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# Dilepton Production in $e^- p$ and $e^+ e^-$ Colliders

Jyoti Agrawal, Paul H. Frampton and Daniel Ng

*Institute of Field physics*

*Department of Physics and Astronomy*

*University of North Carolina*

*Chapel Hill, North Carolina 27599-3255*

In an $e^- p$ collider, a striking signature for a dilepton gauge boson is $e^- p \to e^+ \mu^- \mu^- + anything$ ; this cross-section is calculated by using the helicity amplitude technique. At HERA, with center-of-mass energy $\sqrt{s} = 314GeV$, a dilepton mass above $150GeV$ is inaccessible but at LEPII-LHC, with a center-of-mass energy $\sqrt{s} = 1790GeV$, masses up to 650 GeV can be discovered. In an $e^+ e^-$ collider, the signature is $e^+ e^- \to 2e^+ 2\mu^-$ or $2e^- 2\mu^+$ . The cross-sections of this process are also calculated for the center-of-mass energies $\sqrt{s} = 200, 500$ and $1000GeV$.

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## I. INTRODUCTION

In a recent paper [1], two of the present authors (P.H.F. and D.N.) have studied the phenomenology of dilepton gauge bosons predicted by certain simple extensions of the standard model of strong and electroweak interactions. In particular, the existence of a $SU(2)_L$ doublet $(X^{--}, X^-)$ of vector gauge bosons with lepton number $L = 2$ is a plausible prediction of a general class of theories in which the electroweak $SU(2) \otimes U(1)$ gauge group is expanded to $SU(3) \otimes U(1)$ (e.g. Ref [2]). The crucial theoretical and practical question is then what is the mass scale $M_X$ ?

In Ref. [1], a lower bound of $M_X > 120 GeV$ was established by studying $e^+e^- \to e^+e^-$ as well as the "wrong" muon decay $\mu^- \to e^- \nu_e \bar{\nu}_\mu$ ; the s-channel resonance in $e^-e^- \to X^{--} \to \mu^-\mu^-$ was also computed. For polarized muons, a stronger limit [3] of $M_X > 230$ GeV was estimated. Since certain assumption about the couplings of the dilepton were made in Refs. [1,2] we shall here entertain a more general range $100 GeV < M_X < 1 TeV$ , although it should be borne in mind that the lower end is probably already excluded by existing data.

In the present paper, we shall focus on some striking signatures of lepton number violating processes in electron-proton and electron-positron colliders. A light dilepton gauge boson as anticipated in Ref. [2] couples democratically to the three lepton family associated with $e$,$\mu$ and $\tau$. Total lepton number $L = L_e + L_\mu + L_\tau$ is conserved but the separate flavors of lepton $L_e$, $L_\mu$, $L_\tau$ are violated. This is different from the minimal standard model where $L_e$, $L_\mu$, $L_\tau$ are necessarily separately conserved. This in turn means that there exist dramatic signatures for light (below $1TeV$) dileptons which violate $L_e$, $L_\mu$ and hence have no background events from standard model processes; such evidence for a dilepton gauge boson will be accessible to the next generation of $e^-p$ and $e^+e^-$ colliders as we shall show by explicit estimates of the relevant cross sections.

In an electron-proton collider one may see the process $e^-p \to e^+\mu^-\mu^- + anything$ with zero background from the Standard Model. This is relevant to the HERA collider at DESY in Hamburg, Germany; presently beginning operation with $30GeV$ electrons colliding on

$820 GeV$ protons ($\sqrt{s} = 314 GeV$) with luminosity [4] $1.6 \times 10^{31} cm^{-2} s^{-1} (0.5 fb^{-1} yr^{-1})$. In the future an $e^- p$ collider is planned at CERN with 100 GeV electrons on 8000 GeV protons (LEPII-LHC) ($\sqrt{s} = 1790 GeV$) and luminosity $2 \times 10^{32} cm^{-2} s^{-1} (6 fb^{-1} yr^{-1})$.

In an $e^+e^-$ collider, one may see the background-free process $e^+e^- \to 2e^+2\mu^-$ or $2e^-2\mu^+$ . This is relevant to LEPII and the Next Linear Collider (NLC) with the center-of-mass energies $\sqrt{s} = 200, 500$ and $1000 GeV$ and luminosities [4] $1.7 \times 10^{31}, 1 \times 10^{32}$ and $1 \times 10^{33} cm^{-2} s^{-1}$ $(0.5, 3$ and $30 fb^{-1} yr^{-1})$.

The outline of this paper is as follows. In Sect. II, we compute the amplitude of the Feynman diagrams of the above processes. In Sect. III, the cross-sections are calculated for $e^- p$ and $e^+e^-$ colliders. In Sect. IV, there are some concluding remarks. Appendix A contains the analysis of production of a real on-shell dilepton; we used this to check our computations.

## II. FEYNMAN DIAGRAMS AND HELICITY AMPLITUDES

### A. Preliminary

For the processes we consider in this paper, it is most convenient to calculate Feynman diagrams using the method of helicity amplitudes, particularly when the external particles are taken to be massless, which is a sensible approximation in the present case. The formalism can be found in Ref. [5]. The outer product of a massless spinor with momentum $p$ and helicity $\lambda (= \pm 1)$ is

$$u_\lambda(p) \bar{u}_\lambda(p) = \omega_\lambda \not{p} \ , \ \omega_\lambda = \frac{1}{2}(1 + \lambda \gamma_5) \ . \tag{2.1}$$

Let us define two four-vectors $k_0^\mu$ and $k_1^\mu$ with the following properties:

$$k_0 \cdot k_0 = 0 \ , \ k_1 \cdot k_1 = -1 \ , \ k_0 \cdot k_1 = 0 \ . \tag{2.2}$$

Hence any massless spinors with momentum $p$ and helicity $\lambda$ can be constructed from $u_-(k_0)$ by the following relations,

$$u_+(k_0) = \not{k}_1 u_-(k_0)\ ,\ u_\lambda(p) = \not{p} u_{-\lambda}(k_0)/\sqrt{2p\cdot k_0}\ . \tag{2.3}$$

The expressions in Eq.(2.3) can be verified by substituting into Eq.(2.1). From the second equation, we have $u_\lambda(-p) = iu_\lambda(p)$. Therefore, there is an (unobservable) overall phase when we replace an antifermion spinor by a fermion spinor.

For massless spinors, there are only two non-zero invariant products which are defined as follows,

$$s(p,q) = \bar{u}_+(p)u_-(q) = -s(q,p)\ , t(p,q) = \bar{u}_-(p)u_+(q) = [s(q,p)]^*\ . \tag{2.4}$$

In fact, it is enough to derive the expression of $s$ by using Eqs.(2.1)-(2.3). We obtain

$$\begin{aligned} s(p,q) &= \bar{u}_-(k_0)\ \not{p}\ \not{q}\ \not{k}_1\ u_-(k_0)/\sqrt{4(k_0\cdot p)(k_0\cdot q)} \\ &= Tr[\not{p}\ \not{q}\ \not{k}_1\ \not{k}_0\ \omega_+]/\sqrt{4(k_0\cdot p)(k_0\cdot q)}\ . \end{aligned} \tag{2.5}$$

The expression for $t(p,q)$ can be obtained from the second equation in Eq.(2.4). To calculation the invariant quantity $s(p,q)$, we can choose $k_0$ and $k_1$ to be, for example,

$$k_0 = (1,1,0,0)\ ,\ k_1 = (0,0,1,0)\ . \tag{2.6}$$

With the help of Eqs.(2.5) and (2.6), $s(p,q)$ are given by

$$s(p,q) = (p^y + ip^z)\left[\frac{q^0 - q^x}{p^0 - p^x}\right]^{1/2} - (q^y + iq^z)\left[\frac{p^0 - p^x}{q^0 - q^x}\right]^{1/2}\ . \tag{2.7}$$

Using Eqs.(2.1) and (2.4), we can derive the following useful formulae:

$$\gamma^\mu u_\pm(p)\bar{u}_\pm(q)\gamma_\mu = -2u_\mp(q)\bar{u}_\mp(p)\ , \tag{2.8a}$$

$$\gamma^\mu u_+(p)\bar{u}_-(q)\gamma_\mu = 2\omega_- t(q,p)\ , \tag{2.8b}$$

$$\gamma^\mu u_-(p)\bar{u}_+(q)\gamma_\mu = 2\omega_+ s(q,p)\ . \tag{2.8c}$$

Therefore, we can express any amplitude with external massless fermions in terms of the invariant quantities $s$ and $t$. For more general applications of the helicity amplitude technique involving massive particles, the reader is recommended to read Ref. [5]. For the purpose of this paper, however, the above preliminary introduction is sufficient.

### B. The amplitudes of $e^-q \to e^+2\mu^-q$

In this section, we will compute the helicity amplitudes for the process $e^-q \to e^+2\mu^-q$ . The Feynman diagrams are shown in Fig. 1. Using the Feynman Rules given in Ref [1], the corresponding amplitudes are given by

$$Amp(a) = \left(\frac{g_{3l}}{\sqrt{2}}\right)^2 e^2 Q_q \frac{-1}{(p_2-p_4)^2} \frac{-1}{(p_5+p_6)^2 - M_X^2 + iM_X\Gamma_X} \times \frac{1}{(p_3+p_5+p_6)^2} M(a) \,, \tag{2.9a}$$

$$Amp(b) = \left(\frac{g_{3l}}{\sqrt{2}}\right)^2 e^2 Q_q \frac{-1}{(p_2-p_4)^2} \frac{-1}{(p_5+p_6)^2 - M_X^2 + iM_X\Gamma_X} \times \frac{1}{(-p_1+p_5+p_6)^2} M(b) \,, \tag{2.9b}$$

$$Amp(c) = 2\left(\frac{g_{3l}}{\sqrt{2}}\right)^2 e^2 Q_q \frac{-1}{(p_2-p_4)^2} \frac{-1}{(p_1-p_3)^2 - M_X^2 + iM_X\Gamma_X} \times \frac{-1}{(p_5-p_6)^2 - M_X^2 + iM_X\Gamma_X} M(c) \,, \tag{2.9c}$$

$$Amp(d) = \left(\frac{g_{3l}}{\sqrt{2}}\right)^2 e^2 Q_q \frac{-1}{(p_2-p_4)^2} \frac{-1}{(p_1-p_3)^2 - M_X^2 + iM_X\Gamma_X} \times \frac{1}{(p_1-p_3-p_5)^2} M(d) \,, \tag{2.9d}$$

$$Amp(e) = \left(\frac{g_{3l}}{\sqrt{2}}\right)^2 e^2 Q_q \frac{-1}{(p_2-p_4)^2} \frac{-1}{(p_1-p_3)^2 - M_X^2 + iM_X\Gamma_X} \times \frac{1}{(p_1-p_3-p_6)^2} M(e) \,, \tag{2.9e}$$

where

$$M(a) = \bar{u}(p_4)\gamma_\alpha u(p_2)\bar{u}(p_6)\gamma_\mu\gamma_5 C\bar{u}^T(p_5)v^T(p_3)C\gamma^\mu\gamma_5(\not{p}_3+\not{p}_5+\not{p}_6)\gamma^\alpha u(p_1) \,, \tag{2.10a}$$

$$M(b) = \bar{u}(p_4)\gamma_\alpha u(p_2)\bar{u}(p_6)\gamma_\mu\gamma_5 C\bar{u}^T(p_5)\, v^T(p_3)C\gamma^\alpha(-\not{p}_1+\not{p}_5+\not{p}_6)\gamma^\mu\gamma_5 u(p_1) \,, \tag{2.10b}$$

$$M(c) = \bar{u}(p_4)\gamma^\alpha u(p_2)\bar{u}(p_6)\gamma^\mu\gamma_5 C\bar{u}^T(p_5)v^T(p_3)C\gamma^\beta\gamma_5 u(p_1)$$
$$\times [(p_2 - p_4 + p_5 + p_6)_\beta g_{\mu\alpha} + (-p_5 - p_6 - p_1 + p_3)_\alpha g_{\mu\beta}$$
$$+(p_1 - p_3 - p_2 + p_4)_\mu g_{\alpha\beta}] \ , \tag{2.10c}$$

$$M(d) = \bar{u}(p_4)\gamma_\alpha u(p_2)\bar{u}(p_6)\gamma^\alpha(\not{p}_1 - \not{p}_3 - \not{p}_5)\gamma^\mu\gamma_5 C\bar{u}^T(p_5)v^T(p_3)C\gamma_\mu\gamma_5 u(p_1) \ , \tag{2.10d}$$

$$M(e) = \bar{u}(p_4)\gamma_\alpha u(p_2)\bar{u}(p_6)\gamma^\mu\gamma_5(\not{p}_1 - \not{p}_3 - \not{p}_6)\gamma^\alpha C\bar{u}^T(p_5)v^T(p_3)C\gamma_\mu\gamma_5 u(p_1) \ . \tag{2.10e}$$

$\Gamma_X$ is the total width of $X^{--}$ which decays into $e^-e^-, \mu^-\mu^-$ and $\tau^-\tau^-$ democratically. After some Dirac matrix manipulation, Eq.(2.10c) can be rewritten as

$$M(c) = -M(a) - M(c) \ . \tag{2.11}$$

For massless spinors, we can replace $v(p)$ by $u(p)$. Therefore, we can decompose $M(a)-(e)$ into various helicities as follows:

$$M_{\pm\pm\pm}(a) = \begin{bmatrix} \bar{u}_+(p_4)\gamma_\alpha u_+(p_2) \\ \bar{u}_-(p_4)\gamma_\alpha u_-(p_2) \end{bmatrix} \begin{bmatrix} \bar{u}_+(p_6)\gamma_\mu u_+(p_5) \\ -\bar{u}_-(p_6)\gamma_\alpha u_-(p_5) \end{bmatrix}$$
$$\times \begin{bmatrix} \bar{u}_+(p_3)\gamma^\mu(\not{p}_3 + \not{p}_5 + \not{p}_6)\gamma^\alpha u_+(p_1) \\ -\bar{u}_-(p_3)\gamma^\mu(\not{p}_3 + \not{p}_5 + \not{p}_6)\gamma^\alpha u_-(p_1) \end{bmatrix} , \tag{2.12a}$$

$$M_{\pm\pm\pm}(b) = \begin{bmatrix} \bar{u}_+(p_4)\gamma_\alpha u_+(p_2) \\ \bar{u}_-(p_4)\gamma_\alpha u_-(p_2) \end{bmatrix} \begin{bmatrix} \bar{u}_+(p_6)\gamma_\mu u_+(p_5) \\ -\bar{u}_-(p_6)\gamma_\mu u_-(p_5) \end{bmatrix}$$
$$\times \begin{bmatrix} \bar{u}_+(p_3)\gamma^\alpha(-\not{p}_1 + \not{p}_5 + \not{p}_6)\gamma^\mu u_+(p_1) \\ -\bar{u}_-(p_3)\gamma^\alpha(-\not{p}_1 + \not{p}_5 + \not{p}_6)\gamma^\mu u_-(p_1) \end{bmatrix} , \tag{2.12b}$$

$$M_{\pm\pm\pm}(c) = -M_{\pm\pm\pm}(a) - M_{\pm\pm\pm}(b) \ , \tag{2.12c}$$

$$M_{\pm\pm\pm}(d) = \begin{bmatrix} \bar{u}_+(p_4)\gamma_\alpha u_+(p_2) \\ \bar{u}_-(p_4)\gamma_\alpha u_-(p_2) \end{bmatrix} \begin{bmatrix} \bar{u}_+(p_6)\gamma^\alpha(\not{p}_1 - \not{p}_3 - \not{p}_5)\gamma^\mu u_+(p_5) \\ -\bar{u}_-(p_6)\gamma^\alpha(\not{p}_1 - \not{p}_3 - \not{p}_5)\gamma^\mu u_-(p_5) \end{bmatrix}$$
$$\times \begin{bmatrix} \bar{u}_+(p_3)\gamma_\mu u_+(p_1) \\ -\bar{u}_-(p_3)\gamma_\mu u_-(p_1) \end{bmatrix} , \tag{2.12d}$$

$$M_{\pm\pm\pm}(e) = \begin{bmatrix} \bar{u}_+(p_4)\gamma_\alpha u_+(p_2) \\ \bar{u}_-(p_4)\gamma_\alpha u_-(p_2) \end{bmatrix} \begin{bmatrix} \bar{u}_+(p_6)\gamma^\alpha(\not{p}_1 - \not{p}_3 - \not{p}_6)\gamma^\mu u_+(p_5) \\ -\bar{u}_-(p_6)\gamma^\alpha(\not{p}_1 - \not{p}_3 - \not{p}_6)\gamma^\mu u_-(p_5) \end{bmatrix} \times \begin{bmatrix} \bar{u}_+(p_3)\gamma_\mu u_+(p_1) \\ -\bar{u}_-(p_3)\gamma_\mu u_-(p_1) \end{bmatrix} . \tag{2.12e}$$

Since $M_{\lambda_1\lambda_2\lambda_3} = M^*_{-\lambda_1-\lambda_2-\lambda_3}$, we need calculate only the helicity amplitudes,$M_{+\pm\pm}(a) - (e)$, explicitly in terms of $s$ and $t$. The results are given in Table 1.

## III. CROSS-SECTIONS

Since the violation of $L_e$ and $L_\mu$ conservation is clearly evidenced by the processes $e^-q \to e^+2\mu^-q$ and $e^+e^- \to 2e^+2\mu^-$ or $2e^-2\mu^+$ , it is totally free of the minimal standard model background. Before we proceed, let us justify neglecting the Feynman diagrams in which the photon of Figs. 1(a)-(e) is replaced by a $Z$-boson. Aside from the suppression due to the mass in the $Z$ propagator, the axial vector couplings of electron and Z-boson do not contribute in this process because of the Fermi statistics, see Ref. [1]. Only the vector coupling of $Z$ contributes, but it is proportional to $g_v = (\frac{1}{4} - sin^2\theta_W) \simeq 0.02$. The three boson coupling of $X^{--}-X^{++}-Z$ is also proportional to $g_v$ from the group theory. Therefore, the $Z$-boson contributes at most 0.5% to the processes and it can be safely neglected.

### A. $e^-p$ colliders

To evaluate the production cross-section for the process $e^-p \to e^+\mu^-\mu^- + anything$ in the electron-proton colliders, we use EHLQ [6] parton structure functions (set 1), $F_q(x)$ for quark q. Hence the production cross-section for the process is given by

$$\sigma(M_X) = \int_0^1 dx \sum_q F_q(x, Q^2)\hat{\sigma}(\sqrt{\hat{s}} = xs, M_X) , \tag{3.1}$$

where $\hat{\sigma}$ is the elementary cross-section of the process $e^-q \to e^+2\mu^-q$ ; $x$ is the fractional momentum of the proton carried by the quark $q$, hence $\sqrt{\hat{s}}$ is the center of mass energy available for $e^-q \rightarrow e^+2\mu^-q$ . $Q^2$, defined to be $-(p_2 - p_4)^2$, is the scale for the structure functions for quarks. The result for $\sigma(M_X)$ are shown in Fig. 2 for the cases $\sqrt{s} = 314GeV$ (HERA) and $1790GeV$ (LEPII-LHC).

For HERA, the planned luminosity is $1.6 \times 10^{31}cm^{-2}s^{-1}$ giving an annual integrated luminosity of $0.5fb^{-1}yr^{-1}$. From Fig. 2, we see that there will be less than one event per year if the mass of the dilepton is heavier than $120GeV$. The situation become hopeless for $M_X > 150GeV$ without an upgrade in energy and/or luminosity. For example, an up-grade in center of mass energy up to $400GeV$ will allow, for the same luminoisity, discovery of dileptons up to about $200GeV$. We thus conclude, given the mass bounds mentioned in the introduction, that the chance of HERA discovering such a dilepton state is very marginal.

At LEPII-LHC with $\sqrt{s} = 1790GeV$ the prospects for dilepton discovery are far better. The expected luminosity is about $2 \times 10^{32}cm^{-2}s^{-1}$ and hence annual integrated luminosity $6fb^{-1}yr^{-1}$. Requiring at least 2 events per year for $e^-p \rightarrow e^+\mu^-\mu^- + anything$ , we can detect $M_X$ up to $650GeV$.

### B. $e^+e^-$ colliders

At an $e^+e^-$ collider, dilepton signatures include $e^+e^- \rightarrow 2e^+2\mu^-$ or $2e^-2\mu^+$ . This calculation is quite similar to $e^-q \rightarrow e^+2\mu^-q$ described above and we include also the charge-conjugation of the corresponding Feynman diagrams. We have computed the result for the center-of-mass energies $\sqrt{s} = 200GeV$(LEPII), $500GeV$ and $1000GeV$ (possible NLC energies). The results are displayed in Fig. 3. Requiring at least 2 events per year, we can detect $M_X$ up to 180,450 and 950 GeV in $e^+e^-$ colliders with energies $\sqrt{s} = 200, 500$ and $1000GeV$ assuming the integrated luminosities [4] to be $0.5, 3$ and 30 $fb^{-1}yr^{-1}$ respectively.

The amplitude-squared for the real production of the dilepton is given in the Appendix A. The production cross-sections are also calculated and compared with the curves in Figs. (2) and (3). We find that the contribution from the Feynman diagrams Figs. 1 (d) and (e) are at most 10% relative to that of other diagrams in a wide range of dilepton mass

$M_X$ except at the high values of $M_X$ in which the curves have longer tails. Therefore, it is important to include Figs. 1 (d) and (e) in our calculation in order to provide a better estimation for the maximum $M_X$ being probed in high energy colliders.

## IV. CONCLUSION

We have considered a direct search for doubly-charged dilepton $X^{--}(X^{++})$ by lepton-number violating processes in $e^-p$ and $e^+e^-$ colliders. The mass of $X^{--}$ ranging from 100 to $1000GeV$ is expected from the theory of $SU(15)$. The striking signature for a dilepton gauge boson is $e^-p \to e^+\mu^-\mu^- + anything$ in an $e^-p$ collider and $e^+e^- \to 2e^+2\mu^-$ or $2e^-2\mu^+$ in an $e^+e^-$ collider. The chance of discovering a dilepton at HERA is very marginal unless $M_X$ is less than $150GeV$. The direct discovery of such a dilepton state depends on future colliders such as LEPII-LHC and NLC at which interesting mass ranges will be explored.

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## FIGURES

FIG. 1. Feynman diagrams for $e^- q \to e^+ \mu^- \mu^- q$

FIG. 2. Cross-sections for the process $e^- p \to e^+ \mu^- \mu^- + anything$ with $Q^2 > 25 GeV^2$ at the center-of-mass energies $\sqrt{s} = 314 GeV$ (solid line) and $\sqrt{s} = 1790 GeV$ (dashed line)

FIG. 3. Cross-sections for the process $e^+ e^- \to 2e^- 2\mu^+$ or $2e^+ 2\mu^-$ with $Q^2 > 25 GeV^2$ at the center-of-mass energies $\sqrt{s} = 200 GeV$ (solid line), $\sqrt{s} = 500 GeV$ (dashed line) and $\sqrt{s} = 1000 GeV$ (dotted line)

## TABLES

TABLE I. Helicity amplitudes for the Feynman diagrams shown in Fig. 1

| $M$ | $+++$ | $++-$ | $+-+$ | $+--$ |
|---|---|---|---|---|
| $M(a)$ | $4t(p_2,p_1)s(p_6,p_3)$ | $-4t(p_5,p_3)s(p_4,p_1)$ | $-4t(p_2,p_1)s(p_5,p_3)$ | $4t(p_6,p_3)s(p_4,p_1)$ |
| | $\times[t(p_5,p_3)s(p_4,p_3)$ | $\times[t(p_2,p_3)s(p_6,p_3)$ | $\times[t(p_6,p_3)s(p_4,p_3)$ | $\times[t(p_2,p_3)s(p_5,p_3)$ |
| | $+t(p_5,p_6)s(p_4,p_6)]$ | $+t(p_2,p_5)s(p_6,p_5)]$ | $+t(p_6,p_5)s(p_4,p_5)]$ | $+t(p_2,p_6)s(p_5,p_6)]$ |
| $M(b)$ | $4t(p_5,p_1)s(p_4,p_3)$ | $-4t(p_2,p_3)s(p_6,p_1)$ | $-4t(p_6,p_1)s(p_4,p_3)$ | $4t(p_2,p_3)s(p_5,p_1)$ |
| | $\times[t(p_2,p_5)s(p_6,p_5)$ | $\times[t(p_5,p_6)s(p_4,p_6)$ | $\times[t(p_2,p_6)s(p_5,p_6)$ | $\times[t(p_6,p_5)s(p_4,p_5)$ |
| | $-t(p_2,p_1)s(p_6,p_1)]$ | $-t(p_5,p_1)s(p_4,p_1)]$ | $-t(p_2,p_1)s(p_5,p_1)]$ | $-t(p_6,p_1)s(p_4,p_1)]$ |
| $M(c)$ | | $-M(a)-M(b)$ | | |
| $M(d)$ | $4t(p_1,p_5)s(p_4,p_6)$ | $-4t(p_3,p_5)s(p_4,p_6)$ | $-4t(p_2,p_6)s(p_3,p_5)$ | $4t(p_2,p_6)s(p_1,p_5)$ |
| | $\times[t(p_2,p_1)s(p_3,p_1)$ | $\times[-t(p_2,p_3)s(p_1,p_3)$ | $\times[-t(p_1,p_3)s(p_4,p_3)$ | $\times[t(p_3,p_1)s(p_4,p_1)$ |
| | $-t(p_2,p_5)s(p_3,p_5)]$ | $-t(p_2,p_5)s(p_1,p_5)]$ | $-t(p_1,p_5)s(p_4,p_5)]$ | $-t(p_3,p_5)s(p_4,p_5)]$ |
| $M(e)$ | $4t(p_2,p_5)s(p_3,p_6)$ | $-4t(p_2,p_5)s(p_1,p_6)$ | $-4t(p_1,p_6)s(p_4,p_5)$ | $4t(p_3,p_6)s(p_4,p_5)$ |
| | $\times[-t(p_1,p_3)s(p_4,p_3)$ | $\times[t(p_3,p_1)s(p_4,p_1)$ | $\times[t(p_2,p_1)s(p_3,p_1)$ | $\times[-t(p_2,p_3)s(p_1,p_3)$ |
| | $-t(p_1,p_6)s(p_4,p_6)]$ | $-t(p_3,p_6)s(p_4,p_6)]$ | $-t(p_2,p_6)s(p_3,p_6)]$ | $-t(p_2,p_6)s(p_1,p_6)]$ |

## APPENDIX A:

In this appendix, we will present the calculation of the real production of dilepton $X^{--}$. If there is sufficient center-of-mass energy and the dilepton is light enough, it will be possible to produce a real dilepton in the final state. This limit of light dilepton mass provides, in any case, a useful check on all the calculations given in the main text. Clearly, for a light dilepton, the calculation based on the Feynman diagrams given in Figure 1 with a Breit-Wigner form of the dilepton propagator should agree with a real dilepton calculation using three-body (rather than four-body) phase space. It is because of the fact that Figs. 1(a)-(c) are dominant over Figs. 1 (d) and (e). The success of this comparison gives us confidence that the four-body phase space calculation in the main text is reliable. Only the first three diagrams in Fig. 1 are relevant. The amplitude is

$$Amp = Q_q e^2 \frac{g_{3l}}{\sqrt{2}} \epsilon^\mu(p) a_{\mu\alpha} b^\alpha \frac{1}{(p_2 - p_4)^2} , \tag{A1}$$

where

$$\begin{aligned} a_{\mu\alpha} = & v^T(p_3) C \gamma_\mu \gamma_5 \frac{\not{p}_3 + \not{p}}{(p_3+p)^2} \gamma_a u(p_1) + v^T(p_3) C \gamma_\alpha \frac{-\not{p}_1 + \not{p}}{(-p_1+p)^2} \gamma_\mu \gamma_5 u(p_1) \\ & + v^T(p_3) C \gamma_\beta \gamma_5 u(p_1) \frac{-2}{(p_1-p_3)^2 - M_X^2} \left[ (p_2 - p_4 + p)_\beta g_{\mu\alpha} \right. \\ & \left. + (-p - p_1 + p_3)_\alpha g_{\mu\beta} + (p_1 - p_3 - p_2 + p_4)_\mu g_{\alpha\beta} \right] , \end{aligned} \tag{A2}$$

and

$$b^\alpha = \bar{u}(p_4) \gamma^\alpha u(p_2) , \tag{A3}$$

where $p_1$ and $p_3$ are the momenta for the electron and positron; $p_2$ and $p_4$ are the momenta for the initial and final quarks; $p$ and $\epsilon^\mu(p)$ are the momenta and polarization vector for the dilepton $X^{--}$. Here $eQ_q$ is the quark electric charge and $g_{3l}/\sqrt{2}$ is the coupling constant for the $X^{++} - e - e$ interaction. We have neglected the unimportant width of $X^{--}$ in the propagator. Notice that $(p_2 - p_4)^\mu a_{\mu\alpha} = 0$ because of electromagnetic gauge invariance. Using momentum conservation and Dirac algebra (see Eq. (2.11) in the text), $a_{\mu\alpha}$ in the Eq. (2) can be rewritten as

$$
\begin{aligned}
a_{\mu\alpha} &= \left[\frac{1}{(p_3+p)^2}+\frac{2}{(p_1-p_3)^2-M_X^2}\right] v^T(p_3)C\gamma_\mu(\not p_3+\not p)\gamma_\alpha\gamma_5 u(p_1) \\
&+\left[\frac{1}{(-p_1+p)^2}+\frac{2}{(p_1-p_3)^2-M_X^2}\right] v^T(p_3)C\gamma_\alpha(-\not p_1+\not p)\gamma_\mu\gamma_5 u(p_1)\ .
\end{aligned} \tag{A4}
$$

$p^\mu a_{\mu\alpha}$ is not zero because the dilepton is not coupled to a conserved current; in fact it is given explicitly by

$$
p^\mu a_{\mu\alpha} = 2\frac{(p_2-p_4)^2}{(p_1-p_3)^2-M_X^2} v^T(p_3)C\gamma_\alpha\gamma_5 u(p_1)\ . \tag{A5}
$$

Using the polarization sum $\sum \epsilon^\mu(p)\epsilon^\nu(p) = -g^{\mu\nu}+p^\mu p^\nu/M_X^2$, the amplitude-squared is given by

$$
|Amp|^2 = Q_q e^4\left(\frac{g_{3l}}{\sqrt{2}}\right)^2 \frac{1}{(p_2-p_4)^4}\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{M_X^2}\right) a_{\mu\alpha}a^*_{\nu\beta} b^\alpha b^{\beta *}\ . \tag{A6}
$$

Therefore $|Amp|^2$, with the help of Eq.(5), can be calculated to be

$$
\begin{aligned}
|Amp|^2 = Q_q^2 e^4\left(\frac{g_{3l}}{\sqrt{2}}\right)^2 \frac{64}{(p_2-p_4)^4} \Bigg[ & \left(\frac{1}{(p_3+p)^2}+\frac{2}{(p_1-p_3)^2-M_X^2}\right)^2 \\
& \times\Big[2\, p_3.p\ (p_1.p_2\ p.p_4 + p_1.p_4\ p.p_2) - M_X^2\ (p_1.p_2\ p_3.p_4 + p_1.p_4\ p_3.p_2)\Big] \\
& + \left(\frac{1}{(-p_1+p)^2}+\frac{2}{(p_1-p_3)^2-M_X^2}\right)^2 \\
& \times\Big[2\, p_1.p\ (p_3.p_2\ p.p_4 + p_3.p_4\ p.p_2) - M_X^2\ (p_1.p_2\ p_3.p_4 + p_1.p_4\ p_3.p_2)\Big] \\
& +2\left(\frac{1}{(p_3+p)^2}+\frac{2}{(p_1-p_3)^2-M_X^2}\right)\left(\frac{1}{(-p_1+p)^2}+\frac{2}{(p_1-p_3)^2-M_X^2}\right) \\
& \times\Big[-M_X^2\ p_1.p_3\ p_2.p_4 + 2\, p_1.p\ p_3.p_2\ p_3.p_4 - 2\, p_3.p\ p_1.p_2\ p_1.p_4 \\
& +(2\, p_1.p_3 + p_1.p - p_3.p)(p_1.p_2\ p_3.p_4 + p_1.p_4\ p_3.p_2) \\
& +p_1.p_3\ (p_1.p_2\ p.p_4 + p_1.p_4\ p.p_2 - p.p_2\ p_3.p_4 - p.p_4\ p_3.p_2)\Big] \\
& +2\left(\frac{1}{(p_1-p_3)^2-M_X^2}\right)^2 \frac{(p_2-p_4)^4}{M_X^2}\ (p_1.p_2\ p_3.p_4 + p_1.p_4\ p_3.p_2)\Bigg]\ .
\end{aligned} \tag{A7}
$$

The above equation is used to calculate the production of a real dilepton in the $e^-p$ and $e^+e^-$ collders. We then compared this result with Figs. (2) and (3). We find agreement for light dilepton mass with the curves in Figs. (2) and (3) and that, as expected, the full calculation allowing a virtual dilepton gives an extra contribution in the tail of high $M_X$ values.