Quartification with $T^{'}$ Flavor

David A. Eby[1*], Paul H. Frampton[1†], Xiao-Gang He[2,3‡], and Thomas W. Kephart[4§]

[1] *Department of Physics and Astronomy,*

University of North Carolina, Chapel Hill, NC 27599-3255, USA

[2] *Department of Physics and Center for Theoretical Sciences,*

National Taiwan University, Taipei, Taiwan

[3] *INPAC, Department of Physics, Shanghai Jiao Tong University, Shanghai, China*

[4] *Department of Physics and Astronomy,*

Vanderbilt University, Nashville, Tennessee 37235, USA

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Abstract

In the simplest (non-quiver) unified theories, fermion families are often treated sequentially and a flavor symmetry may act similarly. As an alternative with non-sequential flavor symmetry, we consider a model based on the group $(T^{'} \times Z_2)_{global} \times [SU(3)^4]_{local}$ which combines the predictions of $T^{'}$ flavor symmetry with the features of a unified quiver gauge theory. The model accommodates the relationships between mixing angles separately for neutrinos, and for quarks, which have been previously predicted with $T^{'}$. This quiver unification theory makes predictions of several additional gauge bosons and bifundamental fermions at the TeV scale.

PACS numbers:

* daeby@physics.unc.edu

† paul.h.frampton@gmail.com

‡ hexg@phys.ntu.edu.tw

§ tom.kephart@gmail.com

I. INTRODUCTION

In order to address the question of masses and mixing angles which occur for quarks and leptons in the standard model, one promising direction is to introduce a flavor symmetry that commutes with the standard model gauge group. By judicious assignments of the particles to specific representations of the flavor symmetry, one can obtain relations between parameters in the model. The flavor symmetry may treat the fermion families differently so that the simplest approaches to gauge unification are inapplicable. The present article will show how to combine the flavor group ($T^{'}$), which has been studied previously [1–8], with a quiver unified quartification $SU(3)^4$ gauge group [9], while successfully keeping results previously obtained without unification, such as the Cabibbo angle [6], as well as tribimaximal mixing for neutrinos [10–15]. The quiver unification has the advantage of implying further relationships between the gauge couplings.

II. THE MODEL

We first consider a quartification ($SU(3)^4$) model with bifundamental chiral fermions in the usual arrangement of bifundamentals, but find we can not make the necessary charge assignments to recover the requisite $T^{'}$ family symmetry. This will lead us to add a subquiver of fermions to accommodate $T^{'}$ quartification.

Quartification, from its inception by Joshi and Volkas [16], has historically been used for gauge-coupled unification without supersymmetry and for leptonic color models [9, 17–21]. Many of these models have adapted the same unification techniques as the first GUT theories [22]. There have been several significant milestones in this approach (and several different preferred unification scales) including partial unification [16], complete unification [17], and intermediate symmetry breaking [18]. We choose a different style of unification compared with prior work on quartification, one predicated upon the mechanism in Refs. [23, 24], that by embedding

$$SU(3)_C \times SU(2)_L \times U(1)_Y \ , \tag{1}$$

in $SU(3)^N$ we naturally achieve unification in the TeV region. This is accomplished by replacing the logarithmic evolution of couplings, with the use of group theoretic factors.

The quartification gauge group is the quasi-simple

$$SU(3)_C \times SU(3)_L \times SU(3)_\ell \times SU(3)_R \ , \tag{2}$$

with couplings equal up to numerical group theory factors [23, 24]. Let the family symmetry be

$$T' \times Z_2 \ , \tag{3}$$

with the minimal anomaly-free bifundamental chiral fermions:

$$3[(3,\bar{3},1,1) + (\bar{3},1,1,3) + (1,3,\bar{3},1) + (1,1,3,\bar{3})] \ . \tag{4}$$

We shall assign the leptons to irreps as follows [6] :

$$\left.\begin{array}{l} (13\bar{3}1)_3 \supset \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \\ (13\bar{3}1)_2 \supset \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \\ (13\bar{3}1)_1 \supset \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \end{array}\right\} L_L(3,+1) \quad \begin{array}{lll} (113\bar{3})_3 \supset \tau_R^- \ (1_1,-1) & and & N_R^{(1)} \ (1_1,+1) \\ (113\bar{3})_2 \supset \mu_R^- \ (1_2,-1) & and & N_R^{(2)} \ (1_2,+1) \\ (113\bar{3})_1 \supset e_R^- \ (1_3,-1) & and & N_R^{(3)} \ (1_3,+1) \ . \end{array} \tag{5}$$

For the left handed quarks we make the assignment

$$\begin{array}{lll} (3\bar{3}11)_3 \supset \begin{pmatrix} t \\ b \end{pmatrix}_L & \mathcal{Q}_L & (\mathbf{1_1},+1) \\ \left.\begin{array}{l} (3\bar{3}11)_2 \supset \begin{pmatrix} c \\ s \end{pmatrix}_L \\ (3\bar{3}11)_1 \supset \begin{pmatrix} u \\ d \end{pmatrix}_L \end{array}\right\} & Q_L & (\mathbf{2_1},+1) \ . \end{array} \tag{6}$$

Finally, we need assignments for the six right-handed quarks. They were assigned to

$$\begin{array}{lc} t_R & (\mathbf{1_1}, +1) \\ b_R & (\mathbf{1_2}, -1) \\ \left.\begin{array}{l} c_R \\ u_R \end{array}\right\} \mathcal{C}_R & (\mathbf{2_3}, -1) \\ \left.\begin{array}{l} s_R \\ d_R \end{array}\right\} \mathcal{S}_R & (\mathbf{2_2}, +1) \ , \end{array} \tag{7}$$

under $T' \times Z_2$ in Ref. [6] (FKM). However, this assignment is inapplicable here as t_R and b_R are both in the same irrep $(\bar{3}113)_3$, despite having different T' assignments (likewise for the first and second families). Without additional states, we are able to assign only three of the six right-handed quarks.

We therefore add an anomaly-free sub-quiver representation

$$3[(\bar{3}, 1, 3, 1)^{'} + (1, 1, \bar{3}, 3)^{'} + (3, 1, 1, \bar{3})^{'}] \ , \tag{8}$$

and reassign *all* fermions with $Z_2 = -1$, including the corresponding subset in Eq. (5) and Eq. (7), to this sub-quiver:

$$\begin{array}{lc} b_R & \subset (\bar{3}, 1, 3, 1)^{'}_{3} \\ \mathcal{C}_R & \subset (\bar{3}, 1, 3, 1)^{'}_{1,2} \\ \tau^-_R & \subset (1, 1, \bar{3}, 3)^{'}_{3} \\ \mu^-_R & \subset (1, 1, \bar{3}, 3)^{'}_{2} \\ e^-_R & \subset (1, 1, \bar{3}, 3)^{'}_{1} \ . \end{array} \tag{9}$$

III. YUKAWA COUPLINGS

We introduce notation in which the $SU(3)$ groups (C, R, ℓ, L) in superscripts are assigned to the fundamental 3, while those in subscripts are assigned to the anti-fundamental $\bar{3}$. The $SU(3)$ groups not denoted in subscript or superscript are designated as singlets in this representation. Additionally, the T' assignment will be listed in parenthesis with the Z_2 charge is given as superscript.

With this stated, the lepton Yukawas are denoted:

$$\Sigma_{i=1}^{i=3} Y_D^{(i)} L_\ell^L(3^+) N_R^{\ell(i)}(1_i^+) H_L^R(3^+) \tag{10}$$

and

$$\Sigma_{i=1}^{i=3} Y_\ell^{(i)} L_\ell^L(3^+) \ell_R^{\ell(i)}(1_i^+) H_L^R(3^-) \ . \tag{11}$$

The quark Yukawa couplings are then given as:

$$\begin{aligned}
& Y_t \mathcal{Q}_L^C(1_1^+) t_C^R(1_1^+) H_R^L(1_1^+) + \\
& Y_b \mathcal{Q}_L^C(1_1^+) b_C^\ell(1_2^-) H_\ell^L(1_3^-) + \\
& Y_{\mathcal{QS}} \mathcal{Q}_L^C(1_1^+) \mathcal{S}_C^R(2_2^+) H_R^L(2_3^+) + \\
& Y_{\mathcal{C}} Q_L^C(2_1^+) \mathcal{C}_C^\ell(2_3^-) H_\ell^L(3^-) + \\
& Y_{\mathcal{S}} Q_L^C(2_1^+) \mathcal{S}_C^R(2_2^+) H_R^L(3^+) \ ,
\end{aligned} \tag{12}$$

where the $T^{'}$ representations with superscript $Z_2 = +$ are in the original quiver and all those with superscript $Z_2 = -$ are in the sub-quiver.

The Higgs scalar sector is sufficient to break to the standard model and replicate the mixing matrices for $T^{'}$ found previously. Note that, for example, the Cabibbo angle in Ref. [6] follows because after breaking of $SU(3)_\ell \times SU(3)_R$ the $H(3^-)s$ have a common representation, and can thus act as the appropriate messenger between the charged leptons and the first two families of quarks. The $T^{'}$ doublet (2_3^+) of Higgs allows reproduction of the successful CKM matrix derived in Ref. [25].

The Higgs vacuum expectation values (hereafter VEVs) follow a form highly similar to that in [6], using the same superscript and subscript notation as above. We put the neutral member of the Higgs doublet at $\alpha_L = 3$ and the corresponding VEV for $(T^{'} = 1_1, Z_2 = +)$ as

$$< H_{R(\alpha_R=1)}^{L(\alpha_L=3)}(1, 3, 1, \bar{3}; 1_1^+) >= \frac{m_t}{Y_t} \ , \tag{13}$$

while we put the third family Higgs VEV at $\alpha_R = 1$ and the VEV for $(T^{'} = 1_3, Z_2 = -)$ as

$$< H_{\ell(\alpha_\ell=1)}^{L(\alpha_L=3)}(1, 3, \bar{3}, 1; 1_3^-) >= \frac{m_b}{Y_b} \ , \tag{14}$$

with an $\alpha_\ell = 1$ assignment in the ℓ-sector. There remain three more VEVs, which are $T^{'}$ nonsinglets, so we now indicate their direction in $T^{'}$ - space to be:

$$< H_R^L(2_3^+) > \quad \propto \quad (1, 1) \tag{15}$$

$$< H_L^R(3^-) > \quad \propto \quad (\frac{m_\tau}{Y_\tau}, \frac{m_\mu}{Y_\mu}, \frac{m_e}{Y_e}) \tag{16}$$

$$< H_L^R(3^+) > \quad \propto \quad (1, -2, 1) \ . \tag{17}$$

This collection of five Higgs VEVs can break both the gauge group to the standard model and achieve the quark and lepton masses as previously derived in Ref. [6] and elaborated on in Refs. [2, 15]. In the most general potential involving all the scalar fields, there is such a surfeit of parameters that stationarization of such a potential can, in general, always allow a stable global minimum corresponding to the VEVs assumed in Eqs. (13 - 17).

IV. DISCUSSION

We have constructed a consistent quiver unified framework, based on $(T^{'} \times Z_2)_{global} \times [SU(3)^4]_{local}$ which subsumes the mixing angle predictions for the leptons and quarks previously made using $T^{'}$ flavor symmetry. Its quiver unification predicts additional gauge bosons and bifundamental fermions at the TeV scale. The production and decay of the lightest Higgs at LHC can be such as to facilitate discovery of $H \to \gamma\gamma$ as was the case in Ref. [26].

This model illustrates how non-family-sequential flavor symmetry $(T^{'} \times Z_2)$, while incompatible with a simple GUT model like $SU(5)$, can be wedded successfully to $SU(3)^4$ quiver unification.

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