

Dark Matter from Binary Tetrahedral Flavor Symmetry

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Abstract

The minimal renormalizable $T' \times Z_2$ model (MRT'M) is slightly extended in its Higgs scalar sector such that the abelian part of the flavor symmetry enlarges to $(Z_2 \times Z'_2)$. All standard model and original MRT'M states will transform trivially under Z'_2 . Inspired by the Valencia group's A_4 model building, we propose a T' WIMP candidate as the lightest Z'_2 odd scalar. This extension of the prior MRT'M model maintains the successful predictions for the neutrino mixing matrix and the Cabibbo angle, and provides an attractive candidate for dark matter (Φ_{WIMP}) with $M_\Phi \simeq 780$ GeV.

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I. INTRODUCTION

In recent theoretical cosmology, the most prominent issue is the dark side of the universe consisting of two distinct sectors: dark matter and dark energy. The latter sector is more difficult to explain and may require a modification to the fundamental theory of gravity or even redefining gravity as an emergent property of the universe, like spacetime, rather than a fundamental force. Exciting new experimental data has led to a rapidly evolving viewpoint on gravity over the past few years and will hopefully lead to a better consensus.

Dark matter is much more approachable and is likely to be solved more easily. It is, simply put, invisible matter which clumps like luminous matter. Its existence has been known for 78 years, since 1933 [1], while dark energy was discovered only 12 years ago in 1998 [2, 3]. The most popular candidate for dark matter is a Weakly Interacting Massive Particle (WIMP) [4] with a mass range of $10^{2\pm 1}$ GeV and a typical weak interaction cross section with standard model particles.

There are alternative dark matter candidates such as the invisible axion with mass between 10^{-6} eV and 10^{-3} eV [5–8], and Intermediate-Mass Black Holes (IMBHs) with mass between $3 \times 10^2 M_\odot$ and $3 \times 10^5 M_\odot$ [9, 10]. The fact that these candidate masses range over 77 orders of magnitude is indicative of the uncertainty present in the problem. Unfortunately the well-known galactic halo dark matter profiles found from numerical simulations are insensitive to the dark matter mass because of scale invariance [11].

Nevertheless, the WIMP is especially attractive because it naturally gives the observed relic density. This is well known and will be discussed later. The most popular candidate for a WIMP was, at one time, the neutralino appearing in the supersymmetric extension of the standard model [4]. However, it has long been clear that a WIMP candidate does not require the assumption of supersymmetry [12]. One non-supersymmetric example of such a WIMP is the subject of the present article.

All particles in the minimal standard model are badly suited for the role of dark matter. Nevertheless, the standard model has 28 free parameters when we include massive neutrinos. Of these, no less than 22 arise from the masses and mixings of the quarks and leptons, 12 masses and 10 mixing angles. The most popular approach towards explaining these 22

parameters is by hypothesizing a flavor symmetry, G_F , which commutes with the standard model gauge group. A promising choice for G_F is one of the finite non-abelian groups, T' , the binary tetrahedral flavor symmetry [13–22].

The history of using T' as a flavor symmetry is lengthy and fascinating. First used in Ref. [13] in 1994, it was implemented solely as a symmetry for quarks, because neutrinos were still believed to be massless. After neutrino masses and mixings were discovered [23], the PMNS mixing matrix for leptons was carefully measured and turned out to be very different from the CKM mixing matrix for quarks. A number of different theories developed in response to this first evidence of physics beyond the minimal standard model [24–28]. Eventually a useful approximation to the empirical PMNS matrix was determined to be the tribimaximal (TBM) one suggested in Ref. [29].

In the early 2000s, a purely leptonic flavor symmetry based on $A_4 = T$, the tetrahedral group, was introduced by Ref. [30] to underpin TBM mixing. Further investigation revealed that A_4 could not be extended to quarks because a viable CKM matrix could not be obtained [31]. It was then realized that although A_4 is not a subgroup of its double cover [19], T' , nevertheless from the viewpoint of the Kronecker products used in particle theory model building [14], A_4 behaves *as if* it were a subgroup. This observation provided a watershed where T' could act as a successful flavor symmetry for quarks and leptons.

II. VALENCIA MECHANISM

An ingenious new mechanism has been discovered by a group based in Valencia [32, 33], working on A_4 model building. Their implementation used the flavor symmetry group A_4 , whose double cover is central to our present work. It involved adding a small number of extra scalar fields, one of which, by virtue of a discrete Z_2 analogous to R-symmetry in the MSSM, gives rise to stable dark matter.

Their original model assigned all standard model leptons as different singlets of A_4 with the right-handed neutrinos and one of the newly added Higgs as the only A_4 triplets (the model's other Higgs was an A_4 singlet). These assignments were unconventional as most A_4 models, like the T' model discussed in later sections, utilize triplets in the lepton assignments.

In Ref. [32, 33] a particular generator of A_4 was used to give rise to a Z_2 subgroup of A_4 that stabilized the WIMP. This Z_2 group established a particle sector that is discrete from the standard model particles and inaccessible except via the weak force and gravity.

Since A_4 alone has proved incapable of accommodating quarks in a like manner to leptons [31], the Valencia group relegated the quark sector to future work. An alternative approach, that we pursue, is to use T' to replace A_4 , allowing the incorporation of quarks, a prediction of the Cabibbo angle, and controllable deviations from TBM mixing angles.

III. $(T' \times Z_2 \times Z'_2)$ MODEL

To accommodate the quark sector, we adopt the $(T' \times Z_2)$ model formulated in Ref. [18] and further analyzed in Ref. [20, 21]. This section will establish an extended model including elements of the Valencia Mechanism by incorporating a second Z'_2 , while also adding scalar fields and heavy right-handed neutrinos that are odd under Z'_2 ; the lightest odd scalar will be the dark matter WIMP. This model is a modification of the Minimal Renormalizable T' Model (MRT'M) from Ref. [20] with a global symmetry of $(T' \times Z_2 \times Z'_2)$ restricting the Yukawa couplings. One key difference from Ref. [32, 33] is that Z'_2 will not be subgroup of T' .

The quark assignments below are unchanged from Ref. [18], denoting $\mathcal{Q}_L = \begin{pmatrix} t \\ b \end{pmatrix}_L$, $Q_L = \begin{pmatrix} c \\ s \end{pmatrix}_L$ & $\begin{pmatrix} u \\ d \end{pmatrix}_L$, $\mathcal{C}_R = c_R$ & u_R , and $\mathcal{S}_R = s_R$ & d_R . By setting all quarks even under Z'_2 , past T' predictions are preserved.

Quarks	\mathcal{Q}_L	Q_L	t_R	b_R	\mathcal{C}_R	\mathcal{S}_R
T'	1 ₁	2 ₁	1 ₁	1 ₂	2 ₃	2 ₂
Z_2	+	+	+	-	-	+
Z'_2	+	+	+	+	+	+

The lepton sector of Ref. [18] is retained unchanged, even under Z'_2 , again keeping all the previous successes in Ref. [20, 21]. Inspired by Ref. [32, 33], we have incorporated an additional triplet of right-handed neutrinos, N_T . This triplet is odd under Z'_2 , and is summarized with the other lepton assignments below.

Leptons	L_L	τ_R	μ_R	e_R	$N_R^{(1)}$	$N_R^{(2)}$	$N_R^{(3)}$	N_T
T'	3	1_1	1_2	1_3	1_1	1_2	1_3	3
Z_2	+	-	-	-	+	+	+	+
Z'_2	+	+	+	+	+	+	+	-

The Higgs sector is mostly the same as in Ref. [18], Z'_2 -even, with a new Z'_2 -odd, T' -triplet, H''_3 . The five Higgs irreps of T' are shown in the following table. Note that all of these scalars are doublets under the gauge group $SU(2)_L$.

Higgs	H_{1_1}	H_{1_3}	H_3	H'_3	H''_3
T'	1_1	1_3	3	3	3
Z_2	+	-	+	-	+
Z'_2	+	+	+	+	-

The resultant Yukawa couplings are:

$$\begin{aligned}
\mathcal{L}_Y = & M_0 N_T N_T + M_1 N_R^{(1)} N_R^{(1)} + M_{23} N_R^{(2)} N_R^{(3)} + \\
& Y_e L_L e_R H'_3 + Y_\mu L_L \mu_R H'_3 + Y_\tau L_L \tau_R H'_3 + \\
& Y_1 L_L N_R^{(1)} H_3 + Y_2 L_L N_R^{(2)} H_3 + Y_3 L_L N_R^{(3)} H_3 + \\
& Y_4 L_L (N_T H''_3)_3 + Y_5 L_L (N_T H''_3)_{3'} + \\
& Y_t (\mathcal{Q}_L t_R H_{1_1}) + Y_b (\mathcal{Q}_L b_R H_{1_3}) + \\
& Y_C (\mathcal{Q}_L \mathcal{C}_R H'_3) + Y_S (\mathcal{Q}_L \mathcal{S}_R H_3) + h.c. .
\end{aligned} \tag{1}$$

It is interesting to note that the terms with the new right-handed neutrino triplet, N_T , and new Higgs, H''_3 , involve $(3 \times 3 \times 3)$ under T' , which contains two (1_1) singlets [34], and hence produces just two additional Yukawa couplings. This will prove important to our implementation of the Type-I seesaw mechanism. The same Yukawa couplings, with Y_4 and/or Y_5 complexified, will naturally lead to leptogenesis ¹.

¹ It is notable that one decay mode of the triplet N_T is into a light neutrino and dark matter.

IV. DARK MATTER AND NEUTRINO PREDICTIONS

A. Dark Matter Candidate

The T' WIMP candidate is the lightest state with an assignment of $Z'_2 = -1$. The Z'_2 odd states are N_T and H_3'' . The neutrino triplet, N_T , is expected to be very heavy from the seesaw mechanism discussed in the Appendix A. It decays into an H_3'' and a lepton, making it a good candidate for the leptogenesis mechanism [35].

The WIMP candidate is therefore a superposition of the CP-even neutral scalars contained in H_3'' , which has three $SU(2)_L$ doublets:

$$H_3''(1) = \begin{pmatrix} h_1^+ \\ h_1^0 + iA_1 \end{pmatrix}, \quad H_3''(2) = \begin{pmatrix} h_2^+ \\ h_2^0 + iA_2 \end{pmatrix}, \quad H_3''(3) = \begin{pmatrix} h_3^+ \\ h_3^0 + iA_3 \end{pmatrix}. \quad (2)$$

This set includes 6 charged scalars, 3 neutral CP-even scalars, and 3 neutral CP-odd scalars. Our dark matter candidate will be a superposition of the three real Z'_2 -odd, CP-even, neutral scalar states:

$$\Phi_{WIMP} = \alpha h_1^0 + \beta h_2^0 + \gamma h_3^0. \quad (3)$$

An evaluation of the dark matter candidate coefficients, α , β , and γ , requires knowledge of the coefficients in the Higgs scalar potential, shown in Appendix B, and is beyond the scope of this paper.

B. Relic Density and WIMP Mass

A common tool for estimating the mass of a dark matter candidate (M_Φ) is the relic density. This approach uses both particle and cosmological inputs as well as model properties to estimate the annihilation cross section and particle density after freeze out. We will follow the general treatment outlined in Ref. [36]. Starting with,

$$\frac{n_{DM}(T)}{s(T)} \approx \left(\frac{1}{M_P T_f \langle \sigma_{AV} \rangle} \right) \sqrt{\frac{180}{\pi g_C}}, \quad (4)$$

and noting a weak hypercharge of $Y = 1/2$ and the Planck mass of $M_P = 1.22 \times 10^{19}$ GeV. Also, in the relevant temperature range ($T_f \ll M_\Phi$), we can safely approximate $M_\Phi/T_f \approx 26$.

Focusing on an approximation of the most significant instance of elastic scattering, we will adopt the equation seen in Ref. [32],

$$\langle\sigma_{Av}\rangle \simeq \frac{3g_2^4 + g_Y^4 + 6g_2^2g_Y^2 + 4\lambda^2}{128\pi M_\Phi^2} \quad (5)$$

where $g_2 = \sqrt{(4\pi\alpha)/(1 - (M_W/W_Z)^2)}$ and $g_Y = \sqrt{4\pi\alpha}M_Z/M_W$. Rather than solve the Higgs scalar potential (detailed in Appendix B), we make the assumption that the quartic coupling constant, λ , yields a very small contribution. This allows us to simplify the relic density to the form,

$$\frac{n_{DM}(T)}{s(T)} \approx \left(\frac{1248M_\Phi M_W^4 (M_Z^2 - M_W^2)^2}{\alpha^2 M_P M_Z^4 (M_Z^4 + 4M_Z^2 M_W^2 - 2M_W^4)} \right) \sqrt{\frac{5}{\pi^3 g_C}}. \quad (6)$$

Data from WMAP 7 indicates $\Omega_{DM}h^2 = 0.110 \pm 0.006$ (i.e. $n_{DM}/s = 0.40 \pm 0.02 \text{ eV}/M_\Phi$) [37] and from Ref. [38] indicates $M_W = 80.399 \text{ GeV}$ and $M_Z = 91.1876 \text{ GeV}$. Finally we note that g_C , the degrees of freedom, is based (at this temperature range) on $g = (g_B + \frac{7}{8}g_F)$ from the numbers of bosons and fermions. The degrees of freedom can also be split into $g_{SM} = 104.125$ for the standard model (using Majorana neutrinos unlike the more commonly referenced 106.75 assuming Dirac neutrinos) and $1 \lesssim n \lesssim 37.25$ for our model's additions. Taking the maximally allowed degrees of freedom results in $g_C \approx 141.375$ and $M_\Phi \approx 0.78 \text{ TeV}$.

C. Neutrino Mixing

By relying on an exterior Z'_2 group rather than a subgroup of T' , the quark assignments and couplings have been left identical to that of previous T' models. This serves to preserve the predictions of the Cabibbo angle [18],

$$\tan 2\Theta_{12} = \frac{\sqrt{2}}{3}. \quad (7)$$

Also preserved is the MRT' M seesaw mechanism relation (detailed in Appendix A) between reactor and atmospheric neutrino mixing angles, made possible by perturbing the Cabibbo angle closer to its empirical value [20].

$$\theta_{13} = \sqrt{2} \left(\frac{\pi}{4} - \theta_{23} \right). \quad (8)$$

Current best fit estimates of the neutrino mixing angles θ_{13} and θ_{23} , are based on a measurement of $2 \sin^2 \theta_{23} \sin^2 2\theta_{13}$ (commonly listed as $\sin^2 2\theta_{13}$ under the assumption of maximal θ_{23}). By combining Eq. (8) with the recent integration of experimental data from

T2K+MINOS+DC [39], we can solve for unique sets of these two neutrino angles. Based on measurements of $0.015 < 2 \sin^2 \theta_{23} \sin^2 2\theta_{13} < 0.15$ at 95% CL with a best fit value of 0.08 (assuming normal hierarchy), we predict the following values,

$$\theta_{23} = 38^\circ \begin{smallmatrix} +4^\circ \\ -3^\circ \end{smallmatrix}, \quad \theta_{13} = 9^\circ \begin{smallmatrix} +5^\circ \\ -5^\circ \end{smallmatrix}. \quad (9)$$

The values in Eq. (9) are significant deviations from TBM values and should soon be verified by neutrino experiments.

V. DISCUSSION

The use of T' flavor symmetry has previously led to interesting predictions for the CKM and PMNS mixing matrices for quarks and leptons, respectively. Of special interest is the fact that T' flavor symmetry predicts a unique relationship between these two mixing matrices that enter the W^\pm mixings of weak interactions. More accurate experimental measurements of the mixing angles, particularly θ_{13} , are presently underway, and it will be interesting to discover whether the T' predictions are corroborated.

There is wide expectation that the LHC will shortly, perhaps within a year, discover the Higgs boson (H). Of special significance to the present model are the H production cross section and the H partial decay widths. These depend theoretically on the Yukawa coupling of H to the fermions, the quarks, and leptons. In the minimal standard model (MSM) these Yukawa couplings are all simply proportional to the fermion mass, implying that the H production by gluon fusion is dominated by a one-loop top quark triangle with dominant decay modes of bottom quarks and tau leptons. In the T' model, the Yukawa couplings do not follow this pattern, and significant deviations from the minimal standard model are expected. Although everything else about the MSM has withstood close scrutiny, the Yukawa couplings are not geometrical like the gauge couplings, and appear as the most vulnerable piece of the MSM Lagrangian.

In this article, we have shown how the T' model can be adapted to include a WIMP dark matter candidate without modifying any prior predictions. Hopefully, new data from the LHC and assorted neutrino experiments will soon allow us to confirm this T' model.

Appendix A: Generalized Type-I Seesaw Mechanism

At this point we can state that the vacuum expectation values (VEVs) of our model's Higgs are as follows,

$$\begin{aligned}
 \langle H_3 \rangle &= (V_1, V_2, V_3), \\
 \langle H'_3 \rangle &= \left(\frac{m_\tau}{Y_\tau}, \frac{m_\mu}{Y_\mu}, \frac{m_e}{Y_e} \right), \\
 \langle H''_3 \rangle &= (0, 0, 0), \\
 \langle H_{1_1} \rangle &= \left(\frac{m_t}{Y_t} \right), \quad \langle H_{1_3} \rangle = \left(\frac{m_b}{Y_b} \right).
 \end{aligned}
 \tag{A1}$$

H'_3 is tied to the charged lepton masses and remains disconnected from the neutrinos assuming the charged leptons are mass eigenstates. H''_3 must have at least one component without a VEV in order to create stable dark matter but must also have 3 identical values in order for Z'_2 to commute with $T' \times Z_2$, hence three zeroes. H_3 remains in a general form that we will further specify by using the seesaw mechanism.

We will begin with the tribimaximal form of the PMNS mixing matrix, an analog of the CKM matrix for neutrinos.

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = U_{TBM} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \quad U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}
 \tag{A2}$$

The PMNS matrix can be used to diagonalize the neutrino mass matrix, and by using the tribimaximal form we can predict the preferred symmetry.

$$\begin{aligned}
 M_{\nu,diag} &= U_{TBM}^T M_\nu U_{TBM}, \\
 M_\nu &= U_{TBM} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{TBM}^T
 \end{aligned}
 \tag{A3}$$

The resultant matrix, presented here in simplified form, will then be compared with the same matrix derived via other means.

$$M_\nu = \begin{pmatrix} -A + B + C & A & A \\ A & B & C \\ A & C & B \end{pmatrix}
 \tag{A4}$$

Next we will implement a generalized Type-I Seesaw Mechanism (the (3, 6) form defined by 3 families and 6 $SU(2)$ singlet field) [40], first noting the key equation in Ref. [41] showing another way to determine M_ν ,

$$M_\nu = M_D M_N^{-1} M_D^T. \quad (\text{A5})$$

The Dirac and Majorana mass matrices below are based on a generalized form of those used in Ref. [42]. Due to the 6 right-handed neutrino states, the Majorana matrix enlarges to 6×6 , while the Dirac matrix becomes 3×6 . The zero elements of the Dirac mass matrix are caused by VEV zeroes of H_3'' .

$$M_D = \begin{pmatrix} 0 & 0 & 0 & Y_1 V_1 & Y_2 V_3 & Y_3 V_2 \\ 0 & 0 & 0 & Y_1 V_3 & Y_2 V_2 & Y_3 V_1 \\ 0 & 0 & 0 & Y_1 V_2 & Y_2 V_1 & Y_3 V_3 \end{pmatrix}, \quad M_N = \begin{pmatrix} M_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{23} \\ 0 & 0 & 0 & 0 & M_{23} & 0 \end{pmatrix} \quad (\text{A6})$$

For simplicity we will set $x_1 \equiv Y_1^2/M_1$ and $x_{23} \equiv Y_2 Y_3/M_{23}$. By following Eq. (A5) we find the symmetric form of the light neutrino mass matrix,

$$M_\nu = \begin{pmatrix} x_1 V_1^2 + 2x_{23} V_2 V_3 & x_1 V_1 V_3 + X_{23}(V_2^2 + V_1 V_3) & x_1 V_1 V_2 + x_{23}(V_3^2 + V_1 V_2) \\ & x_1 V_3^2 + 2x_{23} V_1 V_2 & x_1 V_2 V_3 + x_{23}(V_1^2 + V_2 V_3) \\ & & x_1 V_2^2 + 2x_{23} V_1 V_3 \end{pmatrix} \quad (\text{A7})$$

After comparing Eq. (A7) to its symmetric counter part, Eq. (A4), we attempt to solve for the VEVs of H_3 . One possibility that preserves an acceptable form of the neutrino masses is $\langle H_3 \rangle = (-2, 1, 1)$. These values can then be plugged into the eigenvalues of Eq. (A4) resulting in the parameterized values of the left-handed neutrino masses.

$$\begin{aligned} m_1 &= B + C - 2A = -9x_{23} \\ m_2 &= A + B + C = 0 \\ m_3 &= B - C = 6x_1 + 3x_{23} \end{aligned} \quad (\text{A8})$$

These solutions show that the addition of the neutrino triplet to the MRT'M does not change the results of the seesaw mechanism and preserves the predictions of Ref. [18, 20, 21].

Appendix B: The Higgs Scalar Potential

Included below is the Higgs scalar potential up to quartic order, consisting of 218 terms including 77 hermitian conjugates. We will use $1_{1,2,3}$, to represent the three singlet representations of T' ; additionally 3_1 and 3_2 will be used to distinguish the two triplet products of two contracted T' triplets.

We have studied assiduously the set of equations $\partial V/\partial v_i$, where the v_i are the VEVs, and the related requirements for a local minimum of positive Hessian eigenvalues. We find, after careful calculation, that the VEVs in Eq. (A1) are allowed without fine tuning.

Without further assumptions, one cannot determine the superposition coefficients α , β , and γ of Eq. (3). It may be fruitful to seek an additional assumption to increase our model's predictivity. For the dedicated reader who wishes to pursue this interesting question, we provide below the complete Higgs potential.

$$\begin{aligned}
V = & \mu_{H_1}^2 H_{11}^\dagger H_{11} + \mu_{H_3}^2 H_{13}^\dagger H_{13} + \mu_{H_3}^2 H_3^\dagger H_3 + \mu_{H_3'}^2 H_3'^\dagger H_3' + \mu_{H_3''}^2 H_3''^\dagger H_3'' + \lambda_1 [H_{11}^\dagger H_{11}]_{11}^2 \\
& + \lambda_2 [H_{13}^\dagger H_{13}]_{11}^2 + \lambda_3 [H_{11}^\dagger H_{11}]_{11} [H_{13}^\dagger H_{13}]_{11} + \lambda_4 [H_{11}^\dagger H_{13}]_{12} [H_{11} H_{13}]_{13} + \lambda_5 [H_{13}^\dagger H_{13}]_{13} [H_{13} H_{13}]_{12} \\
& + \lambda_6 [H_{11}^\dagger H_{11}]_{11} [H_3^\dagger H_3]_{11} + \lambda_7 [H_{11}^\dagger H_{11}]_{11} [H_3'^\dagger H_3']_{11} + \lambda_8 [H_{11}^\dagger H_{11}]_{11} [H_3''^\dagger H_3'']_{11} \\
& + \lambda_9 [H_{13}^\dagger H_{13}]_{11} [H_3^\dagger H_3]_{11} + \lambda_{10} [H_{13}^\dagger H_{13}]_{11} [H_3'^\dagger H_3']_{11} + \lambda_{11} [H_{13}^\dagger H_{13}]_{11} [H_3''^\dagger H_3'']_{11} \\
& + \lambda_{12} ([H_{11}^\dagger H_{11}]_{11} [H_3 H_3]_{11} + h.c.) + \lambda_{13} ([H_{11}^\dagger H_{11}]_{11} [H_3' H_3']_{11} + h.c.) + \lambda_{14} ([H_{11}^\dagger H_{11}]_{11} [H_3'' H_3'']_{11} + h.c.) \\
& + \lambda_{15} ([H_{13}^\dagger H_{13}]_{13} [H_3 H_3]_{12} + h.c.) + \lambda_{16} ([H_{13}^\dagger H_{13}]_{13} [H_3' H_3']_{12} + h.c.) + \lambda_{17} ([H_{13}^\dagger H_{13}]_{13} [H_3'' H_3'']_{12} + h.c.) \\
& + \lambda_{18} [H_{11}^\dagger H_3]_3 [H_3^\dagger H_{11}]_3 + \lambda_{19} [H_{11}^\dagger H_3']_3 [H_3'^\dagger H_{11}]_3 + \lambda_{20} [H_{11}^\dagger H_3'']_3 [H_3''^\dagger H_{11}]_3 \\
& + \lambda_{21} [H_{13}^\dagger H_3]_3 [H_3^\dagger H_{13}]_3 + \lambda_{22} [H_{13}^\dagger H_3']_3 [H_3'^\dagger H_{13}]_3 + \lambda_{23} [H_{13}^\dagger H_3'']_3 [H_3''^\dagger H_{13}]_3 \\
& + \lambda_{24} ([H_{13}^\dagger H_3']_3 [H_3^\dagger H_{11}]_3 + h.c.) \\
& + \lambda_{25} ([H_{11}^\dagger H_3]_3 [H_3^\dagger H_3]_{31} + h.c.) + \lambda_{26} ([H_{11}^\dagger H_3']_3 [H_3^\dagger H_3]_{31} + h.c.) \\
& + \lambda_{27} ([H_{11}^\dagger H_3]_3 [H_3^\dagger H_3]_{32} + h.c.) + \lambda_{28} ([H_{11}^\dagger H_3']_3 [H_3^\dagger H_3]_{32} + h.c.) \\
& + \lambda_{29} ([H_{11}^\dagger H_3]_3 [H_3'^\dagger H_3']_{31} + h.c.) + \lambda_{30} ([H_{11}^\dagger H_3']_3 [H_3'^\dagger H_3']_{31} + h.c.) \\
& + \lambda_{31} ([H_{11}^\dagger H_3]_3 [H_3'^\dagger H_3']_{32} + h.c.) + \lambda_{32} ([H_{11}^\dagger H_3']_3 [H_3'^\dagger H_3']_{32} + h.c.) \\
& + \lambda_{33} ([H_{11}^\dagger H_3]_3 [H_3''^\dagger H_3'']_{31} + h.c.) + \lambda_{34} ([H_{11}^\dagger H_3']_3 [H_3''^\dagger H_3'']_{31} + h.c.) \\
& + \lambda_{35} ([H_{11}^\dagger H_3]_3 [H_3''^\dagger H_3'']_{32} + h.c.) + \lambda_{36} ([H_{11}^\dagger H_3']_3 [H_3''^\dagger H_3'']_{32} + h.c.)
\end{aligned}$$

$$\begin{aligned}
& +\lambda_{37}([H_{1_1}^\dagger H_3]_3[H_3^{\prime\prime\dagger} H_3^{\prime\prime}]_{3_1} + h.c.) + \lambda_{38}([H_{1_1}^\dagger H_3^{\prime\prime}]_3[H_3^\dagger H_3^{\prime\prime}]_{3_1} + h.c.) \\
& +\lambda_{39}([H_{1_1}^\dagger H_3^\dagger]_3[H_3^{\prime\prime} H_3^{\prime\prime}]_{3_1} + h.c.) + \lambda_{40}([H_{1_1}^\dagger H_3^{\prime\prime\dagger}]_3[H_3 H_3^{\prime\prime}]_{3_1} + h.c.) \\
& +\lambda_{41}([H_{1_1}^\dagger H_3]_3[H_3^{\prime\prime\dagger} H_3^{\prime\prime}]_{3_2} + h.c.) + \lambda_{42}([H_{1_1}^\dagger H_3^{\prime\prime}]_3[H_3^\dagger H_3^{\prime\prime}]_{3_2} + h.c.) \\
& +\lambda_{43}([H_{1_1}^\dagger H_3^\dagger]_3[H_3^{\prime\prime} H_3^{\prime\prime}]_{3_2} + h.c.) + \lambda_{44}([H_{1_1}^\dagger H_3^{\prime\prime\dagger}]_3[H_3 H_3^{\prime\prime}]_{3_2} + h.c.) \\
& +\lambda_{45}([H_{1_3}^\dagger H_3']_3[H_3^{\prime\dagger} H_3']_{3_1} + h.c.) + \lambda_{46}([H_{1_3}^\dagger H_3^{\prime\dagger}]_3[H_3' H_3']_{3_1} + h.c.) \\
& +\lambda_{47}([H_{1_3}^\dagger H_3']_3[H_3^{\prime\dagger} H_3']_{3_2} + h.c.) + \lambda_{48}([H_{1_3}^\dagger H_3^{\prime\dagger}]_3[H_3' H_3']_{3_2} + h.c.) \\
& +\lambda_{49}([H_{1_3}^\dagger H_3']_3[H_3^\dagger H_3]_{3_1} + h.c.) + \lambda_{50}([H_{1_3}^\dagger H_3]_3[H_3^{\prime\dagger} H_3]_{3_1} + h.c.) \\
& +\lambda_{51}([H_{1_3}^\dagger H_3^{\prime\dagger}]_3[H_3 H_3]_{3_1} + h.c.) + \lambda_{52}([H_{1_3}^\dagger H_3^{\prime\dagger}]_3[H_3' H_3]_{3_1} + h.c.) \\
& +\lambda_{53}([H_{1_3}^\dagger H_3']_3[H_3^\dagger H_3]_{3_2} + h.c.) + \lambda_{54}([H_{1_3}^\dagger H_3]_3[H_3^{\prime\dagger} H_3]_{3_2} + h.c.) \\
& +\lambda_{55}([H_{1_3}^\dagger H_3^{\prime\dagger}]_3[H_3 H_3]_{3_2} + h.c.) + \lambda_{56}([H_{1_3}^\dagger H_3^{\prime\dagger}]_3[H_3' H_3]_{3_2} + h.c.) \\
& +\lambda_{57}([H_{1_3}^\dagger H_3']_3[H_3^{\prime\dagger} H_3^{\prime\prime}]_{3_1} + h.c.) + \lambda_{58}([H_{1_3}^\dagger H_3^{\prime\prime}]_3[H_3^{\prime\dagger} H_3^{\prime\prime}]_{3_1} + h.c.) \\
& +\lambda_{59}([H_{1_3}^\dagger H_3^{\prime\dagger}]_3[H_3^{\prime\prime} H_3^{\prime\prime}]_{3_1} + h.c.) + \lambda_{60}([H_{1_3}^\dagger H_3^{\prime\prime\dagger}]_3[H_3' H_3^{\prime\prime}]_{3_1} + h.c.) \\
& +\lambda_{61}([H_{1_3}^\dagger H_3']_3[H_3^{\prime\prime\dagger} H_3^{\prime\prime}]_{3_2} + h.c.) + \lambda_{62}([H_{1_3}^\dagger H_3^{\prime\prime}]_3[H_3^{\prime\dagger} H_3^{\prime\prime}]_{3_2} + h.c.) \\
& +\lambda_{63}([H_{1_3}^\dagger H_3^{\prime\dagger}]_3[H_3^{\prime\prime} H_3^{\prime\prime}]_{3_2} + h.c.) + \lambda_{64}([H_{1_3}^\dagger H_3^{\prime\prime\dagger}]_3[H_3' H_3^{\prime\prime}]_{3_2} + h.c.) \\
& +\lambda_{65}[H_3^\dagger H_3]_{1_2}[H_3^\dagger H_3]_{1_3} + \lambda_{66}[H_3^\dagger H_3^\dagger]_{1_2}[H_3 H_3]_{1_3} + \lambda'_{66}[H_3^\dagger H_3^\dagger]_{1_3}[H_3 H_3]_{1_2} \\
& +\lambda_{67}[H_3^\dagger H_3]_{1_1}^2 + \lambda_{68}[H_3^\dagger H_3^\dagger]_{1_1}[H_3 H_3]_{1_1} + \lambda_{69}([H_3^\dagger H_3]_{3_1}[H_3^\dagger H_3]_{3_1} + h.c.) \\
& \quad +\lambda_{70}[H_3^\dagger H_3]_{3_1}[H_3^\dagger H_3]_{3_2} + \lambda_{71}[H_3^\dagger H_3^\dagger]_{3_1}[H_3 H_3]_{3_2} \\
& +\lambda_{72}[H_3^{\prime\dagger} H_3']_{1_2}[H_3^{\prime\dagger} H_3']_{1_3} + \lambda_{73}[H_3^{\prime\dagger} H_3^{\prime\dagger}]_{1_2}[H_3' H_3']_{1_3} + \lambda'_{73}[H_3^{\prime\dagger} H_3^{\prime\dagger}]_{1_3}[H_3' H_3']_{1_2} \\
& +\lambda_{74}[H_3^{\prime\dagger} H_3']_{1_1}^2 + \lambda_{75}[H_3^{\prime\dagger} H_3^{\prime\dagger}]_{1_1}[H_3' H_3']_{1_1} + \lambda_{76}([H_3^{\prime\dagger} H_3']_{3_1}[H_3^{\prime\dagger} H_3']_{3_1} + h.c.) \\
& \quad +\lambda_{77}[H_3^{\prime\dagger} H_3']_{3_1}[H_3^{\prime\dagger} H_3']_{3_2} + \lambda_{78}[H_3^{\prime\dagger} H_3^{\prime\dagger}]_{3_1}[H_3' H_3']_{3_2} \\
& +\lambda_{79}[H_3^{\prime\prime\dagger} H_3^{\prime\prime}]_{1_2}[H_3^{\prime\prime\dagger} H_3^{\prime\prime}]_{1_3} + \lambda_{80}[H_3^{\prime\prime\dagger} H_3^{\prime\prime\dagger}]_{1_2}[H_3^{\prime\prime} H_3^{\prime\prime}]_{1_3} + \lambda'_{80}[H_3^{\prime\prime\dagger} H_3^{\prime\prime\dagger}]_{1_3}[H_3^{\prime\prime} H_3^{\prime\prime}]_{1_2} \\
& +\lambda_{81}[H_3^{\prime\prime\dagger} H_3^{\prime\prime}]_{1_1}^2 + \lambda_{82}[H_3^{\prime\prime\dagger} H_3^{\prime\prime\dagger}]_{1_1}[H_3^{\prime\prime} H_3^{\prime\prime}]_{1_1} + \lambda_{83}([H_3^{\prime\prime\dagger} H_3^{\prime\prime}]_{3_1}[H_3^{\prime\prime\dagger} H_3^{\prime\prime}]_{3_1} + h.c.) \\
& \quad +\lambda_{84}[H_3^{\prime\prime\dagger} H_3^{\prime\prime}]_{3_1}[H_3^{\prime\prime\dagger} H_3^{\prime\prime}]_{3_2} + \lambda_{85}[H_3^{\prime\prime\dagger} H_3^{\prime\prime\dagger}]_{3_1}[H_3^{\prime\prime} H_3^{\prime\prime}]_{3_2}
\end{aligned}$$

$$\begin{aligned}
& +\lambda_{86}[H_3^\dagger H_3]_{1_1}[H_3'^\dagger H_3']_{1_1} + \lambda_{87}([H_3^\dagger H_3']_{1_1}[H_3^\dagger H_3']_{1_1} + h.c.) \\
& +\lambda_{88}[H_3^\dagger H_3'^\dagger]_{1_1}[H_3 H_3']_{1_1} + \lambda_{89}([H_3^\dagger H_3^\dagger]_{1_1}[H_3' H_3']_{1_1} + h.c.) \\
& +\lambda_{90}[H_3^\dagger H_3]_{1_2}[H_3'^\dagger H_3']_{1_3} + \lambda_{91}([H_3^\dagger H_3']_{1_2}[H_3^\dagger H_3']_{1_3} + h.c.) \\
& +\lambda_{92}[H_3^\dagger H_3'^\dagger]_{1_2}[H_3 H_3']_{1_3} + \lambda_{93}([H_3^\dagger H_3^\dagger]_{1_2}[H_3' H_3']_{1_3} + h.c.) \\
& +\lambda'_{92}[H_3^\dagger H_3'^\dagger]_{1_3}[H_3 H_3']_{1_2} + \lambda'_{93}([H_3^\dagger H_3^\dagger]_{1_3}[H_3' H_3']_{1_2} + h.c.) \\
& +\lambda_{94}([H_3^\dagger H_3]_{3_1}[H_3'^\dagger H_3']_{3_1} + h.c.) + \lambda_{95}([H_3^\dagger H_3']_{3_1}[H_3^\dagger H_3']_{3_1} + h.c.) \\
& +\lambda_{96}[H_3^\dagger H_3]_{3_1}[H_3'^\dagger H_3']_{3_2} + \lambda_{97}([H_3^\dagger H_3']_{3_1}[H_3^\dagger H_3']_{3_2} + h.c.) \\
& +\lambda_{98}[H_3^\dagger H_3'^\dagger]_{3_1}[H_3 H_3']_{3_2} + \lambda_{99}([H_3^\dagger H_3^\dagger]_{3_1}[H_3' H_3']_{3_2} + h.c.) \\
& +\lambda_{100}[H_3^\dagger H_3]_{1_1}[H_3''^\dagger H_3'']_{1_1} + \lambda_{101}([H_3^\dagger H_3^\dagger]_{1_1}[H_3'' H_3'']_{1_1} + h.c.) \\
& +\lambda_{102}[H_3^\dagger H_3'^\dagger]_{1_1}[H_3 H_3']_{1_1} + \lambda_{103}([H_3^\dagger H_3^\dagger]_{1_1}[H_3'' H_3'']_{1_1} + h.c.) \\
& +\lambda_{104}[H_3^\dagger H_3]_{1_2}[H_3''^\dagger H_3'']_{1_3} + \lambda_{105}([H_3^\dagger H_3^\dagger]_{1_2}[H_3'' H_3'']_{1_3} + h.c.) \\
& +\lambda_{106}[H_3^\dagger H_3'^\dagger]_{1_2}[H_3 H_3']_{1_3} + \lambda_{107}([H_3^\dagger H_3^\dagger]_{1_2}[H_3'' H_3'']_{1_3} + h.c.) \\
& +\lambda'_{106}[H_3^\dagger H_3''^\dagger]_{1_3}[H_3 H_3']_{1_2} + \lambda'_{107}([H_3^\dagger H_3^\dagger]_{1_3}[H_3'' H_3'']_{1_2} + h.c.) \\
& +\lambda_{108}([H_3^\dagger H_3]_{3_1}[H_3''^\dagger H_3'']_{3_1} + h.c.) + \lambda_{109}([H_3^\dagger H_3^\dagger]_{3_1}[H_3'' H_3'']_{3_1} + h.c.) \\
& +\lambda_{110}[H_3^\dagger H_3]_{3_1}[H_3''^\dagger H_3'']_{3_2} + \lambda_{111}([H_3^\dagger H_3^\dagger]_{3_1}[H_3'' H_3'']_{3_2} + h.c.) \\
& +\lambda_{112}[H_3^\dagger H_3''^\dagger]_{3_1}[H_3 H_3']_{3_2} + \lambda_{113}([H_3^\dagger H_3^\dagger]_{3_1}[H_3'' H_3'']_{3_2} + h.c.) \\
& +\lambda_{114}[H_3^\dagger H_3']_{1_1}[H_3''^\dagger H_3'']_{1_1} + \lambda_{115}([H_3^\dagger H_3^\dagger]_{1_1}[H_3'' H_3'']_{1_1} + h.c.) \\
& +\lambda_{116}[H_3^\dagger H_3''^\dagger]_{1_1}[H_3' H_3'']_{1_1} + \lambda_{117}([H_3^\dagger H_3^\dagger]_{1_1}[H_3'' H_3'']_{1_1} + h.c.) \\
& +\lambda_{118}[H_3^\dagger H_3']_{1_2}[H_3''^\dagger H_3'']_{1_3} + \lambda_{119}([H_3^\dagger H_3^\dagger]_{1_2}[H_3'' H_3'']_{1_3} + h.c.) \\
& +\lambda_{120}[H_3^\dagger H_3''^\dagger]_{1_2}[H_3' H_3'']_{1_3} + \lambda_{121}([H_3^\dagger H_3^\dagger]_{1_2}[H_3'' H_3'']_{1_3} + h.c.) \\
& +\lambda'_{120}[H_3^\dagger H_3''^\dagger]_{1_3}[H_3' H_3'']_{1_2} + \lambda'_{121}([H_3^\dagger H_3^\dagger]_{1_3}[H_3'' H_3'']_{1_2} + h.c.) \\
& +\lambda_{122}([H_3^\dagger H_3']_{3_1}[H_3''^\dagger H_3'']_{3_1} + h.c.) + \lambda_{123}([H_3^\dagger H_3^\dagger]_{3_1}[H_3'' H_3'']_{3_1} + h.c.) \\
& +\lambda_{124}[H_3^\dagger H_3']_{3_1}[H_3''^\dagger H_3'']_{3_2} + \lambda_{125}([H_3^\dagger H_3^\dagger]_{3_1}[H_3'' H_3'']_{3_2} + h.c.) \\
& +\lambda_{126}[H_3^\dagger H_3''^\dagger]_{3_1}[H_3' H_3'']_{3_2} + \lambda_{127}([H_3^\dagger H_3^\dagger]_{3_1}[H_3'' H_3'']_{3_2} + h.c.)
\end{aligned}$$

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