

The Timing of Pay

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Abstract

There exists large and persistent variation in not only *how*, but *when* employees are paid, a fact unexplained by existing theory. This paper develops a simple model of optimal pay timing for firms. When workers have self-control problems, they under-save and experience volatile consumption between paychecks. Thus, pay whose delivery matches the timing of workers' consumption needs will reduce wage costs. The model also explains why pay timing should be regulated (as it is in practice): although the worker benefits from a timing profile that smoothes her consumption, her lack of self-control induces her to attempt to undo the arrangement, either by renegotiating with her employer or taking out payday loans. Regulation of pay timing and consumer borrowing is required to counter these efforts, helping the worker help herself.

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JEL: H53, I38, J31, J33

Pay him his wages each day before sunset, because he is poor and is counting on it. Otherwise he may cry to the Lord against you, and you will be guilty of sin.

– Deuteronomy 24:15

Wages can vary along three dimensions. *Level* differences, such as a car salesman earning \$40,000 versus a librarian earning \$30,000, are usually attributed to workers having different value marginal products or outside options.¹ *Structure* differences, such as a bartender being paid mostly in tips versus a salaried postal worker, typically arise in response to incentive or information problems. *Timing* differences, the subject of this paper, are variations in the temporal patterns of when pay, for a given level and structure, is disbursed to employees. Examples would include a farm issuing laborers weekly, instead of monthly, paychecks, a bank awarding bonuses to its tellers around Christmas, or a university spreading out a professor’s nine month salary over twelve months.

In contrast to an extensive theoretical literature on the first two dimensions, there is a comparative absence with respect to pay timing. This paper is an initial attempt to address this void.

Our analysis is motivated by two facts. First, under standard assumptions, the timing of wage payments should not matter—workers can save or borrow to create any timing profile they desire—but the data suggest otherwise. The timing of bonuses is an example: in many cases, employers temporarily boost wages to coincide with holidays (Christmas bonuses in North America), vacation (summer bonuses in Greece), or job transitions. The goal, it seems, is to minimize the *time* between when money is delivered, and when it is spent. Another common example of timing is pay frequency, i.e., how often workers are regularly paid for their efforts. Figures 1, 2 and 3 show that the variation in U.S. worker pay frequency is large and non-random, varying with the worker’s education, financial sophistication, and income. Rather than being arbitrary or irrelevant, pay timing mechanisms appear to be addressing a fundamental economic problem—one rooted in time.

Second, pay timing is often regulated. In the U.S., 45 states explicitly legislate pay frequency, often by type of work. For example, with the exception of executive, administrative, and professional workers, the state of Maryland requires firms to issue paychecks at least twice a month. Pay timing is also regulated internationally. In many countries, holiday bonuses are mandatory. The Mexican *aguinaldo* and Indonesian *Tunjangan Hari Raya*, for example, are bonuses paid at Christmas and

¹It has also been noted that wage levels can serve incentive (Lazear and Rosen, 1981, Shapiro and Stiglitz, 1984) or signaling (Hayes and Schaefer, 2009) functions.

Ramadan, respectively. As of this writing, Greek workers are still by law awarded “14 months” of pay per year, with one additional month’s pay delivered at Christmas, one-half month’s at Easter, and the balance during the summer holidays. Other examples abound.

These observations set the bar for any plausible theory: pay timing should influence worker welfare, and should benefit from regulation. We propose a simple framework that yields both implications.

Consider a savings problem involving a present-biased worker.² When she receives a paycheck, she faces a strong urge to consume a large fraction of it immediately, even though she knows this will leave her poor in future periods. Although she recognizes her own self-control problems, she cannot stick to a pre-determined consumption schedule. Consequently, her realized consumption path will not maximize her *ex ante* welfare.

Because time is the culprit, it follows that her employer can improve her welfare by closing the gap between when she receives money and when she would prefer, *ex ante*, to spend it. Essentially, the firm chooses a timing profile that reduces the worker’s reliance on her own (inadequate) ability to commit to a future spending path. Moreover, to the extent that the worker understands this *ex ante*, a well-timed pay profile will reduce the overall wage the worker is willing to accept, and therefore firm costs. Basic calculations suggest that the welfare benefits—and therefore wage savings—can be large, depending on the worker’s lack of self-control. For example, a worker with logarithmic utility and a one-period discount factor that is 30% less than the long-run discount factor would demand a 4% premium to be paid monthly instead of weekly.

Although very simple, this benchmark model easily explains many, if not most, of the empirical patterns related to pay timing. Analyzed over longer horizons, holiday, vacation, and signing bonuses are all shown to help workers save for large, relatively infrequent expenditures. Over shorter horizons, the model also applies to more regular expenditures such as monthly bills, and can thus explain cross-sectional patterns in pay frequency. Moreover, the model’s predictions also line up broadly with the cross-sectional evidence. Workers who make less—and therefore have less of a savings buffer with which to smooth consumption—should be paid more frequently, a finding overwhelmingly true in the data. Also, to the extent that the accumulation of financial assets or

²Numerous experimental and field studies point to people having time-varying discount rates. See Laibson (1997), Barro (1999), O’Donoghue and Rabin (1999), Jovanovic and Stolyarov (2000), Harris and Laibson (2001), Gul and Pesendorfer (2001), Fernández-Villaverde and Mukherji (2002), and Laibson, Repetto and Tobacman (forthcoming).

education proxies for self-control, the data also confirm the model’s predictions.

Having established conditions under which pay timing matters for welfare, we then move to our second question: “Why is regulation needed?” This question is relevant because the results above, being derived from a firm’s optimization problem, would not appear to require legislative intervention. The reason, as in almost all models of time-inconsistency, is due to the incentive to renegotiate. Specifically, a worker with self-control problems will always want to “sell” the firm her future wages, even at a discount, because of her high short-run discount rate. Assuming that there is any room for such renegotiation (i.e., that the worker will not quit after receiving an advance), the firm will agree. Thus, in order for the benefits of better timing—e.g., holiday bonuses or frequent regular paychecks—to accrue to workers, a commitment device is needed. The law provides such a device. This prediction is consistent with the ubiquity of pay timing regulation, from laws governing pay frequency in the U.S. (see Table 1), to the dozens of international laws specifying mandatory bonuses at certain times.³

We conclude our analysis by allowing the worker another way to undo the firm’s desired timing profile: borrowing. To this point, we have neglected credit markets, not because they are not attractive to workers with self-control problems, but precisely the opposite. Because of their desire to overconsume, present-biased consumers will quickly exhaust all or most of their debt capacity, thus collapsing the problem to the no-borrowing case. This argument is consistent with recent empirical work: Lusardi, Schneider, and Tufano (2011) find that nearly half of Americans in 2009 were either certainly or probably unable to raise \$2000 in 30 days, suggesting that credit constraints are a severe problem for a large fraction of US citizens. Among the strongest predictors of “financial fragility” are low educational attainment and a lack of financial education, two variables that we find correlate strongly with pay frequency. For workers with remaining debt capacity, we are interested in whether pay timing still affects welfare, and in particular, whether well-placed regulations on credit markets can make a difference.

For two reasons, we focus on the interaction between *payday loans* and paycheck frequency. First, as their name suggests, payday loans are collateralized directly by a worker’s paycheck and thus, when used in series, are capable of continually altering the firm’s chosen timing profile. Second,

³Of course, the existence of such laws does not prevent some amount of renegotiation from occurring, but the scope for such activities is likely reduced. Some firms may develop reputations for not renegotiating wage timing profiles, making regulation unnecessary, but regulation is still likely to be relevant for firms lacking this ability.

the high interest rates often charged by payday vendors often make them lenders of last resort, and thus, likely apply to a large group of workers who are otherwise credit constrained.

The most important result of this analysis is that *perpetual* payday loan usage can in fact improve worker welfare, even: 1) with relatively high interest rates, and 2) without stochastic consumption shocks. Although perhaps counter-intuitive, the intuition is that the longer the gap between paychecks, the more volatile a time-inconsistent worker's consumption profile. Allowing the worker access to a payday loan at the end of the pay cycle thus delivers consumption when it is most needed, similar to the rationale given by, e.g., Morse (2011), who documents the beneficial aspects of payday lending after natural disasters. What our analysis shows however, is that for a worker with present bias, *each pay cycle may bring its own predictable mini-crisis*, meaning that habitual payday loan access can smooth consumption over the long term, with large aggregate effects.

Importantly, any welfare improvement requires that payday loans be capped in both amount (relative to the worker's check) and when they may be accessed in the pay cycle. Moreover, both are a function of the worker's prevailing pay timing profile. The connection between pay frequency and payday loans is, we believe, both novel and important for policy. Inspection of state-by-state payday laws and pay frequency laws reveals little relation, suggesting possibilities for policy improvements.

We view our paper as making three contributions. The first is emphasizing pay timing broadly as an important competitive and policy choice, one which we think has large welfare implications for the millions whose consumption appears tied to paycheck receipt.⁴ The second is to introduce a simple theoretical framework consistent not only with pay timing mattering at all (under standard assumptions it will not), but also with the cross-sectional evidence on pay frequency and regulation. The third is to highlight the connection between an employer's choice of pay timing and the worker's attempts to undo it with payday lending, with a particular eye on policy implications. Because both payday lending and pay frequency are often regulated, our analysis indicates that joint regulation is worthwhile.

The goal of parsimony is worth emphasizing. While our model appears to reconcile the empirical patterns related to pay timing and its regulation, it certainly does not imply that *all* aspects of pay timing result from firms or governments attempting to accommodate time-inconsistency. For example, the timing of performance or signing bonuses for CEOs clearly is not meant to smooth

⁴For example, see Parker (1999), Shea (1995), Souleles (1999), and Stevens (2003, 2006, 2008).

consumption and, likewise, the nature of the job can dictate when cash is exchanged, e.g., when a construction job is completed and no future interactions are expected. There may also be situations where mutual distrust between workers and firms dictates frequent pay, even without time inconsistency. These alternatives/exceptions notwithstanding, we believe that a single, simple model that explains many facts is superior to a set of customized models explaining the same facts, especially as a starting point for further research.

It is also worth noting that even simpler models—particularly those with credit constraints *but no self-control problems*—often have trouble offering good explanations for pay timing. Problems arise on both theoretical and empirical fronts. Theoretically, note that a time-consistent worker will have smooth consumption regardless of the frequency of pay, so that after a few periods of savings, consumption and pay timing are not linked. Empirically, we observe significant regulation of pay-timing, a fact difficult to reconcile with a model lacking commitment problems.

To our knowledge, our treatment of pay timing is novel, and there are many interesting extensions that we do not model.⁵ Perhaps the most interesting concerns the worker’s problem in coordinating the receipt and disbursements of payments. While in our model we take the worker’s consumption needs as given, workers actually have considerable leeway in timing their payments *to* firms to match the receipt of wages *from* firms. For example, many lenders allow workers to “choose the due date” of loan payments (most likely so that payments come due shortly after workers receive paychecks) and utilities often give customers the option of paying equal amounts throughout the year, allowing them to better balance their monthly expenditures. This coordination problem also implies that workers who function largely in a credit-based economy should be paid monthly, as most bills are due monthly, while workers functioning in a cash-based economy should be paid much more frequently. It also implies that creditors have an incentive to match the frequency of due-dates with the most common frequency of pay for their customers. This seems to be valid empirically, as landlords in lower income areas are more likely to charge rent on a weekly basis, consistent with data in Figures 2 and 3 indicating that fully 20% of workers receiving weekly pay did not graduate from high school, and have lower incomes. We do not address these issues directly, but they immediately

⁵While several papers have identified justifications for moving pay earlier or later in an employee’s tenure (e.g., Lazear and Rosen 1981, Holmstrom 1982, Van Wesep 2010), the passage of time in these and similar models is incidental, while in ours it is the central focus. These papers are concerned with moving money before or after certain realizations of information and therefore concern pay structure—the dependence of pay on outcomes and information—not timing.

follow from the broader observation that the timing of payments matters.

Section I describes a wide set of stylized facts related to pay timing. Section II introduces the model and features results concerning the timing of bonuses and frequency of pay. In Section III, we show that when the worker and firm can renegotiate contracts, the problem unravels, admitting a role for regulation that enforces contract terms. In Section IV we consider the effect of payday lending on welfare, showing that it is an imperfect substitute for more frequent pay. It can benefit workers by shortening the pay cycle, but only if the amount of a loan is capped. Section V considers how changing/relaxing the assumptions in our model would change its empirical implications, and Section VI addresses some interesting issues in the provision of government assistance. Section VII concludes. Where not in the text, proofs can be found in the appendix.

I Stylized Facts Related to Pay Timing

To motivate the model, we begin with a brief discussion of several mechanisms that alter the timing of wages and/or expenditures. This is not intended as an exhaustive summary, but simply meant to both illustrate the prevalence of such devices, and give specific examples of the mechanisms our model predicts.

I.1 Holiday and Vacation Bonuses

It is common, and sometimes required, for firms to pay special bonuses to workers around holidays, vacations, or other occasions. Although common in the U.S., holiday and vacation pay is more standardized abroad, with many countries explicitly legislating extra pay to coincide with periods of high marginal value of consumption. For example, the *aguinaldo* is a mandatory bonus awarded to workers in many Latin American countries including Argentina, Costa Rica, Guatemala, Honduras, Mexico and Uruguay. In Mexico, the *aguinaldo* occurs in December and must be equal to at least 15 days of pay, while in Uruguay two *aguinaldos* are required, one in the amount of two weeks' pay, due typically on or about December 22, and an equal amount due on June 22. The Indonesian analog is the *Tunjangan Hari Raya*, whose timing coincides with the end of the fasting month of Ramadan. In Europe, Belgium, Germany, Greece, France, and Holland all mandate that employers structure payments to coincide with holidays or vacations. Perhaps the most celebrated of these is Greece's

“14th salary” tradition of paying two months of annual bonuses at three different times—one month at Christmas, one-half month at Easter, and the remainder in July.

I.2 Pay Frequency

Table 1 indicates that the vast majority of states in the U.S. (45/51, including Washington D.C.) explicitly regulate pay frequency. Only Alabama, Florida, Montana, Nebraska, Pennsylvania, and South Carolina lack standards specifying the minimum time between paychecks. As the totals indicate, approximately half of states specify a semi-monthly (or bi-weekly) frequency, with half again requiring pay at the monthly frequency. Weekly pay, the highest frequency we observe, is mandated in six states. As our analysis of the model below shows, the benefits of pay frequency are larger for less wealthy workers, i.e., those who live “paycheck to paycheck.” Indeed, inspection of state regulations reveals a number of cases where low wage jobs are singled out—even within the same state—as requiring higher frequency. For instance, “executive” (in District of Columbia and Georgia) and “professional” (in Illinois and Maryland) jobs are often exempted from the rules, or subject to less stringent frequency requirements. As a more specific example, the New York Labor Law (Section 191—*Frequency of Payments*) spells out different pay frequencies for 1) manual and/or railroad workers, paid at least weekly, 2) clerical workers, paid at least semi-monthly, and 3) commission salesman, paid at least monthly.

Aggregate data on pay frequency, obtained from the U.S. Census Bureau Survey of Income and Program Participation (1996), confirms a similar pattern. Figure 3 shows that pay frequency is monotonically related to monthly income, with workers paid monthly earning approximately 45% more than workers paid weekly. Likewise, Figure 1 shows pay frequency for two alternative measures of wealth and/or financial sophistication, “some stock ownership” or “some certificate of deposit (CD) ownership.” Perhaps not surprisingly, the results here mimic those in the previous figure, with higher pay frequency associated with lower rates of financial market participation.

Our theory shows that more present-biased workers should be paid more frequently. Although present-bias is difficult to observe directly, there are good reasons to think it correlates with educational attainment. Investing in education requires one to forgo immediate consumption, and studying requires a postponement of immediate gratification. Individuals with present-bias are therefore less likely to attain high levels of education. Figure 2 presents the cumulative distribution

of pay frequency by educational attainment. Over 20% of workers paid weekly lack a high school diploma, versus only 6% of workers paid monthly. Moreover, for any level of educational attainment X , a greater fraction of workers paid weekly have education less than or equal to X than workers paid bi-weekly. The same follows for bi-weekly versus monthly.

I.3 Mechanisms Initiated by Workers

One of the theory’s key assumptions is that the appropriate timing of wage payments ultimately benefits workers, and that workers are “sophisticated” enough to recognize the corresponding improvement in utility. Perfect sophistication is not required for the model’s implications to be relevant, but support for the idea that workers are at least somewhat aware that pay timing can affect their long-run utility can be gleaned by examining their own, rather than their employers’, financial decisions.

Workers sometimes choose to smooth pay that would otherwise vary in a way unrelated to marginal utility. As an example familiar to academics, most U.S. universities and schools pay salaries based on a nine month calendar year. Yet, many professors and teachers choose to spread pay equally over twelve months, despite a loss of interest income. Likewise, utilities frequently offer customers the option of paying equal amounts throughout the year, allowing them to better balance their monthly expenditures. Workers also choose to convert smooth pay to pay that varies with marginal utility, for example by requesting that an employer withhold more in tax than is expected to be owed, thereby creating a type of savings account (also with a loss of interest). These savings allow workers to save for large purchases such as a car (Adams, Einav and Levin, 2009). Certain installment loans allow the borrower a “month off” at her discretion, presumably in a month in which she has a high need for cash. In such a case, her net pay is higher when it is more needed.

II The Model

Our model is devoted to understanding the impact of time-inconsistency on contract design. Screening, signaling and motivating clearly play a role in contract design, and many papers have developed theories of contracting that are designed to perform these three tasks. On the other hand, there are many empirically common, but seemingly mundane, variations in contracts that are not well ex-

plained by these three more analyzed justifications. We show that many of these follow immediately from workers' time-inconsistency.

To focus on precisely this driver of contract design, we model a setting in which there is neither moral hazard nor adverse selection, and there is no risk in the productive technology. We begin with a very narrow question: what is the optimal contract when the worker may neither borrow nor save? We then allow the worker to save and show that our result does not depend on the no-saving requirement. The results in these sections have a variety of implications for contract design which are largely consistent with the stylized facts. We then turn to the question of how frequently workers should be paid.

II.1 The optimal contract when no saving is allowed

A firm must hire a worker for T periods and offer an initial present value of utility to the worker of \bar{u} . Once hired, the worker will remain with the firm throughout the duration of the contract: there are no individual rationality constraints after the first (the contract is enforceable in court).⁶ The worker has period utility of $u_t(c_t)$ where u has the standard properties: $u'_t(c) > 0$, $u''_t(c) < 0$, and $u'(0) = \infty$. Let the firm pay a wage of w_t . The firm's problem is to design a contract that minimizes its expenses subject to the worker accepting the contract—it is assumed to be profitable at the optimal contract. Let the worker have $\beta - \delta$ preferences with $0 < \beta, \delta \leq 1$. That is, the worker's utility in period t is

$$U_t = u_t(c_t) + \beta \sum_{s=1}^{T-t} \delta^s u_{t+s}(c_{t+s}). \quad (1)$$

Suppose the worker can neither borrow nor save. Then $w_t = c_t$, so the optimal contract satisfies

$$\min_{\{w_t\}_{t=1 \dots T}} \sum_{t=1}^T \delta^{t-1} w_t \quad (2)$$

$$s.t. \ u_1(w_1) + \beta \sum_{t=2}^T \delta^{t-1} u_t(w_t) \geq \bar{u}. \quad (3)$$

⁶While this assumption is in violation of the 13th ammendment of the US constitution, it could be removed at the expense of parsimony. Allowing the worker to quit at any time could change pay timing profiles in which large payments are made early in the life of the contract, but would not affect those where larger payments are deferred. For example, workers starting a job in November might not be granted a Christmas bonus, but workers starting in February would. It is empirically common, in fact, for bonuses to have such a pro-rating provision. If we assume a positive separation cost, then we would find that higher separation costs allow more front-loaded contracts.

Proposition 1 *When neither saving nor borrowing is allowed, the optimal contract features wages that equate the marginal utility of the wage in every period except the first, which features a higher payment:*

$$u'_t(w_t) = u'_s(w_s) \text{ for } t, s \neq 1 \quad (4)$$

$$u'_1(w_1) = \beta u'_t(w_t) \text{ for } t \neq 1. \quad (5)$$

This baseline contract has straightforward intuition: the optimal consumption stream for the period-one type (the one who must accept or reject the contract offer) is the one that equates marginal utility in all periods except the first, in which marginal utility is lower by the factor β . Note that standard, time-consistent preferences feature $\beta = 1$, so marginal utility of consumption would normally be equated across all periods. When the worker can neither borrow nor save, the optimal contract must deliver wages precisely when they are valuable to the worker.

Corollary 1 *Suppose utility does not vary over time so $u_t(c) = u(c)$ for all t . Then the optimal contract is a flat wage plus a signing bonus. If marginal utility is the same in all periods except during the holidays, when it is higher, then the optimal contract would specify a flat wage, a signing bonus, and a holiday bonus.*

Corollary 1 gives an interesting description of the optimal contract: it includes a flat wage and a signing bonus and, perhaps, an end-of-year/holiday bonus. The flat wage and holiday bonus direct pay so that it is high when marginal utility is high. At the outset, the worker is impatient and would greatly value pay up front. This allows the firm to pay a lower wage in exchange for offering a signing bonus. As is standard, moving pay forward in time when the worker has a lower discount factor can be efficient. With $\beta - \delta$ preferences, the worker only has a lower discount factor between the present and one period in the future, so it is only advantageous to move pay ahead in that one period.

Corollary 2 *If the contract is signed in period 0 and no wage can be paid in that period, the optimal contract does not include a signing bonus.*

If the contract is signed well before the worker joins the firm, a payment at signing may not be possible, or at least might not apply to the worker's initial individual rationality constraint.⁷ In this case, we should expect no signing bonus since the worker's period zero discount factor between period one and two is δ , equal to the firm's.

II.2 The optimal contract when saving is allowed

If saving is allowed, the previous contract remains optimal because the present-biased worker will never choose to save. Of course, if the worker has time-consistent preferences and can save at interest rate $r = \frac{1}{\delta} - 1$, the previous contract is only one example of an optimal contract. Any contract that specifies payments that have the same cumulative total as the preceding contract at every time t is equivalent. To see this, let $\{w_t^*\}_{t=1\dots T}$ be the wage schedule in the first-best contract and let $R = 1 + r$ be the interest factor. Let $\bar{w}_t = \sum_{s=1}^t R^{t-s} w_s^*$ be the value at time t of wages paid in the first-best contract up to time t . Then any contract $\{w'_t\}_{t=1\dots T}$ where $\sum_{s=1}^t R^{t-s} w'_s \geq \bar{w}_t$ for all t is equivalent for the worker. She can always save to replicate the first-best contract. For example, the contract could offer the entire wage in period one so that $w'_1 = \sum_{s=1}^T \delta^{s-1} w_s^*$ and $w'_t = 0$ for $t \neq 1$, and allow the worker to save.

The case where $\beta = 1$, with a collection of equivalent contracts, is a knife-edge case. When $\beta < 1$ the worker will not save sufficiently, and the more that the contract requires saving, the less value she will get from the contract. To see that the first best contract, described in Proposition 1, is uniquely optimal, consider a small deviation in which the worker is paid one extra dollar in period t and $\frac{1}{\delta}$ dollars less in period $t + 1$. These contracts would normally be equivalent since, in theory, the worker could save the dollar and match the previous consumption schedule, which is optimal for the time-zero worker. When $\beta < 1$, however, the worker in period zero realizes that her period t self will consume, rather than save, the dollar. After all, her period t self has a discount factor for period t to any future period that is strictly between δ and $\beta\delta$, so she would like to choose consumption in the two periods so that $u'_t(c_t) < u'_{t+1}(c_{t+1})$. Since the first-best contract sets $u'_t(c_t) = u'_{t+s}(c_{t+s})$, her period t marginal benefit from shifting consumption from $t + s$ to t is greater than unity: she will consume a dollar delivered early. Her period zero self has discount factor δ between the two periods, and is therefore less happy with the consumption stream that will ensue if the contract

⁷See Van Wesep (2010) for a more detailed discussion of this issue.

pays money earlier. *When employees are time-inconsistent, they are only able to efficiently save if the contract saves for them.*

Note that this implies that this contract should be offered to all of a firm’s employees, regardless of the distribution of β among them. Employees that are not present biased are not harmed by this pay schedule, and those that are present biased are strictly helped. It is therefore easy for a firm that does not know the degree to which present-bias is prevalent among its workers to choose an optimal contract.

II.3 The Frequency of Pay

The benchmark analysis so far has shown that pay, when delivered to coincide with periods of high marginal utility, will reduce the firm’s overall wage costs. We have been intentionally agnostic, however, about what constitutes a “pay period” as, indeed, the model suggests that the firm would pay the worker continuously. This is clearly unrealistic—in practice, on the vast majority of days, workers are paid nothing. To accommodate this feature and increase the realism of the model, we now incorporate a cost each time the worker is paid. As we will see, this feature delivers optimal *pay frequency* as balancing the physical costs of paying more often and the welfare benefits derived in the last section.⁸ Moreover, the model can now also speak to the types of workers we expect to be paid more or less frequently.

Let the cost of making a wage payment in a given period be $c > 0$. This is a reduced form way to state that higher frequency is costly for some reason, whether it be lost interest on the float, administrative costs, un-modeled moral hazard or screening costs, etc.⁹ For simplicity, assume that

⁸The optimality of periodic pay here derives from a similar trade-off as is found in Clower constrained models (e.g., Clower 1967; Lucas, 1980) in which consumption must be funded with cash, which may only be transferred from a savings account at a positive fixed cost. In these models, withdrawals from savings occur at fixed, periodic intervals. Huberman (1988) provides a theoretical foundation for the use of periodic actions/transactions even when the benefits or costs of action are stochastic. We therefore believe that our conclusions regarding optimal pay frequency do not depend heavily on our assumptions of periodic pay and determinism.

⁹To delve more deeply in what such a cost may be, let us consider administrative costs of paying workers. In order for paychecks to be sent out, an employee of the firm must check that the bank account from which the checks will draw has sufficient funds. If funds are unavailable, other cash must be transferred into the account. If no other cash is available, some assets must be liquidated. This may require effort from a CFO or some higher authority. If accounts receivable become cash in a less-than-predictable way, then this process may be fairly costly. Regardless of the precise mechanism that causes paying workers to be costly, and regardless of whether the particular process we describe is accurate at every firm, it should be clear that there is some cost of more frequent paychecks. As evidence that the cost of making wage payments includes a fixed cost, and is not solely proportional to the number of workers, note that firms offering pay to some workers on a monthly basis also tend to offer pay to less educated workers on a semi-monthly, rather than bi-weekly, basis. Firms that pay some workers weekly tend to pay others bi-weekly, not semi-monthly. This practice minimizes the number of days per month on which at least one wage payment is made,

the contract is signed in period zero and payments begin in period one, and let $u_t = u$ for all t . We look for contracts with periodic payments: payment every period, every two, every three, etc.

Clearly, the optimal contract when $\beta = 1$ is to pay the full present value of the contract at the outset. There is only one costly wage payment and the worker can save to smooth consumption herself. Her consumption from a contract paying one payment in period one is shown in Figure 4. As we can see, as β decreases, the problem of over-consumption early in the contract becomes increasingly severe. Therefore, $\beta = 1$ is a knife-edge case: for any $\beta < 1$, payments should be periodic. As β declines, the worker is less able to save efficiently and therefore benefits from more frequent payment.

Suppose that payments are made every $F \in \mathcal{F}$ periods, where $\mathcal{F} = \{1, 2, \dots, \bar{F}\} \cup T$ and $T = \bar{F}!$. This assumption will ensure that F divides T for all possible F under consideration.¹⁰ Then the worker must save for $F - 1$ periods with the wage payment in the first. Let w^F be the payment made every F periods and let s_i be the amount saved entering period i within the pay-period. That is, s_1 is the worker's savings at the start of in the period in which she is paid, s_2 is her savings at the start of the next period, etc., and s_F equals the savings in the ultimate period before she is paid again. To see how valuable a contract with frequency F is to a worker, we can calculate her consumption in each of the F periods between payments. The problem is identical after each payment since T is a multiple of F .

We derive our results below using backward induction from the date before the next paycheck, but a brief application of previous $\beta - \delta$ results to our setting is useful. Harris and Laibson (2001)'s *Hyperbolic Euler Relation*, adapted to the model at hand, can be written as

$$\frac{u'(c_{F-i-1})}{u'(c_{F-i})} = 1 - (1 - \beta) \frac{dc_{F-i}}{ds_{F-i}}, \quad (6)$$

where $\frac{dc_{F-i}}{ds_{F-i}}$ is the fraction of the marginal saved dollar that is applied to consumption in the following period. Because $\frac{dc_{F-i}}{ds_{F-i}} > 0$, $\frac{u'(c_{F-i-1})}{u'(c_{F-i})}$ is less than unity, consumption is decreasing over time within a pay-period. Since the worker will not save in the period before a new paycheck,¹¹

suggesting some fixed cost.

¹⁰We will sometimes refer to F as the ‘‘pay frequency.’’ While the frequency is literally $1/F$, we simply refer to it as F for readability. ‘‘Higher frequency’’ corresponds to a lower value of F .

¹¹As we will see, the worker's marginal utility in the period preceding a paycheck is greater than in that following a paycheck, so zero saving is optimal. That is, the consumer is at a corner solution of Equation (8) in Harris and Laibson (2001).

$\frac{dc_F}{ds_F} = 1$. Equation (6) can also be written iteratively as

$$u'(c_{F-i}) = u'(c_F) \times \prod_{j=1}^i \left(1 - (1 - \beta) \frac{dc_{F-j}}{ds_{F-j}} \right). \quad (7)$$

While this equation shows relative consumption, the budget constraint can pin down levels:

$$\sum_{i=0}^{F-1} \delta^i c_{F-i} = wF. \quad (8)$$

Note that as the length of the pay-period F increases, because u' is decreasing, the level of consumption at the beginning of a pay-period is increasing in F and the level of consumption at the end is decreasing.

We henceforth make two simplifying assumptions. First, we set $\delta = 1$ to simplify the analysis. Given that T is finite, the qualitative results are not dependent upon δ so long as it is positive. Setting $\delta = 1$ is useful so that we can talk simply about “weekly pay” etc. without being concerned with discount and interest rates.¹² Second, we let $u(c) = \log(c)$ so that we can find analytical solutions for the consumer’s problem. We believe, though we clearly cannot make an exhaustive search, that most concave utility functions would deliver similar qualitative results.

Proposition 2 *Consumption in period i of a pay-period of length F is given by*

$$c_1 = w \times \frac{F}{1 + (F - 1)\beta} \quad (9)$$

$$c_i = w \times \frac{F}{1 + (F - i)\beta} \times \left[\prod_{j=1}^{i-1} \frac{(F - j)\beta}{1 + (F - j)\beta} \right] \text{ for } i \in \{2, 3, \dots, F\}. \quad (10)$$

Proof. The worker in period $F - 1$ will choose consumption to $\max_{c_{F-1}} \log(c_{F-1}) + \beta \log(s_{F-1} - c_{F-1})$, which yields $c_{F-1} = \frac{1}{\beta} c_F$. In terms of savings at time $F - 1$, we get $c_{F-1} = \frac{1}{1+\beta} s_{F-1}$ and $c_F = \frac{\beta}{1+\beta} s_{F-1}$. In period $F - 2$ the worker’s problem is

$$\max_{c_{F-2} \leq s_{F-2}} \left[\log(c_{F-2}) + \beta \left(\log \left(\frac{1}{1 + \beta} s_{F-1} \right) + \log \left(\frac{\beta}{1 + \beta} s_{F-1} \right) \right) \right], \quad (11)$$

¹² $\delta = 1$ is also approximately correct, since we are comparing pay frequencies on the order of weeks, and weekly discount factors are likely very close to unity.

which can be written $\max_{c_{F-2} \leq s_{F-2}} [\log(c_{F-2}) + X + 2\beta \log(s_{F-2} - c_{F-2})]$, where $X = \beta(\log(\beta) - 2\log(1+\beta))$, which does not depend upon c_{F-2} . This yields $c_{F-2} = \frac{1}{1+2\beta}s_{F-2}$, $c_{F-1} = \frac{2\beta}{1+2\beta}\frac{1}{1+\beta}s_{F-2}$ and $c_F = \frac{2\beta}{1+2\beta}\frac{\beta}{1+\beta}s_{F-2}$. Assume that the worker consumes her entire savings in the period before her next paycheck. We will confirm that this assumption holds below. Then the savings at the time of the paycheck is $s_0 = 0$ and the initial payment is wF . Completing the iteration therefore yields $c_1 = \frac{1}{1+(F-1)\beta}wF$, $c_2 = \frac{(F-1)\beta}{1+(F-1)\beta}\frac{1}{1+(F-2)\beta}wF$, $c_3 = \frac{(F-1)\beta}{1+(F-1)\beta}\frac{(F-2)\beta}{1+(F-2)\beta}\frac{1}{1+(F-3)\beta}wF$ et cetera, up to $c_F = \frac{(F-1)\beta}{1+(F-1)\beta}\frac{(F-2)\beta}{1+(F-2)\beta}\dots\frac{\beta}{1+\beta}wF$. This can be written generally as

$$c_1 = \frac{wF}{1 + (F - 1)\beta} \quad (12)$$

$$c_i = \frac{wF}{1 + (F - i)\beta} \times \left[\prod_{j=1}^{i-1} \frac{(F - j)\beta}{1 + (F - j)\beta} \right] \text{ for } i \in \{2, 3, \dots, F\}. \quad (13)$$

We now check that the worker would prefer to consume her entire savings in the period before a paycheck. We begin with the period before the final paycheck. The worker's marginal propensity to consume in the period of her final paycheck is $\frac{dc_1}{ds_1} = \frac{1}{1+(F-1)\beta}$, so Equation (7) implies that the worker's ideal ratio of marginal utility in the period before a paycheck to the next period is $\frac{1/c_F}{1/c_1} = 1 - (1-\beta)\frac{dc_1}{ds_1}$, unless the worker is at a corner solution, in which case $\frac{c_1}{c_F} > 1 - (1-\beta)\frac{1}{1+(F-1)\beta}$. Since $c_1 > c_F$, it immediately follows that $\frac{c_1}{c_F} > 1 - (1-\beta)\frac{1}{1+(F-1)\beta}$, so the worker indeed prefers to spend her entire savings in the period before her final paycheck. The same analysis applies to all previous paychecks iteratively. ■

Corollary 3 *The marginal propensity to consume in a period is decreasing in β and in the time to the next paycheck. In the limit where $\beta = 0$, the marginal propensity to consume is one. In the limit where $\beta = 1$ and the worker is time-consistent, the marginal propensity to consume is the inverse of the number of periods before the next paycheck.*

Proof. The derivative of c_{F-i} with respect to s_{F-i} is

$$\frac{dc_{F-i}}{ds_{F-i}} = \frac{1}{1 + i\beta}, \quad (14)$$

which is decreasing in β and i . Inserting $\beta = 0$ or $\beta = 1$ yields a marginal propensity to consume of one or $\frac{1}{1+i}$ respectively.

Naturally, more time-inconsistent people, who have a lower value of β , have a higher marginal propensity to consume. As $\beta \rightarrow 0$, consumption responds to saving one for one and future saving is nil. When there is more time before the next paycheck, however, consumption responds less to saving. Because every worker, even a time-inconsistent one, prefers smooth consumption, each must save more when there is more time before the next paycheck. Essentially, since a time i self knows that her medium-term self will overconsume relative to her time i preferences, she is more willing to save so that her longer-term self has greater consumption. This beneficence toward her longer-term self is partly mitigated by her time-inconsistency, and consumption is always decreasing during a pay-cycle. Note also that when $\beta = 1$, the standard result concerning the marginal propensity to consume holds. If there are i periods remaining in the pay cycle, then a time-consistent worker spreads a marginal dollar equally over those periods, and the current one, so the marginal propensity to consume is $1/(1+i)$. ■

The consumption choices described in Equation (10) are shown in Figure 5 for workers with $\beta = 0.9$ and $\beta = 0.7$. The worker consumes the most on payday and consumes less each subsequent week before the next paycheck. The difference between consumption from the week before payday to the week after is roughly 22% when $\beta = 0.9$. For $\beta = 0.7$, the problem becomes severe, with consumption in the week before payday roughly half of consumption in the week after.

Two implications immediately follow from Proposition 2. First, since per-period pay is fixed at w and utility is concave, the *long-run* utility over the pay-period is decreasing in F , where long-run utility corresponds to what the worker's utility would be if β were equal to one. Longer pay-periods imply more varied consumption, which is utility destroying for the worker. Second, when workers are more time-inconsistent (β is lower), the divergence between the high initial consumption and low terminal consumption within a pay-period is larger.

Together, these imply that *worker utility is decreasing as pay becomes less frequent, with the reduction in utility being larger when β is lower*. If the firm wishes to reduce the frequency of pay and still meet the worker's individual rationality (IR) constraint, it must increase the per-period wage. This trade-off will lead to an optimal frequency. As β decreases, the optimal choice of F decreases. Let $\hat{w}(F, \beta)$ be the per-period wage that must be paid to a β type, given frequency F . This is defined by the worker IR constraint binding in period zero,¹³ $\bar{u} = \beta \times (T/F) \times \sum_{t=1}^F u(c_t)$,

¹³Recall that in this special case, we have allowed $\delta = 1$ which makes the exposition simple. There is no qualitative

where total consumption in each pay-period is given by the budget constraint (8) and relative consumption is given by (7). Let $\Delta\hat{w}(F, \beta) = \hat{w}(F, \beta) - \hat{w}(F - 1, \beta)$. Proposition 2 implies that $\Delta\hat{w}$ is positive and decreasing in β . Therefore:

Proposition 3 *The optimal length of pay-period is increasing in β , with a limit of T as $\beta \rightarrow 1$.*

To get a sense of magnitudes, Figure 6 shows the additional weekly wage that must be paid as the pay-period lengthens for a time-consistent type as well as a time inconsistent type with $\beta = 0.7$. As the pay-period lengthens from one week to four weeks, for example, a time inconsistent worker must be paid an additional 4%. This may seem small, but if wages are large fraction of a firm's cost, then 4% of wages would constitute a substantial fraction of profit margins. With more time-inconsistent workers, the impact will be magnified.

A firm may not know its workers' levels of β and even if it does, may wish to choose a pay frequency so that all workers within a given job class have the same pay frequency even if there is heterogeneity in the level of β . Nonetheless, proposition 3 provides clear guidance as to which job classes should be paid more frequently. Suppose that the firm can observe some attribute of the worker θ , which provides some information regarding the worker's likely value of β . In particular, suppose that the value of β for a worker is a random variable with cumulative distribution $G(\beta; \theta), \beta \in (0, 1]$. Let $\theta \in (a, b)$ where $-\infty < a < b < \infty$, and let $G(\beta; \theta_1)$ first order stochastically dominate $G(\beta; \theta_2)$ if and only if $\theta_1 > \theta_2$. This implies that as θ increases, the expected value of β for the worker increases. Proposition 3 immediately implies the following corollary:

Corollary 4 *The optimal length of pay-period is increasing in θ .*

Any observable worker attributes that are associated with present bias will cause the firm to pay the worker more frequently. In practice, this means that if the distribution of present-bias differs depending upon job class, we will expect firms to vary pay frequencies by job class as well.

To summarize the results so far, pay should be timed so that workers receive pay precisely when they desire consumption, thereby eliminating the need for them to save or borrow. If paying workers entails a cost, then pay may be infrequent, but when workers are particularly present-biased, the firm must simply bear those costs and deliver pay frequently.

difference if we allow $\delta \in (0, 1)$ so long as the interest rate is given by $r = (1/\delta) - 1$.

These results, however, are subject to an important caveat that, to this point, we have left unmentioned. Once the timing of pay is set, the $t > 0$ worker has every incentive to attempt to undo the pay schedule. In the next two sections we consider two ways that she may achieve this. The first is to renegotiate with the firm and get “an advance” on a future paycheck. Indeed, renegotiation will cause our results thus far to unravel, implying a need for regulation. The second is to get an advance from a third-party—a payday loan. Because this is *ex ante* harmful for the worker as well, regulators can help workers by illegalizing payday lending. We show, however, that an even better policy is to permit capped payday loans.

III Renegotiation and Regulation

Thus far we have assumed that the firm and worker can commit to not renegotiate the contract after it has been signed. This assumption has bite: the period-one worker has different time preferences than the period zero worker and, given the contract terms we derive above, would be willing to sacrifice disproportionate future income to obtain an advance. Large and on-going firms may be able to establish and maintain reputations for refusing to renegotiate, thus decreasing future wage bills, but for many smaller companies renegotiation may be too tempting to avoid. As we show in this section, this fact suggests a role for regulation. A regulator, by mandating holiday bonuses, certain pay frequencies, etc., can maximize social welfare and solve the renegotiation problem.

III.1 Renegotiation

We begin by deriving the optimal renegotiation-proof contract. As we will see, there is a set of such contracts, all yielding identical consumption paths. The firm essentially pays the full amount at the outset, and the worker consumes from that buffer savings over T periods. Equivalent contracts pay the worker some cash later, but always deliver sufficient cash by each period so that this consumption path is feasible.

Proposition 4 *There are a continuum of optimal renegotiation-proof contracts, each inducing the same consumption path as a contract offering all pay in period one.*

With renegotiation, the worker and firm both know that when a future present-biased version of the worker requests an advance, there will be an interest rate such that the firm will accede.

A renegotiation-proof contract must therefore be designed so that the worker never asks for an advance, implying that the worker's consumption stream with any renegotiation-proof contract is identical to that which obtains when the worker is simply paid the full amount up-front. Note that this argument follows for holiday bonuses and any other compensation that is deferred so that the worker has money when it is more needed.

III.2 Regulation

The above analysis implies that both the firm and the worker would prefer, at time zero, for an outside entity to prevent renegotiation of contracts. In practice, this is a government, and the enforcement is not against renegotiation *per se*, but in favor of requiring disbursements at particular times. Not surprisingly, the contract required by the regulator would be identical to those we have already derived as the optimal contract for the firm, under the assumption that it could commit. This is because the social planner's problem and the firm's problem are identical, up to a constant. The regulator would trade off the cost to a firm of paying more frequently with the benefit to a worker of being paid more frequently. The regulator could not observe how present-biased each individual is, of course, but could perhaps form estimates based upon observables (e.g., worker education or job class). Indeed, as we have discussed, regulators in 45 US states require wages to be paid at a minimum frequency that typically depends upon job class. Jobs that typically require more education or financial savvy, or that are higher paid, require less frequent pay.

III.3 Firms vs. Financial Intermediaries in Timing Pay

A question that may arise is why firms, *per se*, should be the ones to adjust the worker's pay timing. One could imagine, for example, banks or payday loan companies offering a similar service. Perhaps workers would make deposits at the beginning of a pay-period and then receive periodic disbursements to correlate with rent, bills, or other expenditures. Such a third-party arrangement may be beneficial for a number of reasons, including financial institutions having lower transaction costs, or perhaps possessing a reputation for being hard-nosed (non) renegotiators.

There is nothing in the model that prevents external parties from stepping in and, indeed, because payroll services are often outsourced (especially for medium and small companies), this might be happening to some degree in practice. We are not aware of any cases where true third

parties such as banks alter paycheck delivery, although Christmas Clubs (where workers voluntarily deposit amounts that cannot be withdrawn until Christmas) have been offered by banks since the Great Depression. Moreover, while we can only speculate why similar mechanisms do not seem to be offered with respect to pay frequency, one reason could be that pay timing is likely most important for financially unsophisticated workers (see Figure 1).¹⁴ If workers do not trust banks, or even lack bank accounts, such an arrangement may not be feasible. However, none of this changes the main message of the paper—it is not particularly important *who* specifically conducts the timing-welfare calculation, as long as *someone* does.

IV The Effect of Payday Loans

If the worker is unable to change the pay timing profile by getting an advance from the firm, she may instead attempt to get an advance from a third-party—a payday loan. A payday loan is a loan that may be taken from a lender some number of days before a worker receives her paycheck, where said paycheck is pledged as collateral. Payday loans are often expensive, if the cost of the loan is calculated as an interest rate, but are nonetheless common in many areas of the United States.¹⁵ The fact that such loans are both expensive and common is evidence in favor of our assumption thus far that many workers do not have access to standard credit lines, and also suggests the importance of understanding their welfare implications.

This section requires a discussion of worker welfare. Until now, we have avoided this because the relevant criterion is implied by the firm’s cost-minimization problem. Whichever “self” the firm is contracting with, that self’s utility is the relevant criterion. Once we wish to say, however, that a worker is “better off” if payday loans are disallowed, we need a welfare criterion. We adopt the oldest/standard criterion in this literature, which is the worker’s “long-run” utility. The worker is better off with consumption schedule A than consumption schedule B if the worker’s utility would be higher with A than B if $\beta = 1$.¹⁶

¹⁴According to a 2009 FDIC survey, 7.7% of households have no account at a bank or insured financial institution. Over 25% either lack a bank account or have an account but continue to use high cost financial service providers. These households are likely to disproportionately overlap with the set of households for which pay timing is important.

¹⁵For example, a loan for \$400 taken one week prior to payday may involve a \$20 fee, implying a 5% weekly interest rate.

¹⁶This criterion has been adopted by many authors, including O’Donoghue and Rabin (1999) and Gruber and Koszegi (2001) and, in our context, yields identical welfare implications as that proposed in Gul and Pessendorfer (2004). Alternative methods of welfare analysis are possible: Bernheim and Rangel (2009), for example, abandon the

We begin by considering the case where payday loans are uncapped, meaning that the maximum loan value is equal to the size of the paycheck. We restrict attention to the case where a loan may be taken out one period before payday, though most of our policy prescriptions have clear analogues in the more general case.¹⁷ Let the interest rate on payday loans be $r \in [0, \frac{1-\beta}{\beta})$ and define $R = \frac{1}{1+r}$.

IV.1 Uncapped payday loans and welfare

If $F = 1$ and pay is every period then payday loans are identical to credit cards with a debt ceiling equal to the size of the paycheck. We therefore focus, in this section, on the case where $F \geq 2$.

Proposition 5 *When payday loans are uncapped and paychecks are infrequent, $F \geq 2$, the availability of payday debt shifts the consumption cycle forward by one period, increasing consumption in the first F periods and decreasing it in the final F periods. Specifically, in all but the final period of each pay-period, the worker's consumption:*

1. *In the first $F - 1$ periods is identical to what would obtain without payday lending and with paychecks of size wF but pay periods of length $F - 1$.*
2. *In the final $F + 1$ periods is identical to what would obtain without payday lending and with paychecks of size wF but pay periods of length $F + 1$.*
3. *In the periods between is identical to the case without payday lending, but shifted one period earlier.*

In the final period of each pay-period, consumption is R times what obtains when payday loans are not available.

Figure 7 shows the consumption cycle described in Proposition 5, with $r = 0$, $F = 4$ and $\beta = 0.7$. For the bulk of the contract term, the consumption pattern is identical to the case with no payday lending, but shifted forward by one period. The worker knows that her future self will take a large payday loan, so will consume her entire paycheck a period earlier than when no payday loans are available. Payday lending therefore simply shifts consumption forward one period. This raises

concept of preferences entirely, but also provide a choice-theoretic ground for the long-run criterion.

¹⁷In practice, payday loans can often be taken out by consumers two or more weeks before the next paycheck. We do not precisely connect the concept of a "period" in this model to weeks or days in the real world. We could easily change the allowable types of payday loan, but the qualitative trade-offs we analyze here would remain.

consumption somewhat at the start of the contract and decreases it somewhat at the end. From the perspective of the period-one worker, payday lending could be beneficial since consumption is higher in period one, but the overall welfare effect is small.

From the perspective of the period-zero worker, when $r = 0$, payday lending is harmful because consumption is slightly less even, but this effect is small as well. Payday lending is essentially irrelevant when pay is infrequent, interest rates are low, and loans can be as large as a paycheck. As interest rates increase, the worker is unambiguously worse off. Consumption in most periods is unchanged as r increases, but consumption in the period in which the payday loan is taken out decreases with no corresponding increase elsewhere.

Corollary 5 *As the interest rate on payday loans increases, consumption in the period before payday decreases so that the amount repaid remains constant. Consumption in other periods of the pay-period is unaffected.*

IV.2 Capped payday loans and welfare

Payday loans can, however, be beneficial when the size of the payday loan is capped at an amount less than the worker would like to borrow.

Proposition 6 *Payday lending can improve long-run utility if the loan amount is capped. The optimal cap is either 0 or $\frac{T-F}{1+T-F}Rw$, which is approximately equal to Rw when T is large.*

Figure 8 shows the consumption pattern for three representative levels of the cap, assuming $r = 0$. There are two effects on consumption when the cap is increased. First, some consumption is shifted from the final F periods of the contract to the first $F - 1$. This is a negative effect of a higher cap. Second, the variance in consumption in the middle periods of the contract decreases as the cap approaches w and increases again for higher values of the cap. Therefore, welfare in the middle periods of the contract is concave in the level of the cap, attaining a maximum when the cap is w . The optimal value of the cap trades off these effects. Because T is large compared to F , the first effect is small relative to the second, yielding an optimal cap close to w . As $T \rightarrow \infty$, the optimal cap tends to w .

To see why long-run utility is concave in the size of the cap, we must consider how the cap affects consumption during the pay-cycle. When the cap is low, increasing the cap does not affect

the consumption cycle in the middle of the contract. Consider a typical pay-period and a cap C . The worker will have borrowed C before the pay-period, and will also have access to a payday loan of size C in the final period of the pay-period. Her cash available for consumption in the pay-period is therefore $wF - C + C = wF$. Her options are unchanged, so her behavior is unchanged. We see that for a cap of both 20% of weekly pay and 60% of weekly pay, her consumption cycle in the middle of the contract is identical.

If the cap gets large, however, things are somewhat different. She begins the pay-period with $wF - C$ and knows that she will borrow C in the final period of the pay-period. By spreading $wF - C$ over $F - 1$ periods, she consumes less than C/β in the penultimate period of the pay-period. She would like to increase consumption in the penultimate period and reduce it in the final period, but she cannot access the payday loan until that final period. Therefore, she spreads her paycheck, net of the cap, over $F - 1$ periods, knowing that in the final period of the pay-period she will again borrow C . The payday loan *effectively changes the wage stream* from one payment of wF every F periods to a payment of $wF - C$ that must cover $F - 1$ periods, and a payment of C every F periods, just prior to the paycheck.

When the cap equals w , increasing the cap would increase her consumption in the period before a paycheck, but this would raise her consumption that period to a level higher than her average consumption in other periods, and therefore would reduce welfare. Another way to view the effect of a payday loan cap is shown in Figure 9. This figure plots consumption in the first, second, third and fourth periods of a contract with $F = 4$ for a worker with $\beta = 0.7$. These are consumption levels in the middle of a contract: we ignore here the effect of the payday lending cap on the first and last pay-periods. We see that when the cap is small, there is no effect on consumption. Only when the cap is so large that the worker in period 3 of a pay-period would like to borrow does the consumption change as we adjust the cap. Then, consumption in the period before payday begins to rise and consumption in the other periods begins to fall.¹⁸ Eventually, the cap is large enough that it does not bind.

¹⁸There is a small bump in the consumption curves when the “no borrowing except before payday” constraint binds. This is because the worker saves early in the pay-period, knowing that her self at the end of the pay-period will receive some of that savings. Once the constraint binds, her saving will be consumed earlier and her discount rate is therefore higher, reducing saving and increasing consumption. This also accounts for the small negative hit to average utility when the cap is 0.7, shown in Figure 10.

We can see that the consumption over four periods is the same as with no payday lending or a small cap, but with the highest consumption immediately before payday and lower consumption in subsequent periods. The variation in consumption is clearly lowest when the cap equals per-period pay.

Long-run utility as a function of the cap is plotted in Figure 10 for three types of contract. For lower levels of β , consumption during a pay-period is less smooth, so capped payday loans are more beneficial. For longer-term contracts, payday loans are more beneficial as well. This is because the benefit of smoother consumption in the middle cycles of the contract occurs for a longer time and is therefore more able to offset the cost of shifting consumption from the end to the start of the contract. Note also that as F increases, consumption is more volatile within a pay-period, so capped payday loans are more beneficial for the worker.

If we allow the interest rate r to increase, then payday lending is less valuable for the worker. Her self in period F of each pay-period will continue to take out payday loans, at the expense of her long-run self. While the shape of long-run utility as a function of the cap is the same as in Figure 10, the peak is at a lower cap as we increase r . Interestingly, while the optimal cap decreases as the interest rate on payday loans increases, the repayment amount is approximately w regardless of the interest rate.

IV.3 Implications for payday lending policy

To summarize the implications from this section, payday loans can be either good or bad, depending on *when* the consumer accesses them, and *how much* she is allowed to borrow. First, we have found that payday loans improve welfare when their use is restricted to times when consumption is very low, which, in our model occurs: 1) at the end of the pay cycle, and 2) when paychecks arrive infrequently. Second, we have shown that granting unlimited access to payday loans does nothing to smooth consumption—in fact, it delivers the opposite result—but that limiting access to periods directly before a paycheck and capping the loan amount can produce welfare improvements.

This dual prediction is useful in both a descriptive and prescriptive sense. Descriptively, it is noteworthy that a consensus on the welfare impacts of payday lending is far from unanimous, both within and outside of academia. As of the writing of this manuscript, a bill sponsored by North Carolina Senator Kay Hagan (D) would place federal restrictions on payday amounts. “By

reigning in payday lenders, we will protect consumers from racking up endless, long-term debt that can ultimately cause a family to declare bankruptcy,” she said.¹⁹ Her position is buttressed by a number of academic studies that document negative welfare effects for users of payday loans. For example, Skiba and Tobacman (2009) find that even though the average payday loan amount is small (around \$300), borrowers were substantially more likely to declare personal bankruptcy in the months following a loan. Likewise, Melzer (2009) finds that payday borrowers experience a wide variety of negative welfare-reducing events: increased likelihood of being late on bill payments, delaying medical care, forgoing the filling of prescriptions, etc.

On the other hand, the selection issues surrounding payday lending are considerable: because consumers are not randomly selected to have access to loans, nor to borrow, the possibility that payday loans are simply a proxy for (imperfectly observed) financial distress is concerning. Indeed, some empirical studies have documented positive effects of payday lending. For example, Morse (2009) examines payday loans within California from 1996-2002, finding that “payday loan communities” were less impacted by natural disasters than those lacking such establishments. A natural reconciliation of these findings—and one to which our analysis directly speaks—is that payday loans help consumers, provided that they are used for the right purposes. In Morse’s study of natural disasters, payday loans are used in presumed states of (very) high marginal utility; more generally, payday loans may facilitate consumption in less dire situations. Our study suggests exactly this, although what constitutes an appropriate use of payday lending can be summarized by when it is accessed during the pay cycle.

Together, this analysis suggests a clear policy implication: payday loans and similar instruments are beneficial only when the loan amount is capped, and are particularly beneficial at the end of the pay cycle. However, actual policy appears to reflect only this first implication. As shown in the final three columns of Table 1, when payday loans are allowed, they are usually capped, both in term and amount. However, these limits appear to bear little systematic relation to pay frequency, which the theory indicates should be closely connected. Specifically, the theory indicates that payday loans strictly decrease welfare if workers can access them too early in the pay cycle. This will of course be the case if payday loan terms are longer than the pay cycle, allowing workers to perpetually access payday loans, rather than only immediately preceding a paycheck.

¹⁹See http://hagan.senate.gov/?p=press_release&id=551.

The vast majority of states (41), however, allow the term of a payday loan to span an entire pay cycle. As a particularly egregious example, Vermont requires workers to be paid weekly (and within six days of the end of a pay-period), but payday lending remains virtually unregulated, both in term and amount. The model predicts that in such a case, workers simply access the maximum amount they are permitted as early as possible, and experience lower consumption over their lifetimes. The prohibition of payday lending in Massachusetts, a state that also mandates frequent (weekly) pay, is more consistent with the theory, but this is the exception rather than the rule. In light of our analysis, this is a puzzling finding, and could perhaps shed light on the conflicting findings for the welfare effects of payday lending discussed above. When pay is delivered frequently to employees, payday lending can have little other effect than to encourage perpetual indebtedness, rather than to deliver consumption near the end of the pay cycle. A cross-sectional analysis of payday lending's effects on welfare, particularly as it relates to pay frequency laws, would buttress support for the resulting policy implications.

V Could alternative assumptions concerning behavior generate our results?

There are several assumptions we have made to get our results. We have assumed that the worker is time-inconsistent and sophisticated, and we have assumed that workers are largely credit-constrained. In this section we discuss which assumptions are necessary and which could be altered. We discuss first the difference between the optimal contract implied by time-inconsistency versus that implied by impatience, in which the worker's discount rate is higher than the firm's. Impatience cannot generate our results. We then discuss the importance of sophistication in our model and how we could change the model if the worker were naïve. We finish by showing that credit constraints for the worker are not, on their own, sufficient to generate the contractual implications we derive above.

V.1 Time-inconsistency versus impatience

If the worker is simply impatient, rather than time-inconsistent, her discount factor, δ_w , is less than the firm's discount factor, δ_F . She prefers to front-load consumption and the firm is happy to

deliver. Because an impatient worker is not time-inconsistent, there are many contracts that deliver the first-best consumption stream. One such contract would be the firm simply giving a lump-sum at the start of the contract. Our results are therefore clearly inconsistent with impatience alone. With time-inconsistency, the firm benefits by helping the worker help herself: the worker is willing to accept a lower wage in order to have smoother future consumption.

V.2 Sophistication versus naïveté

The worker signing the contract is willing to accept a lower average wage in exchange for better timed wages because her future self would not make the saving decisions that her current self would prefer. This clearly requires the worker to be sophisticated in her knowledge of herself. That is, she must know that her future self will make different consumption choices if given the option and therefore value a contract that removes that option.²⁰

One could imagine, however, a slightly different set-up that does not require worker sophistication, but where pay timing is still important. For example, suppose that the firm knows that the worker is time-inconsistent but that the worker does not. Suppose further that at the end of a pay cycle, workers desperate for cash are poorly rested (e.g., from taking on extra work), poorly nourished, or in some other way less productive. The firm would then prefer a wage profile that keeps the worker from experiencing periods of low consumption. That is, it may be productivity improvements—not wage savings—that make timing important. We could construct a model in which a firm must hire a naïve worker and show an equivalent result to Proposition 1.

We therefore do not believe that our broader point—firms employing time-inconsistent workers must consider pay timing—is dependent upon the assumption of worker sophistication. We choose in this paper to allow for sophisticated workers and to focus on wage savings from proper timing, but allowing for naïveté would have been equally possible.

V.3 Credit constraints without time-inconsistency

It seems plausible that credit constraints alone could generate pay timing as we describe above. Workers who live “paycheck to paycheck” would suffer from infrequent pay since they would ex-

²⁰In a consumption-saving environment, Ali (2010) establishes a learning-theoretic basis for the assumption of sophistication.

perience periods of very low consumption in the run-up to a paycheck. In fact, credit constraints alone cannot generate our results. As we have shown, when $\beta = 1$, any wage schedule $\{w'_t\}_{t=1\dots T}$ such that $\sum_{s=1}^t R^{t-s} w'_s \geq \bar{w}_t$ is equally valuable for the worker. Paying the full amount due in the contract up-front is as at least as good as any other mechanism. While the contract with a signing bonus, holiday bonus, etc., is an example of a first-best contract, it is one of many.

We have shown that pay should be more frequent for more time-inconsistent workers, but this also would not follow from credit constraints alone. One might argue the following: “Suppose it’s two weeks before payday. If I were paid semi-monthly, then I would receive part of the check now and part later. This would relieve a credit constraint since I’m poor now, and therefore increase my utility.” But we must then ask why the worker is poor before payday. Indeed, on payday, a time-consistent worker would be indifferent between a paycheck that pays the full amount immediately or half now and half in two weeks. The problem of credit constraints only arises before the first payment in a contract or because of time-inconsistency.

VI The optimal timing of government assistance

Foley (2009) finds that crimes motivated by money are more frequent at the end of a welfare payment cycle: as recipients run out of money over the course of the month, many turn to crime to supplement income. Shapiro (2005) finds that nourishment decreases over the month following a welfare payment. Both results are consistent with recipients being time-inconsistent, and this should be no surprise.²¹ A low value of β causes procrastination and an inability to put long-term goals ahead of short term rewards, thus making it costly for time-inconsistent people to acquire an education. A low value of β also makes them less able to show up for work on time, follow orders, etc. The demand for current consumption, moreover, prevents the accumulation of wealth. The set of people requiring government assistance is therefore likely to include a disproportionate number possessing time-inconsistency.

Figure 5 shows how time-inconsistency can cause periods of severe under-consumption. The welfare pay cycle in most states is monthly, implying that recipients with log utility and $\beta = 0.7$ consume only half as much in the week before payday as in the week after. This dearth of

²¹Both results are also consistent with impatience.

consumption could tempt some recipients to commit crime to supplement their meager savings in these periods. Even if a recipient's average utility over the course of the month is only roughly 4% lower than if she were paid weekly, as is shown in Figure 6, her utility in the week before payday is far lower because of her time-inconsistency. We do not model the decision to commit crime explicitly, but it should be clear where the temptation arises.

The large majority of recipients of government assistance do not turn to crime, but they are punished with low nourishment at the end of the pay cycle. Better designed welfare payments could improve their livelihoods while reducing crime and saving taxpayer money. We argue that a better system for government assistance would pay recipients when they need the money. A social worker could work with the recipient to set out when large bills like rent and utilities are due, for example, and these amounts would be directly deposited on the day the bills are due. The remainder of the monthly stipend could be divided by the days in the month and deposited each day into the recipient's personal account. This would deliver money when it is needed and not require the recipient to plan/save for an entire month. A plan that pays weekly, with bonus payments on the day large bills are due, would probably do similarly well.

VII Conclusion

We develop a model of optimal pay timing for time-inconsistent workers. Because these workers have difficulty saving, firms pay them so that consuming the entire paycheck leads to constant marginal utility over time: higher or lower pay coincides with periods of higher or lower marginal utility, respectively. This can be implemented, for example, by a fixed wage plus a bonus for the holidays, summer vacations, signing, or severance. We also analyze the case where paying more frequently is costly, and show that time-inconsistent workers experience feast-famine consumption cycles within a pay-period. When the time-inconsistency problem is more severe, the *ex ante* cost of these cycles is also more severe, so pay should be more frequent. This is consistent with broad empirical patterns we see in practice. Insofar as education and wealth correlate negatively with time-inconsistency, more educated and more wealthy workers should be, and are, paid less frequently.

Given the optimal contract agreed-upon *ex ante*, the worker will attempt to undo the contract *ex post* by either renegotiating the contract with the firm or seeking a payday loan. We show that

the firm will be happy to renegotiate, for a price, thus preventing the optimal contract from being implemented. This suggests a role for regulators in which they require the firm to pay workers according to the optimal pay schedule. Indeed, regulation of holiday bonuses and pay frequency is common both in the U.S. and internationally.

We also show that payday lenders will be happy to lend to the worker, though this fact can be beneficial or harmful. The worker's consumption cycles can be at least partly mitigated with payday loans, which deliver cash to the worker in periods when consumption would otherwise be low, and remove cash in periods where consumption would otherwise be high. We show that this is only the case when the amount of the payday loan is capped. Specifically, the maximum allowable loan amount should equal the per-period pay multiplied by the number of periods before payday when the money is borrowed. Essentially, payday loans are an inferior substitute for more frequent pay: when pay is frequent, payday loans act as revolving debt and encourage life-long indebtedness. When pay is infrequent, however, capped payday loans deliver money when it is most needed, and can improve worker welfare. We provide prescriptive suggestions for legislation involving pay frequency and consumer lending regulation.

We have attempted in this paper to provide a simple, parsimonious model in which many features of contracts that we observe in practice result from a simple cost-minimization problem. We do not consider issues of moral hazard or risk in the production process, nor do we address the use of contracts to screen workers. In discussing policy implications for payday lending, we do not model the broader planner's problem. This is for three reasons. First, these additional complexities are unnecessary for our results and would serve to make less clear the connection between time-inconsistent preferences and pay timing. Second, as this paper is a first attempt to tackle issues of pay timing, it is important to establish a baseline against which other results can be compared. Third, working with time-inconsistent preferences quickly becomes analytically infeasible as additional complexity is added. That said, extensions of the model are an area for future work.

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VIII Appendix

Proof of Proposition 1. The firm's problem can be written as the following Lagrangian

$$\mathcal{L} = - \sum_{t=1}^T \delta^{t-1} w_t + \lambda \left[u_1(w_1) + \beta \sum_{t=2}^T \delta^{t-1} u_t(w_t) - \bar{u} \right]. \quad (15)$$

The first order constraints are

$$1 = \lambda u_1'(w_1) \quad (16)$$

$$1 = \lambda \beta u_t'(w_t) \text{ for } t = \{2, 3, \dots, T\} \quad (17)$$

$$\bar{u} = u_1(w_1) + \beta \sum_{t=2}^T \delta^{t-1} u_t(w_t). \quad (18)$$

Equation (17) yields $u_t'(w_t) = u_s'(w_s)$ for $t, s \neq 1$ and combining Equations (16) and (17) yields $u_1'(w_1) = \beta u_t'(w_t)$ for $t \neq 1$. ■

The following is a heuristic derivation of Equation (7), which follows closely to the derivation of the Quasi-Hyperbolic Euler Relation in Harris and Laibson (2001). Consumption in period $F - i$ is chosen to $\max_{c \leq s_{F-i}} (u(c) + \beta \delta U_{F-i+1}(\frac{1}{\delta}(s_{F-i} - c)))$, which yields the first order condition

$$u'(c_{F-i}) = \beta U'_{F-i+1}(s_{F-i+1}). \quad (19)$$

The marginal utility of consumption that period equals the marginal utility of saving, discounted by β . Also,

$$U_{F-i}(s_{F-i}) = u(c_{F-i}) + \delta U_{F-i+1}(s_{F-i+1}).$$

Marginal continuation utility can be found by taking a derivative of each side with respect to s_{F-i} :

$$U'_{F-i}(s_{F-i}) = u'(c_{F-i}) \left(\frac{dc_{F-i}}{ds_{F-i}} \right) + \delta U'_{F-i+1}(s_{F-i+1}) \left(\frac{ds_{F-i+1}}{ds_{F-i}} \right).$$

Since $s_{F-i+1} = \frac{1}{\delta} (s_{F-i} - c_{F-i})$ we can re-write this as

$$\begin{aligned}
U'_{F-i}(s_{F-i}) &= u'(c_{F-i}) \frac{dc_{F-i}}{ds_{F-i}} + U'_{F-i+1}(s_{F-i+1}) \left(1 - \frac{dc_{F-i}}{ds_{F-i}}\right) \\
&= \left(u'(c_{F-i}) - U'_{F-i+1}(s_{F-i+1})\right) \frac{dc_{F-i}}{ds_{F-i}} + U'_{F-i+1}(s_{F-i+1}) \\
&= U'_{F-i+1}(s_{F-i+1}) \left(1 - (1 - \beta) \frac{dc_{F-i}}{ds_{F-i}}\right),
\end{aligned}$$

which implies

$$\frac{U'_{F-i}(s_{F-i})}{U'_{F-i+1}(s_{F-i+1})} = 1 - (1 - \beta) \frac{dc_{F-i}}{ds_{F-i}}.$$

Since $\frac{dc_{F-i}}{ds_{F-i}} < 0$, marginal continuation utility is increasing in time since the last paycheck.

Plugging in $u'(c_{F-i}) = \beta U'_{F-i+1}(s_{F-i+1})$ yields

$$\frac{u'(c_{F-i-1})}{u'(c_{F-i})} = 1 - (1 - \beta) \frac{dc_{F-i}}{ds_{F-i}}.$$

Proof of Proposition 3. The per period wage, \hat{w} , multiplied by the number of periods per pay-period, F , is the size of the paycheck. c is the fixed cost of paying a check, so the total cost per pay-period to the firm is $\hat{w}(F, \beta) \times F + c$. Since there are T/F pay periods, the firm's problem is to choose F to

$$\min_F \frac{T}{F} (\hat{w}(F, \beta) \times F + c).$$

Since \hat{w} is increasing in F and the rate of increase is decreasing in β , $\Delta \hat{w}$ is positive and decreasing in β . This yields

$$\frac{\partial \hat{w}(F, \beta)}{\partial F} F^{*2} = c$$

Since $\frac{\partial \hat{w}(F, \beta)}{\partial F}$ is larger for lower β , the optimal time between paychecks F must be smaller. As $\beta \rightarrow 1$, $\frac{\partial \hat{w}(F, \beta)}{\partial F} \rightarrow 0$ since the consumption choices specified in Equation (7) do not vary with F . For any positive c , therefore, $F^* \rightarrow \infty$ as $\beta \rightarrow 1$. The maximum allowable value of F is \bar{F} , so $F \rightarrow \bar{F}$ as $\beta \rightarrow 1$. ■

Proof of Proposition 2. The worker in period $F - 1$ will choose consumption to

$$\max_{c_{F-1}} \log(c_{F-1}) + \beta \log(s_{F-1} - c_{F-1}),$$

which yields

$$c_{F-1} = \frac{1}{\beta} c_F.$$

In terms of savings at time $F - 1$, we get

$$\begin{aligned} c_{F-1} &= \frac{1}{1 + \beta} s_{F-1} \\ c_F &= \frac{\beta}{1 + \beta} s_{F-1}. \end{aligned}$$

In period $F - 2$ the worker's problem is

$$\max_{c_{F-2} \leq s_{F-2}} \left[\log(c_{F-2}) + \beta \left(\log \left(\frac{1}{1 + \beta} s_{F-1} \right) + \log \left(\frac{\beta}{1 + \beta} s_{F-1} \right) \right) \right],$$

which can be written

$$\max_{c_{F-2} \leq s_{F-2}} [\log(c_{F-2}) + X + 2\beta \log(s_{F-2} - c_{F-2})]$$

where $X = \beta(\log(\beta) - 2\log(1 + \beta))$, which does not depend upon c_{F-2} . This yields

$$\begin{aligned} c_{F-2} &= \frac{1}{1 + 2\beta} s_{F-2} \\ c_{F-1} &= \frac{2\beta}{1 + 2\beta} \frac{1}{1 + \beta} s_{F-2} \\ c_F &= \frac{2\beta}{1 + 2\beta} \frac{\beta}{1 + \beta} s_{F-2}. \end{aligned}$$

Assume that the worker consumes her entire savings in the period before her next paycheck. We will confirm that this assumption holds below. When we iterate back to period 1, the savings at

the time of the paycheck is $s_0 = 0$ and the initial payment is wF . Given F we have

$$\begin{aligned}
c_1 &= \frac{1}{1 + (F-1)\beta} wF \\
c_2 &= \frac{(F-1)\beta}{1 + (F-1)\beta} \frac{1}{1 + (F-2)\beta} wF \\
c_3 &= \frac{(F-1)\beta}{1 + (F-1)\beta} \frac{(F-2)\beta}{1 + (F-2)\beta} \frac{1}{1 + (F-3)\beta} wF \\
&\dots \\
c_F &= \frac{(F-1)\beta}{1 + (F-1)\beta} \frac{(F-2)\beta}{1 + (F-2)\beta} \dots \frac{\beta}{1 + \beta} wF,
\end{aligned}$$

which can be written generally as

$$c_1 = \frac{wF}{1 + (F-1)\beta} \quad (20)$$

$$c_i = \frac{wF}{1 + (F-i)\beta} \times \left[\prod_{j=1}^{i-1} \frac{(F-j)\beta}{1 + (F-j)\beta} \right] \text{ for } i \in \{2, 3, \dots, F\}. \quad (21)$$

We now check that the worker would prefer to consume her entire savings in the period before a paycheck. We begin with the period before the final paycheck. The worker's marginal propensity to consume in the period of her final paycheck is $\frac{dc_1}{ds_1} = \frac{1}{1+(F-1)\beta}$, so Equation (7) implies that the worker's ideal ratio of marginal consumption in the period before a paycheck to the next period is

$$\begin{aligned}
\frac{1/c_F}{1/c_1} &= 1 - (1-\beta) \frac{dc_1}{ds_1} \\
\Rightarrow \frac{c_1}{c_F} &= 1 - (1-\beta) \frac{1}{1 + (F-1)\beta},
\end{aligned}$$

unless the worker is at a corner solution, in which case

$$\frac{c_1}{c_F} > 1 - (1-\beta) \frac{1}{1 + (F-1)\beta}.$$

Since $c_1 > c_F$, it immediately follows that $\frac{c_1}{c_F} > 1 - (1-\beta) \frac{1}{1+(F-1)\beta}$, so the worker indeed prefers to spend her entire savings in the period before her final paycheck. The same analysis applies to all previous paychecks iteratively. ■

Proof of Proposition 3. The per period wage, \hat{w} , multiplied by the number of periods per

pay-period, F , is the size of the paycheck. c is the fixed cost of paying a check, so the total cost per pay-period to the firm is $\widehat{w}(F, \beta) \times F + c$. Since there are approximately T/F pay periods,²² the firm's problem is to choose F to

$$\min_F \frac{T}{F} (\widehat{w}(F, \beta) \times F + c).$$

Since \widehat{w} is increasing in F and the rate of increase is decreasing in β , $\Delta \widehat{w}$ is positive and decreasing in β . This yields

$$\frac{\partial \widehat{w}(F, \beta)}{\partial F} F^2 = c$$

Since $\frac{\partial \widehat{w}(F, \beta)}{\partial F}$ is larger for lower β , the optimal time between paychecks F must be smaller. As $\beta \rightarrow 1$, $\frac{\partial \widehat{w}(F, \beta)}{\partial F} \rightarrow 0$ since the consumption choices specified in Equation (7) do not vary with F . For any positive c , therefore, $F^* \rightarrow \infty$ as $\beta \rightarrow 1$. ■

Proof of Proposition 4.

We show the result using backward induction. Suppose the worker has savings of s_{T-1} entering period $T - 1$ and is to be paid w_{T-1} that period and w_T in the final period. The worker's discount factor between periods $T - 1$ and T is $\beta < 1$ whereas the firm's is unity. Suppose $\frac{w_T}{w_{T-1} + s_{T-1}} > \beta$. Then the worker would be willing to pay a positive interest rate to the firm for an advance on the final paycheck, and the firm would be willing to accept a positive interest rate. Any contract where this will result is therefore not renegotiation-proof. Therefore, $\frac{w_T}{w_{T-1} + s_{T-1}} \leq \beta$. Regardless of the wages and savings that allow that inequality to hold, consumption in the final two periods is given by $\frac{c_T}{c_{T-1}} = \beta$. This argument follows iteratively for prior periods along lines shown in the proof of proposition 2 and is therefore omitted here. ■

Proof of Proposition 5. Payday lending allows the worker to choose consumption in the period before a paycheck just like every other period. Suppose that in the final period before her final paycheck, she borrows B , so that $B(1 + r)$ repaid when her final paycheck arrives. Then her

²² "Approximately", because T/F may not be integer valued. The actual number of payments T/F rounded down. As T gets large, fixing F , the difference between T/F and the actual number of pay-periods, as a proportion of total costs, goes to zero.

consumption in the final pay-period satisfies

$$c_1 = \frac{wF - B(1+r)}{1 + (F-1)\beta} \quad (22)$$

$$c_i = \frac{wF - B(1+r)}{1 + (F-i)\beta} \times \prod_{j=1}^{i-1} \frac{(F-j)\beta}{1 + (F-j)\beta} \text{ for } i \in \{2, 3, \dots, F\}. \quad (23)$$

Her borrowing choice is given by

$$\begin{aligned} B^* &= \arg \max_B \left[\log(s_F + B) + \beta \log \left(\frac{wF - B(1+r)}{1 + (F-1)\beta} \right) \right. \\ &\quad \left. + \beta \sum_{t=2}^F \log \left(\frac{wF - B(1+r)}{1 + (F-i)\beta} \prod_{j=1}^{i-1} \frac{(F-j)\beta}{1 + (F-j)\beta} \right) \right] \\ &= \arg \max_B [\log(s_F + B) + \beta F \log(wF - B(1+r))]. \end{aligned}$$

Then borrowing equals

$$B^{T/F} = \frac{RwF - \beta F s_F}{1 + \beta F},$$

where the superscript notes that it is the amount borrowed before the T/F (read: final) paycheck.

Her period $F-1$ self (in the penultimate pay-period) must decide whether to save. We assume she does not save, and confirm this below. If she does not save, then $c_{F-1}^{T/F-1} = s_{F-1}^{T/F-1}$ while her next-period consumption equals $B^{T/F} = \frac{RwF}{1+\beta F}$. In the final pay cycle, when she will not be able to take a payday loan at the end, her consumption is as before except with lower initial wealth (the repayment amount of $\frac{wF}{1+\beta F}$ must be repaid at the start of the pay cycle):

$$c_1^{T/F} = \frac{\beta F}{1 + \beta F} \frac{wF}{1 + (F-1)\beta} \quad (24)$$

$$= \frac{wF}{1 + (F-1)\beta} \times \prod_{j=0}^0 \frac{(F-j)\beta}{1 + (F-j)\beta} \quad (25)$$

$$c_i^{T/F} = \frac{wF}{1 + (F-i)\beta} \times \left[\prod_{j=0}^{i-1} \frac{(F-j)\beta}{1 + (F-j)\beta} \right] \text{ for } i \in \{2, 3, \dots, F\}. \quad (26)$$

In the second through $T/F-1$ cycles, her consumption is identical to the case without payday lending, but the cycle begins one period earlier (the proof follows immediately from the proofs thus

far shown).

$$c_1^k = \frac{(F-1)\beta}{1+(F-1)\beta} \frac{wF}{1+(F-2)\beta} \quad (27)$$

$$c_i^k = \frac{wF}{1+(F-i)\beta} \times \left[\prod_{j=1}^{i-1} \frac{(F-1-j)\beta}{1+(F-1-j)\beta} \right] \text{ for } i \in \{2, 3, \dots, F-1\} \quad (28)$$

$$c_F^k = \frac{RwF}{1+(F-1)\beta} \text{ for } k < T/F - 1 \quad (29)$$

$$c_F^k = \frac{RwF}{1+\beta F} \text{ for } k = T/F - 1, \quad (30)$$

for $k \in \{2, 3, \dots, T/F - 1\}$. The first pay-period is different, however. In this period, there is no outstanding debt when the first paycheck arrives. Since the worker will take a payday loan in period F , consumption in the first $F-1$ periods is identical to the case where pay is wF but checks arrive every $F-1$ periods.

$$c_1^1 = \frac{wF}{1+(F-2)\beta} \quad (31)$$

$$c_i^1 = \frac{wF}{1+(F-1-i)\beta} \times \left[\prod_{j=1}^{i-1} \frac{(F-1-j)\beta}{1+(F-1-j)\beta} \right] \text{ for } i \in \{2, 3, \dots, F-1\} \quad (32)$$

$$c_F^1 = \frac{RwF}{1+(F-1)\beta}. \quad (33)$$

This all assumes that the worker will choose $s_F = 0$ in the final period of each pay-period. We check that she would indeed prefer not to save into the final period of the penultimate pay-period. She chooses s_F to maximize:

$$s_F = \arg \max_{s \geq 0} \left[\begin{aligned} & \log(s_{F-1} - s) + \beta \log\left(\frac{s+RwF}{1+\beta F}\right) + \beta \log\left(\frac{wF - \frac{RwF - \beta F s}{1+\beta F}(1+r)}{1+(F-1)\beta}\right) \\ & + \beta \sum_{t=2}^F \log\left(\frac{wF - \frac{RwF - \beta F s}{1+\beta F}(1+r)}{1+(F-i)\beta} \prod_{j=1}^{i-1} \frac{(F-j)\beta}{1+(F-j)\beta}\right) \end{aligned} \right],$$

where $s_{F-1} = \frac{wF}{1+\beta} \times \left[\prod_{j=1}^{F-2} \frac{(F-1-j)\beta}{1+(F-1-j)\beta} \right]$.²³ Plugging in and reducing, we get

$$s_F = \arg \max_{s \geq 0} \left[\log \left(\frac{wF}{1+\beta} \times \left[\prod_{j=1}^{F-2} \frac{(F-1-j)\beta}{1+(F-1-j)\beta} \right] - s \right) + \beta \log(s + Rwf) + \beta F \log \left(\frac{wF + (1+r)s}{1+\beta F} \beta F \right) \right]$$

The derivative of utility with respect to the amount saved is

$$-\frac{1}{\frac{wF}{1+\beta} \times \left[\prod_{j=1}^{F-2} \frac{(F-1-j)\beta}{1+(F-1-j)\beta} \right] - \hat{s}} + \beta \frac{1}{\hat{s} + RF} - \beta F \frac{\frac{\beta F}{1+\beta F}(1+r)}{F - \frac{RF - \beta F \hat{s}}{1+\beta F}(1+r)},$$

where $\hat{s} = s/w$. This derivative is less than zero if and only if

$$\frac{R(1+\beta)}{\prod_{j=1}^{F-2} \frac{(F-1-j)\beta}{1+(F-1-j)\beta}} + \beta(F-1) > 0,$$

which it clearly is. This means that the worker would prefer to borrow in the penultimate period and repay that amount with some of her payday loan, but this is not possible. Therefore, she sets $s_F = 0$, as we previously assumed.

A similar calculation can be performed for pay-periods prior to the final one. For brevity, we do not show those here. ■

Proof of Proposition 6. Consumption in period F equals the cap unless the worker has saved. There will be a threshold C^* such that when $C < C^*$ the worker saves for the final period in a pay-period: $s_F > 0$. When $C > C^*$, the worker will not save so $s_F = 0$. This threshold is lower in the first pay-period since consumption is higher in period $F-1$ in pay-period one than in future pay periods, making the marginal cost of saving lower. The marginal benefit of saving is the same in all cases since consumption in period F is always C .

We solve for the consumption paths in four cases. First, we find consumption when $s_F = 0$, but the cap binds. She would like to borrow $\frac{RwF}{1+\beta F}$ but if $C < \frac{RwF}{1+\beta F}$ she may only borrow up to the cap.

²³This is because we are confirming that $s_F = 0$ is optimal given the consumption and saving choices at other time periods. In particular, the amount saved for period $F-1$ equals the amount consumed in period $F-1$ under that assumption, so we set $s_{F-1} = c_{F-1}$. Again, this is only correct if indeed $s_F = 0$ is optimal.

Her consumption in the final F periods is therefore given by

$$c_1^{T/F} = \frac{wF - C(1+r)}{1 + (F-1)\beta} \quad (34)$$

$$c_i^{T/F} = \frac{wF - C(1+r)}{1 + (F-i)\beta} \times \left[\prod_{j=1}^{i-1} \frac{(F-j)\beta}{1 + (F-j)\beta} \right] \text{ for } i \in \{2, 3, \dots, F\}. \quad (35)$$

In the middle pay periods, $k \in \{2, 3, \dots, T/F - 1\}$, her consumption is

$$c_1^k = \frac{wF - C(1+r)}{1 + (F-2)\beta} \quad (36)$$

$$c_i^k = \frac{wF - C(1+r)}{1 + (F-1-i)\beta} \times \left[\prod_{j=1}^{i-1} \frac{(F-1-j)\beta}{1 + (F-1-j)\beta} \right] \text{ for } i \in \{2, 3, \dots, F-1\} \quad (37)$$

$$c_F^k = C, \quad (38)$$

and in the first pay-period, her consumption is

$$c_1^1 = \frac{wF}{1 + (F-2)\beta} \quad (39)$$

$$c_i^1 = \frac{wF}{1 + (F-1-i)\beta} \times \left[\prod_{j=1}^{i-1} \frac{(F-1-j)\beta}{1 + (F-1-j)\beta} \right] \text{ for } i \in \{2, 3, \dots, F-1\} \quad (40)$$

$$c_F^1 = C. \quad (41)$$

The derivation follows nearly identical lines to the earlier proofs and is omitted for brevity.

This has assumed that the cap was sufficiently low that the worker will borrow up to the cap, but sufficiently high that she will not save from period $F-1$ to period F in a pay-period. Given that the cap binds, there are two thresholds separating when the worker will set $s_F = 0$ versus $s_F > 0$. One concerns the saving decision at the end of the first pay-period, denoted C^{*1} , and one concerns the saving decision in all other pay-periods, denoted C^{*k} . If $C < C^{*k}$, then the worker sets $s_F > 0$ in all pay-periods; if $C^{*k} \leq C < C^{*1}$, then $s_F = 0$ in all but the first pay-period and $s_F > 0$ in the first; and if $C > C^{*1}$, then $s_F = 0$ always. We now show that long-run utility is decreasing in C when $C < C^{*k}$ (thus achieving a maximum at $C = 0$) and is concave in C when $C > C^{*k}$, reaching a maximum at $C = w$.

Case 1: Let $C < C^{*k}$. In these cases, consumption in the middle pay-periods is the same as

when no payday lending is permitted, except during period F of each pay-period, in which case it is lower by a factor of R . Consumption in the first $F - 1$ periods is higher than it otherwise would be and consumption in the last F is lower. Increasing C , therefore, increases the overconsumption in the first pay-period and decreases underconsumption in the final pay-period, and weakly decreases consumption in between. Increasing C is clearly harmful for long-run utility.

Therefore, in this case, the value of C that maximizes long-run utility is $C = 0$.

Case 2: Let $\frac{wF}{1+(F-2)\beta} > C > C^{*1}$. Then $s_F = 0$ and $c_F = C$. Long-run utility in the first pay-period equals

$$U_1^{LR}(C) = \log\left(\frac{wF}{1+(F-2)\beta}\right) + \sum_{i=2}^{F-1} \log\left(\frac{wF}{1+(F-1-i)\beta} \times \left[\prod_{j=1}^{i-1} \frac{(F-1-j)\beta}{1+(F-1-j)\beta}\right]\right) + \log(C),$$

and the derivative of this utility with respect to C equals $\frac{1}{C}$. Long-run utility in each of the second through second-to-last pay-periods equals

$$U_{k \in \{2,3,\dots,T/F-1\}}^{LR} = \log\left(\frac{wF - C(1+r)}{1+(F-2)\beta}\right) + \sum_{i=2}^{F-1} \log\left(\frac{wF - C(1+r)}{1+(F-1-i)\beta} \times \left[\prod_{j=1}^{i-1} \frac{(F-1-j)\beta}{1+(F-1-j)\beta}\right]\right) + \log(C),$$

and the derivative of this utility with respect to C equals $-\frac{(1+r)(F-1)}{wF-C(1+r)} + \frac{1}{C}$. Finally, long-run utility in the last pay-period equals

$$U_{T/F}^{LR}(C) = \log\left(\frac{wF - C(1+r)}{1+(F-1)\beta}\right) + \sum_{i=2}^F \log\left(\frac{wF - C(1+r)}{1+(F-i)\beta} \times \left[\prod_{j=1}^{i-1} \frac{(F-j)\beta}{1+(F-j)\beta}\right]\right),$$

and the derivative of this utility with respect to C equals $-\frac{(1+r)F}{wF-C(1+r)}$. Summing the derivatives of the utilities in each pay-period yields the derivative of long-run utility with respect to C . Setting this equal to zero, we find the optimal cap:

$$C^{FB} = \frac{T - F}{T - F + 1} \times Rw$$

Because $T = \bar{F}!$, $C^{FB} \approx Rw$. Also note that the second derivative of U^{LR} is negative: $\frac{d^2}{dC^2} U^{LR} = -\frac{T-1}{C^2} - \frac{(\frac{T}{F}-2) \times (F-1) + F}{(wF-C)^2} < 0$, so the objective is concave.

Case 3: If $C > \frac{wF}{1+(F-2)\beta}$, then consumption is higher in the first $F - 1$ periods and lower in

the last F . Utility is independent of C , because it is a non-binding constraint. Therefore, utility is equal to the previous case, letting $C = \frac{w^F}{1+(F-2)\beta}$. We know that this is less than the long-run utility that the worker would obtain with $C = \left(\frac{T-F}{1+T-F}\right)w$, so a C in this range cannot be optimal.

Case 4: If $C^{*k} < C < C^{*1}$, then increasing C would increase consumption in the first F periods (because $s_F > 0$) at the expense of the last periods. In the middle, however, consumption is smoother, as $c_F = C$. A similar analysis to Case 2 above shows that U^{LR} is increasing in C and therefore maximized (within this range) at $C = C^{*1}$. Because we know that U^{LR} is even higher at $C = \left(\frac{T-F}{1+T-F}\right)w$, no choice of C in this range can be optimal.

We are therefore left with potential optimal values of C equal to zero or $\left(\frac{T-F}{1+T-F}\right)Rw \approx Rw$.

■

IX Tables and Figures

Pay frequency	<i>4x/month</i>	<i>2x/month</i>	<i>1x/month</i>	<i>No rule</i>	Total
PDL caps					
<i>Prohibited</i>	1	4	0	0	5
<i>Small (<500)</i>	0	7	2	2	11
<i>Large (500+)</i>	2	11	8	3	24
<i>Uncapped</i>	3	5	2	1	11
<hr/> Total	6	27	12	6	51

Table 1: This table reports the number of states mandating a variety of pay frequencies and caps on payday loan sizes. Pay frequency rules are assigned to the closest group if they do not precisely align with any of the groups. For example, a state requiring pay at least every 35 days would be assigned to the 1x/month group. Two states have payday loan caps that are proportional to the borrower’s income. These have been assigned to the “small” cap group, based on a multiplication of the capped proportion and the median income in those states.

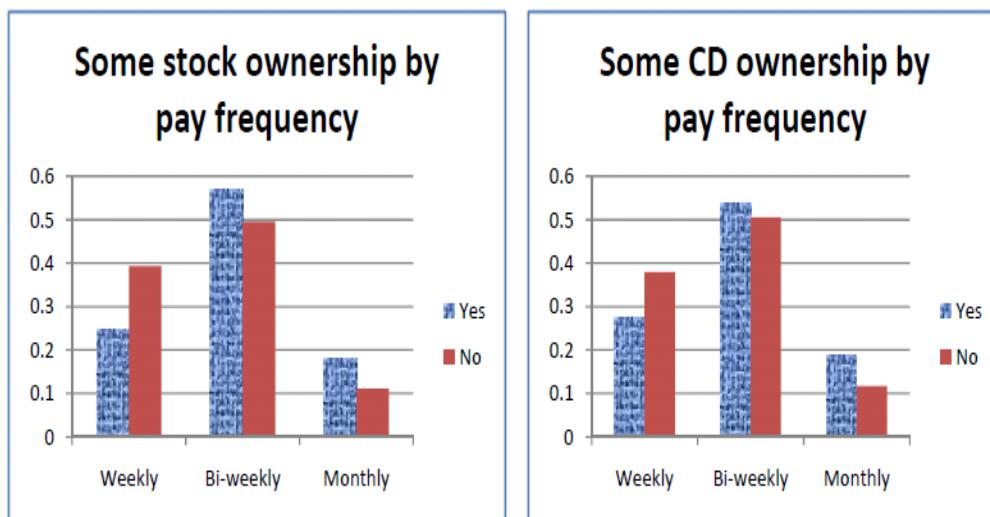


Figure 1: Data are found in the US Census Bureau’s Survey of Income and Program Participation, 1996. For clarity, we restrict attention to workers who report being paid weekly, bi-weekly, or monthly. Of all surveyed employees that own at least some stock, 22% are paid weekly and 19% monthly, with the remainder paid bi-weekly. Of surveyed employees that do not own stock, 40% are paid weekly while only 10% are paid monthly. Stock owners are nearly twice as likely to be paid monthly and almost half as likely to be paid weekly. Similar, though less extreme, numbers as associated with CD ownership.

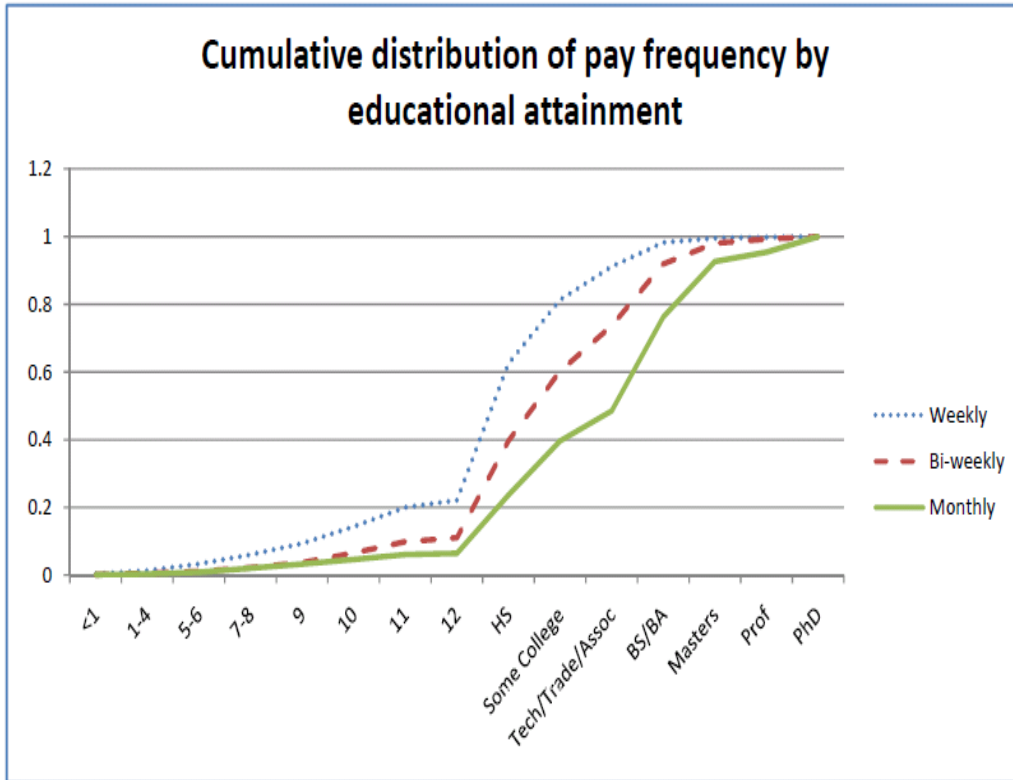


Figure 2: Data are found in the US Census Bureau’s Survey of Income and Program Participation, 1996. For clarity, we restrict attention to workers who report being paid weekly, bi-weekly, or monthly. This figure reports cumulative distributions of educational attainment, stratified by the worker’s pay frequency.

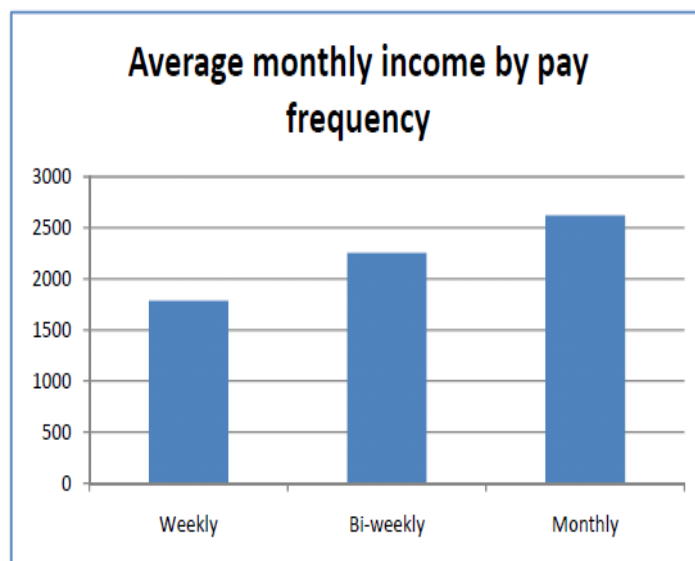


Figure 3: Data are found in the US Census Bureau’s Survey of Income and Program Participation, 1996. For clarity, we restrict attention to workers who report being paid weekly, bi-weekly, or monthly.

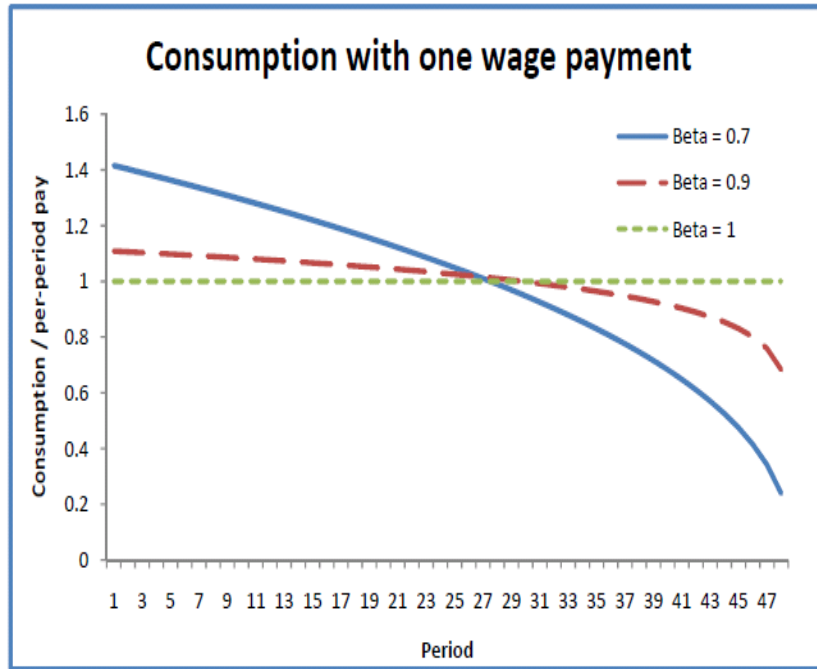


Figure 4: These curves show consumption for a worker that is paid with one up-front payment when $T = 48$. This consumption path is the outcome of any renegotiation-proof contract.

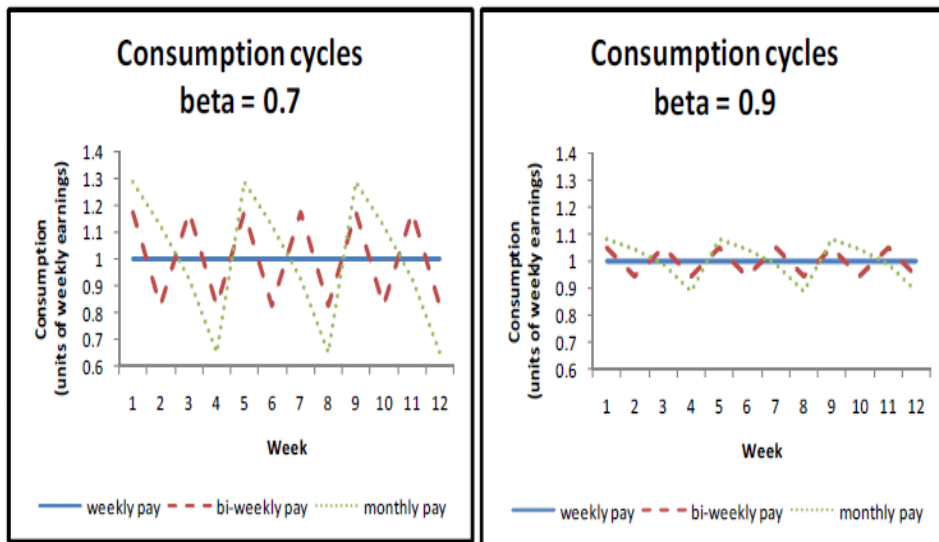


Figure 5: These graphs plot consumption for workers with log utility, $\beta = 0.7$, and $\beta = 0.9$, paid weekly, bi-weekly or quad-weekly. Workers always consume more soon after the paycheck arrives and less prior to the next check. This cyclical pattern is more extreme the longer the time between pay-periods, resulting in lower time-zero utility, and is more severe for workers with greater present-bias (lower β). Consumption within a pay-period is concave down: the undersaving problem is most severe immediately before the next paycheck arrives.

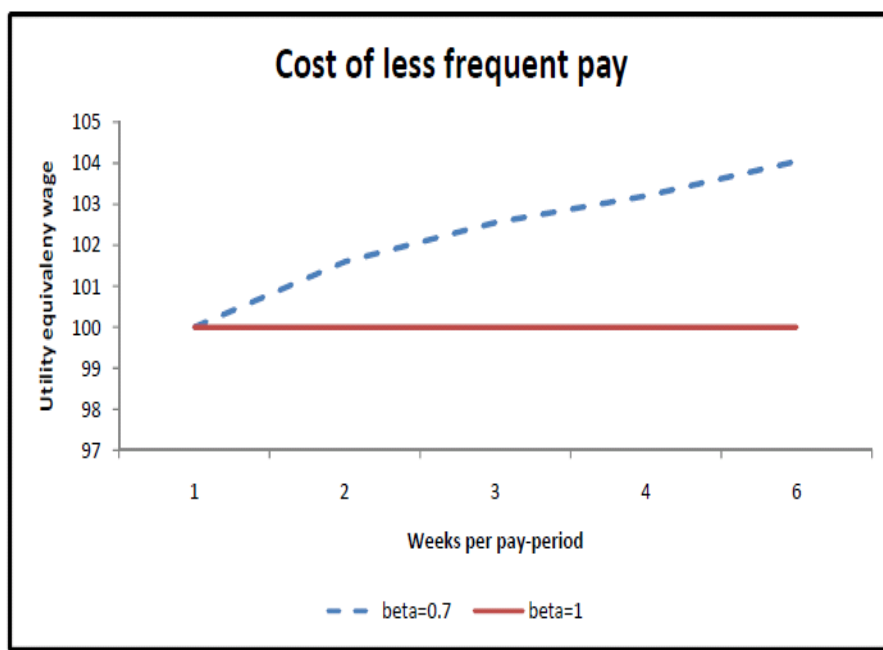


Figure 6: As the length of pay-period increases, the wage must increase for present-biased workers. A worker with $\beta = 0.7$ being paid quad-weekly must be paid 4% more than one paid weekly.

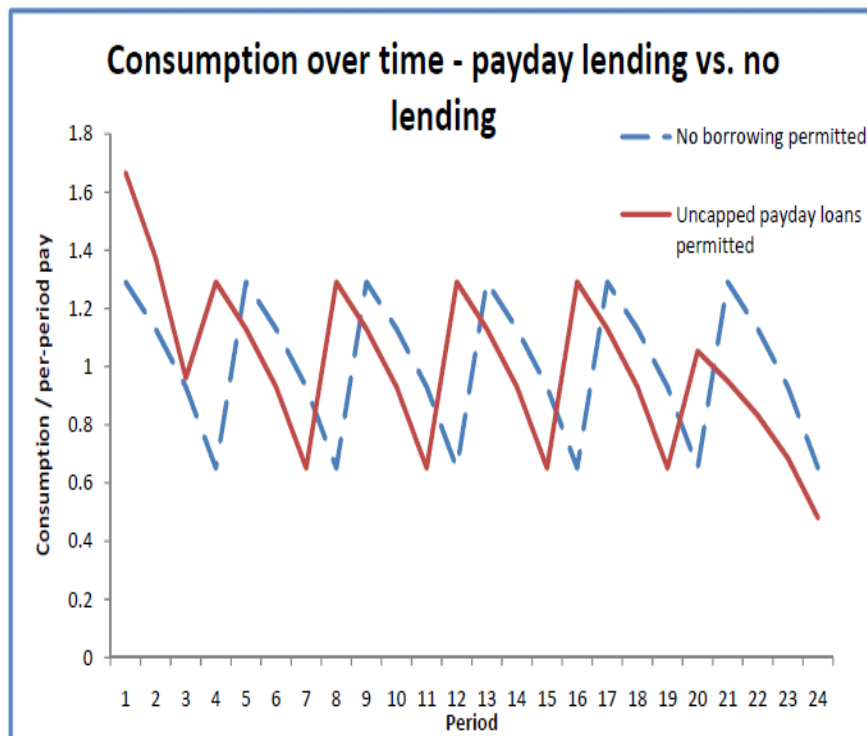


Figure 7: These curves describe consumption choices of consumers with $\beta = 0.7$, paid every four periods, with an interest rate of 0%. When payday loans are unavailable, consumption follows the same pattern in every four week pay-period. When payday loans are available, consumption is higher in the first pay-period at the expense of the last. The consumption cycle advances to begin when the payday loan is available rather than on payday.

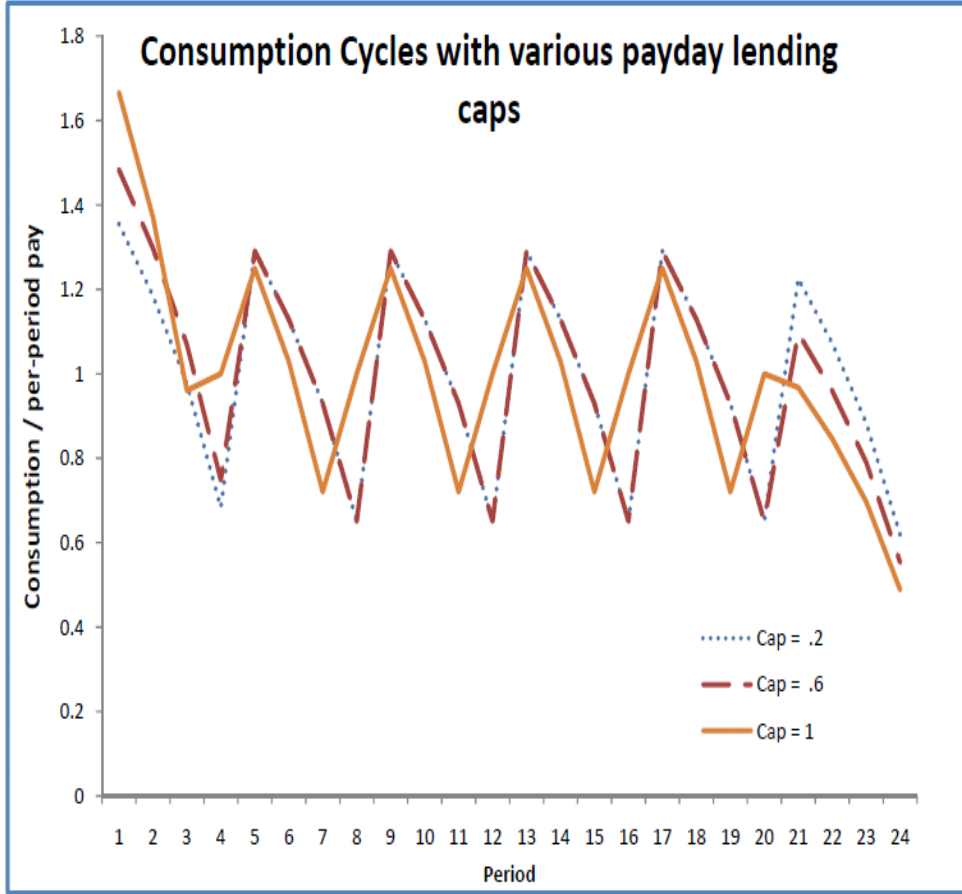


Figure 8: These curves describe consumption choices of consumers with $\beta = 0.7$, paid every four periods, with an interest rate of 0%. Regardless of the size of the cap on payday loans, as that cap increases, consumption in the first pay-period increases, at the expense of consumption in the last. When caps are low, changes in the size of the cap do not affect pay cycles in the middle of the contracting term. When caps are high, increases in the cap increase consumption in the period before payday and decrease consumption in all other periods. So long as the cap is less than per-period pay, increasing the cap decreases the variance in consumption. Once the cap is greater than per-period pay, increasing the cap increases the variance in consumption.

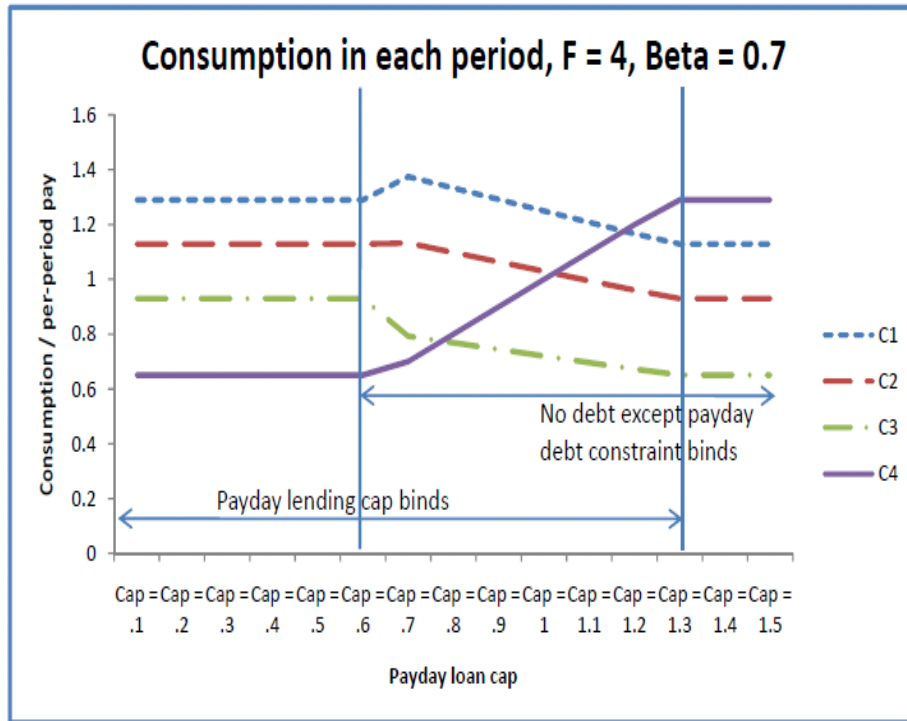


Figure 9: This figure plots consumption in each period of a pay-period when $F = 4$ and $\beta = 0.7$, where C_i is consumption in period i of the pay-period. When the payday lending cap is low, the worker will save for the final period, so her consumption in each period is independent of the cap. Consumption is high on payday and declines through the pay-period. Once the payday lending cap is high enough, she ceases to save from the third to fourth period in the pay-period. Her consumption in the fourth period increases with the cap and consumption in other periods decreases. The variance in her consumption is minimized when the cap is equal to one period's pay. When the cap is high, it does not bind. Consumption in the four periods is identical to when no payday lending is permitted, except that it is now highest in the period before payday and lower in the other three periods.

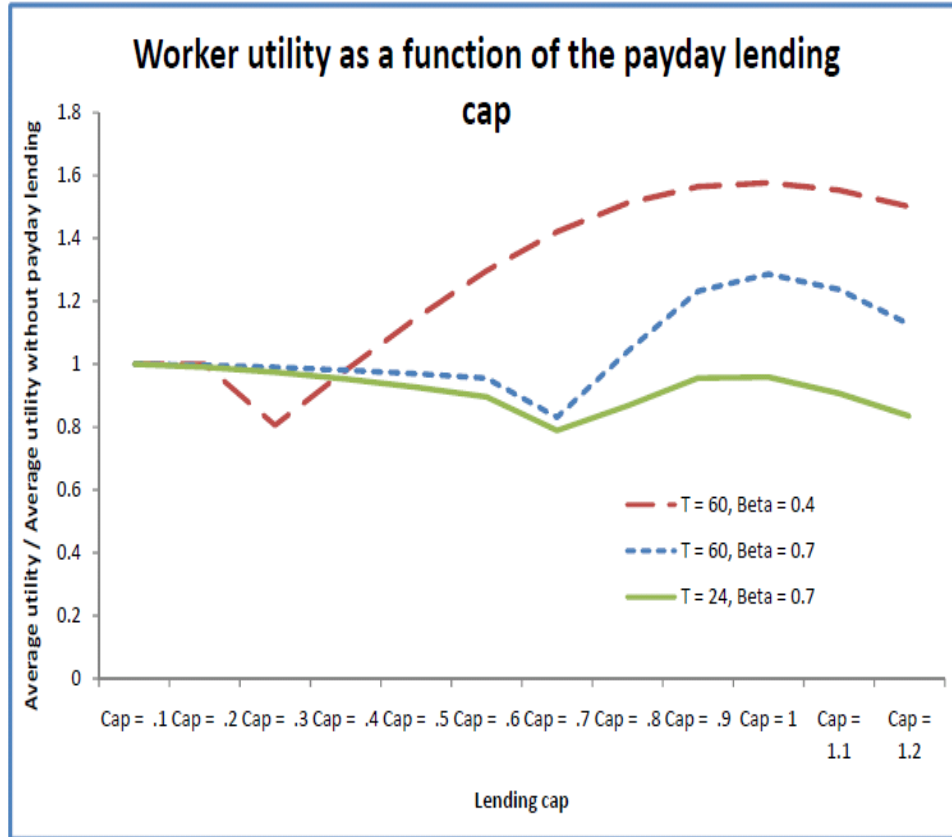


Figure 10: These curves plot long-run worker utility over the life of the contract for three types of worker/contract pair. In all cases $\delta = 1$, the interest rate equals 0%, and pay-periods are four periods long. Because long-run utility depends on the contract and worker parameters, in each case, worker utility is normalized to long-run utility when payday loans are not allowed. As the contract length increases, the benefit of capped payday loans increases because the cost (higher consumption in the first pay-period, when consumption is already high, and lower consumption in the last, where consumption is already low) is independent of contract length and the benefit (less variable consumption in the middle cycles of the contract) is increasing in contract length. As β increases and workers are more patient, the benefit of capped payday lending decreases. The kink at which increasing the cap improves welfare shifts represents the cap at which saving in the penultimate period of a pay-period is optimally set at zero. Since saving increases in β , that kink shifts right as β increases.