(62)	(4.48)	(35)	(17)	(1.49)	(370)	(94)	(12.9)
9	1386	–592	*1447*	*–506*	1675	–748	71	68	5.99	*–22*	*8*	*3.31*	10	15	3.76
	(266)	(150)	*(142)*	*(85)*	(272)	(161)	(260)	(146)	(7.09)	*(27)*	*(12)*	*(1.69)*	(188)	(111)	(4.70)

Table 5

CONTINUED

	D: By Model Type and State Dependence Condition														
	Predictive Validity						*Parameter Bias*								
	λ = *0*		λ = *.75*		λ = *1*		λ = *0*			λ = *.75*			λ = *1*		
Model	*BIC*	*Log-Likelihood*	*BIC*	*Log-Likelihood*	*BIC*	*Log-Likelihood*	*Preference Constants*	*Response Coefficients*	*Loyalty Coefficients*	*Preference Constants*	*Response Coefficients*	*Loyalty Coefficients*	*Preference Constants*	*Response Coefficients*	*Loyalty Coefficients*
1	2672	–1308	2280	–1116	2063	–994	–51	–60	—	–38	–38	—	–50	–21	—
	(173)	(81)	(286)	(142)	(319)	(159)	(46)	(11)	—	(58)	(16)	—	(48)	(20)	—
2	1339	–571	1538	–715	1826	–871	–59	–22	3.82	–56	–10	4.27	–61	–12	2.31
	(237)	(140)	(242)	(123)	(184)	(94)	(39)	(11)	(0.62)	(43)	(14)	(1.02)	(41)	(17)	(1.38)
3	1401	–670	1581	–734	1836	–854	–427	–26	—	–48	–17	—	–56	–11	—
	(210)	(106)	(212)	(102)	(176)	(90)	(274)	(12)	—	(54)	(12)	—	(68)	(16)	—
4	2421	–1241	2090	–964	1968	–897	–21	–56	—	–2	–31	—	–21	–14	—
	(144)	(117)	(270)	(154)	(346)	(196)	(26)	(9)	—	(32)	(16)	—	(52)	(22)	—
5	1956	–1040	1804	–915	1807	–856	–57	–49	—	–34	–19	—	–46	–4	—
	(183)	(87)	(150)	(78)	(138)	(80)	(42)	(11)	—	(59)	(11)	—	(48)	(12)	—
6	*1130*	*–492*	1754	–791	1891	–857	27	55	—	–8	–17	—	–14	–7	—
	(191)	*(100)*	(196)	(126)	(276)	(168)	(208)	(141)	—	(40)	(21)	—	(39)	(27)	—
7	1319	–548	1521	–697	1777	–831	*–56*	*–17*	*3.39*	*–50*	*–4*	*3.83*	*–52*	*–3*	*1.29*
	(231)	(132)	(247)	(126)	(170)	(94)	*(41)*	*(10)*	*(0.68)*	*(46)*	*(13)*	*(1.06)*	*(43)*	*(14)*	*(1.12)*
8	1306	–493	1486	–643	1770	–793	52	7	7.26	–21	6	5.21	–31	4	2.74
	(174)	(100)	(218)	(133)	(225)	(147)	(376)	(90)	(12.6)	(44)	(44)	(2.02)	(61)	(57)	(4.36)
9	1299	–476	*1478*	*–622*	*1731*	*–749*	43	41	6.58	27	36	5.17	–11	15	1.31
	(171)	(90)	*(218)*	*(135)*	*(200)*	*(147)*	(275)	(144)	(7.26)	(171)	(111)	(3.12)	(35)	(50)	(1.35)

Notes: Italics indicate the best model for each data condition. Model 1: zero-order logit; Model 2: logit with GL (1983) loyalty; Model 3: logit with Fader–Lattin (1993) loyalty; Model 4: heterogeneity in preferences and market response (Kamakura and Russell 1989); Model 5: heterogeneity in choice sets (Siddarth, Bucklin, and Morrison 1995); Model 6: habit, state dependence, and heterogeneity (Roy, Chintagunta, and Haldar 1996); Model 7: choice set Model 5 with GL loyalty; Model 8: heterogeneity Model 4 with GL loyalty; Model 9: heterogeneity, state dependence, and choice sets.

3.47. This is again the bias predicted by Table 1, Panel C (1.36 + 2.11 = 3.47). More generally, the between-condition differences among all three measures (preference, market response, and loyalty) in Table 5, Panels B, C, and D, can be reproduced by examining the regression models in Table 1, Panels A, B, and C.

In Table 5, Panel C, we show the means for model type by heterogeneity condition. Model 7 has the best predictive validity when there is no heterogeneity, as we would expect. Model 9 has the best predictive validity when there is discrete heterogeneity, also as expected. However, Model 7 has the best predictive validity when there is continuous heterogeneity, which implies that latent-class models (e.g., Model 9) are not effective in recovering continuous heterogeneity distributions. This pattern of model performance also holds up with parameter bias: Model 7 is best in the no-heterogeneity and continuous-heterogeneity conditions, and Model 9 best in the discrete-heterogeneity condition. The models with underspecified discrete heterogeneity (e.g., Models 2, 5, and 7) have greater than 100% biases in preference constants. This is consistent with the regression results in Table 1, Panel B.

Comparing Models 2 and 8 in the no-heterogeneity condition in Table 5, Panel C, we note that there are spurious heterogeneity effects in some conditions, because these two models do not produce the same results. If there were no spurious heterogeneity effects, Models 2 and 8 would produce the same results in the no-heterogeneity condition, as Models 2 and 7 did in the no-choice-set-effects condition in Table 5, Panel B. Although the BIC value is not lower for Model 8 than for Model 2 in the no-heterogeneity condition, the parameter biases for the two models are very different, which indicates that the two models do not find the same solutions.

In Table 5, Panel D, we show the means and standard deviations by state dependence condition. When state dependence is first order (λ = 0), Model 6 (Roy, Chintagunta, and Haldar 1996) has the best fit. This is the only model equipped with a lagged dependent variable, which is the most parsimonious and most effective way to model first-order state dependence. However, the parameter biases for Model 6 in this condition have much larger standard deviations than some other models. For the most typical state dependence condition (λ = .75), Model 9 has the best predictive validity, and Model 7 has the best parameter bias. The average loyalty coefficient for Model 7 is 3.83, which is close to its true value of 4.00. When there is no state dependence (λ = 1), Model 9 again has the best predictive validity (despite overspecifying the state dependence), and Model 7 again has the best overall parameter bias. Notice that the average loyalty coefficient for Model 7 is 1.29, compared with the true value of zero in this condition. This is because the presence of serial correlation in some conditions inflates the loyalty coefficient. Model 2 has an even larger loyalty coefficient (2.31), because it underspecifies serial correlation and choice sets, both of which produce positive bias in the loyalty coefficient when underspecified.

In summary, we show in Table 5 that the logit model with choice set effects and GL loyalty (Model 7) has the best overall parameter bias numbers across data conditions, whereas Model 9 (latent-class logit with GL loyalty and choice sets) is best in terms of predictive validity. However, there is evidence that Model 9 achieves its stellar predictive performance (and not-so-stellar parameter bias variances) by means of spurious effects, which we examine more closely in the next section.

Analysis of Spurious Effects

In Table 6, we show fit and prediction statistics (Panel A) and the bias in parameters (Panel B) for each model fit to selected data conditions. The boldface text in each column indicates the correct model for the data condition a priori; italics indicate the best model according to predictive validity. Of the 81 experimental conditions, we examine only 6 of the conditions in Table 6 to determine the degree of spurious effects in choice models. There are three replications in each cell, so the numbers in Table 6 are averages across the three replications. In contrast, the results of Table 1 are based on the entire experimental design containing three replications in each of 81 cells.

When the data contain no effects, the zero-order logit model is preferred according to BIC, as we would expect. Although there is no strong evidence of spurious effects in this condition, we note that the latent-class models did not appear to find the same solution as the homogeneous models. The validation log-likelihood is slightly lower for the latent-class models. None of the models (with the possible exception of Model 3) shows serious parameter bias.

When the data contain only state dependence (λ = .75), Model 2 is the preferred model. Model 2 has the best BIC value and the best log-likelihood value (shared with Model 7, which nests Model 2 and finds approximately the same solutions). The parameter bias averages 5% for the preference constants and 3% for the market response coefficients, and the average loyalty coefficient is 3.58 (–10.5% bias). Model 8 nests Model 2 and should find the same solution in the absence of heterogeneity, but it does not. This suggests some spurious heterogeneity, though not enough that BIC favors Model 8 over Model 2. Model 8 overspecifies heterogeneity in preferences and market response, but the loyalty coefficient is biased substantially (52.75%) upward. We might expect spurious heterogeneity to explain some of the state dependence, thereby producing downward bias in the loyalty coefficient. This finding does not hold for Model 9, so we refrain from attempting to draw conclusions.

Also in data condition 2, Models 4 and 5 (heterogeneity and choice sets, both without GL loyalty) fit better than the logit model without GL loyalty (Model 1), which suggests spurious effects. However, the spurious choice set effects disappear when the GL loyalty variable is included (compare BIC for Models 2 and 7), as do the spurious heterogeneity effects (compare BIC for Models 2 and 8). Thus, it is important that heterogeneity and choice set components not be included in models unless there is a GL loyalty variable. It is common practice in the literature to model heterogeneity but not state dependence (Chintagunta 1994; Dillon et al. 1994; Kamakura and Russell 1989), but this study does not support that practice.

When there is only serial correlation (data condition 3), we expect Model 6 to be best because it is intended to model serial correlation, but this is not the case. Model 2 has the best fit and predictive accuracy, though state dependence is grossly overstated because of the serial correlation (the true coefficient is zero, whereas the average

Table 6
MEANS FOR SELECTED EFFECTS

A: Parameter Bias—Average Percent Bias in Preferences, Market Response Coefficients, and Loyalty Coefficients × 100

Data Contains Only

Model	*(1) No Effects* Preference Constants	Response Coefficients	Loyalty Coefficients	*(2) State Dependence (λ = .75)* Preference Constants	Response Coefficients	Loyalty Coefficients	*(3) Serial Correlation (ρ = .90)* Preference Constants	Response Coefficients	Loyalty Coefficients	*(4) Discrete Heterogeneity* Preference Constants	Response Coefficients	Loyalty Coefficients	*(5) SBM Choice Set Usage* Preference Constants	Response Coefficients	Loyalty Coefficients	*(6) Effects 2, 3, 4, and 5* Preference Constants	Response Coefficients	Loyalty Coefficients
1	***–3***	***–3***	—	40	–16	—	21	6	—	–111	0	—	–46	–43	—	–110	–51	—
2	–4	–3	.00	***5***	***3***	***3.58***	*–8*	*21*	*3.17*	–110	1	.74	–79	–30	3.44	–107	–10	4.94
3	–18	–3	—	26	0	—	5	19	—	–169	1	—	–53	–25	—	–107	–19	—
4	2	4	—	46	–6	—	29	22	—	***16***	***9***	—	–90	–38	—	–47	–47	—
5	–3	–2	—	39	–5	—	19	17	—	–111	2	—	***–22***	***–2***	—	–109	–18	—
6	6	9	—	43	3	—	**31**	**30**	—	18	9	—	28	27	—	–59	–31	—
7	–3	–2	–.06	6	3	3.56	–8	21	3.17	–110	1	.69	–22	–2	.00	–109	2	3.93
8	9	10	–.08	59	112	6.11	9	39	3.18	15	9	.22	–64	–21	2.89	–57	–11	6.14
9	8	12	–.18	51	17	3.77	20	45	3.20	17	10	.02	–14	27	–.20	*–40*	*20*	*5.70*

B: Predictive Validity—Average BIC from Estimation Sample and Log-Likelihood from Validation Sample

Data Contains Only

Model	*(1) No Effects* BIC	Log-Likelihood	*(2) State Dependence (λ = .75)* BIC	Log-Likelihood	*(3) Serial Correlation (ρ = .90)* BIC	Log-Likelihood	*(4) Discrete Heterogeneity* BIC	Log-Likelihood	*(5) SBM Choice Set Usage* BIC	Log-Likelihood	*(6) Effects 2, 3, 4, and 5* BIC	Log-Likelihood
1	***1719***	***–773***	1785	–867	1578	–778	1913	–941	2353	–1142	2706	–1332
2	1734	–773	***1459***	***–680***	*1386*	*–657*	1920	–939	1950	–933	1519	–726
3	1764	–773	1498	–693	1414	–665	1947	–942	1914	–867	1537	–737
4	1772	–792	1775	–823	1579	–759	***1534***	***–607***	2310	–1063	2489	–1158
5	1734	–773	1700	–843	1533	–759	1925	–941	***1712***	***–768***	1747	–920
6	1793	–795	1664	–785	**1527**	**–716**	1556	–608	2200	–1037	1860	–850
7	1748	–774	1474	–681	1402	–657	1935	–939	1724	–771	1459	–684
8	1796	–799	1525	–700	1428	–675	1563	–607	1976	–890	1413	–584
9	1827	–799	1557	–700	1456	–680	1595	–608	1773	–754	*1316*	*477*

Notes: Boldface indicates the correct model for the data condition a priori; italics indicate best model according to predictive validity. Model 1: zero-order logit; Model 2: logit with GL (1983) loyalty; Model 3: logit with Fader–Lattin (1993) loyalty; Model 4: heterogeneity in preferences and market response (Kamakura and Russell 1989); Model 5: heterogeneity in choice sets (Siddarth, Bucklin, and Morrison 1995); Model 6: habit, state dependence, and heterogeneity (Roy, Chintagunta, and Haldar 1996); Model 7: choice set Model 5 with GL loyalty; Model 8: heterogeneity Model 4 with GL loyalty; Model 9: heterogeneity, state dependence, and choice sets.

estimated coefficient is 3.17). Model 7 finds the same solution as Model 2, which indicates no spurious choice set effects, but Model 8 does not, which is again indicative of some spurious heterogeneity.

In data condition 4, there is discrete heterogeneity. Model 4 is correctly specified and is the preferred model. The parameter bias figures are good (16% for preference constants, 9% for response coefficients). Model 2, which models state dependence but underspecifies heterogeneity, estimates a brand loyalty coefficient of .74 (when the true value is zero). However, the loyalty variable does not significantly improve the fit of the model (compare BICs for Models 1 and 2). This is consistent with Table 1, Panel C, which shows that underspecified heterogeneity has no significant effect on the loyalty coefficient. This finding again suggests that there is not likely to be spurious state dependence when heterogeneity is underspecified. Previous research (e.g., Keane 1997; Roy, Chintagunta, and Haldar 1996) suggests that underspecified heterogeneity exaggerates state dependence, but this is contrary to our findings. Our study suggests that the elaborate heterogeneity structures used in those studies were instead explaining what was truly state dependence. In any case, the finding that the GL loyalty measure does not significantly exaggerate state dependence when there is underspecified heterogeneity is new and important, and it reflects positively on the use of the GL loyalty measure.

The small amount of bias in response coefficients when heterogeneity is underspecified deserves some mention. This result may be conservative. When there is discrete heterogeneity in data and a homogeneous logit model is estimated, the estimated coefficients are roughly averages of the segment-specific coefficients (each segment contains roughly 50% of the sample). For example, if the segments have true coefficients 1 and 5 (notice that both coefficients have the same sign), as is the case for one of the marketing-mix variables, the estimated coefficient may be near three. One segment will have a positive bias, and the other will have a negative bias. The biases will at least partially offset each other when averaged across consumers. After averaging these biases across four predictors, the average bias across segments may be near zero. In contrast to the small biases for market response coefficients, biases for the preference constants were much larger. Consider another example in which the true coefficients of the two segments have opposite signs (say, –2 and 2), as the preference constants did. No matter what the value of the estimated coefficient, the bias will be near –100%. The biases near zero for response coefficients and the biases near 100% for preference constants for the underspecified models in data condition 4 therefore may be heavily dependent on the true parameter values chosen for the analysis. Regardless, the analysis indicates that severe parameter bias is possible when there is underspecified discrete heterogeneity. We would not be comfortable assuming that underspecified discrete heterogeneity produces no bias in response coefficients. It must produce bias at the individual level, though it may not be apparent after averaging across consumers and predictors, depending on the true values of the coefficients.

Data in condition 5 contain promotion expansion (SBM) choice set formation. As expected, Model 5 is preferred in this condition. Parameter biases for this model are reasonable, but not excellent (–22% for preference constants and –2% for market response coefficients). When choice set usage is underspecified and state dependence is overspecified (Model 2), the GL loyalty variable grossly exaggerates the degree of state dependence, with an estimated coefficient of 3.44 and a true value of zero. However, Model 7 (which contains a loyalty variable) finds the same solution as Model 5, with a loyalty coefficient estimate of zero, so choice set usage and state dependence are not likely to be confused if both are modeled. There also appears to be spurious heterogeneity when choice sets are underspecified (compare BICs for Models 1 and 4).

Data condition 6 combines state dependence (λ = .75), serial correlation (ρ = .90), discrete heterogeneity, and SBM choice set usage. This data condition represents the worst-case scenario for applied choice modelers. There is no a priori best model for these data, because all models are underspecified. Model 9 is the best overall model according to BIC, and it has the lowest bias in preference constants. Notice that, even with the best model, some of the biases are high in this condition: –40% for preference constants, 20% for market response parameters, and 42.5% for the loyalty coefficient.

SUMMARY OF FINDINGS

We summarize the main findings of this study as follows: First, underspecified choice set effects may result in substantial bias. Bias in market response parameters resulting from underspecified promotion expansion (Siddarth, Bucklin, and Morrison 1995) choice set effects was –27%, whereas bias in preference constants was –24% (though not significant), and the brand loyalty coefficient increases by 1.36. No serious bias resulted when choice set effects were generated using the more compensatory strategy suggested by Bronnenberg and Vanhonacker (1996). However, the bulk of empirical evidence at this juncture suggests that true choice set effects are more likely to involve noncompensatory screening such as that suggested by Siddarth, Bucklin, and Morrison (1995). Overspecified choice set effects did not produce any bias in any data condition.

Second, underspecified heterogeneity in preferences and market response results in significant parameter bias if there are discrete segments of consumers. Our analysis showed –105% bias in preference constants when the heterogeneity is underspecified. However, biases in response coefficients (9%) and the loyalty coefficient (a .71 decrease in the coefficient) were not statistically significant, though the result for response coefficients may be conservative. When the distribution of coefficients is assumed to be normal (not discrete), the bias resulting from underspecification is not significant. Whether real-world applications have discrete or continuous distributions of preferences therefore becomes a crucial question, one in need of research. At this time, however, there is some speculation that the assumption of discrete heterogeneity distributions may not be realistic. Overspecification of heterogeneity results in spurious heterogeneity effects, especially when other components (e.g., state dependence, choice sets) are underspecified.

Third, underspecified state dependence has serious consequences for bias in response coefficients and fit. When state dependence was first order, we observed a –40% bias in response coefficients. When state dependence was generated with the commonly indicated value of λ = .75, bias in response coefficients was –18%.

Fourth, underspecification of serial correlation has serious consequences only at extreme levels ($\rho = .90$) and only for the loyalty coefficient. Presence of serial correlation caused the loyalty coefficient to be inflated by 1.39. There was no bias in preference constants or market response parameters. Therefore, there seems to be less incentive to build models to control for serial correlation in residuals.

Fifth, the logit model with choice set effects and GL loyalty has the best overall parameter bias. Latent-class logit models with GL loyalty and choice set effects produced the best fit and forecasting performance of any models. Unfortunately, the parameter biases for the latter models had high variation, partially as a result of their tendency to produce spurious effects. High variation in biases is undesirable because it indicates that sometimes the model estimates a drastically wrong solution.

Sixth, underspecified state dependence results in spurious heterogeneity, but underspecified heterogeneity does not result in exaggerated state dependence. On the contrary, our regression model estimates that underspecified heterogeneity causes the loyalty coefficient to be understated, though not significantly so. Spurious heterogeneity was less of a problem when the GL loyalty variable was included in the specification, though it did not disappear completely. It is therefore important that the GL loyalty variable be included in heterogeneity models. It is common practice in the literature to model heterogeneity and not state dependence, but this results in spurious heterogeneity.

Seventh, underspecified choice set effects result in exaggerated state dependence, and underspecified state dependence results in spurious choice set effects. However, choice set models with GL loyalty do not have difficulty distinguishing choice set effects from other unobserved effects. Without the GL loyalty variable, choice set effects and state dependence may be confused.

Eighth, underspecified choice set effects result in exaggerated heterogeneity, but underspecified heterogeneity does not result in spurious choice set effects. The model intended to account for both heterogeneity and choice set effects (Model 9) did not recover parameters as well as expected, though the fit of the model was excellent.

Ninth, the GL loyalty measure recovered state dependence well in all conditions. It does not exaggerate state dependence when there is underspecified heterogeneity, as is commonly conjectured. However, the GL loyalty variable exaggerates state dependence when there is underspecified choice set usage or serial correlation.

Tenth, when correctly specified, most models recover parameters reasonably well. However, when there are multiple unobserved effects present in the data, even the most suitable models have difficulty recovering parameters. For example, when there were state dependence effects, serial correlation, discrete heterogeneity, and SBM choice set effects present in the data, even the best model (Model 9) had a –40% bias in preferences, a 20% bias in response coefficients, and a 43% bias in the loyalty coefficient. This level of bias may or may not be acceptable given the objective of the study. There is certainly room for improvement.

Eleventh, in general, it is difficult to infer parameter bias on the basis of predictive accuracy. The correlation matrix of four of the measures across experimental conditions is shown in Table 7. For example, across experimental conditions, the correlation of BIC and bias in market response coefficients is –.3698. That BIC and log-likelihood are negatively correlated makes sense because we prefer small values of BIC and large values of log-likelihood. It would be particularly difficult to predict bias in preference constants on the basis of BIC or log-likelihood. Thus, it may not be sufficient to infer parameter bias (or lack thereof) on the basis of a model's fit and forecasting capability. Spurious effects often result in excellent fit but poor parameter estimates.

CONCLUSIONS

In this study, we report the results of a large-scale simulation experiment. The major goals were to assess the parameter bias of today's commonly used choice models and to assess the potential for obtaining spurious state dependence, heterogeneity in preferences and market response, serial correlation, and choice set effects when these models are used.

The first major conclusion is that examination of parameter bias provides some important insights beyond those provided by the traditional predictive validity approach to model selection. We encountered cases in which traditional model selection criteria such as BIC chose models with extremely poor parameter bias, and vice versa. In general, correlations between measures of parameter bias and predictive validity were low. This finding emphasizes the importance of using simulation methods to verify that new models are indeed recovering the values of known parameters instead of relying only on predictive validity in a scanner-panel data setting. It is our contention that simulation methods and scanner data analysis both provide essential information for choosing appropriate models and that neither is adequate if used alone.

A second major conclusion is that the potential for parameter bias in choice model applications appears to be high. Underspecifying state dependence, heterogeneity, and choice set effects can result in substantial parameter bias. Overspecifying heterogeneity introduces the possibility of spurious heterogeneity effects and significant parameter bias, especially when other components are underspecified.

A third major group of results sometimes confirms and sometimes contradicts conventional wisdom about the effects of spurious effects in choice models. For example, underspecified state dependence results in spurious hetero-

Table 7

CORRELATION MATRIX ACROSS EXPERIMENTAL CONDITIONS

	Preference Bias	*Response Bias*	*BIC*	*Validation Log-Likelihood*
Preference bias	1.0000			
Response bias	.3512	1.0000		
BIC	–.0169	–.3698	1.0000	
Validation log-likelihood	.0444	.3626	–.9582	1.0000

geneity, but underspecified heterogeneity does not result in exaggerated state dependence. Likewise, underspecified choice set effects result in spurious heterogeneity, but underspecified heterogeneity does not result in spurious choice set effects. However, underspecified choice set effects result in exaggerated state dependence, and vice versa. One finding was that spurious effects are minimized by including a GL loyalty variable in all model specifications. In general, the GL loyalty specification was shown to perform well in a variety of settings, despite frequent criticism of the construct in the literature.

In addition, the regression models in Table 1 can be used to decide which model to use, given a set of assumptions about the likely effects in the data. For example, if the researcher believes that there are state dependence and choice set effects in the data, Table 1 can be used to predict the likely biases in preference constants, market response parameters, and the loyalty coefficient for each of the models. The model with the most appealing biases could then be used to model the data. Thus, the study can be used to make specific recommendations regarding model specification. In general, logit models with choice sets and GL-type loyalty variables have the best parameter bias under a wide range of circumstances. Another finding is that models with latent-class heterogeneity obtain the correct solution with much less certainty than other models, though their predictive validity is excellent. More work must be done to improve the coefficient estimates for such models. The study suggests that GL loyalty variables should always be included in choice models unless there is no chance of state dependence. Finally, serial correlation in the error terms produces bias only in the loyalty coefficient, so research in this area probably does not have a strong justification.

Finally, this study suggests some possible modeling strategies that can reduce the potential for biases and spurious effects. As discussed previously, combining simulation methods and scanner data methods has great appeal. For a typical new model study, researchers could use the scanner data as the basis for generating artificial choices. The choices would be generated such that they contain known components (including data with and without the new component being modeled), and the new model could be applied along with several carefully selected benchmark models. The new model must be shown to have acceptable parameter bias before it is applied to actual scanner data. The results of the scanner data analysis can also be compared with those of the simulation analysis to look for common patterns in the findings.

Given the important role of simulation in model specification issues, there are surely other important factors that affect parameter bias and predictive validity. First, the performance of models with continuous distributions for coefficients is in need of study. Perhaps some of the findings observed for the models in this study would not hold for such models. Second, frequency of promotions, brand preference concentration, correlation between independent variables, and parametric versus semiparametric model specifications, among others, should be investigated. Third, in the context of latent-class models, the effects of total sample size; distribution of consumers in segments; and distribution of heterogeneity on parameter bias, prediction accuracy, and percentage of correct allocation of subjects to segments could be investigated. Fourth, another important issue, which may require both simulated data and actual data, is the nature of heterogeneity in scanner-panel applications. Our study suggests that the proper modeling strategy depends on whether heterogeneity is continuous or discrete. Perhaps coefficient distributions are multimodal. We hope our work will motivate such efforts.

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