## From spacetime foam to holographic foam cosmology

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## Abstract

Due to quantum fluctuations, spacetime is foamy on small scales. For maximum spatial resolution of the geometry of spacetime, the holographic model of spacetime foam stipulates that the uncertainty or fluctuation of distance l is given, on the average, by  $(ll_P^2)^{1/3}$  where  $l_P$  is the Planck length. Applied to cosmology, it predicts that the cosmic energy is of critical density and the cosmic entropy is the maximum allowed by the holographic principle. In addition, it requires the existence of unconventional (dark) energy/matter and accelerating cosmic expansion in the present era. We will argue that a holographic foam cosmology of this type has the potential to become a full fledged competitor (with distinct testable consequences) for scalar driven inflation.

This paper is dedicated to Rafael Sorkin to celebrate his contributions to physics; to appear on the Sorkin60 website.

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Before last century, spacetime was regarded as nothing more than a passive and static arena in which events took place. Early last century, Einstein's general relativity changed that viewpoint and promoted spacetime to an active and dynamical entity. Nowadays, following Wheeler[1], some of us also believe that space is composed of an ever-changing geometry and topology called spacetime foam, and that the foaminess is due to quantum fluctuations [2] of spacetime. In this paper, we will consider the holographic model of spacetime foam, and apply it to the cosmos of the present era.

The magnitude of spacetime fluctuations can be quantified by the average uncertainty  $\delta l$  in the measurement of distance l. In principle,  $\delta l$  can depend on both l and the Planck length  $l_P \equiv (\hbar G/c^3)^{1/2} \sim 10^{-33}$  cm, and hence can be written as  $\delta l \gtrsim l^{1-\alpha} l_P^{\alpha}$ , with  $\alpha \sim 1$ parametrizing the spacetime foam models. We begin by recalling a simple argument [3, 4]for the choice of  $\alpha = 2/3$ . Consider mapping the geometry of spacetime for a volume of spatial extent l and temporal extent l/c. One way is to fill space with clocks, exchanging signals with each other and measuring the signals' times of arrival. This process is a kind of computation and is hence constrained by the Margolus-Levitin theorem [5] in quantum computation, that bounds the rate of elementary logical operations that can be performed within the volume by the amount of energy that is available for computation, which is  $Mc^2$ for the present setup with M being the total mass of the clocks, divided by Planck's constant. To avoid black hole formation, M must be less than l/G, corresponding to an energy density  $\rho \lesssim (ll_P)^{-2}$ . (Here and henceforth we neglect multiplicative constants of order unity, and set  $c = 1 = \hbar$ .) It follows that the number of elementary operations or events that can occur in this spacetime volume is bounded by  $l^2/l_P^2$ . In other words, if one regards the elementary events partitioning the spacetime volume into "cells", then the number of cells is bounded by the surface area of the spatial volume, and each cell occupies a spacetime volume of  $l^4/(l^2/l_P^2)=l^2l_P^2$  on the average.

The maximum spatial resolution of the geometry is obtained if each clock ticks once in time l. Then each cell occupies a spatial volume of  $l^2 l_P^2 / l = l l_P^2$ , yielding an average separation between neighboring cells of  $l^{1/3} l_P^{2/3}$ . This spatial separation can now be interpreted as the average minimum uncertainty  $\delta l$  in the measurement of a distance l, i.e.,  $\delta l \gtrsim l^{1/3} l_P^{2/3}$ .

Two remarks are in order. First, in the above argument, maximal spatial resolution is possible only if the maximum energy density  $\rho \sim (ll_P)^{-2}$  is available (to map the geometry of the spacetime region) without causing a gravitational collapse. Secondly, since, on the average, each cell occupies a spatial volume of  $ll_P^2$ , a spatial region of size l can contain no more than  $l^3/(ll_P^2) = (l/l_P)^2$  cells. Hence, the above argument also shows that this spacetime foam model (yielding the maximum spatial resolution of the geometry of spacetime) corresponds to the case of maximum number of bits of information  $l^2/l_P^2$  in a spatial region of size l, that is allowed by the holographic principle<sup>[6]</sup>, according to which, the maximum amount of information stored in a region of space scales as the area of its two-dimensional surface, like a hologram. For good reason, this spacetime foam model[7] has come to be known, in recent years, as the holographic model[8]. (We have just illustrated how a holographic description arises from local small scale spacetime foam physics!) Alternatively, the holographic principle can also be derived by using the above argument as follows: Consider a spatial region of size l containing particles each of which carries one bit of information. Heisenberg's uncertainty principle dictates that each particle/bit has a momentum greater than  $l^{-1}$ . Now matter can embody the maximum amount of information when it is converted to energetic, massless particles. In that case, each particle/bit has an energy no less than  $l^{-1}$ . But, as shown above, the maximum amount of energy inside the spatial region is bounded by  $l^3 \times (ll_P)^{-2} = ll_P^{-2}$ . Hence the maximum number of bits in a spatial region of size *l* is bounded by  $ll_P^{-2}/l^{-1} = (l/l_P)^2$ .

So far, we have confined our attention to a static spacetime region with low spatial curvature. The whole discussion can be generalized to the case of an expanding universe by the substitution of l by  $H^{-1}$  in the expressions for energy and entropy densities, where His the Hubble parameter. Thus, applied to cosmology, the holographic model of spacetime foam predicts that (1) the cosmic energy density is critical  $\rho \sim (H/l_P)^2$ , and (2) the universe of Hubble size  $R_H$  contains  $\sim HR_H^3/l_P^2$  bits of information. We call this cosmology the holographic foam cosmology (HFC). (For earlier discussions of holographic cosmology, see [9].)

It is instructive to compare the holographic model we have discussed above in the mapping of the geometry of spacetime with the one that corresponds to spreading the spacetime cells uniformly in both space and time. For the latter case, each cell has the size of  $(l^2 l_P^2)^{1/4} =$  $l^{1/2} l_P^{1/2}$  both spatially and temporally, i.e., each clock ticks once in the time it takes to communicate with a neighboring clock. Since the dependence on  $l^{1/2}$  is the hallmark of a random-walk fluctuation, this spacetime foam model corresponding to  $\delta l \gtrsim (ll_P)^{1/2}$  is called the random-walk model[10]. Compared to the holographic model, the random-walk model predicts a coarser spatial resolution, i.e., a larger distance fluctuation, in the mapping of spacetime geometry. It also yields a smaller bound on the information content in a spatial region, viz.,  $(l/l_p)^2/(l/l_P)^{1/2} = (l/l_P)^{3/2}$ . We further note that the spatial resolution for the random-walk model, unlike the holographic model, does not require the maximum total energy because the clocks can tick less frequently than once in the amount of time  $l^{1/2}l_P^{1/2}$  (so long as each clock ticks at least once in the entire time duration of l.)

We are now in a position to draw a useful conclusion [3, 11] from the *observed* cosmic critical density in the present era (consistent with the prediction of the HFC)  $\rho \sim H_0^2/G \sim$  $(R_H l_P)^{-2}$  (about 10<sup>-9</sup> joule per cubic meter), where  $H_0$  is the present Hubble parameter of the observable universe. Treating the whole universe as a computer [3, 12], one can apply the Margolus-Levitin theorem to conclude that the universe computes at a rate  $\nu$  up to  $\rho R_H^3 \sim R_H l_P^{-2}$  (~ 10<sup>106</sup> op/sec), for a total of  $(R_H/l_P)^2$  (~ 10<sup>122</sup>) operations during its lifetime so far. If all the information of this huge computer is stored in ordinary matter, then we can apply standard methods of statistical mechanics to find that the total number I of bits is  $[(R_H/l_P)^2]^{3/4} = (R_H/l_P)^{3/2}$  (~ 10<sup>92</sup>). It follows that each bit flips once in the amount of time given by  $I/\nu \sim (R_H l_P)^{1/2}$  (~ 10<sup>-14</sup> sec). On the other hand, the average separation of neighboring bits is  $(R_H^3/I)^{1/3} \sim (R_H l_P)^{1/2}$  (~ 10<sup>-3</sup> cm). Hence, assuming only ordinary matter exists to store all the information in the universe results in the conclusion that the time to communicate with neighboring bits is equal to the time for each bit to flip once. It follows that the accuracy to which ordinary matter maps out the geometry of spacetime corresponds exactly to the case of events spread out uniformly in space and time discussed above for the case of the random-walk model of spacetime foam.

Interestingly, it has been shown [11] that the sharp images of distant quasars or active galactic nuclei observed at the Hubble Space Telescope have ruled out the random-walk model. More specifically, the presence of an Airy ring in the image of the quasar-like object PKS1413+135 [13] indicates that the amount of light scattering that could be caused by spacetime foam is much less than that predicted by the random-walk model. From the demise of the random-walk model and the fact that ordinary matter only contains an amount of information dense enough to map out spacetime at a level consistent with the random-walk model, one can now infer that spacetime must be mapped to a finer spatial accuracy than that which is possible with the use of ordinary matter. The natural conclusion [11] one draws is that unconventional (dark[14]) energy/matter exists! Note that

this argument does not make use of the evidence from recent cosmological (supernovae, cosmic microwave background, and galaxy clusters) observations. On the other hand, the demise of the random-walk model and the existence of dark energy/matter are consistent with but do not necessarily mean the vindication of the holographic model. Fortunately, in the next few years, the Very Large Telescope Interferometers in Chile could test [11] the holographic model by observing more distant quasars with their large apertures and long baselines.

But for now, the fact that our universe is observed to be at or very close to its critical density must be taken as solid albeit indirect evidence in favor of the holographic model[11] because, as discussed above, it is the only model that requires, for its consistency, the maximum energy density without causing gravitational collapse. Henceforth we will concentrate on the holographic model. What can be said about the unconventional (dark) energy? According to the HFC, the cosmic density is  $\rho \sim (H/l_P)^2 \sim (R_H l_P)^{-2}$  and the cosmic entropy is given by  $I \sim HR_H^3/l_P^2 \sim (R_H/l_P)^2$ . Hence the average energy carried by each bit is  $\rho R_H^3/I \sim R_H^{-1}$  (~ 10<sup>-31</sup> eV). Such long-wavelength [15] bits or "particles" carry negligible kinetic energy. Since pressure (energy density) is given by kinetic energy minus (plus) potential energy, a negligible kinetic energy means that the pressure of the unconventional energy is roughly equal to minus its energy density, leading to accelerating cosmic expansion. This scenario is very similar to that of quintessence [16], but it has its origin in local small scale physics — specifically, the holographic spacetime foam.

According to the HFC, it takes each unconventional bit the amount of time  $I/\nu \sim R_H$ to flip. Thus, on the average, each bit flips once over the course of cosmic history. Compared to the conventional bits carried by ordinary matter, these bits are rather passive and inert. This is understandable since each unconventional bit has, at its disposal, only such a minuscule amount of energy. But together they supply the missing mass of the universe. Accelerating the cosmic expansion is a relatively simple task, computationally speaking. It is also interesting to note [3] that if the universe contains only ordinary matter, then computationally it is a supreme parallel computer with its subregions working almost independently, as each bit flips once in the same amount of time it takes to communicate with its neighboring bits. But if the universe contains only unconventional energy which encodes the maximum number of bits allowed by the holographic principle, then it can be regarded as a supreme serial computer which operates as a single unit, as each bit flips once in the amount of time it takes for a light signal to cross the whole universe. The universe actually contains both types of matter/energy, though apparently with vastly more bits stored in dark energy/matter than in ordinary matter.

Due to the enormous number of the unconventional bits, neighboring bits are separated from each other, on the average, by only  $(R_H^3/I)^{1/3} \sim R_H^{1/3} l_P^{2/3}$ . But with a wavelength comparable to the Hubble radius, these bits/particles practically sit on top of one another (similar to overlapping wave functions in superfluid or superconductor with large coherent lengths), leading to a rather uniform distribution of energy density which, we recall, is given by the geometric mean of the infrared and ultraviolet energy scales  $\rho \sim (H/l_P)^2 \sim (R_H l_P)^{-2}$ . In this regard, the unconventional energy plays a role akin to that of an effective cosmological "constant"  $\Lambda \sim R_H^{-2}$ . Such an effective cosmological constant [17] was shown to arise in theories like unimodular gravity by Sorkin and others [18], and in recent years, in other different contexts as well [19, 20]. Thomas [20] has also argued that, in an expanding universe, holographic contributions to the cosmological constant are at most of the same order as the energy density of the dominant matter component, thus ameliorating the coincidence problem, and, at the same time, providing a technically natural solution to the cosmological constant problem.

In this paper we have only applied the HFC to the present cosmic era, predicting an accelerating expansion. Whether the HFC can explain the early universe has yet to be seen. Here we conclude on an optimistic note with the following observations. One of the HFC's main features, that the cosmic energy is of critical density, is a hallmark of the inflationary universe paradigm. Thus the flatness problem is automatically solved. Another requirement on the HFC is for the model to provide sufficient density perturbations to account for the observed structure in the universe. We beleive this is possible, as the model already contains the essence of a k-essence model. Turning to the horizon problem, we note that spacetime foam physics is, in a nutshell, black hole physics in a quantum setting, hence it is intimately related to wormhole physics. But it has been argued in [21] that wormholes in a Friedmann-Robertson-Walker universe can be used to solve the horizon problem. Thus it is possible that the HFC can meet its many challenges, solving naturally the classical cosmological problems and predicting new phenomena. Further work along these lines is warranted.

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