

Towards a holographic theory of cosmology – threads in a tapestry

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Dedicated to the memory of Hendrik van Dam (1934-2013)

Abstract

In this Essay we address several fundamental issues in cosmology: What is the nature of dark energy and dark matter? Why is the dark sector so different from ordinary matter? Why is the effective cosmological constant non-zero but so incredibly small? What is the reason behind the emergence of a critical acceleration parameter of magnitude $10^{-8}cm/sec^2$ in galactic dynamics? We suggest that the holographic principle is the linchpin in a unified scheme to understand these various issues.

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1 Introduction: A holographic theory of cosmology

Some alert readers may have already noticed a resemblance between the title of this Essay and that of S. Glashow’s Nobel Lecture [1] “Towards a unified theory – threads in a tapestry.” This resemblance is not a coincidence, for like elementary particle physics, the study of cosmology is like a patchwork quilt. But whereas the patchwork quilt has become a tapestry for the former, the various threads have yet to be coherently woven for the latter. However now there is reason for optimism: we may have found a powerful guiding principle behind nature’s intricate design, yielding (eventually) a beautiful tapestry of gravity and matter. We are referring to the holographic principle [2, 3], an important by-product of the synthesis of quantum mechanics and general relativity, according to which, the maximum amount of information stored in a region of space scales as the area of its two-dimensional surface, like a hologram. The holographic principle is arguably the most important concept in quantum gravity, playing a role similar to the gauge principle in particle physics.

In this Essay we will apply the holographic principle to address a few fundamental issues in gravity and cosmology. One of the key issues in cosmology is to understand the nature of dark energy and dark matter and why the dark sector is so different from ordinary matter. Another issue is to explain the twin puzzles of why our universe is at or very close to its critical density and why the (effective) cosmological constant is nonzero and so small. At the (smaller) galactic scale, there are the issues of the observed flat rotation curves and the emergence of a critical acceleration parameter separating the regime where Newtonian dynamics works well from that where it appears to fail. We liken the resolution of all these issues to finding the right threads in a tapestry — interwoven coherently, with one thread logically leading to another.

2 From spacetime foam to cosmological constant Λ

As will be shown shortly, all the aforementioned issues are linked to the quantum nature of spacetime. Thus it behooves us to start by examining how foamy spacetime is, or, in other words, how large the quantum fluctuations of spacetime are.[4, 5] Let us consider mapping out the geometry of spacetime for a spherical volume of radius l over the amount of time $2l/c$ it takes light to cross the volume.[6] One way to do this is to fill the space with clocks, exchanging signals with the other clocks and measuring the signals’ times of arrival. The total number of operations, including the ticks of the clocks and the measurements of signals, is bounded by the Margolus-Levitin theorem [7] which stipulates that the rate of operations cannot exceed the amount of energy E that is available for the operation divided by $\pi\hbar/2$. This theorem, combined with the bound on the total mass of the clocks to prevent black hole formation, implies that the total number of operations that can occur in this spacetime volume is no bigger than $2(l/l_P)^2/\pi$, where $l_P = \sqrt{\hbar G/c^3}$ is the Planck length. To maximize spatial resolution, each clock must tick only once during the entire time

period. If we regard the operations as partitioning the spacetime volume into “cells”, then on the average each cell occupies a spatial volume no less than $\sim l^3/(l^2/l_P^2) = ll_P^2$, yielding an average separation between neighboring cells no less than $\sim l^{1/3}l_P^{2/3}$. [8] This spatial separation can be interpreted as the average minimum uncertainty in the measurement of a distance l , that is, $\delta l \gtrsim l^{1/3}l_P^{2/3}$,² in *agreement* with the result found in the Wigner-Salecker gedanken experiment³ to measure the fluctuation of a distance l .

We make two observations: [11, 12] First, maximal spatial resolution (corresponding to $\delta l \sim l^{1/3}l_P^{2/3}$) is possible only if the maximum energy density $\rho \sim (ll_P)^{-2}$ is available to map the geometry of the spacetime region, without causing a gravitational collapse. Secondly, since, on the average, each cell occupies a spatial volume of ll_P^2 , a spatial region of size l can contain no more than $\sim l^3/(ll_P^2) = (l/l_P)^2$ cells. Hence, this result for spacetime fluctuations corresponds to the case of maximum number of bits of information l^2/l_P^2 in a spatial region of size l , that is allowed by the holographic principle[2, 3].

It is straightforward to generalize [11] the above discussion for a static spacetime region with low spatial curvature to the case of an expanding universe by the substitution of l by H^{-1} in the expressions for energy and entropy densities, where H is the Hubble parameter. (Henceforth we adopt $c = 1 = \hbar$ for convenience unless stated otherwise for clarity.) Thus, applied to cosmology, the above argument leads to the prediction that (1) *the cosmic energy density has the critical value* $\rho \sim (H/l_P)^2$, and (2) the universe of Hubble size R_H contains $\sim HR_H^3/l_P^2 \sim (R_H/l_P)^2$ bits of information. It follows that the average energy carried by each particle/bit is $\rho R_H^3/I \sim R_H^{-1}$. Such long-wavelength *constituents of dark energy give rise to a more or less uniformly distributed cosmic energy density and act as a dynamical cosmological constant with the observed small but nonzero value* $\Lambda \sim 3H^2$.⁴ [8] Later we will show that these “particles”/bits have exotic statistical properties.

3 From Λ to MoNDian dark matter

The dynamical cosmological constant (originated from quantum fluctuations of spacetime) can now be shown to give rise to a critical acceleration parameter in galactic dynamics. The argument [14] is based on a simple generalization of E. Verlinde’s recent proposal of entropic gravity [15, 16]. Consider a particle with mass m approaching a holographic screen

²One way to detect this minute fluctuation is to look for blurry images of distant quasars in powerful telescope interferometers. [9]

³In the Wigner-Salecker experiment [10, 4], a light signal is sent from a clock to a mirror (at a distance l away) and back to the clock in a timing experiment to measure l . From the jiggling of the clock’s position alone, the uncertainty principle yields $(\delta l)^2 \gtrsim \hbar l/mc$, where m is the mass of the clock. On the other hand, the clock must be large enough not to collapse into a black hole; this requires $\delta l \gtrsim 4Gm/c^2$. We conclude that the fluctuation of a distance l scales as $\delta l \gtrsim l^{1/3}l_P^{2/3}$. [4, 5]

⁴Here we will not address the old cosmological constant problem of why it is not of the Planck scale. See Ref. [13] for possible solutions.

at temperature T . Using the first law of thermodynamics to introduce the concept of entropic force $F = T \frac{\Delta S}{\Delta x}$, and invoking Bekenstein's original arguments [17] concerning the entropy S of black holes, $\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x$, we get $F = 2\pi k_B \frac{mc}{\hbar} T$. In a deSitter space with cosmological constant Λ , the net Unruh-Hawking temperature, [18, 19, 20] as measured by a non-inertial observer with acceleration a relative to an inertial observer, is $T = \frac{\hbar}{2\pi k_B c} [\sqrt{a^2 + a_0^2} - a_0]$, [21] where $a_0 \equiv \sqrt{\Lambda/3}$. Hence the entropic force (in deSitter space) is given by $F = m[\sqrt{a^2 + a_0^2} - a_0]$. For $a \gg a_0$, we have $F/m \approx a$ which gives $a = a_N \equiv GM/r^2$, the familiar Newtonian value for the acceleration due to the source M . But for $a \ll a_0$, $F \approx m \frac{a^2}{2a_0}$, so the terminal velocity v of the test mass m in a circular motion with radius r should be determined from $ma^2/(2a_0) = mv^2/r$. In this small acceleration regime, the observed flat galactic rotation curves (v being independent of r) now require $a \approx (a_N a_0^3)^{\frac{1}{4}}$. But that means $F \approx m\sqrt{a_N a_0}$. This is the celebrated modified Newtonian dynamics (MoND) scaling [22, 23, 24], discovered by Milgrom who introduced the critical acceleration parameter a_0 by hand to phenomenologically explain the flat galactic rotation curves. Lo and behold, *we have recovered MoND with the correct magnitude for the critical galactic acceleration parameter* $a_0 \sim 10^{-8} \text{ cm/s}^2$. From our perspective, MoND is a *classical* phenomenological consequence of *quantum* gravity (with the \hbar dependence in $T \propto \hbar$ and $S \propto 1/\hbar$ cancelled out). [14] As a bonus, we have also recovered the observed Tully-Fisher relation ($v^4 \propto M$).

Having generalized Newton's 2nd law, we [14] can now follow the second half of Verlinde's argument [15] to generalize Newton's law of gravity $a = GM/r^2$ by considering an imaginary quasi-local (spherical) holographic screen of area $A = 4\pi r^2$ with temperature T . Invoking the equipartition of energy $E = \frac{1}{2} N k_B T$ with $N = Ac^3/(G\hbar)$ being the total number of degrees of freedom (bits) on the screen, as well as the Unruh temperature formula and the fact that $E = M_{total} c^2$, we get $2\pi k_B T = G M_{total}/r^2$, where $M_{total} = M + M_d$ represents the *total* mass enclosed within the volume $V = 4\pi r^3/3$, with M_d being some unknown mass, i.e., dark matter. For $a \gg a_0$, consistency with the Newtonian force law $a \approx a_N$ implies $M_d \approx 0$. But for $a \ll a_0$, consistency with the condition $a \approx (a_N a_0^3)^{\frac{1}{4}}$ requires $M_d \approx (\frac{a_0}{a})^2 M \sim (\sqrt{\Lambda}/G)^{1/2} M^{1/2} r$. This yields the dark matter mass density ρ_d profile given by $\rho_d(r) \sim M^{1/2}(r_v)(\sqrt{\Lambda}/G)^{1/2}/r^2$, for an ordinary (visible) matter source of radius r_v with total mass $M(r_v)$.⁵

Thus dark matter indeed exists! And the MoND *force law* derived above, at the galactic scale, *is simply a manifestation of dark matter!* [25] Dark matter of this kind can behave *as if* there is no dark matter but MoND. Therefore, we call it "MoNDian dark matter". Intriguingly the dark matter profile we have obtained relates, at the galactic scale, dark matter (M_d), dark energy (Λ) and ordinary matter (M) to one another. Moreover, our theory, unlike the MoND scheme, is compatible with cosmology, if one properly uses a fully relativistic source (including MoNDian dark matter) at the cluster and cosmic scales.[14]

⁵ This result can be compared with the distribution associated with an isothermal Newtonian sphere in hydrostatic equilibrium (used by some dark matter proponents): $\rho(r) = \sigma(r^2 + r_0^2)^{-1}$. Asymptotically the two expressions agree with σ identified as $\sim M^{1/2}(r_v)(\sqrt{\Lambda}/G)^{1/2}$.

4 Infinite statistics for the dark sector

Why is the dark sector so different from ordinary matter? The reason, as we will show in this section, is that *the quanta constituting the dark sector obey, not the familiar Fermi or Bose statistics as for ordinary matter, but rather an exotic statistics known as the infinite statistics.* [12]

First consider the $N \sim (R_H/l_P)^2$ “particles” constituting dark energy at temperature $T \sim R_H^{-1}$ (the average particle energy) in a volume $V \sim R_H^3$ that is the whole Hubble volume. Let us assume that the “particles” obey the familiar Boltzmann statistics. A standard calculation (for the relativistic case) yields the partition function $Z_N = (N!)^{-1}(V/\lambda^3)^N$, where $\lambda = (\pi)^{2/3}/T$, and the entropy $S = N[\ln(V/N\lambda^3) + 5/2]$. But now since $V \sim \lambda^3$, the entropy S becomes nonsensically negative unless $N \sim 1$ which is equally nonsensical because $N \sim (R_H/l_P)^2 \gg 1$. However, if the N inside the log term for S somehow is absent, then we have a manifestly non-negative $S \sim N \sim (R_H/l_P)^2$. That is the case if the “particles” are distinguishable and nonidentical, for then the Gibbs $1/N!$ factor is absent from the partition function Z_N . But the only known consistent statistics in greater than two space dimensions without the Gibbs factor is infinite statistics (sometimes called “quantum Boltzmann statistics”) [26, 27], as described by the Cuntz algebra (a curious average of the bosonic and fermionic algebras) $a_i a_j^\dagger = \delta_{ij}$. Thus the “particles” constituting dark energy obey infinite statistics. [12, 28] ⁶

Next, to show that the quanta of MoNDian dark matter also obey this exotic statistics, we [30] first reformulate MoND via an effective gravitational dielectric medium, motivated by the analogy [31] between Coulomb’s law in a dielectric medium and Milgrom’s law for MoND. We start with the nonlinear electrostatics embodied in the Born-Infeld theory [32], and write the corresponding gravitational Hamiltonian density as $H_g = b^2 \left(\sqrt{1 + D_g^2/b^2} - 1 \right) / (4\pi)$, where D stands for the electric displacement vector and b is the maximum field strength in the Born-Infeld theory. With $A_0 \equiv b^2$ and $\vec{A} \equiv b \vec{D}_g$, the Hamiltonian density becomes $H_g = \left(\sqrt{A^2 + A_0^2} - A_0 \right) / (4\pi)$. If we invoke energy equipartition ($H_g = \frac{1}{2} k_B T_{\text{eff}}$) and the Unruh temperature formula ($T_{\text{eff}} = \frac{\hbar}{2\pi k_B c} a_{\text{eff}}$), and apply the equivalence principle (in identifying, at least locally, the local accelerations \vec{a} and \vec{a}_0 with the local gravitational fields \vec{A} and \vec{A}_0 respectively), then the effective acceleration a_{eff} is identified as $a_{\text{eff}} \equiv \sqrt{a^2 + a_0^2} - a_0$. But this, in turn, implies that the Born-Infeld inspired force law takes the form (for a given test mass m) $F_{\text{BI}} = m \left(\sqrt{a^2 + a_0^2} - a_0 \right)$, which is precisely the MoNDian force law!

To be a viable cold dark matter candidate, the quanta of the MoNDian dark matter must be much heavier than $k_B T_{\text{eff}}$ since T_{eff} , with its quantum origin (being proportional to \hbar), is a very low temperature. Now recall that the equipartition theorem in general states that the average of the Hamiltonian is given by $\langle H \rangle = -\frac{\partial \log Z(\beta)}{\partial \beta}$, where $\beta^{-1} = k_B T$. To obtain

⁶Our result for the $N \sim (R_H/l_P)^2$ quanta of dark energy obeying infinite statistics has received support from Ref. [29] which shows that the entropy bound of infinite statistics obeys the area law.

$\langle H \rangle = \frac{1}{2} k_B T$ per degree of freedom, even for very low temperature, we require the partition function Z to be of the Boltzmann form $Z = \exp(-\beta H)$. But this is precisely the case of infinite statistics. [30]

5 Conclusion: Threads in a tapestry of holography

In summary, by examining the microscopic fluctuations of spacetime we have found that our universe is naturally at or close to its critical density. The application of the holographic principle then yields an effective dynamical cosmological constant of the observed value. Next we have provided an entropic/holographic interpretation behind Milgrom's modification of Newton's laws and have uncovered a critical galactic acceleration parameter of the correct magnitude whose value is intimately related to the dynamical cosmological constant. We have also explained how Milgrom's MoND can be viewed as a phenomenological manifestation of dark matter with a curious mass profile that connects, at the galactic scale, the dark matter content to the ordinary matter content and dark energy. In principle this dark matter mass profile can be checked by observations. [33] Last but not least, we have shown that the quanta of the dark sector obey infinite statistics; this may explain why the dark sector is so different from ordinary matter.

The last result could be profound. But, if true, it also makes an analysis of the dark sector considerably more difficult. The reason is that a theory of particles obeying infinite statistics, unlike ordinary quantum field theories, is not local⁷. [27] On the other hand, such a theory of MoNDian dark matter would be fundamentally quantum gravitational and thus would give very unusual and distinct yet-to-be explored particle phenomenology.

Now if indeed the quanta of the dark sector obey infinite statistics, then we may wonder whether quantum gravity is actually the origin of particle statistics and whether the underlying statistics is infinite statistics. Here is an intriguing thought [30]: Is it possible that ordinary particles that obey Bose or Fermi statistics are actually some sort of collective degrees of freedom? (For a discussion of constructing bosons and fermions out of particles obeying infinite statistics, see Ref. [34] and [35].)

Using the holographic principle as our beacon, we have taken some small yet tightly logical steps towards a comprehensive understanding of how our universe works – from the foaminess of spacetime to the critical cosmic energy density, from the dynamical cosmological constant via the holographic principle to the critical acceleration parameter in local galactic dynamics, and from dark matter with MoNDian scaling to the dark sector obeying infinite statistics. All these various issues of cosmology have been found to be inter-related – like inter-connecting patches of a quilt, woven together. Yet this is obviously work in progress.

⁷The fields are not local, neither in the sense that their observables commute at spacelike separation nor in the sense that their observables are pointlike functionals of the fields. The expression for the number operator is both nonlocal and non-polynomial in the field operators, and so is the Hamiltonian.

New threads will have to be added, loose ones to be tightend, and some old ones to be overwoven. But we are hopeful that the end product will be a magnificent tapestry.

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