# First-principles calculations of the structural, electronic, vibrational and magnetic properties of $\mathrm{C}_{60}$ and $\mathrm{C}_{48} \mathrm{~N}_{12}$ : a comparative study 

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#### Abstract

We perform first-principles calculations of the structural, electronic, vibrational and magnetic properties of the $\mathrm{C}_{48} \mathrm{~N}_{12}$ azafullerene and $\mathrm{C}_{60}$. Full geometrical optimization shows that $\mathrm{C}_{48} \mathrm{~N}_{12}$ is characterized by several distinguishing features: only one nitrogen atom per pentagon, two nitrogen atoms preferentially sitting in one hexagon, $S_{6}$ symmetry, 6 unique nitrogen-carbon and 9 unique carbon-carbon bond lengths. The Mulliken charge analysis indicates that the doped nitrogen atoms in $\mathrm{C}_{48} \mathrm{~N}_{12}$ exist as electron acceptors and three-fourths of the carbon atoms as electron donors. Electronic structure calculations of $\mathrm{C}_{48} \mathrm{~N}_{12}$ show that the highest occupied molecular orbital (HOMO) is a doubly degenerate level of $a_{g}$ symmetry and the lowest unoccupied molecular orbital (LUMO) is a nondegenerate level of $a_{u}$ symmetry. The calculated binding energy per atom and HOMO-LUMO energy gap of $\mathrm{C}_{48} \mathrm{~N}_{12}$ are about 1 eV smaller than those of $\mathrm{C}_{60}$. For both $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$, the total energies calculated with STO-3G, 3-21G and 6-31G basis sets differ from the 6-31G* basis set results by about $1.5 \%, 0.6 \%$ and $0.05 \%$, respectively. Because of electron correlations, the HOMO-LUMO gap decreases about 5 eV and the binding energy per atom increases about 2 eV . Our vibrational frequency analysis predicts that $\mathrm{C}_{48} \mathrm{~N}_{12}$ has in total 116 vibrational modes: 58 modes are infraredactive ( 29 doubly-degenerate and 29 non-degenerate modes) and 58 modes are Raman-active ( 29 doubly-degenerate unpolarized and 29 non-degenerate polarized). It is found that $\mathrm{C}_{48} \mathrm{~N}_{12}$ exhibits eight ${ }^{13} \mathrm{C}$ and two ${ }^{15} \mathrm{~N}$ NMR (nuclear magnetic resonance) spectral signals. In comparison to isolated carbon or nitrogen atoms, an enhancement in the dipole polarizability is found due to the delocalized $\pi$ electrons in $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$. The average second-order hyperpolarizability of $\mathrm{C}_{48} \mathrm{~N}_{12}$ is about $55 \%$ larger than that of $\mathrm{C}_{60}$. In addition, the effects of basis sets are discussed in detail, and the different methods for calculating nuclear magnetic shielding tensors are compared. Our detailed study of $\mathrm{C}_{60}$ reveals the importance of electron correlations and the choice of basis sets in the first-principles calculations. Our best-calculated results for $\mathrm{C}_{60}$ with the B3LYP hybrid density functional theory are in excellent agreement with experiment and demonstrate the desired efficiency and accuracy needed for obtaining quantitative information on the structural, electronic and vibrational properties of these molecules. Our results suggest that $\mathrm{C}_{48} \mathrm{~N}_{12}$ has potential applications as semiconductor components, nonlinear optical materials, and possible building blocks for molecular electronics and photonic devices.


## I. INTRODUCTION

Graphite is a stable and abundant solid form of pure carbon [1]. In this form, three valence electrons of each carbon atom form three strong $s p^{2}$ trigonal bonds to three nearest neighbors with an equal distance of 0.142 nm , while the fourth valence electron from different carbon atoms interacts by weak $\pi$ bonds perpendicular to successive sheets with an inter-plane distance of 0.34 nm [2]. Diamond is another slightly less stable and less abundant crystallographic form of pure carbon [1], where each carbon atom is covalently bonded to four neighbors via
$s p^{3}$ hybridization at the apexes of a regular tetrahedron. In 1985, a fascinating molecule, named $\mathrm{C}_{60}$ (a truncated icosahedron with 20 hexagonal and 12 pentagonal faces, and 60 vertices, each of which is at the intersection of two hexagonal and one pentagonal faces) was discovered by Kroto et al. [3], and a new form of pure carbon, named fullerenes [4], was born.

Fullerenes can crystallize in a variety of threedimensional structures [5,6], being made from an even number of three-coordinated $\mathrm{sp}^{2}$ carbon atoms that arrange themselves into 12 pentagonal faces and any number $(>1)$ of hexagonal faces [4]. The macroscopic syn-
thesis of soot [5], which contains $\mathrm{C}_{60}$ and other fullerenes in large compounds, plus the straightforward purification techniques of the soot which make the pure fullerene materials available, have led to extensive studies of fullerenes [7-10,12].

Doped fullerenes have also attracted a great deal of interest due to their remarkable structural, electronic, optical and magnetic properties [7-10,12]. For example, the doped fullerenes can exhibit large third-order optical nonlinearities $[9,10]$ and be ideal candidates as photonic devices including all-optical switching, data processing, and eye and sensor protection [9,10]. Another example is alkali-doped $C_{60}$ crystals, which can become superconducting $[13,14]$, for example, at a critical temperature $T_{c}=30 \mathrm{~K}[13]$. In addition to the endohedral doping (inside fullerenes) and exohedral doping (outside fullerenes), there is another type of doping, named substitutional doping, where one or more carbon atoms of fullerene are substituted by other atoms [7-12], due to the unique structural and electronic properties of fullerenes. Because boron and nitrogen bracket carbon in the periodic table, much attention has been paid to alternate boron- and/or nitrogen-doped compounds [7-12]. Over the past 10 years, boron and nitrogen atoms have been successfully used to replace carbon atoms of $\mathrm{C}_{60}$ and synthesize many kinds of heterofullerenes, $\mathrm{C}_{60-\mathrm{m}-\mathrm{n}} \mathrm{N}_{\mathrm{m}} \mathrm{B}_{\mathrm{n}}$ [ $7-10,12,15-18]$. In 1995, a very efficient method of synthesizing $\mathrm{C}_{59} \mathrm{~N}$ was reported [16]. This method has led to a number of detailed studies of the physical and chemical properties of $\mathrm{C}_{59} \mathrm{~N}$ [8-10,12,17]. Very recently, $\mathrm{C}_{60}$ with more than one nitrogen atom replacing carbon atoms in the cage has been synthesized by Hultman et al. [18], and the existence of a novel $\mathrm{C}_{48} \mathrm{~N}_{12}$ aza-fullerene [18-20] was reported. Hence, it would be interesting and useful to investigate and predict the structural, electronic, vibrational and magnetic properties of this aza-fullerene by performing detailed first-principles calculations. This forms main purpose of the present paper.

Fullerenes have been challenging molecules for firstprinciples calculations because of their size [21,22]. Recent advances in ab initio methods and parallel computing have brought a substantial improvement in capabilities for predicting the properties of large molecules. The coupled cluster method has been used to predict phenomena in $\mathrm{C}_{20}$ [24]. Other first-principles methods, which are less demanding in terms of computation cost than the coupled cluster method, have been used for much larger fullerenes and carbon nanotubes. For example, $\mathrm{C}_{60}$ [25-30] has been studied with self-consistent field (SCF) and Moller-Plesset second-order (MP2) theory, $\mathrm{C}_{240}$ [31] and carbon nanotubes [32] with density functional theory (DFT) $[33,34]$, and $\mathrm{C}_{540}[35]$ with the Hartree-Fock (HF) method.
$\mathrm{C}_{60}$ has the highest symmetry $I_{h}$ in the point group, two types of carbon-carbon (CC) bonds (one single bond C-C shared by adjacent five- and six-membered carbon rings and one double bond $\mathrm{C}=\mathrm{C}$ by adjacent sixmembered carbon rings), and two kinds of bond angles
(one angle between two adjacent C-C bonds, and another one between a $\mathrm{C}=\mathrm{C}$ bond and a adjacent $\mathrm{C}-\mathrm{C}$ bond) [7]. In section II, we perform full geometry optimizations of $\mathrm{C}_{48} \mathrm{~N}_{12}$ as well as $\mathrm{C}_{60}$ with both DFT and restricted HF (RHF) methods. It is found that the $\mathrm{C}_{48} \mathrm{~N}_{12}$ aza-fullerene has several distinguishing features: only one nitrogen atom per pentagon, two nitrogen atoms preferentially sitting in one hexagon, $\mathrm{S}_{6}$ symmetry, six unique nitrogen-carbon (NC) and nine unique CC bonds. The Mulliken charge analysis shows that the doped nitrogen atoms in $\mathrm{C}_{48} \mathrm{~N}_{12}$ exist as electron acceptors and threefourths of the carbon atoms as electron donors. Our best CC bond lengths and radius of $\mathrm{C}_{60}$ calculated with B3LYP/6-31G* are in excellent agreement with experiment.

Total energy calculations of the optimized $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ are discussed in section III. It is found that the highest occupied molecular orbital (HOMO) is a doubly degenerate level with $a_{g}$ symmetry and the lowest unoccupied molecular orbital (LUMO) is a nondegenerate level with $a_{u}$ symmetry. The calculated HOMO-LUMO energy gap of $\mathrm{C}_{48} \mathrm{~N}_{12}$ is about 1 eV smaller than that of $\mathrm{C}_{60}$. For both molecules, the total energies calculated with STO-3G, 3-21G and 6-31G basis sets differ from the $6-31 \mathrm{G}^{*}$ results by about $1.5 \%, 0.6 \%$ and $0.05 \%$, respectively, and the HOMO-LUMO gaps decrease about 5 eV due to electron correlations. For $\mathrm{C}_{60}$, our calculated results are in agreement with other groups' calculations, and our best HOMO-LUMO energy gap calculated with B3LYP/6-31G* is in agreement with experiment.

When an external electric field is applied to a molecule, its charges are redistributed and dipoles are induced or reoriented [36]. The relation between the dipole moment $\mathbf{P}$ and the applied field $\mathbf{G}$ can be written as [9]

$$
\begin{equation*}
\mathbf{P}=\mathbf{P}_{0}+\alpha \mathbf{G}+\frac{\beta}{2} \mathbf{G}^{2}+\frac{\gamma}{6} \mathbf{G}^{3}+\ldots \tag{1}
\end{equation*}
$$

where $\mathbf{G}$ is the electric field, $\mathbf{P}_{0}$ is the permanent dipole moment, $\alpha$ is the dipole polarizability, $\beta$ is the first-order hyperpolarizability, and $\gamma$ is the second-order hyperpolarizability. The static dipole polarizability (SDP) measures the ability of the valence electrons to find a configuration which screens a static external field [9]. It has been shown that molecules with many delocalized valence electrons should display large SDPs [9,37]. The first- and secondorder hyperpolarizabilities play a key role in the description of nonlinear optical phenomena since a time-varying polarization can act as the source of new components of the electromagnetic field [36]. In section IV, we calculate the SDPs and first- and second-order hyperpolarizabilities of $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$. In comparison to isolated carbon or nitrogen atoms, we find an enhancement in the SDP due to the delocalized $\pi$ electrons in $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$. The calculated SDP for $C_{60}$ is in agreement with experiment. The average second-order hyperpolarizability of $\mathrm{C}_{48} \mathrm{~N}_{12}$ is about $55 \%$ larger than that of $\mathrm{C}_{60}$.

When a material is doped, its mechanical, electronic, magnetic and optical properties change [7,9-11]. The
ability to control such induced changes is vital to progress in material science. Raman and infrared (IR) spectroscopic techniques $[38,39]$ are useful experimental tools to investigate how doping modifies the structural and dynamical properties of the pristine material and to understand the physical origin of such induced changes. Over the past 10 years, both techniques have been used widely to study the vibrational properties of $\mathrm{C}_{60}$ [40-45], its derivative compounds [46-57], and (doped) carbon nanotubes [58-64]. It has been shown that $\mathrm{C}_{60}$ has in total 46 vibrational modes including 4 IR-active [40-43] and 10 Raman-active vibrational modes [44,45]. These studies have offered a good guide to the phonon spectrum in the solid state of these materials. In section V, we perform a vibrational analysis and calculate the infrared (IR) intensities of $\mathrm{C}_{48} \mathrm{~N}_{12}$. Fifty eight IR-active (i.e., 29 doubly-degenerate and 29 non-degenerate modes) and 58 Raman-active (i.e., 29 doubly-degenerate unpolarized and 29 non-degenerate polarized) frequencies are determined. The best vibrational frequencies and IR results for $\mathrm{C}_{60}$ calculated with B3LYP are in excellent agreement with experiment and demonstrate the desirable efficiency and accuracy of this theory for obtaining quantitative information on the vibrational properties of these materials. Comparison with other groups' calculations of $\mathrm{C}_{60}$ is made and discussed.

High resolution NMR [65-67] gives spectra which can be analyzed to yield parameters such as the nuclear magnetic shielding $\sigma$ [68] and the nuclear spin-spin coupling $J$ [69], which characterize molecular systems and structures. Both $\sigma$ and $J$ are determined by the electronic environments of the nuclei involved. A satisfactory theoretical description of the distribution of electrons in a molecule can lead to reliable predictions of $\sigma$ and $J$, which have a number of applications, such as the identification of the conformation or structure of the species present in a given sample. In section VI the GIAO (gauge-including atomic orbital) [70] and the CSGT (continuous set of gauge transformations) [71] methods are utilized for calculating the nuclear magnetic shielding tensor $\sigma$ in $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ at both the HF and DFT levels of theory. Eight ${ }^{13} \mathrm{C}$ and two ${ }^{15} \mathrm{~N}$ NMR spectral signals are predicted for $\mathrm{C}_{48} \mathrm{~N}_{12}$. Our best calculated NMR results for $\mathrm{C}_{60}$ are in excellent agreement with experiment.

Finally, we end in section VII by giving a summary and outlook on the potential applications of $\mathrm{C}_{48} \mathrm{~N}_{12}$.

## II. OPTIMIZED GEOMETRIC STRUCTURE

The geometries of both $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ were fully optimized by using the Gaussian 98 program [72,73], where we have employed both RHF and DFT methods. Also we discuss the effects of basis sets by considering STO-3G, $3-21 \mathrm{G}, 6-31 \mathrm{G}$ and $6-31 \mathrm{G}^{*}[74-79]$. For the DFT method, we use the B3LYP hybrid functional [80].


FIG.1: Geometric structure of $\mathrm{C}_{48} \mathrm{~N}_{12}$. The site numbers $\{5,9,14,21,26,30,35,39,45,50,55,60\}$ are for nitrogen atoms, while the others are for carbon atoms.

In Fig.1, we present the geometry of $\mathrm{C}_{48} \mathrm{~N}_{12}$. The $a b$ initio calculations show that $\mathrm{C}_{48} \mathrm{~N}_{12}$ has only one nitrogen atom per pentagon and two nitrogen atoms preferentially sit in one hexagon. The symmetry of $\mathrm{C}_{48} \mathrm{~N}_{12}$ is the $S_{6}$ point group $[19,20]$. The optimized distances (or radii) $\mathrm{R}_{i}$ from the $i$ th atom to the density center of the molecule are listed in Table I. We find that there are 10 unique radii for $\mathrm{C}_{48} \mathrm{~N}_{12}$, which suggest that $\mathrm{C}_{48} \mathrm{~N}_{12}$ is an ellipsoidal structure and has 10 unique sites ( 2 for N sites and 8 for C sites), while $\mathrm{C}_{60}$ has an equal radius for each carbon atom. As shown in section VI, the 10 unique sites of $\mathrm{C}_{48} \mathrm{~N}_{12}$ can be identified by NMR experiments. Comparing the B3LYP results with the RHF results shows that the radii are increased by up to $2 \%$ due to the electron correlation. Comparing the $6-31 \mathrm{G}$ and 6 $31 \mathrm{G}^{*}$ 's results shows that adding polarization functions decreases the radius of carbon sites but increases the radius of nitrogen sites. In comparison with the results of STO-3G and the split valence basis sets, we find that increasing the basis size would lead to a decreased radius. For $\mathrm{C}_{60}$, the radius for each carbon site calculated by using B3LYP and the $6-31 \mathrm{G}^{*}$ basis set is $3.5502 \AA$, which is in excellent agreement with experiment $(\mathrm{R}=$ $3.55 \AA$ ) [86], the LDA (local density approximation) calculation ( $\mathrm{R}=3.537 \AA$ ) with a pseudopotential approach (PPA) [87], and the LDA-based Car-Parrrinello molecular dynamics ( CPMD ) simulation ( $\mathrm{R}=3.55 \AA$ ) [88]. The success of the B3LYP calculation of the $\mathrm{C}_{60}$ radius demonstrates the importance of electron correlation for
an accurate description of a molecular geometrical structure.

The calculated net Mulliken charges $\mathrm{Q}_{\mathrm{i}}$ of carbon and nitrogen atoms in $\mathrm{C}_{48} \mathrm{~N}_{12}$ are also listed in Table I. There are two unique types of nitrogen atoms in the structure. The net Mulliken charges $\mathrm{Q}_{\mathrm{i}} / q\left(q=1.6 \times 10^{-19}\right)$ on both types of N are negative, for example, -0.5953 C and -0.6002 C with B3LYP/6-31G*, -0.7803 C and -0.7829 C with RHF/6-31G*. The net Mulliken charges of the carbon atoms in $\mathrm{C}_{48} \mathrm{~N}_{12}$ separate into two groups: $1 / 4$ of carbon atoms with negative $\mathrm{Q}_{\mathrm{i}}$ and the remaining 3/4 with positive $\mathrm{Q}_{\mathrm{i}}$. Although the Mulliken charge analysis cannot estimate the atomic charges quantitatively, their signs can be estimated [89]. From these results, we find that the doped nitrogen atoms and one-fourth of the carbon atoms exist as electron acceptors, and threefourths of the carbon atoms as electron donors. It should be mentioned that we also performed calculations of net

Mulliken charges of carbon and boron atoms in $\mathrm{C}_{48} \mathrm{~B}_{12}$ [90]. We found that the doped boron atoms exist as electron donors and all carbon atoms as electron acceptors [90]. Therefore, $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{48} \mathrm{~B}_{12}$ have opposite electronic polarizations, while $\mathrm{C}_{60}$ is isotropic. In the case of doping into silicon, the V family in the periodic table (for example, phosphorous) exists as a donor, while the III family (for example, boron) exists as an electron acceptor. Thus, the B- or N -substituted doping in $\mathrm{C}_{60}$ differs greatly from that for silicon. This is due to the unique structural and electronic properties of $\mathrm{C}_{60}$ [7]. With respect to the electron correlation and the choice of basis sets, we find that the absolute value of the net Mulliken charge $Q_{i}$ for each atom in $\mathrm{C}_{48} \mathrm{~N}_{12}$ increases with an increase of the basis size, but decreases due to the electron correlation or by adding polarization functions to a given basis set.

Table I: Net Mulliken charge $\mathrm{Q}_{\mathrm{i}}\left(q=1.6 \times 10^{-19}\right)$ and radius $\mathrm{R}_{\mathrm{i}}(1 \AA=0.1 \mathrm{~nm})$ at the site number $n_{i}$ in $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ calculated by RHF and B3LYP methods with a variety of Pople-style basis sets.

| Method | Fullerene | Site Number $\left\{\mathrm{n}_{\mathrm{i}}\right\}$ | Atom | STO-3G |  | 3-21G |  | 6-31G |  | 6-31G* |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\begin{gathered} \mathrm{R}_{\mathrm{i}} \\ {[\AA]} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{Q}_{\mathrm{i}} / q \\ {[\mathrm{C}]} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{R}_{\mathrm{i}} \\ {[\AA]} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{Q}_{\mathrm{i}} / q \\ {[\mathrm{C}]} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{R}_{\mathrm{i}} \\ {[\AA]} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{Q}_{\mathrm{i}} / q \\ {[\mathrm{C}]} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{R}_{\mathrm{i}} \\ {[\AA]} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{Q}_{\mathrm{i}} / q \\ {[\mathrm{C}]} \\ \hline \end{gathered}$ |
| RHF | $\mathrm{C}_{48} \mathrm{~N}_{12}$ | \{ 1, 13, 16, 31, 38, 51\} | C | 3.5452 | 0.1073 | 3.5059 | 0.3856 | 3.5082 | 0.3546 | 3.4988 | 0.2926 |
|  |  | $\{2,12,29,32,37,52\}$ | C | 3.5591 | -0.0238 | 3.5299 | -0.0897 | 3.5271 | -0.0515 | 3.5162 | -0.0621 |
|  |  | $\{3,11,28,33,36,53\}$ | C | 3.5602 | -0.0192 | 3.5353 | -0.0609 | 3.5275 | -0.0344 | 3.5389 | -0.0480 |
|  |  | \{ 4, 15, 27, 34, 40, 54\} | C | 3.5331 | 0.0942 | 3.4954 | 0.3905 | 3.4999 | 0.3701 | 3.4716 | 0.3167 |
|  |  | $\{5,14,30,35,39,55\}$ | N | 3.6974 | -0.2406 | 3.5691 | -0.9541 | 3.5685 | -0.9552 | 3.6151 | -0.7803 |
|  |  | $\{6,18,24,42,48,58\}$ | C | 3.4292 | 0.0601 | 3.4229 | 0.2907 | 3.4299 | 0.2758 | 3.3880 | 0.2209 |
|  |  | $\{7,19,23,43,47,57\}$ | C | 3.4023 | 0.0893 | 3.3874 | 0.3363 | 3.3983 | 0.3058 | 3.3409 | 0.2962 |
|  |  | $\{8,20,22,44,46,56\}$ | C | 3.4723 | 0.0718 | 3.4599 | 0.3191 | 3.4577 | 0.3522 | 3.4416 | 0.2433 |
|  |  | $\{9,21,26,45,50,60\}$ | N | 3.5870 | -0.2344 | 3.4985 | -0.9878 | 3.4951 | -0.9913 | 3.5082 | -0.7829 |
|  |  | $\{10,17,25,41,49,59\}$ | C | 3.4851 | 0.0952 | 3.4586 | 0.3703 | 3.4631 | 0.3737 | 3.4222 | 0.3036 |
|  | $\mathrm{C}_{60}$ | $\{1,2,3,4,5,6, \ldots, 60\}$ | C | 3.5473 | 0 | 3.5238 | 0 | 3.5300 | 0 | 3.5226 | 0 |
| B3LYP | $\mathrm{C}_{48} \mathrm{~N}_{12}$ | $\{1,13,16,31,38,51\}$ | C | 3.5871 | 0.0827 | 3.5302 | 0.2770 | 3.5365 | 0.2147 | 3.5171 | 0.1961 |
|  |  | $\{2,12,29,32,37,52\}$ | C | 3.5923 | -0.0188 | 3.5469 | -0.0422 | 3.5458 | -0.0175 | 3.5275 | -0.0125 |
|  |  | $\{3,11,28,33,36,53\}$ | C | 3.6019 | -0.0193 | 3.5533 | -0.0358 | 3.5492 | -0.0173 | 3.5395 | -0.0298 |
|  |  | $\{4,15,27,34,40,54\}$ | C | 3.5951 | 0.0682 | 3.5381 | 0.2861 | 3.5408 | 0.2486 | 3.5190 | 0.2266 |
|  |  | $\{5,14,30,35,39,55\}$ | N | 3.7175 | -0.1883 | 3.5918 | -0.7543 | 3.5945 | -0.6690 | 3.6187 | -0.5953 |
|  |  | $\{6,18,24,42,48,58\}$ | C | 3.5092 | 0.0550 | 3.4592 | 0.2522 | 3.4706 | 0.1985 | 3.4348 | 0.1919 |
|  |  | $\{7,19,23,43,47,57\}$ | C | 3.4882 | 0.0679 | 3.4328 | 0.2635 | 3.4465 | 0.2103 | 3.4049 | 0.2124 |
|  |  | $\{8,20,22,44,46,56\}$ | C | 3.5431 | 0.0618 | 3.4917 | 0.2637 | 3.4946 | 0.2623 | 3.4720 | 0.1998 |
|  |  | $\{9,21,26,45,50,60\}$ | N | 3.6302 | -0.1814 | 3.5336 | -0.7787 | 3.5304 | -0.7047 | 3.5335 | -0.6002 |
|  |  | $\{10,17,25,41,49,59\}$ | C | 3.5554 | 0.0723 | 3.5056 | 0.2686 | 3.5087 | 0.2548 | 3.4807 | 0.2110 |
|  | $\mathrm{C}_{60}$ | $\{1,2,3,4,5,6, \ldots, 60\}$ | C | 3.6034 | 0 | 3.5555 | 0 | 3.5615 | 0 | 3.5502 | 0 |

Table II: Bond lengths ( $\mathrm{L}, 1 \AA=0.1 \mathrm{~nm}$ ) in $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ calculated by using B3LYP methods with a variety of Pople-style basis sets, where ( $n_{i}, n_{j}$ ) denotes the site number pair that forms a bond.

| Fullerene | Bond | $\left(n_{i}, n_{j}\right)$ | STO-3G |  | 3-21G |  | 6-31G |  | 6-31G* |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\begin{aligned} & \mathrm{L}_{d f t} \\ & {[\AA]} \end{aligned}$ | $\begin{gathered} \mathrm{L}_{r h f} \\ {[\AA]} \end{gathered}$ | $\begin{gathered} \mathrm{L}_{d f t} \\ {[\hat{\AA}]} \end{gathered}$ | $\begin{gathered} \hline \mathrm{L}_{r h f} \\ {[\AA]} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{L}_{d f t} \\ {[\hat{\AA}]} \end{gathered}$ | $\begin{gathered} \hline \mathrm{L}_{r h f} \\ {[\AA]} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{L}_{d f t} \\ {[\AA]} \end{gathered}$ | $\begin{gathered} \hline \mathrm{L}_{r h f} \\ {[\AA]} \\ \hline \end{gathered}$ |
| $\mathrm{C}_{48} \mathrm{~N}_{12}$ | CC | $(1,2)(12,13)(16,29)$ $(31,32)(37,38)(51,52)$ | 1.4275 | 1.3914 | 1.4103 | 1.3855 | 1.4125 | 1.3880 | 1.4061 | 1.3836 |
|  | CC | $\begin{aligned} & (1,52)(2,31)(12,38) \\ & (13,29)(16,37)(32,51) \end{aligned}$ | 1.4371 | 1.4151 | 1.4171 | 1.4083 | 1.4216 | 1.4134 | 1.4155 | 1.4024 |
|  | NC | $\begin{gathered} (1,5)(13,14)(16,30) \\ (31,35)(38,39)(51,55) \end{gathered}$ | 1.4742 | 1.4601 | 1.432 | 1.4206 | 1.4315 | 1.4164 | 1.4300 | 1.4272 |
|  | CC | $\begin{gathered} (2,3)(11,12)(28,29) \\ (32,33)(36,37)(52,53) \end{gathered}$ | 1.4701 | 1.4662 | 1.4517 | 1.4524 | 1.4488 | 1.4478 | 1.4455 | 1.4521 |
|  | CC | $\begin{gathered} (3,4)(11,15)(27,28) \\ (33,34)(36,40)(53,54) \end{gathered}$ | 1.4092 | 1.3590 | 1.3947 | 1.3620 | 1.3971 | 1.3652 | 1.3901 | 1.3588 |
|  | CC | $\begin{aligned} & (3,10)(11,59)(17,33) \\ & (25,36)(28,49)(41,53) \end{aligned}$ | 1.4559 | 1.4566 | 1.4320 | 1.4350 | 1.4346 | 1.4367 | 1.4314 | 1.4368 |
|  | NC | $\begin{aligned} & (4,5)(14,15)(27,30) \\ & (39,40)(34,35)(54,55) \end{aligned}$ | 1.4707 | 1.4553 | 1.4307 | 1.4128 | 1.4286 | 1.4108 | 1.4224 | 1.4047 |
|  | CC | $\begin{gathered} (4,46)(8,15)(20,40) \\ (22,54)(27,44)(34,56) \end{gathered}$ | 1.4567 | 1.4589 | 1.4326 | 1.4367 | 1.4354 | 1.4388 | 1.4313 | 1.4372 |
|  | NC | $\begin{gathered} (5,42)(6,35)(14,48) \\ (18,55)(24,30)(39,58) \end{gathered}$ | 1.4749 | 1.4637 | 1.4299 | 1.4294 | 1.4317 | 1.4278 | 1.4287 | 1.4310 |
|  | CC | $\begin{gathered} (6,7)(18,19)(23,24) \\ (42,43)(47,48)(57,58) \end{gathered}$ | 1.4399 | 1.4352 | 1.4211 | 1.4182 | 1.4217 | 1.4185 | 1.4136 | 1.4143 |
|  | CC | $\begin{aligned} & (6,10)(17,18)(24,25) \\ & (58,59)(41,42)(48,49) \end{aligned}$ | 1.4119 | 1.3641 | 1.3963 | 1.3664 | 1.4004 | 1.3712 | 1.3941 | 1.3634 |
|  | CC | $\begin{aligned} & (7,8)(19,20)(22,23) \\ & (43,44)(46,47)(56,57) \end{aligned}$ | 1.4195 | 1.3705 | 1.4021 | 1.3718 | 1.4068 | 1.3761 | 1.4021 | 1.3703 |
|  | NC | $\begin{gathered} (7,60)(9,47)(19,26) \\ (21,57)(23,45)(43,50) \end{gathered}$ | 1.4586 | 1.4419 | 1.4159 | 1.4047 | 1.4189 | 1.4071 | 1.4099 | 1.3958 |
|  | NC | $\begin{gathered} (8,9)(20,21)(22,26) \\ (44,45)(46,50)(56,60) \end{gathered}$ | 1.4510 | 1.4313 | 1.4182 | 1.4083 | 1.4149 | 1.4012 | 1.4084 | 1.4056 |
|  | NC | $\begin{aligned} & (9,10)(17,21) \\ & (25,26) \\ & (41,45)(49,50) \\ & (59,60) \end{aligned}$ | 1.4612 | 1.4360 | 1.4271 | 1.4043 | 1.4226 | 1.3997 | 1.4134 | 1.3919 |
| $\mathrm{C}_{60}$ | $\mathrm{C}=\mathrm{C}$ | $(1,52)(2,31)(3,10) \ldots$ | 1.4130 | 1.3759 | 1.3899 | 1.3671 | 1.3981 | 1.3750 | 1.3949 | 1.3732 |
|  | C-C | $(1,2)(1,5)(2,3) \ldots$ | 1.4773 | 1.4628 | 1.4601 | 1.4529 | 1.4592 | 1.4524 | 1.4539 | 1.4487 |

The optimized CC and NC bond lengths in $\mathrm{C}_{48} \mathrm{~N}_{12}$ are listed in Table II. We find that there are 6 unique NC and 9 unique CC bonds. In comparison with the calculated CC bond lengths for $\mathrm{C}_{60}$ shown in Table II, the CC bond length in $\mathrm{C}_{48} \mathrm{~N}_{12}$, in general, is less than the single CC bond length of $\mathrm{C}_{60}$ due to the redistribution of the electron density. It is also found that the bond length increases due to the electron correlation, but decreases as we increase the basis size or include the polarization function.

In comparison with experimental data available for $\mathrm{C}_{60}$ listed in Table III, we find that the two kinds of bond lengths of $\mathrm{C}_{60}$ calculated by using the B3LYP with a large basis set $6-31 \mathrm{G}^{*}$ are in good agreement with the results measured by X-ray powder diffraction (XRPD) [91], NMR [92,93], gas-phase electron diffraction (GPED) [94] or X-ray crystallography technique (XRCT) [86].

For comparison, Table III also lists the calculated CC bond lengths for $\mathrm{C}_{60}$ with a selection of previous theoretical calculations. Given the low computational cost of Hückel theory, the bond lengths [95] predicted by this theory are remarkably satisfactory. The semiempirical QCFF/PI (quantum-chemical-force-fields for $\pi$ electrons) [96] does not predict as good bond lengths as the Hückel theory since it has been parameterized mainly with respect to frequencies of conjugated hydrocarbons. The CC bond lengths calculated by using the semiempirical MNDO (modified neglect of differential overlap) [97] and the extended Hubbard model (EHM) [98] are a little improved. These theoretical approaches empirically include the effect of electron correlation found in conjugated $\pi$-systems. The HF $[25-27,30]$ and SCFMO (self-consistent field with MO) [29] calculations are in agreement with our RHF results. As listed in Table III,
the calculated MP2 bond distances [28] usually decrease by about $0.01 \AA$ when more $d$ functions are added to the basis set, demonstrating the necessity of including polarization functions in calculations with correlation. Based on the differences between the HF and MP2 data, it is evident that electron correlation effects should be considered in an accurate description of the equilibrium structure of a molecule. The CC bond lengths calculated with the LDA [99-101], LDA-PPA [87], and LDAbased CPMD simulation [88,102] are in agreement with our B3LYP's results and demonstrate the importance of electron correlation effects in giving accurately the equilibrium structure of a molecule.

Table III: Equilibrium CC bond lengths ( $1 \AA=0.1 \mathrm{~nm}$ ) of $\mathrm{C}_{60}$ from previous theoretical predictions and experimental findings. STO-DZP and DNP denote double-zeta STO's and double numerical basis with polarization functions, respectively.

| Method | C-C | C=C | Reference |
| :---: | :---: | :---: | :--- |
|  | $[\AA]$ | $[\AA]$ |  |
| Hückel | 1.436 | 1.418 | $[95]$ |
| QCFF/PI | 1.471 | 1.411 | $[96]$ |
| MNDO | 1.465 | 1.376 | $[97]$ |
| EHM | 1.446 | 1.402 | $[98]$ |
| SCFMO | 1.49 | 1.43 | $[29]$ |
| HF/STO-3G | 1.463 | 1.376 | $[25]$ |
| HF/3-21G | 1.453 | 1.367 | $[30]$ |
| HF/DZ | 1.451 | 1.368 | $[26]$ |
| HF/STO-3G | 1.463 | 1.376 | $[27]$ |
| HF/DZ | 1.451 | 1.368 | $[27]$ |
| HF/DZP | 1.450 | 1.375 | $[27]$ |
| HF/TZP | 1.448 | 1.370 | $[27]$ |
| MP2/DZ | 1.470 | 1.407 | $[28]$ |
| MP2/DZP | 1.451 | 1.412 | $[28]$ |
| MP2/TZP | 1.446 | 1.406 | $[28]$ |
| LDA/STO-DZP | 1.436 | 1.384 | $[101]$ |
| LDA/DZP | 1.445 | 1.395 | $[100]$ |
| LDA/DNP | 1.444 | 1.391 | $[99]$ |
| LDA-PPA | 1.449 | 1.390 | $[87]$ |
| LDA-CPMD | 1.45 | 1.40 | $[88]$ |
| LDA-CPMD | 1.45 | 1.39 | $[102]$ |
| Exp./XRCT | 1.4459 | 1.3997 | $[86]$ |
| Exp./XRPD | 1.455 | 1.391 | $[91]$ |
| Exp./NMR | 1.45 | 1.40 | $[92]$ |
| Exp//NMR | 1.46 | 1.40 | $[93]$ |
| Exp./GPED | 1.458 | 1.401 | $[94]$ |

As mentioned before, $\mathrm{C}_{60}$ has only two kinds of bond angles [7], $108^{\circ}$ (the angle between two adjacent single C-C bonds) and $120^{\circ}$ (the angle between a double $\mathrm{C}=\mathrm{C}$ bond and an adjacent single C-C bond). Fig.2(a-d)
show the distribution of (C-C-C, C-N-C, C-C-N) bond angles in $\mathrm{C}_{48} \mathrm{~N}_{12}$ calculated by using both RHF and B3LYP methods with several different basis sets. Onethird and two-thirds of the bond angles in $\mathrm{C}_{48} \mathrm{~N}_{12}$ fluctuate around $108^{\circ}$ and $120^{\circ}$, respectively. Comparing Fig.2(a) and 2(b) shows that increasing the basis size leads to smaller fluctuations in the bond angle distribution (BAD) around either $108^{\circ}$ or $120^{\circ}$. In comparison with Fig.2(b), Fig.2(c) exhibits enhanced fluctuations in the BAD as polarization functions are added to the 631G basis set. Comparing Fig.2(c) with Fig.2(d), we find that the BAD in $\mathrm{C}_{48} \mathrm{~N}_{12}$ are decreased due the effect of electron correlation.

## III. TOTAL ELECTRONIC ENERGY

We performed total energy calculations of $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ by using both RHF and B3LYP with STO-3G, 3$21 \mathrm{G}, 6-31 \mathrm{G}$ and $6-31 \mathrm{G}^{*}$ basis sets. The results are summarized in Table IV. The orbital energies are shown in Fig.3, where the orbital symmetries are also labeled.

Table IV demonstrates the convergence of the total energy calculations of both RHF and B3LYP methods with respect to the basis sets. For both $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$, the total energies calculated with STO-3G, 3-21G and $6-31 \mathrm{G}$ basis sets differ from the $6-31 \mathrm{G}^{*}$ basis set results by about $1.5 \%, 0.6 \%$ and $0.05 \%$, respectively. For both molecules, comparing the B3LYP and RHF results shows that the HOMO-LUMO energy gap $\Delta$ decreases about 5 eV and the binding energy $E_{b}$ per atom increases about 2 eV because of the electron correlations. The calculated binding energy $E_{b}$ per atom and HOMO-LUMO energy gap $\Delta$ of $\mathrm{C}_{48} \mathrm{~N}_{12}$ are about 1 eV smaller than those of $\mathrm{C}_{60}$.

Because of the valency of the doped nitrogen atoms, the electronic properties of $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ are significantly different. As shown in Fig.3(a) and Fig.3(b), the HOMO for $\mathrm{C}_{60}$ is fivefold-degenerate with $\mathrm{h}_{\mathrm{u}}$ symmetry, the LUMO is threefold-degenerate with $\mathrm{t}_{1 \mathrm{u}}$ symmetry, and the others are threefold-degenerate with $t_{1 g}$ or $t_{2 u}$ symmetry, fourfold-degenerate with $g_{g}$ or $g_{u}$ symmetry, and fivefold-degenerate with $h_{g}$ symmetry. We notice from Fig. 3 that each energy level in $\mathrm{C}_{48} \mathrm{~N}_{12}$ splits since the icosahedral symmetry of $\mathrm{C}_{60}$ is lost by the substitutional doping. For $\mathrm{C}_{48} \mathrm{~N}_{12}$, the HOMO is a doubly degenerate level of $\mathrm{a}_{\mathrm{g}}$ symmetry, the LUMO is a nondegenerate level with $a_{u}$ symmetry, and the others are specified in Fig.3. Considering that $\mathrm{C}_{48} \mathrm{~N}_{12}$ is isoelectronic with $\mathrm{C}_{60}^{-12}$, we find that the filling of the energy levels in $\mathrm{C}_{48} \mathrm{~N}_{12}$ corresponds to a complete filling of the $\mathrm{t}_{1 \mathrm{u}}$ and $\mathrm{t}_{1 \mathrm{~g}}$ levels of $\mathrm{C}_{60}$.


FIG.2: $A b$ initio calculations of C-C-C (open circles) and C-N-C (or C-C-N) (filled circles) bond angles in $\mathrm{C}_{48} \mathrm{~N}_{12}$ : (a) B3LYP/STO-3G; (b) B3LYP/6-31G; (c) B3LYP/6-31G*; (d) RHF/6-31G*. The site numbers are labeled as in Fig.1. The solid lines are the C-C-C bond angles ( $108^{\circ}$ and $120^{\circ}$ ) of $\mathrm{C}_{60}$.

Table IV: Total electronic energy ( $E_{t}$, in eV), LUMO energy ( $E_{\text {lumo }}$, in eV ), HOMO energy ( $E_{\text {homo }}$, in eV), HOMO-LUMO energy gap ( $\Delta$, in eV ), binding energy per atom ( $E_{b}$, in eV ), ionization potential ( $E_{I P}$, in eV ) and electron affinity ( $E_{E A}$, in eV ) of $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ calculated by using RHF and B3LYP methods with a variety of Pople-style basis sets.

| Method | Energy | $\mathrm{C}_{60}$ |  |  |  | $\mathrm{C}_{48} \mathrm{~N}_{12}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | STO-3G | 3-21G | 6-31G | 6-31G* | STO-3G | 3-21G | 6-31G | 6-31G* |
| RHF | $E_{t}$ | -61067.132 | -61471.413 | -61795.062 | -61817.394 | -66397.387 | -66846.232 | -67193.956 | -67223.547 |
|  | $E_{\text {lumo }}$ | 2.688 | -0.632 | -0.709 | -0.117 | 3.797 | 0.499 | 0.214 | 0.643 |
|  | Ehomo | -5.456 | -8.323 | -7.919 | -7.644 | -3.287 | -6.506 | -6.203 | -6.189 |
|  | $\Delta$ | 8.144 | 7.691 | 7.210 | 7.527 | 7.084 | 7.005 | 6.417 | 6.832 |
|  | $E_{b}$ | 5.581 | 4.627 | 4.666 | 4.956 | 4.510 | 3.734 | 3.724 | 4.149 |
|  | $E_{I P}$ | 4.892 | 8.230 | 5.685 | 5.659 | 2.45 | 6.144 | 5.314 | 5.530 |
|  | $E_{E A}$ | 2.280 | 0.881 | 1.187 | 1.125 | 1.947 | 0.146 | 0.209 | 0.267 |
| B3LYP | $E_{t}$ | -61446.662 | -61864.864 | -62194.006 | -62209.002 | -66795.835 | -67263.771 | -67617.031 | -67637.724 |
|  | $E_{\text {lumo }}$ | -1.127 | -3.563 | -3.390 | -3.219 | -0.188 | -2.783 | -2.706 | -2.614 |
|  | $E_{\text {homo }}$ | -4.356 | -6.509 | -6.221 | -5.987 | -2.365 | -4.743 | -4.633 | -4.383 |
|  | $\Delta$ | 3.229 | 2.946 | 2.831 | 2.768 | 2.177 | 1.96 | 1.927 | 1.774 |
|  | $E_{b}$ | 7.715 | 6.786 | 6.843 | 6.982 | 6.888 | 6.140 | 6.088 | 6.365 |
|  | $E_{I P}$ | 3.868 | 7.814 | 8.576 | 7.317 | 2.488 | 6.040 | 5.600 | 5.663 |
|  | $E_{E A}$ | 1.487 | 2.295 | 1.073 | 2.398 | 0.804 | 1.514 | 0.223 | 1.493 |

In Table IV, we list the calculated ionization potential (IP) $E_{I P}$ and electron affinity (EA) $E_{E A}$ for both $\mathrm{C}_{60}$ and $\mathrm{C}_{48} \mathrm{~N}_{12}$. It shows that $\mathrm{C}_{48} \mathrm{~N}_{12}$ is a good electron donor, while $\mathrm{C}_{60}$ is a good electron acceptor. The calculated IP for $\mathrm{C}_{60}$ is in good agreement with the experiments: $(7.54 \pm 0.01) \mathrm{eV}[103],(7.57 \pm 0.01) \mathrm{eV}[104]$, (7.58 $8_{-0.02}^{+0.04} \mathrm{eV}$ [105], and $(7.59 \pm 0.02) \mathrm{eV}$ [106]. The calculated EA for $\mathrm{C}_{60}$ agrees well with the experiments: (2.666 $\pm 0.001) \mathrm{eV}[107],(2.689 \pm 0.008) \mathrm{eV}[108]$.


Fig.3: Orbital energies $\mathrm{E}_{k}$ of the $k$ th eigenstate of $\mathrm{C}_{60}$ and $\mathrm{C}_{48} \mathrm{~N}_{12}$ calculated with (a) B3LYP/6$31 \mathrm{G}^{*}$ and (b) RHF/6-31G*. Open circles are for $\mathrm{C}_{60}$. Filled circles and triangles are for $\mathrm{C}_{48} \mathrm{~N}_{12}$, respectively. The orbital symmetries of energy levels are shown.

For comparison, here we list the total energies of $\mathrm{C}_{60}$ for a selection of previous theoretical calculations. Using the HF methods, Scuseria [27] found that the total energies of $\mathrm{C}_{60}$ calculated with DZ, DZP and TZP basis sets $[25-27,30]$ are $-61745.723 \mathrm{eV},-61826.192 \mathrm{eV}$ and -61832.443 eV , respectively. These results are in agreement with our RHF calculations. Moreover, using the MP2 method, Scuseria and colleagues [28] have shown that the total energies of $\mathrm{C}_{60}$ with DZ, DZP and TZP basis sets are $-61893.398 \mathrm{eV},-61826.192 \mathrm{eV}$ and -61832.443 eV , respectively. The total energies calculated by using the MP2 method do improve the approximation made in HF method. This result demonstrates that electron correlation effects cannot be neglected for an accurate prediction of the total energy of a molecule.

In crystals, the on-site Coulomb interaction between two electrons on the same molecule is given by [109]

$$
\begin{align*}
U & =U_{\text {free }}-2 \mathrm{E}_{p}  \tag{2}\\
U_{\text {free }} & =\mathrm{E}_{I P}-\mathrm{E}_{E A}-\Delta, \tag{3}
\end{align*}
$$

where $\mathrm{E}_{p}=z \alpha e^{2} /\left(2 L^{4}\right)(z$ : the number of nearest neighbors, $L$ : the distance between molecules, $\alpha$ : the dipole polarizability) is the polarization energy and $U_{\text {free }}$ is the on-site Coulomb interaction between two electrons on the free molecule [7]. Our plane-wave pseudopotential calculations performed by using the CASTEP program $[73,110]$ show that the optimized lattice constants for $\mathrm{C}_{60}$ and $\mathrm{C}_{48} \mathrm{~N}_{12}$-based fcc solids (note: $z=12$ for both cases) are all around 1.45 nm , while the experimental lattice constant $=1.4161 \mathrm{~nm}$ for $\mathrm{C}_{60}$ [111]. Based on our first principles results for $E_{I P}, E_{E A}$ and $\Delta$ for $\mathrm{C}_{60}$ and $\mathrm{C}_{48} \mathrm{~N}_{12}$ molecules, we arrive at the value of the onsite Coulomb interaction, $\mathrm{U}=1.84,2.05 \mathrm{eV}$ for $\mathrm{C}_{60}$ and $\mathrm{C}_{48} \mathrm{~N}_{12}$ solids, respectively. The experimental values for $\mathrm{C}_{60}$ are $\mathrm{U}=1.6 \pm 0.2 \mathrm{eV}$ [109] and 1.54 eV [112]. Moreover, our DFT/GGA calculations show that the band gap $E_{\text {gap }}$ for the $\mathrm{C}_{48} \mathrm{~N}_{12}$ solid is about 0.85 eV smaller than that of $\mathrm{C}_{60}$. This is consistent with the $\Delta$-U-W relation [7]

$$
\begin{equation*}
E_{g a p}=\Delta+U-W \tag{4}
\end{equation*}
$$

where $W$ is the bandwidth for the HOMO- or LUMOderived energy bands. Assuming the same $W$ for both $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ solids, we arrive at

$$
\begin{equation*}
E_{g a p}^{C_{48} N_{12}} \approx E_{\text {gap }}^{C_{60}}-0.79 \mathrm{eV} \tag{5}
\end{equation*}
$$

The DFT is known to underestimate the band gap of solids. From experiment, $E_{\text {gap }}=2.3 \pm 0.1 \mathrm{eV}$ [109] and 2.86 eV [112] for the $\mathrm{C}_{60}$ solid. Hence, using the approximate relation given by Eq.(5), we obtain an estimated band gap for the $\mathrm{C}_{48} \mathrm{~N}_{12}$ fcc solid of $E_{g a p}^{C_{48} N_{12}}=1.7 \pm 0.3$ eV . Thus, the $\mathrm{C}_{48} \mathrm{~N}_{12}$ solid, like $\mathrm{C}_{60}$, is a semiconducting material.

Similarly, we find that the $\mathrm{C}_{48} \mathrm{~B}_{12}$-based fcc solid (lattice constant $\approx 1.45 \mathrm{~nm}$ ) is also a semiconducting material [90], having $U=1.9 \mathrm{eV}, E_{\text {gap }}^{C_{48} B_{12}}-E_{\text {gap }}^{C_{60}}=-1.34$ eV , and $E_{\text {gap }}^{C_{48} B_{12}}=1.2 \pm 0.3 \mathrm{eV}$.

## IV. STATIC (HYPER)POLARIZABILITY

The static polarizabilities for $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ are presented in Table V. The B3LYP and RHF results are obtained by using the Gaussian 98 program package $[72,73]$ and the LDA results by using the ADF (Amsterdam Density Functional) program $[73,113,114]$. The ADF program uses basis sets of Slater functions, where a triple zeta valence basis plus polarization is augmented with the field-induced polarization (FIP) functions of Zeiss et al. [115]. Here this basis set is denoted as TZP ++ ([6s4p2d1f] for C and N atoms, and $[4 \mathrm{~s} 2 \mathrm{p} 1 \mathrm{~d}]$ for H atom)
and has recently been used for calculating the secondorder hyperpolarizabilities, $\gamma$, for $\mathrm{C}_{60}, \mathrm{C}_{58} \mathrm{~N}_{2}, \mathrm{C}_{58} \mathrm{~B}_{2}$ and $\mathrm{C}_{58} \mathrm{BN}$ [116].

From the results listed in Table V, we see that the basis set dependence is identical for both B3LYP and RHF cases. As expected, improving the basis set increases the polarizability. Our RHF results for $\mathrm{C}_{60}$ are in good agreement with a previous study [117] which used a similar method and basis sets. The B3LYP values are about $10 \%$ larger than the corresponding RHF values. The LDA results are much larger than both the B3LYP and RHF results. This is expected since the basis set is larger and expected to predict a more accurate polarizability
[116]. Also LDA, in general, predicts a larger polarizability than does with B3LYP [118]. The LDA results for $\mathrm{C}_{60}$ are in good agreement with previous LDA studies $[119-121]$. We find that the polarizability of $\mathrm{C}_{48} \mathrm{~N}_{12}$ is slightly smaller than the polarizability of $\mathrm{C}_{60}$. For $\mathrm{C}_{48} \mathrm{~N}_{12}$ we also find that the $z z$ component is slightly larger than the $x x$ component except when using RHF and a small basis set. In comparison with the SDP of a single carbon or nitrogen atom, we find an enhanced linear polarizability for both $\mathrm{C}_{60}$ and $\mathrm{C}_{48} \mathrm{~N}_{12}$. This is expected since both molecules have many conjugate $\pi$ electrons delocalized over the entire system.

Table V: Static polarizabilities ( $\alpha$, in $\AA^{3}=10^{-30} \mathrm{~m}^{3}$ ) for $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ with RHF and DFT methods and a variety of basis sets. Symmetry relations give for $\mathrm{C}_{60}, \alpha_{x x}=\alpha_{y y}=\alpha_{z z}$, and for $\mathrm{C}_{48} \mathrm{~N}_{12}, \alpha_{x x}=\alpha_{y y}$.

|  | STO-3G |  | 3-21G |  | 6-31G |  | 6-31G* |  | $\frac{\mathrm{TZP}++}{\mathrm{LDA}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | B3LYP | RHF | B3LYP | RHF | B3LYP | RHF | B3LYP | RHF |  |
| $\overline{\mathrm{C}_{60}}$ |  |  |  |  |  |  |  |  |  |
| $\alpha_{x x}$ | 52.5 | 45.2 | 64.9 | 59.5 | 68.3 | 63.2 | 69.5 | 64.7 | 84.7 |
| $\mathrm{C}_{48} \mathrm{~N}_{12}$ |  |  |  |  |  |  |  |  |  |
| $\alpha_{x x}$ | 51.5 | 43.5 | 62.0 | 55.6 | 65.6 | 59.4 | 66.6 | 60.0 | 79.3 |
| $\alpha_{z z}$ | 51.6 | 41.6 | 62.6 | 54.9 | 66.1 | 58.5 | 67.5 | 60.4 | 81.5 |

Table VI: Static 2nd-order hyperpolarizabilities ( $\gamma$, in au. 1 au $=6.235378 \times 10^{-65} \mathrm{C}^{4} \mathrm{~m}^{4} \mathrm{~J}^{-3}$ ) for $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ calculated using LDA and TZP ++ . The average 2 nd-order hyperpolarizability is given by $\bar{\gamma}=\frac{1}{15} \sum_{i, j}\left(\gamma_{i i j j}+\gamma_{i j i j}+\right.$ $\underline{\left.\underline{\gamma_{i j j i}}\right) \text {. Symmetry relations give } \gamma_{x x x x}=\gamma_{y y y y}, \gamma_{x x z z}=\gamma_{y y z z} \text { and } \gamma_{z z x x}=\gamma_{z z y y} \text {. }}$

|  | $\gamma_{x x x x}$ | $\gamma_{x x y y}$ | $\gamma_{z z z z}$ | $\gamma_{x x z z}$ | $\gamma_{z z x x}$ | $\bar{\gamma}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{C}_{60}$ | 137950 | 45983 | 137950 | 45983 | 45983 | 137950 |
| $\mathrm{C}_{48} \mathrm{~N}_{12}$ | 188780 | 62880 | 232970 | 85120 | 84790 | 215222 |

Because $\mathrm{C}_{60}$ is well separated in a crystal [7], one can model the fullerene crystal in an electric field as a collection of isolated dipoles. For such systems, the ClausiusMossotti relation [122] is expected to yield a reasonably accurate relationship between the linear polarizability of an isolated molecule and the dielectric constant of a fullerene crystal, i.e.,

$$
\begin{equation*}
|\alpha|=\frac{3(\epsilon-1)}{4 \pi \rho_{f}(\epsilon+2)} \tag{6}
\end{equation*}
$$

where $\rho_{f}$ is the density of fullerene molecules in a fcc crystal and $\epsilon$ is the dielectric constant of the fullerene crystal. For the purposes of comparison, we note that Hebard et al. [123] and Ecklund [124] measured the dielectric constants of $\mathrm{C}_{60}$ films in the range between 3.9 and 4.0. Based on these measured dielectric constants, one can derive an experimental polarizability of about $84.9 \times 10^{-30} \mathrm{~m}^{3}$ for $\mathrm{C}_{60}$ (the density $\rho_{f}=10^{30} / 712 \mathrm{~m}^{-3}$ is given by Quong and Pederson [125]).Very recently, using the molecular beam deflection technique, Antoine et
al. [126] have measured the electric polarizability of isolated $\mathrm{C}_{60}$ molecules and obtained a value of $(76.5 \pm 8.0) \times$ $10^{-30} \mathrm{~m}^{3}$. Using a new optical technique that uses light forces and a time-of-flight spectrometer, Ballard et al. [127] have made absolute measurements of cluster polarizabilities and determined the optical polarizability of $\mathrm{C}_{60}$ at the fundamental wavelength of a Nd:YAG laser $(\lambda=1064 \mathrm{~nm})$ to be $(79 \pm 4) \times 10^{-30} \mathrm{~m}^{3}$. The experimental results are in good agreement with our LDA results, especially considering that LDA is expected to overestimate the polarizability.

The static first-order hyperpolarizability $\beta$ [36] of the $\mathrm{C}_{48} \mathrm{~N}_{12}$ molecule is also calculated and found to be zero, the same as that for $\mathrm{C}_{60}$ molecule. This is expected since both $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ molecule display inversion symmetries. Consequently, this aza-fullerene cannot produce second-order nonlinear optical interactions.

The static 2nd-order hyperpolarizabilities for $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ molecules are presented in Table VI. For the calculations of the 2nd-order hyperpolarizability,
$\gamma$, we use time-dependent (TD) DFT as described in Ref. [116,120,128,129]. First, the 1st-order hyperpolarizability, $\beta$, is calculated analytically in the presence of a small electric field. Then, the 2nd-order hyperpolarizability can be obtained by a finite-field differentiation of the analytically calculated 1st-order hyperpolarizability. For all the TD-DFT calculations we used the RESPONSE code $[73,128,130]$ implemented in the ADF program $[73,113,114]$. The small difference between $\gamma_{x x z z}$ and $\gamma_{z z x x}$ for $\mathrm{C}_{48} \mathrm{~N}_{12}$ is due to the numerical method adopted for calculating $\gamma$.

For the $\gamma$ value of $\mathrm{C}_{60}$, we find good agreement with previous first-principles results [120,121,131]. A comparison with experiments will not be made for the 2nd-order hyperpolarizability due to large differences in the experimental results $[9,10,120,131]$. We find that all components of the 2nd-order hyperpolarizability for $\mathrm{C}_{48} \mathrm{~N}_{12}$ are larger than for $\mathrm{C}_{60}$. This gives an average 2ndorder hyperpolarizability of $\mathrm{C}_{48} \mathrm{~N}_{12}$ which is about 55 \% larger than the average 2nd-order hyperpolarizability of $\mathrm{C}_{60}$. The $z z z z$ components of the 2nd-order hyperpolarizaility of $\mathrm{C}_{48} \mathrm{~N}_{12}$ is also larger than that found in the donor/acceptor substituted $\mathrm{C}_{58} \mathrm{BN}$ molecule [116].

## V. VIBRATIONAL FREQUENCY ANALYSIS

## A. Theory

To lowest order, IR intensities are proportional to the derivatives of the dipole moment with respect to the vibrational normal modes of the material, evaluated at the equilibrium geometry. In detail, the IR intensity of the $q$ th vibrational mode is given by [132]

$$
\begin{equation*}
I_{I R}^{(q)}=\frac{\rho_{p} \pi}{3 c}\left|\frac{d \mathbf{P}}{d \Xi_{q}}\right|^{2} \tag{7}
\end{equation*}
$$

where $\rho_{p}$ is the particle density, $\Xi_{q}$ is the normal coordinate cooresponding to the $q$ th mode and $c$ is the velocity of light. Since $\left|d \mathbf{P} / d \Xi_{q}\right|$ is the only molecular property entering the formula, it is often referred to as absolute IR intensity.

To obtain the IR data, one must compute the derivatives of the dipole moment with respect to the normal mode coordinates. These can be viewed as directional derivatives in the space of 3 N nuclear coordinates and expressed in terms of derivatives with respect to atomic coordinates, $R_{k}$. For the $i$ th component of the dipole moment $\mathbf{P}(\mathrm{i}=\mathrm{x}, \mathrm{y}, \mathrm{z})$, we have

$$
\begin{equation*}
\frac{d \mathbf{P}_{i}}{d \Xi_{q}}=\sum_{k=1}^{3 N} \frac{\partial \mathbf{P}_{i}}{\partial R_{k}} \xi_{k q} \tag{8}
\end{equation*}
$$

where $\xi_{k q}=\partial R_{k} / \partial \Xi_{q}$ is the $k$ th atomic displacement of the $q$ th normal mode. Then, the necessary derivatives can be expressed in terms of the atomic forces as follows [132,133]

$$
\begin{equation*}
\frac{\partial \mathbf{P}_{i}}{\partial R_{k}}=-\frac{\partial^{2} E}{\partial G_{i} \partial R_{k}}=\frac{\partial F_{k}}{\partial G_{i}} \tag{9}
\end{equation*}
$$

where $E$ is the total energy, $G_{i}$ is the $i$ th component of an assumed external electric field $\mathbf{G}$, and $F_{k}$ is the calculated force on the $k$ th atomic coordinate.

## B. Normal Vibrations in $\mathrm{C}_{60}$ and $\mathrm{C}_{48} \mathrm{~N}_{12}$

Using the Gaussian 98 program [72,73], we calculated the harmonic vibrational frequencies of both $\mathrm{C}_{60}$ and $\mathrm{C}_{48} \mathrm{~N}_{12}$ and considered the effects of the basis sets. It should be mentioned that our frequencies have not been scaled.

Table VII and VIII summarize the vibrational frequencies for $\mathrm{C}_{60}$ calculated by using RHF and B3LYP methods, respectively. As shown by Dresselhaus et al. [134], there are 46 vibrational modes for $\mathrm{C}_{60}$. These modes are classified in even and odd parities and in ten irreducible representatives of the $I_{h}$ point group: the $\left\{a_{g}, a_{u}\right\},\left\{t_{1 g}\right.$, $\left.t_{1 u}, t_{2 g}, t_{2 u}\right\},\left\{g_{g}, g_{u}\right\}$ and $\left\{h_{g}, h_{u}\right\}$ modes are non-, threefold-, fourfold- and fivefold-degenerate, respectively. Tables VII and VIII demonstrate that increasing the basis size improves the accuracy of the predicted vibrational frequencies, but adding polarization functions to the 6 31 G basis set only improves slightly the accuracy of the vibrational frequencies. In choosing a basis set for the first-principles calculation, one must make a compromise between accuracy and CPU time. Our results shows that the minimum calculation can be done in about 4 hours of CPU, while the most expensive calculation requires 12 days of CPU. Without significant computational cost, one can do B3LYP/STO-3G calculation and still obtain results more accurate than any RHF calculations. Going beyond STO-3G for B3LYP calculations requires a drastic increase in CPU time. Suprisingly, going just to $3-21 \mathrm{G}$ provides the most accurate results, while for the bigger basis set $6-31 \mathrm{G}$, the results are worse and adding a polarized function to $6-31 \mathrm{G}$ only slightly improves the results. The $3-21 \mathrm{G}$ basis set gives systematically lower frequencies than the $6-31 \mathrm{G}$ basis set, while the frequencies obtained from the $6-31 \mathrm{G}^{*}$ basis set typically lie between the results of the other two basis sets. In contrast, as discussed in section II, $6-31 \mathrm{G}^{*}$ does provide the most accurate bond lengths. This suggests that the better accuracy of $3-21 \mathrm{G}$ is foirtuitous. Increasing the basis set to $6-31 \mathrm{G}$ stiffens the bonds, while adding the polarization function compensates by softening the bonds. In comparison with the B3LYP results, RHF calculated frequencies are too high due to an incorrect description of bond dissociation, while B3LYP with large basis sets (even the minimum basis set STO-3G) generally gives results in good agreement with the experiments of Wang et al. [42] and Dong et al. [47]. This demonstrates the importance of electron correlation in an accurate description of the vibrational frequencies.

For comparison, Table VII lists the vibrational frequencies of $\mathrm{C}_{60}$ calculated by using various theories, for example, the semi-empirical MNDO [97] and QCFF/PI [96] methods. Of these, the QCFF/PI method, which has been parameterized mainly with respect to vibrational frequencies of conjugated and aromatic hydrocarbons [135], results in the best results although it gives less satisfactory geometry. Such accurate prediction implies that the electronic structures of $\mathrm{C}_{60}$ is not much different from other aromatic hydrocarbons [96]. Häser et al. [28] showed that the approximate harmonic frequencies for the two $a_{g}$ vibrational modes of $\mathrm{C}_{60}$ are $1615 \mathrm{~cm}^{-1}$ and $487 \mathrm{~cm}^{-1}$ for HF/DZP, $1614 \mathrm{~cm}^{-1}$ and $483 \mathrm{~cm}^{-1}$ for HF/TZP, $1614 \mathrm{~cm}^{-1}$ and $437 \mathrm{~cm}^{-1}$ for MP2/DZP, and $1586 \mathrm{~cm}^{-1}$ and $437 \mathrm{~cm}^{-1}$ for MP2/TZP. Their HF calculations are in agreement with our RHF/3-21G results. Their MP2 results are more accurate when obtained with large basis sets, which also demonstrates the importance of electron correlation in predicting the vibrational frequencies.

In addition, there have been a number of 2 nd nearestneighbor force-constant models (FCMs) [136-138] which have been used to calculate the phonon frequencies of $\mathrm{C}_{60}$. None of them yield good agreement with the experimental data. For example, an empirical force field, which has been parameterized with respect to polycyclic aromatic hydrocarbons, is used with Hückel theory and predicts vibrational frequencies of the two $a_{g}$ modes of $1409 \mathrm{~cm}^{-1}$ and $388 \mathrm{~cm}^{-1}$ [137] that are too low. However, the modified FCM (MFCM) by Jishi et al. [139] considered interactions up to the third-nearest neighbors, and the calculated results, as shown in Table VII, are in excellent agreement with the experiments of Wang et al. [42] and Dong et al. [47].

Table VIII also lists the vibrational frequencies of $\mathrm{C}_{60}$ calculated by other DFT methods, for example, LDAPPA [87], LDA [100,101] and DFT-LDA-based CPMD simulations [102]. In general, those calculated results are in good agreement with experiment. Very recently, Choi et al. [141] have performed B3LYP vibrational calculations of $\mathrm{C}_{60}$ with a $3-21 \mathrm{G}$ basis set but involving scaling of the internal force constants (SIFC) $\tilde{K}_{i j}^{i n t}$ by using Pulay's method [142], i.e.,

$$
\begin{equation*}
\tilde{K}_{i j}^{\text {scaled }}=\left(s_{i} s_{j}\right)^{1 / 2} \tilde{K}_{i j}^{i n t} \tag{10}
\end{equation*}
$$

where $\tilde{K}_{i j}^{i n t}$ is the force constant in internal coordinates ( the Gaussian 98 program [72] uses this form), and $s_{i}$ and $s_{j}$ are scaling factors for the $i$ th and $j$ th redundant internal coordinates, respectively. They optimized the scaling factors by minimizing the root-mean-square deviations between the experimental and calculated scaled frequencies. Their results are listed in Table IX. Overall, their scaling procedure improves the accuracy for the 46 vibrational frequencies of $\mathrm{C}_{60}$, especially, for the $a_{g}, h_{g}$ and $t_{1 u}$ vibrational modes.

In Table X and XI, we list the vibrational frequencies for $\mathrm{C}_{48} \mathrm{~N}_{12}$ calculated with RHF and B3LYP methods
and a variety of Pople-style basis sets. In contrast with $\mathrm{C}_{60}$, it is found that there are in total 116 vibrational modes for $\mathrm{C}_{48} \mathrm{~N}_{12}$ because of its lowered symmetry, $S_{6}$. These vibrational modes are classified into 58 doublydegenerate and 58 nondegenerate modes. Among those vibrational modes, there are 58 IR-active (listed in Table X) and 58 Raman-active modes (listed in Table XI). Table X and XI show that the electron correlation or increasing the basis size results in a redshift of the vibrational frequencies. This is similar to that of $\mathrm{C}_{60}$.

## C. IR Intensities in $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$

We perform calculations of IR intensities $I_{I R}$ for both $\mathrm{C}_{48} \mathrm{~N}_{12}$ and $\mathrm{C}_{60}$ by using the Gaussian 98 program [72,73] with RHF and B3LYP methods. The calculated IR intensities for $\mathrm{C}_{60}$ at the corresponding frequencies are listed in Table XII, and those for $\mathrm{C}_{48} \mathrm{~N}_{12}$ are shown in Fig.4.

For $\mathrm{C}_{60}$, we note that its IR spectrum is very simple. Namely, it is composed of 4 IR-active vibrational modes with $t_{1 u}$ symmetry. This is a consequence of the symmetry of the icosahedral group [134]. Carbon clusters of comparable size, but lower symmetry, have many more IR-active frequencies. For example, the graphitene isomer of $\mathrm{C}_{60}$ has $D_{6 h}$ symmetry and 20 IR-active frequencies [97]. Other examples [97] included $\mathrm{C}_{54}$, a planar graphite fragment with $D_{6 h}$ symmetry and 22 IR-active frequencies, and $\mathrm{C}_{50}$, a spheroidal cluster with $D_{5 h}$ symmetry and 22 IR-active frequencies. From Table XII, we see that the IR intensity of a given mode decreases due to the electron correlation and converges with increasing basis size. We find that our intensities calculated with B3LYP agree reasonably with experimental spectrum [140] obtained by in situ high-resolution FTIR measurement of a $\mathrm{C}_{60}$ film.

The IR intensities $I_{I R}$ at the corresponding vibrational frequencies for $\mathrm{C}_{48} \mathrm{~N}_{12}$ are presented in Fig.4(a)(b)(c) for calculations with B3LYP/STO-3G, B3LYP/3-21G and RHF/3-21G, respectively. As discussed above, $\mathrm{C}_{48} \mathrm{~N}_{12}$ has 29 nondegenerate and 29 doubly-degenerate IRactive vibrational modes. Similar to the case of $\mathrm{C}_{60}$, the IR signals separate into two regions, i.e., a high-frequency ( $>1000 \mathrm{~cm}^{-1}$ ) and a low-frequency $\left(\leq 1000 \mathrm{~cm}^{-1}\right)$ region. The IR-active frequencies are redshifted by including the electron correlation or increasing the basis size. The IR intensities decrease after including electron correlations and converge with increasing basis size. The strongest IR spectral lines in both low- and highfrequency regions are the doubly-degenerate modes located, for example, at $440 \mathrm{~cm}^{-1}$ and $1310 \mathrm{~cm}^{-1}$, respectively, for the B3LYP/3-21G case. Since experimental IR spectroscopic data do not directly indicate the specific type of nuclear motion producing each IR peak, we do not give here the normal mode information for each vibrational frequency and the displacements of each nuclei corresponding to each normal mode. In Fig.5, taking

B3LYP/3-21G calculations as an example, we only show the vibrational displacements of sites 1 to 5 ( 4 C sites and 1 N site) for the strongest IR spectral signals in the low-
and high-frequency regions. It is seen that the pentagon structure for the low-frequency case expands slightly and

Table VII: Vibrational frequencies ( $\nu$, in $\mathrm{cm}^{-1}$ ) of $\mathrm{C}_{60}$ calculated with RHF and a variety of Pople-style basis sets. Numbers in the parenthesis are the relative errors to the experimental frequencies, $\nu^{e x p}$, from Wang et al. [42] and Dong et al. [47]. The approximated CPU times for STO-3G, 3-21G, $6-31 \mathrm{G}$ and $6-31 \mathrm{G}^{*}$ basis sets are 5 hours, 12 hours, 18 hours and 39 hours, respectively. Results of other theoretical calculations, for example, QCFF/PI [96], MNDO [97] and MFCM [139], are also listed.

| Mode | STO-3G | $3-21 \mathrm{G}$ | 6-31G | $6-31 \mathrm{G}^{*}$ | QCFF/PI | MFCM | MNDO | Exp. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Even Parity |  |  |  |  |  |  |  |  |
| $a_{g}$ | 1684 (14.5\%) | 1604 (9.1\%) | 1637 (11.3\%) | 1600 (8.9\%) | 1442 (1.9\%) | 1468 (0.1\%) | 1667 (13.4\%) | 1470 |
|  | 553 (11.0\%) | 518 (4.0\%) | 526 (5.7\%) | 527 (5.7\%) | 513 (3.0\%) | 492 (1.2\%) | 610 (22.5\%) | 498 |
| $g_{g}$ | 1802 (18.2\%) | 1667 (9.3\%) | 1697 (11.3\%) | 1687 (10.6\%) | 1585 (3.9\%) | 1521 (0.3\%) | 1650 (8.2\%) | 1525 |
|  | 1531 (12.9\%) | 1426 (5.1\%) | 1450 (6.9\%) | 1441 (6.3\%) | 1450 (6.9\%) | 1375 (1.4\%) | 1404 (3.5\%) | 1356 |
|  | 1203 (11.8\%) | 1095 (1.8\%) | 1142 (6.2\%) | 1132 (5.2\%) | 1158 (7.6\%) | 1056 (1.9\%) | 1235 (14.8\%) | 1076 |
|  | 908 (12.6\%) | 787 (2.3\%) | 878 (9.0\%) | 836 (3.8\%) | 770 (4.5\%) | 805 (0.1\%) | 856 (6.2\%) | 806 |
|  | 653 (5.2\%) | 646 (4.1\%) | 643 (3.5\%) | 619 (0.3\%) | 614 (1.1\%) | 626 (0.8\%) | 579 (6.9\%) | 621 |
|  | 574 (18.0\%) | 529 (9.0\%) | 549 (12.9\%) | 537 (10.5\%) | 476 (2.1\%) | 498 (2.5\%) | 491 (1.0\%) | 486 |
| $h_{g}$ | 1912 (21.2\%) | 1772 (12.3\%) | 1799 (14.0\%) | 1791 (13.5\%) | 1644 (4.2\%) | 1575 (0.2\%) | 1722 (9.1\%) | 1578 |
|  | 1658 (16.2\%) | 1546 (8.4\%) | 1585 (11.1\%) | 1562 (9.4\%) | 1465 (2.7\%) | 1401 (1.8\%) | 1596 (11.8\%) | 1427 |
|  | 1482 (18.4\%) | 1326 (6.0\%) | 1377 (10.1\%) | 1380 (10.3\%) | 1265 (1.1\%) | 1217 (2.7\%) | 1407 (12.5\%) | 1251 |
|  | 1290 (17.2\%) | 1184 (7.5\%) | 1208 (9.8\%) | 1208 (9.7\%) | 1154 (4.8\%) | 1102 (0.1\%) | 1261 (14.5\%) | 1101 |
|  | 886 (14.3\%) | 828 (6.9\%) | 843 (8.6\%) | 840 (8.4\%) | 801 (3.4\%) | 788 (1.7\%) | 924 (19.2\%) | 775 |
|  | 836 (17.6\%) | 761 (7.1\%) | 821 (15.4\%) | 794 (11.7\%) | 691 (2.8\%) | 708 (0.4\%) | 771 (8.4\%) | 711 |
|  | 509 (17.5\%) | 475 (9.7\%) | 496 (14.5\%) | 482 (11.4\%) | 440 (1.6\%) | 439 (1.4\%) | 447 (3.2\%) | 433 |
|  | 302 (10.7\%) | 295 (8.0\%) | 296 (8.3\%) | 289 (5.9\%) | 258 (5.5\%) | 269 (1.5\%) | 263 (3.7\%) | 273 |
| $t_{1 g}$ | 1505 (10.9\%) | 1371 (0.9\%) | 1403 (3.3\%) | 1404 (3.4\%) | 1398 (2.9\%) | 1346 (0.9\%) | 1410 (3.8\%) | 1358 |
|  | 966 (1.0\%) | 940 (3.7\%) | 939 (3.8\%) | 916 (6.1\%) | 975 (0.1\%) | 981 (0.5\%) | 865 (11.4\%) | 976 |
|  | 687 (36.9\%) | 618 (23.1\%) | 670 (33.4\%) | 640 (27.5\%) | 597 (18.9\%) | 501 (0.2\%) | 627 (24.9\%) | 502 |
| $t_{2 g}$ | 1619 (19.1\%) | 1453 (6.9\%) | 1504 (10.6\%) | 1511 (11.1\%) | 1470 (8.1\%) | 1351 (0.7\%) | 1483 (9.0\%) | 1360 |
|  | 912 (0.2\%) | 907 (0.8\%) | 898 (1.8\%) | 868 (5.0\%) | 890 (2.6\%) | 931 (1.8\%) | 919 (0.5\%) | 914 |
|  | 903 (4.3\%) | 701 (19.0\%) | 828 (4.3\%) | 734 (15.1\%) | 834 (3.6\%) | 847 (2.1\%) | 784 (9.4\%) | 865 |
|  | 647 (14.1\%) | 637 (12.3\%) | 634 (11.7\%) | 613 (8.1\%) | 637 (12.3\%) | 541 (4.6\%) | 591 (4.2\%) | 567 |
| Odd Parity |  |  |  |  |  |  |  |  |
| $a_{u}$ | 1111 (2.8\%) | 1112 (2.5\%) | 1097 (4.1\%) | 1061 (7.2\%) | 1206 (5.5\%) | 1142 (0.1\%) | 972 (15.0\%) | 1143 |
| $g_{u}$ | 1701 (17.7\%) | 1562 (8.0\%) | 1607 (11.1\%) | 1597 (10.4\%) | 1546 (6.9\%) | 1413 (2.3\%) | 1587 (9.8\%) | 1446 |
|  | 1527 (16.6\%) | 1406 (7.3\%) | 1447 (10.5\%) | 1437 (9.7\%) | 1401 (6.9\%) | 1327 (1.3\%) | 1436 (9.6\%) | 1310 |
|  | 1113 (14.7\%) | 1031 (6.3\%) | 1052 (8.4\%) | 1050 (8.3\%) | 1007 (3.8\%) | 961 (0.9\%) | 1110 (4.1\%) | 970 |
|  | 922 (0.2\%) | 859 (7.0\%) | 864 (6.5\%) | 826 (10.6\%) | 832 (10.0\%) | 929 (0.5\%) | 914 (1.1\%) | 924 |
|  | 863 (13.5\%) | 738 (2.9\%) | 846 (11.4\%) | 786 (3.4\%) | 816 (7.4\%) | 789 (3.8\%) | 750 (1.3\%) | 760 |
|  | 414 (3.5\%) | 390 (2.4\%) | 400 (0.1\%) | 390 (2.4\%) | 358 (10.5\%) | 385 (3.8\%) | 362 (9.5\%) | 400 |
| $h_{u}$ | 1905 (22.2\%) | 1762 (13.0\%) | 1791 (14.9\%) | 1784 (14.5\%) | 1646 (5.6\%) | 1552 (0.4\%) | 1709 (9.6\%) | 1559 |
|  | 1597 (15.3\%) | 1447 (4.5\%) | 1487 (7.4\%) | 1491 (7.7\%) | 1469 (6.1\%) | 1385 (0.0\%) | 1467 (5.9\%) | 1385 |
|  | 1453 (30.0\%) | 1320 (18.2\%) | 1353 (21.2\%) | 1354 (21.2\%) | 1269 (13.6\%) | 1129 (1.1\%) | 1344 (20.3\%) | 1117 |
|  | 886 (10.5\%) | 793 (1.0\%) | 858 (7.1\%) | 824 (2.8\%) | 812 (1.4\%) | 801 (0.0\%) | 822 (2.6\%) | 801 |
|  | 777 (11.7\%) | 753 (8.2\%) | 761 (9.3\%) | 738 (6.0\%) | 724 (4.0\%) | 700 (0.6\%) | 706 (1.4\%) | 696 |
|  | 640 (13.6\%) | 587 (4.2\%) | 613 (8.9\%) | 592 (5.2\%) | 531 (5.7\%) | 543 (3.6\%) | 546 (3.0\%) | 563 |
|  | 463 (35.0\%) | 455 (32.7\%) | 454 (32.4\%) | 442 (28.7\%) | 403 (17.5\%) | 361 (5.0\%) | 403 (17.5\%) | 343 |
| $t_{1 u}$ | 1637 (14.5\%) | 1553 (8.6\%) | 1587 (11.0\%) | 1549 (8.4\%) | 1437 (0.6\%) | 1450 (1.5\%) | 1628 (13.9\%) | 1429 |
|  | 1396 (18.0\%) | 1245 (5.2\%) | 1287 (8.8\%) | 1297 (9.6\%) | 1212 (2.5\%) | 1208 (2.1\%) | 1353 (14.4\%) | 1183 |
|  | 656 (13.9\%) | 614 (6.5\%) | 623 (8.1\%) | 625 (8.5\%) | 637 (10.6\%) | 589 (2.3\%) | 719 (24.8\%) | 576 |
|  | 627 (18.9\%) | 575 (9.1\%) | 621 (17.8\%) | 599 (13.6\%) | 544 (3.2\%) | 505 (4.2\%) | 577 (9.5\%) | 527 |
| $t_{2 u}$ | 1828 (15.9\%) | 1709 (8.4\%) | 1728 (9.5\%) | 1713 (8.6\%) | 1558 (1.2\%) | 1575 (0.1\%) | 1687 (7.0\%) | 1577 |
|  | 1327 (29.3\%) | 1214 (1.1\%) | 1259 (4.5\%) | 1257 (4.7\%) | 1241 (3.3\%) | 1212 (0.9\%) | 1314 (9.4\%) | 1201 |
|  | 1074 (4.7\%) | 995 (3.1\%) | 1025 (0.1\%) | 1014 (1.2\%) | 999 (2.6\%) | 1025 (0.1\%) | 1134 (10.5\%) | 1026 |
|  | 835 (22.8\%) | 765 (12.6\%) | 830 (22.1\%) | 799 (17.5\%) | 690 (1.5\%) | 677 (0.4\%) | 776 (14.1\%) | 680 |
|  | 393 (10.5\%) | 377 (5.9\%) | 385 (8.0\%) | 372 (4.5\%) | 350 (1.7\%) | 367 (3.1\%) | 348 (2.2\%) | 356 |

Table VIII: Vibrational frequencies $\nu\left(\mathrm{cm}^{-1}\right)$ of $\mathrm{C}_{60}$ calculated by using the B3LYP method with a variety of Pople-style basis sets. Numbers in the parenthesis are the relative errors of the calculated frequencies to the experimental frequencies listed in Table VII. The approximated CPU times for STO-3G, 3-21G, $6-31 \mathrm{G}$ and $6-31 \mathrm{G}^{*}$ basis sets are 4 hours, 5 days, 8 days and 12 days, respectively. The other theoretical results are from Bohnen et al. [87], Dixon et al. [100], Hara et al. [101], and Onida et al. [102].

| Mode | STO-3G | 3-21G | 6-31G | 6-31G* | Hara | Dixon | Onida | Bohnen |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Even Parity |  |  |  |  |  |  |  |  |
| $a_{g}$ | 1549 (5.4\%) | 1501 (2.1\%) | 1524 (3.7\%) | 1504 (2.3\%) | 1531 (4.1\%) | 1525 (3.7\%) | 1447 (1.6\%) | 1475 (0.3\%) |
|  | 502 (0.9\%) | 491 (1.4\%) | 496 (0.4\%) | 489 (1.8\%) | 502 (0.8\%) | 499 (0.2\%) | 482 (3.2\%) | 481 (3.4\%) |
| $g_{g}$ | 1594 (4.5\%) | 1524 (0.1\%) | 1546 (1.4\%) | 1538 (0.9\%) | 1538 (0.9\%) | 1548 (1.5\%) | 1479 (3.0\%) | 1501 (1.6\%) |
|  | 1380 (1.8\%) | 1323 (2.4\%) | 1342 (1.0\%) | 1334 (1.6\%) | 1337 (1.4\%) | 1347 (0.7\%) | 1314 (3.1\%) | 1287 (5.1\%) |
|  | 1127 (4.7\%) | 1062 (1.3\%) | 1099 (2.1\%) | 1093 (1.6\%) | 1123 4.4(\%) | 1122 (4.3\%) | 1047 (2.7\%) | 1037 (3.6\%) |
|  | 788 (2.2\%) | 690 (14.4\%) | 777 (3.7\%) | 754 (6.5\%) | 759 (5.8\%) | 788 (2.2\%) | 781 (3.1\%) | 772 (4.2\%) |
|  | 592 (4.6\%) | 598 (3.7\%) | 594 (4.3\%) | 577 (7.1\%) | 579 (6.8\%) | 573 (7.7\%) | 594 (4.3\%) | 570 (8.2\%) |
|  | 508 (4.6\%) | 484 (0.4\%) | 500 (2.9\%) | 489 (0.7\%) | 486 (0.0\%) | 484 (0.4\%) | 482 (0.8\%) | 480 (1.2\%) |
| $h_{g}$ | 1677 (6.3\%) | 1609 (2.0\%) | 1627 (3.1\%) | 1618 (2.5\%) | 1609 (2.0\%) | 1618 (2.5\%) | 1573 (0.3\%) | 1580 (0.1\%) |
|  | 1500 (5.1\%) | 1436 (0.6\%) | 1466 (2.8\%) | 1455 (1.9\%) | 1475 (3.4\%) | 1475 (3.4\%) | 1394 (2.3\%) | 1422 (0.4\%) |
|  | 1332 (6.5\%) | 1231 (1.6\%) | 1274 (1.8\%) | 1275 (1.9\%) | 1288 (3.0\%) | 1297 (3.7\%) | 1208 (3.4\%) | 1198 (4.2\%) |
|  | 1166 (5.9\%) | 1112 (1.0\%) | 1129 (2.6\%) | 1125 (2.2\%) | 1129 (2.5\%) | 1128 (2.5\%) | 1098 (0.3\%) | 1079 (2.0\%) |
|  | 802 (3.5\%) | 781 (0.8\%) | 788 (1.7\%) | 766 (1.2\%) | 794 (2.5\%) | 788 (1.7\%) | 775 (0.0\%) | 763 (1.5\%) |
|  | 734 (3.3\%) | 678 (4.7\%) | 738 (3.8\%) | 718 (0.9\%) | 711 (0.0\%) | 727 (2.3\%) | 730 (2.7\%) | 716 (0.7\%) |
|  | 449 (3.7\%) | 429 (1.0\%) | 448 (3.5\%) | 436 (0.8\%) | 430 (0.7\%) | 431 (0.5\%) | 435 (0.5\%) | 422 (2.5\%) |
|  | 271 (0.6\%) | 271 (0.8\%) | 272 (0.5\%) | 266 (2.7\%) | 269 (1.5\%) | 261 (4.4\%) | 261 (4.4\%) | 263 (3.7\%) |
| $t_{1 g}$ | 1357 (0.1\%) | 1278 (5.9\%) | 1303 (4.0\%) | 1301 (4.2\%) | 1305 (3.9\%) | 1318 (2.9\%) | 1284 (5.4\%) | 1241 (8.6\%) |
|  | 865 (11.3\%) | 865 (11.4\%) | 858 (12.1\%) | 840 (13.9\%) | 842 (13.7\%) | 830 (15.0\%) | 847 (13.2\%) | 826 (15.4\%) |
|  | 594 (18.3\%) | 544 (8.3\%) | 593 (18.1\%) | 576 (14.8\%) | 565 (12.5\%) | 579 (15.3\%) | 580 (15.5\%) | 563 (10.8\%) |
| $t_{2 g}$ | 1431 (5.2\%) | 1330 (2.2\%) | 1372 (0.8\%) | 1370 (0.7\%) | 1370 (0.7\%) | 1393 (2.4\%) | 1257 (7.6\%) | 1277 (6.1\%) |
|  | 827 (9.5\%) | 839 (8.2\%) | 829 (9.3\%) | 804 (12.0\%) | 809 (11.5\%) | 839 (8.2\%) | 816 (10.7\%) | 800 (12.5\%) |
|  | 809 (6.4\%) | 650 (24.9\%) | 766 (11.4\%) | 743 (14.1\%) | 765 (11.6\%) | 804 (7.1\%) | 789 (8.8\%) | 788 (8.9\%) |
|  | 581 (2.5\%) | 586 (3.3\%) | 582 (2.6\%) | 566 (0.1\%) | 566 (0.2\%) | 551 (2.8\%) | 559 (1.4\%) | 543 (4.4\%) |
| Odd Parity |  |  |  |  |  |  |  |  |
| $a_{u}$ | 994 (13.0\%) | 1013 (11.4\%) | 991 (13.3\%) | 982 (14.1\%) | 968 (15.3\%) | 972 (15.0\%) | 934 (18.3\%) | 973 (14.9\%) |
| $g_{u}$ | 1513 (4.7\%) | 1435 (0.8\%) | 1470 (1.7\%) | 1461 (1.0\%) | 1474 (1.9\%) | 1480 (2.4\%) | 1395 (3.5\%) | 1420 (1.8\%) |
|  | 1381 (5.4\%) | 1303 (0.5\%) | 1338 (2.1\%) | 1333 (1.7\%) | 1345 (2.7\%) | 1359 (3.7\%) | 1289 (1.6\%) | 1259 (3.9\%) |
|  | 1009 (4.1\%) | 972 (0.2\%) | 985 (1.5\%) | 977 (0.7\%) | 989 (2.0\%) | 984 (1.4\%) | 939 (3.2\%) | 937 (3.4\%) |
|  | 815 (11.8\%) | 795 (14.0\%) | 788 (14.7\%) | 787 (14.8\%) | 780 (15.6\%) | 830 (10.2\%) | 796 (13.9\%) | 790 (14.5\%) |
|  | 784 (3.1\%) | 666 (12.4\%) | 775 (2.0\%) | 751 (1.2\%) | 762 (0.3\%) | 762 (0.3\%) | 763 (0.4\%) | 756 (0.5\%) |
|  | 368 (8.1\%) | 359 (10.3\%) | 366 (8.4\%) | 357 (10.7\%) | 355 (11.3\%) | 350 (12.5\%) | 352 (12.0\%) | 348 (13.0\%) |
| $h_{u}$ | 1668 (7.0\%) | 1596 (2.4\%) | 1617 (3.7\%) | 1608 (3.2\%) | 1598 (2.5\%) | 1611 (3.3\%) | 1545 (0.9\%) | 1566 (0.4\%) |
|  | 1427 (3.0\%) | 1340 (3.2\%) | 1371 (1.0\%) | 1369 (11.8\%) | 1371 (1.0\%) | 1389 (0.3\%) | 1314 (5.1\%) | 1291 (6.8\%) |
|  | 1295 (16.0\%) | 1219 (9.1\%) | 1246 (11.6\%) | 1243 (11.3\%) | 1243 (11.3\%) | 1248 (11.7\%) | 1198 (7.3\%) | 1175 (5.2\%) |
|  | 771 (3.7\%) | 719 (10.3\%) | 763 (4.7\%) | 736 (8.1\%) | 742 (7.4\%) | 762 (4.9\%) | 769 (4.0\%) | 750 (6.4\%) |
|  | 696 (0.1\%) | 674 (3.2\%) | 697 (0.1\%) | 679 (2.5\%) | 677 (2.7\%) | 671 (3.6\%) | 672 (3.4\%) | 661 (5.0\%) |
|  | 564 (0.1\%) | 532 (5.5\%) | 555 (1.4\%) | 540 (4.1\%) | 537 (4.6\%) | 541 (3.9\%) | 540 (4.1\%) | 527 (6.4\%) |
|  | 417 (21.5\%) | 419 (22.1\%) | 418 (21.8\%) | 408 (19.0\%) | 411 (19.8\%) | 401 (16.9\%) | 404 (17.8\%) | 388 (13.1\%) |
| $t_{1 u}$ | 1505 (5.3\%) | 1454 (1.7\%) | 1479 (3.5\%) | 1464 (2.4\%) | 1489 (4.2\%) | 1486 (4.0\%) | 1399 (2.1\%) | 1457 (2.0\%) |
|  | 1266 (7.0\%) | 1175 (0.6\%) | 1209 (2.2\%) | 1212 (2.5\%) | 1222 (3.3\%) | 1224 (3.5\%) | 1158 (2.1\%) | 1143 (3.4\%) |
|  | 596 (3.4\%) | 582 (0.9\%) | 587 (1.8\%) | 570 (1.0\%) | 595 (3.3\%) | 591 (2.6\%) | 566 (1.7\%) | 569 (1.2\%) |
|  | 546 (3.6\%) | 508 (3.6\%) | 553 (4.9\%) | 537 (1.8\%) | 528 (0.2\%) | 535 (1.5\%) | 541 (2.7\%) | 514 (2.5\%) |
| $t_{2 u}$ | 1622 (2.8\%) | 1568 (0.6\%) | 1579 (0.2\%) | 1568 (0.6\%) | 1568 (0.6\%) | 1571 (0.4\%) | 1537 (2.5\%) | 1546 (2.0\%) |
|  | 1235 (2.8\%) | 1163 (3.2\%) | 1201 (0.0\%) | 1199 (0.2\%) | 1231 (2.5\%) | 1234 (2.7\%) | 1108 (7.7\%) | 1131 (5.8\%) |
|  | 1002 (2.4\%) | 964 (6.0\%) | 984 (4.1\%) | 966 (5.9\%) | 997 (2.8\%) | 996 (2.9\%) | 936 (8.8\%) | 945 (7.9\%) |
|  | 734 (7.9\%) | 678 (0.3\%) | 742 (9.2\%) | 722 (6.1\%) | 715 (5.1\%) | 726 (6.8\%) | 774 (13.8\%) | 725 (6.6\%) |
|  | 354 (0.5\%) | 345 (3.1\%) | 352 (4.3\%) | 342 (4.0\%) | 343 (3.7\%) | 342 (3.9\%) | 340 (4.5\%) | 343 (3.8\%) |

Table IX: Vibrational frequencies $\nu\left(\mathrm{cm}^{-1}\right)$ of $\mathrm{C}_{60}$ obtained by Choi et al. [141] using B3LYP/3-21G but involving scaling of the internal force constant by using Pulay's method. Numbers in the parenthesis are the relative errors of the calculated frequencies to the experimental frequencies listed in Table VII.

| Even Parity |  |  |  |  |  | Odd Parity |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mode | $\nu$ | Mode | $\nu$ | Mode | $\nu$ | Mode | $\nu$ | Mode | $\nu$ | Mode | $\nu$ |
| $a_{g}$ | 1470 (0.0\%) |  |  | $t_{1 g}$ | 1290 (5.0\%) | $a_{u}$ | 1078 (5.7\%) |  |  | $t_{1 u}$ | 1433 (0.3\%) |
|  | 495 ( 0.6\%) | $h_{g}$ | 1576 (0.1\%) |  | 904 (7.4\%) |  |  |  |  |  | 1180 (0.3\%) |
| $g_{g}$ |  |  | 1427 (0.0\%) |  | 565 (12.5\%) |  |  | $h_{u}$ | 1567 (0.5\%) |  | 577 (0.2\%) |
|  | 1497 (1.8\%) |  | 1251 (0.0\%) |  |  | $g_{u}$ | 1429 (1.2\%) |  | 1343 (3.0\%) |  | 526 (0.2\%) |
|  | 1348 (0.6\%) |  | 1101 (0.0\%) |  |  |  | 1315 (0.4\%) |  | 1214 (8.7\%) | $t_{2 u}$ | 1524 (3.4\%) |
|  | 1040 (3.3\%) |  | 775 (0.0\%) | $t_{2 g}$ | 1340 (1.5\%) |  | 970 (0.0\%) |  | 737 (8.0\%) |  | 1142 (5.0\%) |
|  | 758 (6.0\%) |  | 711 (0.0\%) |  | 831 (9.1\%) |  | 797 (13.7\%) |  | 694 (0.3\%) |  | 955 (6.9\%) |
|  | 592 ( 4.7\%) |  | 431 (0.5\%) |  | 668 (22.8\%) |  | 707 (7.0\%) |  | 535 (5.0\%) |  | 716 (5.3\%) |
|  | 485 ( 0.2\%) |  | 267 (2.2\%) |  | 614 (8.3\%) |  | 354 (11.5\%) |  | 403 (17.5\%) |  | 340 (4.5\%) |

Table X: Fifty eight IR-active frequencies ( $\nu$, in $\mathrm{cm}^{-1}$ ) of $\mathrm{C}_{48} \mathrm{~N}_{12}$ calculated by using RHF and B3LYP methods with a variety of Pople-style basis sets.


Table XI: Fifty eight Raman-active frequencies ( $\nu$, in $\mathrm{cm}^{-1}$ ) for $\mathrm{C}_{48} \mathrm{~N}_{12}$ calculated by using RHF and B3LYP methods with a variety of Pople-style basis sets.

| Doubly-degenerate Modes |  |  |  |  |  |  |  | Non-degenerate Modes |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  | B3LYP |  |  |  | RHF |  |  |  |
| STO-3G | 3-21G | 6-31G | 6-31G* | STO-3G | 3-21G | 6-31G | 6-31G* | STO-3G | 3-21G | 6-31G | 6-31G* | STO-3G | 3-21G | 6-31G | 6-31G* |
| 245 | 248 | 252 | 245 | 263 | 262 | 268 | 261 | 264 | 264 | 268 | 264 | 294 | 288 | 291 | 288 |
| 259 | 261 | 265 | 260 | 286 | 281 | 286 | 281 | 382 | 368 | 389 | 376 | 445 | 406 | 429 | 388 |
| 376 | 368 | 387 | 371 | 429 | 409 | 428 | 413 | 406 | 398 | 415 | 398 | 463 | 415 | 448 | 425 |
| 410 | 388 | 409 | 396 | 470 | 433 | 455 | 440 | 444 | 415 | 442 | 424 | 510 | 455 | 483 | 454 |
| 447 | 429 | 446 | 437 | 514 | 479 | 497 | 488 | 471 | 458 | 477 | 467 | 523 | 491 | 511 | 489 |
| 493 | 450 | 482 | 472 | 583 | 517 | 549 | 540 | 498 | 491 | 500 | 495 | 576 | 521 | 543 | 524 |
| 549 | 568 | 566 | 551 | 604 | 610 | 611 | 592 | 544 | 505 | 544 | 510 | 608 | 566 | 587 | 575 |
| 580 | 596 | 597 | 581 | 652 | 647 | 650 | 629 | 578 | 588 | 592 | 576 | 646 | 637 | 640 | 615 |
| 617 | 613 | 652 | 627 | 725 | 699 | 736 | 706 | 585 | 598 | 602 | 588 | 650 | 651 | 652 | 637 |
| 649 | 629 | 662 | 641 | 773 | 726 | 759 | 741 | 621 | 603 | 616 | 597 | 723 | 655 | 670 | 652 |
| 665 | 671 | 698 | 671 | 780 | 762 | 785 | 757 | 625 | 614 | 652 | 627 | 744 | 705 | 749 | 716 |
| 716 | 695 | 714 | 699 | 831 | 776 | 807 | 788 | 658 | 657 | 684 | 656 | 777 | 760 | 784 | 738 |
| 767 | 760 | 768 | 766 | 848 | 820 | 833 | 821 | 730 | 690 | 709 | 689 | 832 | 779 | 802 | 773 |
| 782 | 776 | 782 | 780 | 871 | 844 | 853 | 840 | 768 | 765 | 773 | 766 | 867 | 825 | 836 | 819 |
| 841 | 865 | 860 | 843 | 935 | 943 | 944 | 918 | 836 | 851 | 844 | 830 | 924 | 918 | 919 | 897 |
| 858 | 887 | 877 | 854 | 961 | 974 | 966 | 933 | 856 | 881 | 869 | 844 | 953 | 965 | 956 | 916 |
| 1124 | 1084 | 1105 | 1093 | 1238 | 1160 | 1192 | 1164 | 1115 | 1080 | 1100 | 1092 | 1216 | 1165 | 1192 | 1175 |
| 1162 | 1123 | 1141 | 1138 | 1281 | 1197 | 1225 | 1215 | 1194 | 1160 | 1173 | 1162 | 1328 | 1244 | 1264 | 1251 |
| 1182 | 1147 | 1163 | 1156 | 1313 | 1238 | 1262 | 1250 | 1221 | 1181 | 1196 | 1189 | 1344 | 1260 | 1281 | 1273 |
| 1274 | 1186 | 1220 | 1223 | 1423 | 1276 | 1325 | 1318 | 1316 | 1241 | 1267 | 1265 | 1400 | 1335 | 1374 | 1348 |
| 1318 | 1243 | 1274 | 1272 | 1476 | 1341 | 1383 | 1377 | 1337 | 1253 | 1280 | 1279 | 1469 | 1347 | 1389 | 1377 |
| 1386 | 1311 | 1337 | 1336 | 1546 | 1421 | 1457 | 1449 | 1367 | 1293 | 1322 | 1320 | 1498 | 1375 | 1424 | 1380 |
| 1452 | 1369 | 1407 | 1404 | 1608 | 1468 | 1517 | 1503 | 1416 | 1347 | 1383 | 1379 | 1534 | 1415 | 1468 | 1429 |
| 1466 | 1388 | 1420 | 1420 | 1647 | 1490 | 1537 | 1540 | 1439 | 1373 | 1415 | 1397 | 1583 | 1437 | 1485 | 1476 |
| 1504 | 1415 | 1452 | 1451 | 1717 | 1541 | 1591 | 1587 | 1481 | 1419 | 1453 | 1441 | 1655 | 1528 | 1578 | 1552 |
| 1571 | 1489 | 1517 | 1515 | 1781 | 1631 | 1671 | 1664 | 1531 | 1440 | 1477 | 1477 | 1721 | 1560 | 1612 | 1602 |
| 1595 | 1516 | 1551 | 1551 | 1831 | 1670 | 1710 | 1720 | 1558 | 1487 | 1511 | 1505 | 1766 | 1612 | 1649 | 1626 |
| 1633 | 1544 | 1578 | 1578 | 1883 | 1708 | 1749 | 1759 | 1581 | 1509 | 1545 | 1530 | 1817 | 1644 | 1687 | 1680 |
| 1662 | 1578 | 1610 | 1610 | 1916 | 1735 | 1778 | 1788 | 1683 | 1592 | 1624 | 1623 | 1946 | 1767 | 1804 | 1809 |

Table XII: RHF and B3LYP calculations of IR intensities ( $I_{I R}$, in $10^{3} \mathrm{~m} / \mathrm{mole}$ ) of $\mathrm{C}_{60}$ with the corresponding vibrational modes and frequencies ( $\nu$, in $\mathrm{cm}^{-1}$ ).

| Method | Mode | STO-3G |  | 3-21G |  | 6-31G |  | 6-31G* |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $I_{I R}$ | $\nu$ | $\overline{I_{I R}}$ | $\nu$ | $I_{I R}$ | $\nu$ | $\overline{I_{I R}}$ | $\nu$ |
| B3LYP | $\mathrm{t}_{1 \mathrm{u}}$ | 10.6 | 1505 | 14.0 | 1454 | 17.2 | 1479 | 15.6 | 1464 |
|  |  | 21.0 | 1266 | 9.2 | 1175 | 10.1 | 1209 | 8.9 | 1212 |
|  |  | 0.5 | 596 | 5.9 | 582 | 8.2 | 587 | 10.7 | 570 |
|  |  | 22.0 | 546 | 28.8 | 508 | 27.7 | 553 | 27.1 | 537 |
| RHF | $\mathrm{t}_{1 \mathrm{u}}$ | 11.5 | 1637 | 16.5 | 1553 | 24.5 | 1587 | 17.1 | 1549 |
|  |  | 25.1 | 1396 | 11.4 | 1245 | 11.8 | 1287 | 10.6 | 1297 |
|  |  | 0.6 | 656 | 6.0 | 614 | 14.7 | 623 | 10.4 | 625 |
|  |  | 35.9 | 627 | 42.1 | 575 | 34.3 | 621 | 48.0 | 599 |



FIG.4: Ab initio calculation of IR intensity ( $I_{I R}$, in $10^{3} \mathrm{~m} / \mathrm{mole}$ ) at its corresponding frequency in $\mathrm{C}_{48} \mathrm{~N}_{12}$ : (a) B3LYP/STO-3G; (b) B3LYP/3-21G; (c) RHF/3-21G. The solid lines are the calculated results for $\mathrm{C}_{60}$.
shows collective vibration along the z-x direction, while the pentagon structure for the high-frequency case contracts, accompanying a large stretching of site 3 along the z direction.


FIG.5: The vibrational displacements of sites 1 to 5 for the strongest IR spectral signals of $\mathrm{C}_{48} \mathrm{~N}_{12}$ in the low-frequency (filled circles) and high-frequency (open circles) regions for the B3LYP/3-21G case.

## VI. NUCLEAR MAGNETIC SHIELDING TENSORS

## A. Theoretical Methods

There are a number of theoretical methods for calculating the second-order magnetic response properties of molecules. It has been shown that gauge-invariant procedures (GIPs) [71,143] are required to predict accurately these properties within a finite basis approximation. In
this paper, we focus on using two GIPs, i.e., the gaugeincluding atomic orbital (GIAO) procedure and the continuous set of gauge transformations (CSGT) procedure, to predict NMR shielding tensors at the Hartree-Fock and DFT levels of theory. GIAO and CSGT achieve gauge-invariance in different ways. The GIAO method uses basis functions having explicit field dependence [70], whereas the CSGT method achieves the gauge-invariance by performing a continuous set of gauge transformations. Ditchfield [144] first adopted the GIAO method to perform quantum chemical NMR shift calculations and the CSGT method was developed by Keith and Bader [71]. In the following, we briefly introduce the two methods. More details can be found, for example, in the works of Cheeseman et al. [145] and Wolinski et al. [146].

It is known that the nuclear magnetic shielding tensor can be written as the mixed second derivative of the energy $E$ with respect to the external magnetic field $\mathbf{B}$ and the magnetic moment $\mathbf{I}$ of nucleus $X$ :

$$
\begin{equation*}
\sigma_{i k}=\frac{\partial^{2} E}{\partial \mathrm{I}_{i} \partial \mathrm{~B}_{k}} \tag{11}
\end{equation*}
$$

where $\mathrm{B}_{k}$ and $\mathrm{I}_{i}$ are the components of the external magnetic field and induced magnetic moment, respectively. The nuclear magnetic shielding isotropy $\sigma$ is defined as [65]

$$
\begin{equation*}
\sigma=\left(\sigma_{x x}+\sigma_{y y}+\sigma_{z z}\right) / 3 \tag{12}
\end{equation*}
$$

and the shielding anisotropy $\Delta \sigma$, an indication of the quality of the magnetic shielding tensor, is defined as [65]

$$
\begin{equation*}
\Delta \sigma=\sigma_{3}-\left(\sigma_{1}+\sigma_{2}\right) / 2 \tag{13}
\end{equation*}
$$

where $\sigma_{1}<\sigma_{2}<\sigma_{3}$ are the eigenvalues of the symmetrized shielding tensor. The nuclear magnetic shielding difference, or say, chemical shift $\delta$, is reported in ppm (i.e., parts per million) and given by [65]

$$
\begin{equation*}
\delta=\left(\sigma^{(\text {reference })}-\sigma^{(\text {sample })}\right) \times 10^{6} \tag{14}
\end{equation*}
$$

where $\sigma^{(\text {reference })}$ and $\sigma^{(\text {sample })}$ denote the shielding isotropies $\sigma$ for the reference and sample, respectively.

In both DFT and HF theory, the expression for the nuclear magnetic shielding tensor of nucleus X is given by [145]

$$
\begin{equation*}
\sigma_{i k}=<\Pi_{i k} O>+<\theta_{k} \frac{\partial O}{\partial \mathrm{~B}_{k}}> \tag{15}
\end{equation*}
$$

with

$$
\begin{equation*}
\Pi_{i k}=\left\langle\Phi_{\mu}\right| \frac{\mathbf{r} \bullet\left(\mathbf{r}-\mathbf{R}_{X}\right) \delta_{i k}-r_{i}\left(\mathbf{r}-\mathbf{R}_{X}\right)_{k}}{2 c^{2}\left|\mathbf{r}-\mathbf{R}_{X}\right|^{3}}\left|\Phi_{\nu}\right\rangle \tag{16}
\end{equation*}
$$

$$
\begin{equation*}
\theta_{k}=\left\langle\Phi_{\mu}\right|-\frac{i\left[\left(\mathbf{r}-\mathbf{R}_{X}\right) \times \nabla\right]_{k}}{c\left|\mathbf{r}-\mathbf{R}_{X}\right|^{3}}\left|\Phi_{\nu}\right\rangle \tag{17}
\end{equation*}
$$

where $c$ is the velocity of light, $O$ is the density matrix, $\Phi_{\mu}$ and $\Phi_{\nu}$ are spin orbitals, $\mathbf{R}_{X}$ is the position vector of nuclear X , and $\mathbf{r}$ is the real space vector. In the above equation, $\partial O / \partial \mathrm{B}_{k}$ is the derivative of the density matrix $O$ with respect to the $k$ th component of the magnetic field $\mathbf{B}$ and is obtained via solution of the coupledperturbed equations [145] for the appropriate perturbation. As gauge-invariance is achieved in different ways, the GIAO and CSGT methods differ at this point in the formation of the coupled-perturbed equations [145].

Table XIII: RHF and B3LYP calculations of the absolute isotropy ( $\sigma$, in ppm ) and anisotropy ( $\Delta \sigma$, in ppm ) of the carbon and nitrogen shielding tensors with a variety of Pople-style basis sets for $\mathrm{C}_{48} \mathrm{~N}_{12}$ aza-fullerene, $\mathrm{C}_{60}$ and tetramethylsilane (TMS) by using the GIAO method. Numbers in the parenthesis for $\mathrm{C}_{60}$ are the relative errors of the calculated ${ }^{13} \mathrm{C}$ NMR shift $\delta$ to the NMR chemical shift $\delta^{e x p .}=142.7 \mathrm{ppm}$ measured by Taylor et al. [149].


## 1. GIAO Method

2. CSGT Method

The GIAO method uses the following explicit fielddependent basis functions for calculating the magnetic shielding tensor:

$$
\begin{equation*}
\left|\Phi_{\mu}(\mathbf{B})\right\rangle=e^{-i\left(\mathbf{B} \times \mathbf{r}_{\mu}\right) \bullet \mathbf{r} /(2 c)}\left|\Phi_{\mu}(0)\right\rangle \tag{18}
\end{equation*}
$$

where $\mathbf{r}_{\mu}$ is the position vector of basis function $\Phi_{\mu}$ and $\Phi_{\mu}(0)$ denotes the usual field-independent basis functions. In this method, three sets of the coupled-perturbed equations [145] are solved, one for each direction of the magnetic field.

In the CSGT method, the gauge-invariance is achieved from accurate calculations of the induced first-order electronic current density $\mathbf{J}^{(1)}(\mathbf{r})$ by performing a gauge transformation for each point in space. The nuclear magnetic shielding tensor is expressed in terms of $\mathbf{J}^{(1)}(\mathbf{r})$, i.e.,

$$
\begin{equation*}
\sigma_{i k}=-\frac{1}{\mathrm{~B} c} \int\left[\frac{\mathbf{R}_{X} \times \mathbf{J}_{k}^{(1)}(\mathbf{r})}{R_{X}^{3}}\right]_{i} d \mathbf{R}_{X} \tag{19}
\end{equation*}
$$

Table XIV: RHF and B3LYP calculations of the absolute isotropy ( $\sigma$, in ppm ) and anisotropy ( $\Delta \sigma$, in ippm) of the carbon and nitrogen shielding tensors with a variety of Pople-style basis sets for $\mathrm{C}_{48} \mathrm{~N}_{12}$ aza-fullerene, $\mathrm{C}_{60}$ and tetramethylsilane (TMS) by using CSGT methods. Numbers in the parenthesis for $\mathrm{C}_{60}$ are the relative errors of the calculated ${ }^{13} \mathrm{C}$ NMR shift $\delta$ to the NMR chemical shift $\delta^{e x p}=142.7 \mathrm{ppm}$ measured by Taylor et al. [149].

| Method | Molecule | Site Numbers $\left\{n_{i}\right\}$ | Nuclei | STO-3G |  | 3-21G |  | 6-31G |  | 6-31G* |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\sigma$ | $\Delta \sigma$ | $\sigma$ | $\Delta \sigma$ | $\sigma$ | $\Delta \sigma$ | $\sigma$ | $\Delta \sigma$ |
| RHF | $\mathrm{C}_{48} \mathrm{~N}_{12}$ | \{1, 13, 16, 31, 38, 51\} | ${ }^{13} \mathrm{C}$ | 45.3 | 125.0 | 60.7 | 151.5 | 30.7 | 168.9 | 41.3 | 170.1 |
|  |  | $\{2,12,29,32,37,52\}$ | ${ }^{13} \mathrm{C}$ | 55.8 | 127.0 | 84.1 | 143.8 | 54.0 | 162.1 | 60.4 | 168.3 |
|  |  | $\{3,11,28,33,36,53\}$ | ${ }^{13} \mathrm{C}$ | 60.0 | 114.0 | 91.2 | 122.4 | 60.9 | 139.1 | 78.9 | 129.8 |
|  |  | $\{4,15,27,34,40,54\}$ | ${ }^{13} \mathrm{C}$ | 52.9 | 119.5 | 68.8 | 138.7 | 41.1 | 155.5 | 39.2 | 160.4 |
|  |  | $\{5,14,30,35,39,55\}$ | ${ }^{13} \mathrm{~N}$ | 78.0 | 134.0 | 120.4 | 154.3 | 86.3 | 176.5 | 128.2 | 136.4 |
|  |  | $\{6,18,24,42,48,58\}$ | ${ }^{13} \mathrm{C}$ | 63.8 | 97.9 | 89.8 | 96.5 | 59.3 | 115.5 | 79.8 | 87.3 |
|  |  | $\{7,19,23,43,47,57\}$ | ${ }^{13} \mathrm{C}$ | 55.5 | 113.5 | 72.2 | 130.9 | 43.2 | 149.5 | 46.5 | 150.5 |
|  |  | $\{8,20,22,44,46,56\}$ | ${ }^{13} \mathrm{C}$ | 58.6 | 118.0 | 84.9 | 125.5 | 52.4 | 150.1 | 73.7 | 135.2 |
|  |  | $\{9,21,26,45,50,60\}$ | ${ }^{13} \mathrm{~N}$ | 74.0 | 144.8 | 116.0 | 170.8 | 72.2 | 203.6 | 112.2 | 173.1 |
|  |  | $\{10,17,25,41,49,59\}$ | ${ }^{13} \mathrm{C}$ | 55.3 | 107.8 | 73.1 | 118.7 | 43.3 | 136.9 | 47.3 | 129.5 |
|  | $\mathrm{C}_{60}$ | $\{1,2,3,4,5,6, \ldots, 60\}$ | ${ }^{13} \mathrm{C}$ | 50.2 | 128.8 | 71.2 | 156.1 | 43.3 | 172.7 | 48.9 | 178.6 |
|  | TMS | carbon site | ${ }^{13} \mathrm{C}$ | 131.9 | 9.5 | 191.0 | 6.3 | 185.5 | 13.1 | 190.6 | 13.7 |
|  | $\mathrm{NH}_{3}$ | nitrogen site | ${ }^{13} \mathrm{~N}$ | 148.1 | 18.6 | 200.2 | 5.1 | 209.2 | 6.3 | 228.4 | 6.6 |
|  | $\mathrm{C}_{60}$ | Calculated ${ }^{13} \mathrm{C}$ NMR shift $\delta$ |  | 81.7 (42.7\%) |  | 119.8 (16.0\%) |  | 142.2 (0.4\%) |  | 141.7 (0.7\%) |  |
| B3LYP | $\mathrm{C}_{48} \mathrm{~N}_{12}$ | $\{1,13,16,31,38,51\}$ | ${ }^{13} \mathrm{C}$ | 58.6 | 86.4 | 66.0 | 114.1 | 39.1 | 126.6 | 45.7 | 131.8 |
|  |  | $\{2,12,29,32,37,52\}$ | ${ }^{13} \mathrm{C}$ | 66.1 | 94.8 | 81.7 | 119.3 | 53.9 | 133.5 | 57.2 | 139.6 |
|  |  | $\{3,11,28,33,36,53\}$ | ${ }^{13} \mathrm{C}$ | 69.3 | 83.5 | 87.2 | 99.0 | 60.3 | 111.1 | 66.3 | 109.4 |
|  |  | $\{4,15,27,34,40,54\}$ | ${ }^{13} \mathrm{C}$ | 64.6 | 84.8 | 75.8 | 104.0 | 49.1 | 118.5 | 54.7 | 117.6 |
|  |  | $\{5,14,30,35,39,55\}$ | ${ }^{13} \mathrm{~N}$ | 78.5 | 116.6 | 104.9 | 142.0 | 74.6 | 152.7 | 96.8 | 142.4 |
|  |  | $\{6,18,24,42,48,58\}$ | ${ }^{13} \mathrm{C}$ | 71.9 | 63.6 | 84.1 | 72.0 | 58.7 | 81.9 | 66.9 | 66.5 |
|  |  | $\{7,19,23,43,47,57\}$ | ${ }^{13} \mathrm{C}$ | 66.5 | 77.7 | 73.8 | 99.7 | 47.5 | 112.2 | 51.0 | 115.3 |
|  |  | $\{8,20,22,44,46,56\}$ | ${ }^{13} \mathrm{C}$ | 70.4 | 81.9 | 83.5 | 101.7 | 56.7 | 115.8 | 65.7 | 118.7 |
|  |  | $\{9,21,26,45,50,60\}$ | ${ }^{13} \mathrm{~N}$ | 81.6 | 112.8 | 103.4 | 140.2 | 69.3 | 156.6 | 86.0 | 148.5 |
|  |  | $\{10,17,25,41,49,59\}$ | ${ }^{13} \mathrm{C}$ | 66.9 | 71.6 | 76.1 | 88.1 | 49.5 | 100.1 | 54.8 | 93.8 |
|  | $\mathrm{C}_{60}$ | $\{1,2,3,4,5,6, \ldots, 60\}$ | ${ }^{13} \mathrm{C}$ | 58.2 | 100.3 | 69.2 | 134.7 | 43.1 | 148.4 | 45.9 | 149.2 |
|  | TMS | carbon site | ${ }^{13} \mathrm{C}$ | 124.9 | 10.0 | 181.7 | 9.0 | 175.0 | 16.5 | 181.6 | 18.0 |
|  | $\mathrm{NH}_{3}$ | nitrogen site | ${ }^{13} \mathrm{~N}$ | 137.7 | 27.1 | 190.0 | 9.6 | 200.4 | 6.0 | 221.5 | 6.5 |
|  | $\mathrm{C}_{60}$ | Calculated ${ }^{13} \mathrm{C}$ NMR shift $\delta$ |  | 66.7 (53.3\%) |  | 112.5 (21.2\%) |  | 131.9 (7.6\%) |  | 135.7 (4.9\%) |  |

where $B$ is the magnitude of magnetic field. Since the basis functions do not depend on the magnetic field, only six sets of the coupled-perturbed equations [145] are left and
solved (three for the components of the angular momentum perturbation using any single gauge origin, and three for the linear momentum perturbation resulting from any
single shift in gauge origin). Furthermore, the shielding tensors are obtained via Becke's multi-center numerical integration scheme [147]

## B. Results

The above mentioned GIAO and CSGT methods have been implemented into the Gaussian 98 program [73,72] and all calculations in this section are performed using this program.

Here we perform RHF and B3LYP hybrid DFT calculations of the nuclear magnetic shielding tensor of carbon and nitrogen atoms in $\mathrm{C}_{48} \mathrm{~N}_{12}, \mathrm{C}_{60}$, and tetramethylsilane (TMS [148]) by using GIAO and CSGT methods with STO-3G, $3-21 \mathrm{G}, 6-31 \mathrm{G}$ and $6-31 \mathrm{G}^{*}$ basis sets. Calculation performed with a specific basis set and ab initio method use geometry optimized with the same basis set and ab initio method (see section II). Detailed results are summarized in Table XIII and XIV. Since no present functional includes a magnetic field dependence, the DFT methods do not provide systematically better shielding results than RHF [72]. Nevertheless, we also list the DFT results in Table XIII and XIV for comparison. The isotropic NMR chemical shifts $\delta$ relative to that of TMS are also shown in Table XIII and XIV. It is seen that the NMR shielding tensors for $\mathrm{C}_{48} \mathrm{~N}_{12}$ are separated into eight groups for carbon atom and two groups for nitrogen atoms, i.e., eight ${ }^{13} \mathrm{C}$ and two ${ }^{15} \mathrm{~N}$ NMR spectral signals are predicted. In contrast, $\mathrm{C}_{60}$ has only one ${ }^{13} \mathrm{C}$ NMR signal which is in agreement with experiment [149]. The experimental values of the absolute shielding constant $\sigma$ for ${ }^{15} \mathrm{~N}$ in $\mathrm{NH}_{3}$ and ${ }^{13} \mathrm{C}$ in TMS, as shown in Table XIII and XIV, are $264.5 \mathrm{ppm}[150,151]$ and 188.1 ppm [148], and our best calculated results for those reference materials are in good agreement with the experiments.

Table XIII and XIV also demonstrate the convergence of the GIAO and CSGT methods with respect to basis set for absolute shielding constants calculated with RHF and B3LYP hybrid DFT methods. For each first principles method, the shielding constants calculated with GIAO and CSGT methods are found to converge to almost the same values for the large basis set $6-31 \mathrm{G}^{*}$. NMR chemical shifts, as opposed to the absolute shielding constants $\sigma$, are measured with high accuracy in applications of NMR spectroscopy. On the other hand, calculated chemical shifts are in better agreement with experiment as relative differences are better represented [145]. Hence, given the absolute shielding constants for ${ }^{13} \mathrm{C}$ and ${ }^{15} \mathrm{~N}$ in reference materials, i.e., TMS and $\mathrm{NH}_{3}$ shown in Table XIII and XIV, we are able to calculate the chemical shifts $\delta$ and compare them with the experiments. For $\mathrm{C}_{60}$, we find that the NMR chemical shifts $\delta$ calculated with the CSGT method at the RHF/6-31G and RHF/6-31G* levels are in agreement with experiment ( $\delta^{e x p}=142.7 \mathrm{ppm}$ measured by Taylor et al. [149]). This suggests that CSGT method would predict NMR spec-
tral signals much better than GIAO. However, CSGT costs more CPU times than GIAO on the same machine, for example, about 5 hours more for RHF/6-31G* and 2 hours for B3LYP/6-31G* in the CSGT method.

To predict accurate NMR chemical shifts for large molecules, it is necessary to assess the accuracy of the available ab initio methods by employing lower levels of theory and by using basis sets as small as possible [145]. The results in Table XIII and XIV indicate that the RHF method yields better chemical shifts than the B3LYP hybrid DFT method, especially, for GIAO method and the minimum basis set STO-3G. As shown in Table XIII, the ${ }^{13} \mathrm{C}$ chemical shift in $\mathrm{C}_{60}$ differs from experiment by 23.5 ppm at the B3LYP level, while it differs by only 6 ppm at the RHF level of theory. Also it is noted that RHF makes positive and negative errors, while B3LYP makes only positive errors.

In addition, nuclear magnetic shielding anisotropies $\Delta \sigma$ are reported in Table XIII and XIV. Although these properties usually cannot be determined experimentally in the gas phase (except in cases where the high symmetry of the molecule together with the calculated diamagnetic contribution to the shielding tensor allows the determination of the anisotropy from the spin rotation [152]), these values are of interest. Anisotropies can be determined in solid state NMR experiments and calculations are often important for a correct assignment. From Table XIII and XIV, we find that the shielding anisotropies $\Delta \sigma$ for ${ }^{15} \mathrm{~N}$ in $\mathrm{NH}_{3}$ calculated with GIAO method with both ab initio theories are in good agreement with experiment, while those calculated with CSGT agree poorly with experiment.

## VII. SUMMARY AND OUTLOOK

In summary, we have performed $a b$ initio calculations of the structures, electronic properties, vibrational frequencies, IR intensities, NMR shielding tensors, linear polarizabilities, first- and second-order hyperpolarizabilities of the $\mathrm{C}_{48} \mathrm{~N}_{12}$ aza-fullerene. Calculated results of $\mathrm{C}_{48} \mathrm{~N}_{12}$ are compared with those of $\mathrm{C}_{60}$ at the same level of theory. It is found that this aza-fullerene has some remarkable features, which are different from and can compete with $\mathrm{C}_{60}$. The detailed studies of $\mathrm{C}_{60}$ show the importance of electron correlations and the choice of basis sets in the ab initio calculations. Our best results for $\mathrm{C}_{60}$ obtained with the B3LYP hybrid DFT method are in excellent agreement with experiment and demonstrate the needed efficiency and accuracy of this method for obtaining quantitative information on the structural, electronic and vibrational properties of these materials.

Laser sources are widely used in the laboratory and industry. However, they are a potential hazard for eyes, thermal cameras and other optical sensors $[10,11,153]$. Development of optical limiters which can suppress undesired radiation and effectively decrease transmittance
at high intensity or fluence is necessary. An ideal optical limiter $[10,153]$ should have reasonable linear transmittance at low input fluence (at least of $70 \%$ ), protecting optical sensors or eyes against laser pulses of any wavelength and pulse duration, and its output energy must remain below the optical damage threshold of sensors or eyes at high fluences. $\mathrm{C}_{60}$ and fullerene derivatives are good candidates for optical limiting applications [10,154,155]. In this paper, we found that the average second-order hyperpolarizability of $\mathrm{C}_{48} \mathrm{~N}_{12}$ is about $55 \%$ larger than that of $\mathrm{C}_{60}$. Hence, it is expected that $\mathrm{C}_{48} \mathrm{~N}_{12}$ is also a good candidate for optical limiting applications.

Non-linear optical (NLO) materials have vast technological applications in telecommunications, optical data storage and optical information processing. The search for materials with such properties is the subject of intense research $[9,10,156]$. Organic molecules, which have high NLO response, could be designed by linking appropriate electron-donor and acceptor groups with a spacer made of one or several units of conjugated molecules such as a polyene or an aromatic chain [156]. Based on such donoracceptor model, we can link donor $\mathrm{C}_{48} \mathrm{~N}_{12}$ and acceptor $\mathrm{C}_{60}$ with a polyene or an aromatic chain and design one kind of organic molecules. In such organic molecules, charge would migrate from $\mathrm{C}_{48} \mathrm{~N}_{12}$ to $\mathrm{C}_{60}$ upon electronic excitation, giving rise to a large dipole moment along the direction connecting the donor/acceptor pair. Thus, large NLO response is expected in these donor/acceptorbased molecules.

Carbon nanotubes are promising materials for building electronic devices, in particular, field effect transistors (FETs) [11,157]. However, single-walled carbon nanotube (SWNT) FETs built from as-grown tubes are unipolar $p$-type, i.e., there is no electron current flow even at large positive gate biases. This behavior suggests the presence of a Schottky barrier at the metal-nanotube contact. Obviously, the capability to produce $n$-type transistors is important technologically, as it allows the fabrication of nanotube-based complementary logic devices and circuits [158,159]. Very recently, Bockrath et al. [160] have reported a controlled chemical doping of individual semiconducting SWNT ropes by reversibly intercalating and deintercalating potassium. Potassium doping could change the carriers in the ropes from holes to electrons [160]. Their experiments open the way toward other experiments that require controlled doping such as making nanoscale p- $n$ junctions. As shown in this paper, $\mathrm{C}_{48} \mathrm{~N}_{12}$ is a good electron donor. We found that incorporating $\mathrm{C}_{48} \mathrm{~N}_{12}$ into a $(10,10)$ semiconducting SWNT [7] would result in -0.26 e charge [90] on the nanotube forming a $n$-type SWNT-based transistor.

To obtain molecular rectification, the LUMO of the acceptor should lie at or slightly above the Fermi level of the electrode and above the HOMO of the donor [161]. Hence it is important to search for desired donor/acceptor pairs which satisfy those conditions. The acceptor/donor pair, $\mathrm{C}_{48} \mathrm{~B}_{12} / \mathrm{C}_{48} \mathrm{~N}_{12}$, actually is demonstrated to be an ideal candidate for molecular electronics [90]. For example,
we have shown that a rectifier formed by $\mathrm{C}_{48} \mathrm{~B}_{12} / \mathrm{C}_{48} \mathrm{~N}_{12}$ pair shows a perfect rectification behavior [90].

As shown before, $\mathrm{C}_{48} \mathrm{~B}_{12}$ and $\mathrm{C}_{48} \mathrm{~N}_{12}$ fcc solids are semiconducting materials. Because both $\mathrm{C}_{48} \mathrm{~B}_{12}$ and $\mathrm{C}_{48} \mathrm{~N}_{12}$ molecules have opposite electronic polarizations, it is possible to use them to build semiconducting materials with opposite electronic polarizations.

Therefore, $\mathrm{C}_{48} \mathrm{~N}_{12}$ could have potential applications as semiconductor components, nonlinear optical materials, and possible building materials for molecular electronics and photonic devices. Efficiently synthesizing $\mathrm{C}_{48} \mathrm{~N}_{12}$ would be of great experimental interest within reach of today's technology.

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[1] G. Benedek, P. Milani, and V. G. Ralchenko (eds.), Nanostructured Carbon for Advanced Applications (Kluwer, Dordrecht, 2001).
[2] A. A. Lucas, F. Moreau, and Ph. Lambin, Rev. Mod. Phys. 74, 1 (2002).
[3] H. W. Kroto, J. R. Heath, S. C. O'Brien, R. F. Curl, and R. E. Smalley, Nature (London) 318, 162 (1985).
[4] R. F. Curl and R. E. Smalley, Sci. Am. 264, 54 (1991).
[5] W. Krätschmer, L. D. Lamb, K. Fostiropoulos, and D. R. Huffman, Nature (London) 347, 354 (1990).
[6] D. R. Huffman, Phys. Today 44, 22 (1991).
[7] M. S. Dresselhaus, G. Dresselhaus, and P. C. Eklund, Science of Fullerenes and Carbon Nanotubes (Academic Press, New York, 1996).
[8] W. Andreoni (ed.), The Physics of Fullerene-Based and Fullerene-Related Materials (Kluwer, New York, 2000)
[9] R. H. Xie, "Nonlinear Optical Properties of Fullerenes and Carbon Nanotubes", in: Handbook of Advanced Electronic and Photonic Materials and Devices, Vol. 9: Nonlinear Optical Materials, H. S. Nalwa (Ed.) (Academic Press, New York, 2000), pp.267-307.
[10] R.H. Xie, Q. Rao, and L. Jensen, "Nonlinear Optics of Fullerenes and Carbon Nanotubes", in: Encyclopedia of Nanoscience and Nanotechnology, H.S. Nalwa (Ed.) (American Scientific Publisher, California, 2003).
[11] R.H. Xie, J. Zhao, and Q. Rao,"Doped Carbon Nanotubes", in: Encyclopedia of Nanoscience and Nanotechnology, H.S. Nalwa (Ed.) (American Scientific Publisher, California, 2003).
[12] E. Osawa (ed.), Perspectives of Fullerene Nanotechnology (Kluwer, New York, 2002).
[13] K. Holczer, O. Klein, S. M. Huang, R. B. Kaner, K. J. Fu, R. L. Whetten, and F. Diederich, Science 252, 1154 (1991).
[14] D. T. On, L. Jiang, K. Kitazawa, A. Fujishima, and K. Hashimoto, J. Phys. Chem. B 103, 3511 (1999).
[15] T. Guo, C. M. Jin, and R. E. Smalley, J. Phys. Chem. 95, 4948 (1991).
[16] J. C. Hummelen, B. Knight, J. Pavlovich, R. González, and F. Wudl, Science 269, 1554 (1995).
[17] W. Andreoni, A. Curioni, K. Holczer, K. Prassides, M. Keshavarz-K, J.C. Hummelen, and F. Wudl, J. Am. Chem. Soc. 118, 11335 (1996).
[18] L. Hultman, S. Stafström, Z. Czigány, J. Neidhardt, N. Hellgren, I. F. Brunell, K. Suenaga, and C. Cooliex, Phys. Rev. Lett. 87, 225503 (2001).
[19] S. Stafström, L. Hultman, and N. Hellgren, Chem. Phys. Lett. 340, 227 (2001).
[20] R.H. Xie, G.W. Bryant, and V.H. Smith, Jr., Chem. Phys. Lett. 368, 486 (2002).
[21] J. Cioslowski, Electronic Structure Calculations on Fullerenes and Their Derivatives (Oxford University Press, New York, 1995).
[22] G. E. Scuseria, Science 271, 942 (1996).
[23] D. B. Cook, Handbook of Computational Quantum Chemistry (Oxford University Press, New York, 1998).
[24] P. R. Taylor, E. Bylaska, J. H. Weare, and R. Kawai, Chem. Phys. Lett. 235, 558 (1995).
[25] R. L. Disch and J. M. Schulmann, Chem. Phys. Lett. 125, 465 (1986).
[26] H. P. Lüthi and J. Almlöf, Chem. Phys. Lett. 135, 357 (1987).
[27] G. Scuseria, Chem. Phys. Lett. 176, 423 (1991).
[28] M. Häser, J. Almlöf, and G. E. Scuseria, Chem. Phys. Lett. 181, 497 (1991).
[29] N. Kurita, K. Kobayashi, H. Kumahora, and K. Tago, Phys. Rev. B 48, 4850 (1993).
[30] J. Hrusák and H. Schwarz, Chem. Phys. Lett. 205, 187 (1993).
[31] D. Bakowies, M. Bühl, and W. Thiel, Chem. Phys. Lett. 247, 491 (1995).
[32] J. Cioslowski, N. Rao, and D. Moncrieff, J. Am. Chem. Soc. 124, 8485 (2002).
[33] P. Hohenberg and W. Kohn, Phys. Rev. 136, B864 (1964).
[34] W. Kohn and L. J. Sham, Phys. Rev. 140, A1133 (1965).
[35] G. E. Scuseria, Chem. Phys. Lett. 243, 193 (1995).
[36] R. W. Boyd, Nonlinear Optics (Academic Press, New York, 1992).
[37] M. Bianchetti, P. F. Buonsante, F. Ginelli, H. E. Roman, R. A. Broglia, and F. Alasia, Phys. Reports 357, 459 (2002).
[38] L. A. Woodward, Introduction to the Theory of Molecular Vibrations and Vibrational Spectroscopy (Oxford

University Press, London, 1972).
[39] N. B. Colthup, L. H. Daly, and S. E. Wiberley, Introduction to Infrared and Raman Spectroscopy (3rd Ed.) (Academic Press, New York, 1990).
[40] W. Krätschmer, K. Fostiropoulos, and D. R. Huffman, Chem. Phys. Lett. 170, 167 (1990).
[41] D. S. Bethune, G. Meijer, W. C. Tang, H. J. Rosen, W. G. Golden, H. Seki, C. A. Brown, and M. S. de Vries, Chem. Phys. Lett. 179, 181 (1991).
[42] K. A. Wang, A. M. Rao, P. C. Eklund, M. S. Dresselhaus, and G. Dresselhaus, Phys. Rev. B 48, 11375 (1993).
[43] M. C. Martin, X. Du, J. Kwon, and L. Mihaly, Phys. Rev. B 50, 173 (1994).
[44] K. A. Wang, Y. Wang, P. Zhou, J. M. Holden, S. L. Ren, G. T. Hager, H. F. Ni, P. C. Eklund, G. Dresselhaus, and M. S. Dresselhaus, Phys. Rev. B 45, R1955 (1992).
[45] K. Lynch, C. Tanke, F. Menzel, W. Brockner, P. Scharff and E. Stumpp, J. Phys. Chem. 99, 7985 (1995).
[46] P. Zhou, K. A. Wang, Y. Wang, P. C. Eklund M. S. Dresselhaus, G. Dresselhaus, and R. A. Jishi, Phys. Rev. B 46, 2595 (1992).
[47] Z. H. Dong, P. Zhou, J. M. Holden, P. C. Eklund, M. S. Dresselhaus, and G. Dresselhaus, Phys. Rev. B 48, R2862 (1993).
[48] S. Guha, J. Menéndez, J. B. Page, G. B. Adams, G. S. Spencer, J. P. Lehman, P. Giannozzi, and S. Baroni, Phys. Rev. Lett. 72, 3359 (1994).
[49] P. J. Horoyski, M. L. W. Thewalt, and T. R. Anthony, Phys. Rev. Lett. 74, 194 (1995).
[50] P. Giannozzi and W. Andreoni, Phys. Rev. Lett. 76, 4915 (1996).
[51] K. Pokhodnia, J. Demsar, A. Omerzu, D. Mihailovic, and H. Kuzmany, Phys. Rev. B 55, 3757 (1997).
[52] A. M. Rao, P. C. Eklund, J-L. Hodeau, L. Marques, and M. Nunez-Regueiro, Phys. Rev. B 55, 4766 (1997).
[53] M. F. Limonov, Yu. E. Kitaev, A. V. Chugreev, V. P. Smirnov, Yu. S. Grushko, S. G. Kolesnik, and S. N. Kolesnik, Phys. Rev. B 57, 7586 (1998).
[54] X. H. Chen, T. Takenobu, T. Muro, H. Fudo, and Y. Iwasa, Phys. Rev. B 60, 12462 (1999).
[55] V. A. Davydov, L. S. Kashevarova, A. V. Rakhmanina, V. M. Senyavin, R. Céolin, H. Szwarc, H. Allouchi, and V. Agafonov, Phys. Rev. B 61, 11936 (2000).
[56] I. D. Hands, J. L. Dunn, and C. A. Bates, Phys. Rev. B 63, 245414 (2001).
[57] A. V. Talyzin, L. S. Dubrovinsky, T. Le Bihan, and U. Jansson, Phys. Rev. B 65, 245413 (2002).
[58] A. M. Rao, P. C. Eklund, S. Bandow, A. Thess, and R. E. Smalley, Nature ( London) 388, 257 (1997).
[59] A. M. Rao, E. Richter, S. Bandow, B. Chase, P. C. Eklund, K. A. Williams, S. Fang, K. R. Subbaswamy, M. Menon, A. Thess, R. E. Smalley, G. Dresselhaus, and M. S. Dresselhaus, Science 275, 187 (1997).
[60] N. Bendiab, E. Anglaret, J. L. Bantignies, A. Zahab, J. L. Sauvajol, P. Petit, C. Mathis, and S. Lefrant, Phys. Rev. B 64, 245424 (2001).
[61] U. D. Venkateswaran, E. A. Brandsen, M. E. Katakowski, A. Harutyunyan, G. Chen, A. L. Loper,
and P. C. Eklund, Phys. Rev. B 65, 054102 (2002).
[62] W. Zhou, S. Xie, L. Sun, D. Tang, Y. Li, Z. Liu, L. Ci, Z. Zou, G. Wang, P. Tan, X. Dong, B. Xu, and B. Zhao, Appl. Phys. Lett. 80, 2553 (2002).
[63] M. E. Itkis, S. Niyogi, M. E. Meng, M. A. Hamon, H. Hu, and R. C. Haddon, Nano Lett. 2, 155 (2002).
[64] K. A. Williams, B. K. Pradhan, P. C. Eklund, M. K. Kostov, and M. W. Cole, Phys. Rev. Lett. 88, 165502 (2002).
[65] I. Ando and G. A. Webb, Theory of NMR Parameters (Academic Press, New York, 1983).
[66] R. J. Abraham, J. Fisher and P. Loftus, Introduction to NMR Spectroscopy (Chichester, New York, 1988).
[67] B. D. N. Rao and M. D. Kemple, NMR as a Structural Tool for Macromolecules: Current Status and Future Directions ( Plenum Press, New York, 1996).
[68] N. S. Ramsey, Phys. Rev. 78, 699 (1950).
[69] N. S. Ramsey, Phys. Rev. 91, 303 (1953).
[70] F. London, J. Phys. Radium (Paris) 8, 397 (1937).
[71] T. A. Keith and R. F. W. Bader, Chem. Phys. Lett. 210, 223 (1993).
[72] Gaussian 98, Revision A.9, M. J. Frisch, G. W. Trucks, H. B. Schlegel, G. E. Scuseria, M. A. Robb, J. R. Cheeseman, V. G. Zakrzewski, J. A. Montgomery, Jr., R. E. Stratmann, J. C. Burant, S. Dapprich, J. M. Millam, A. D. Daniels, K. N. Kudin, M. C. Strain, O. Farkas, J. Tomasi, V. Barone, M. Cossi, R. Cammi, B. Mennucci, C. Pomelli, C. Adamo, S. Clifford, J. Ochterski, G. A. Petersson, P. Y. Ayala, Q. Cui, K. Morokuma, D. K. Malick, A. D. Rabuck, K. Raghavachari, J. B. Foresman, J. Cioslowski, J. V. Ortiz, A. G. Baboul, B. B. Stefanov, G. Liu, A. Liashenko, P. Piskorz, I. Komaromi, R. Gomperts, R. L. Martin, D. J. Fox, T. Keith, M. A. Al-Laham, C. Y. Peng, A. Nanayakkara, M. Challacombe, P. M. W. Gill, B. Johnson, W. Chen, M. W. Wong, J. L. Andres, C. Gonzalez, M. HeadGordon, E. S. Replogle, and J. A. Pople, Gaussian, Inc., Pittsburgh PA, 1998.
[73] Use of this software does not constitute an endorsement or certification by NIST.
[74] J. C. Slater, Phys. Rev. 36, 57 (1930).
[75] W. J. Hehre, R. F. Stewart, and J. A. Pople, J. Chem. Phys. 51, 2657 (1969).
[76] J. S. Binkley and J. A. Pople, J. Am. Chem. Soc. 102, 939 (1980).
[77] W. J. Hehre, R. Ditchfield, and J. A. Pople, J. Chem. Phys. 56, 2257 (1972).
[78] R. Krishnan, J. S. Binkley, R. Seeger, and J. A. Pople, J. Chem. Phys. 72, 650 (1980).
[79] M. J. Frisch, J. A. Pople, and J. S. Binkley, J. Chem. Phys. 80, 3265 (1984).
[80] A. D. Becke, J. Chem. Phys. 98, 5648 (1993).
[81] J. C. Slater, Phys. Rev. 81, 385 (1951).
[82] A. D. Becke, Phys. Rev. A 38, 3098 (1988).
[83] S. H. Vosko, L. Wilk, and M. Nusair , Can. J. Phys. 58, 1200 (1980).
[84] C. Lee, W. Yang, and R. G. Parr, Phys. Rev. B 37, 785 (1988).
[85] J. P. Perdew and Y. Wang, Phys. Rev. B 45, 13244 (1992).
[86] H. B. Bürgi, E. Blanc, D. Schwarzenbach, S. Liu, Y. Lu, M. M. Kappes, and J. A. Ibers, Angew. Chem. Int. Ed. Engl. 41, 640 (1992).
[87] K. P. Bohnen, R. Heid, K. M. Ho, and C. T. Chan, Phys. Rev. B 51, 5805 (1995).
[88] Q. M. Zhang, J. Y. Yi, and J. Bernholc, Phys. Rev. Lett. 66, 2633 (1991).
[89] A. Szabo and N. S. Ostlund, Modern Quantum Chemistry (Macmillan, New York, 1982).
[90] R.H. Xie, G.W. Bryant, J. Zhao, V.H. Smith, Jr., A. Di Carlo, and A. Pecchia, unpublished.
[91] W. I. F. David, R. M. Ibberson, J. C. Matthewman, K. Prassides, T. J. S. Dennis, J. P. Hare, H. W. Kroto, R. Taylor, and D. R. M. Walton, Nature (London) 353, 147 (1991).
[92] C. S. Yannoni, P. P. Bernier, D. S. Bethune, G. Maijer, and J. R. Salem, J. Am. Chem. Soc. 113, 3190 (1991).
[93] R. D. Johnson, G. Meijier, and D. S. Bethune, J. Am. Chem. Soc. 112, 8983 (1990).
[94] K. Hedberg, L. Hedberg, D. S. Bethune, C. A. Brown, H. C. Dorn, R. D. Johnson, and M. de Vries, Science 254, 410 (1991).
[95] R. C. Haddon, L. E. Brus, and K. Raghavachari, Chem. Phys. Lett. 125, 459 (1986).
[96] F. Negri, G. Orlandi, and F. Zerbetto, Chem. Phys. Lett. 144, 31 (1988).
[97] R. E. Stanton and M. D. Newton, J. Phys. Chem. 92, 2141 (1988).
[98] M. Kállay, K. Németh, and P. R. Surján, J. Phys. Chem. 102, 1261 (1998).
[99] X. Q. Wang, C. Z. Wang, and K. M. Ho, Phys. Rev. B 48, 1884 (1993).
[100] D. A. Dixon, B. E. Chase, G. Fitzgerald, and N. Matsuzawa, J. Phys. Chem. 99, 4486 (1995).
[101] T. Hara, J. Onoe, and K. Takeuchi, Phys. Rev. B 63, 115412 (2001).
[102] G. Onida, W. Andreoni, J. Kohanoff, and M. Parrinello, Chem. Phys. Lett. 219, 1 (1994).
[103] I. V. Hertel, H. Steger, J. DeVries, B. Weisser, C. Menzel, B. Kamke, and W. Kamke, Phys. Rev. Lett. 68, 784 (1992).
[104] R. K. Yoo, B. Ruscic, and J. Berkowitz, J. Chem. Phys. 96, 911 (1992).
[105] J. DeVries, H. Steger, B. Kamke, C. Menzel, B. Weisser, W, Kamke, and I. V. Hertel, Chem. Phys. Lett. 188, 159 (1992).
[106] H. Steger, J. Holzapfel, A. Hielscher, W. Kamke, and I. V. Hertel, Chem. Phys. Lett. 234, 455 (1995).
[107] C. Brink, L. H. Andersen, P. Hvelplund, D. Mathur, and J. D. Voldstad, Chem. Phys. Lett. 233, 52 (1995).
[108] X.B. Wang, C.F. Ding, and L.S. Wang, J. Chem. Phys. 110, 8217 (1999).
[109] R. W. Lof, M. A. van Veenendaal, B. Koopmans, H. T. Jonkman, and G. A. Sawatzky, Phys. Rev. Lett. 68, 3924 (1992).
[110] CASTEP, distributed by Accelrys Inc., is a DFT package based on plane-wave pseudopotential technique: M.C.Payne et al., Rev. Mod. Phys. 64, 1045 (1992).
[111] R.M. Fleming, A.P. Ramirez, M.J. Rosseinsky, D.W. Murphy, R.C. Haddon, S.M. Zahurak, and A.V.

Makhija, Nature (London), 352, 787 (1991).
[112] R. Schwedhelm, L. Kipp, A. Dallmeyer, and M. Skibowski, Phys. Rev. B 58, 13176 (1998).
[113] ADF 2002.01., Theoretical Chemistry Vrije Universiteit, Amsterdam (2002).
[114] G. te Velde, F. M. Bickelhaupt, E. J. Baerends, C. Fonseca Guerra, S. J. A. van Gisbergen, J. G. Snijders, and T. Ziegler. J. Comp. Chem., 22, 9312001.
[115] G. D. Zeiss, W. R. Scott, N. Suzuki, and D. P. Chong, Mol. Phys. 37, 1543 (1979).
[116] L. Jensen, P. Th. van Duijnen, J. G. Snijders, and D. P. Chong, Chem. Phys. Lett. 359, 524 (2002).
[117] P. W. Fowler, P. Lazeretti, and R. Zanasi, Chem. Phys. Lett. 165, 79 (1990).
[118] D. J. Tozer and N. C. Handy, J. Chem. Phys. 109, 10180 (1998).
[119] M. R. Pederson and A. A. Quong, Phys. Rev. B 46, 13584 (1992).
[120] S. J. A. van Gisbergen, J. G. Snijders, and E. J. Baerends, Phys. Rev. Lett. 78, 3097 (1997) .
[121] J.-I. Iwata, K. Yabana, and G. F. Bertsch, J. Chem. Phys. 115, 8773 (2001).
[122] C. Kittel, Introduction to Solid State Physics (Wiley, New York, 1976).
[123] A. F. Hebard, R. C. Haddon, R. M. Fleming, and A. R. Korton, Appl. Phys. Lett. 59, 2109 (1991).
[124] P. Ecklund, Bull. Am. Phys. Soc. 37, 191 (1992).
[125] A. A. Quong and M. R. Pederson, Phys. Rev. B 46, 12906 (1992).
[126] R. Antoine, Ph. Dugourd, D. Rayane, E. Benichou, M. Broyer, F. Chandezon, and C. Guet, J. Chem. Phys. 110, 9771 (1999).
[127] A. Ballard, K. Bonin, and J. Louderback, J. Chem. Phys. 113, 5732 (2000).
[128] S. J. A. van Gisbergen, J. G. Snijders, and E. J. Baerends, J. Chem. Phys. 109, 10644 (1998).
[129] S. J. A. van Gisbergen, J. G. Snijders, and E. J. Baerends, J. Chem. Phys. 109, 10657 (1998).
[130] S. J. A. van Gisbergen, J. G. Snijders, and E. J. Baerends, Comput. Phys. Commun. 118, 119 (1999).
[131] D. Jonsson, P. Norman, K. Ruud, H. Ågren, and T. Helgaker, J. Chem. Phys. 109, 572 (1998).
[132] E. B. Wilson, J. C. Decius, and P. C. Cross, Molecular Vibrations (McGraw-Hill, New York, 1955).
[133] M. Cardona, in: Light Scattering in Solids II, Topics in Applied Physics, Vol. 50 (Springer, Berlin, 1982).
[134] G. Dresselhaus, M. S. Dresselhaus, and P. C. Eklund, Phys. Rev. B 45, 6923 (1992).
[135] A. Warshel and M. Karplus, J. Am. Chem. Soc. 94,

5612 (1974).
[136] Z. C. Wu, D. A. Jelski, and T. F. George, Chem. Phys. Lett. 137, 291 (1987).
[137] S. J. Cyvin, E. Brendsdal, B. N. Cyvin, and J. Brunvoll, Chem. Phys. Lett. 143, 377 (1988).
[138] D. E. Weeks and W. G. Harter, J. Chem. Phys. 90, 4744 (1989).
[139] R. A. Jishi, R. M. Mirie, and M. S. Dresselhaus, Phys. Rev. B 45, 13685 (1992).
[140] J. Onoe and K. Takeuchi, Phys. Rev. B 54, 6167 (1996).
[141] C.H. Choi, M. Kertesz, and L. Mihaly, J. Phys. Chem. A 104, 102 (2000).
[142] P. Pulay, G. Fogarasi, G. Pongor, J.E. Boggs, and A. Vargha, J. Am. Chem. Soc. 105, 7037 (1983).
[143] J. Gauss, J. Chem. Phys. 99, 3629 (1993).
[144] R. Ditchfield, Mol. Phys. 27, 789 (1974).
[145] J. R. Cheeseman, M. J. Frisch, G. W. Trucks, and T. A. Keith, J. Chem. Phys. 104, 5497 (1996).
[146] K. Wolinski, J. F. Hilton, and P. Pulay, J. Am. Chem. Soc. 112, 8251 (1990).
[147] A. D. Becke, J. Chem. Phys. 88, 2547 (1988).
[148] A. K. Jameson and C. J. Jameson, Chem. Phys. Lett. 134, 461 (1987).
[149] R. Taylor, J. P. Hare, A. K. Abdul-Sada, and H. W. Kroto, J. Chem. Soc., Chem. Commun., 1423 (1990).
[150] S. G. Kukolich, J. Am. Chem. Soc. 97, 5704 (1975).
[151] C. J. Jameson, A. K. Jameson, D. Oppusungu, S. Wille, and P. M. Burrell, J. Chem. Phys. 74, 81 (1981).
[152] J. Gauss and J. F. Stanton, J. Chem. Phys. 103, 3561 (1995).
[153] L. Vivien, P. Lancon, D. Riehl, F. Hache, and E. Anglaret, Carbon 40, 1789 (2002).
[154] L.W. Tutt and A. Kost, Nature (London) 356, 225 (1991).
[155] C. Liu, G. Zhao, Q. Gong, K. Tang, X. Jin, P. Cui, and L. Li, Opt. Commun. 184, 309 (2000).
[156] A.G.H. Barbosa and M.A.C. Nascimento, Chem. Phys. Lett. 343, 15 (2001), and references therein.
[157] R. Martel, T. Schmidt, H.R. Shea, T. Hertel, and Ph. Avouris, Appl. Phys. Lett. 73, 2447 (1998).
[158] V. Derycke, R. Martel, J. Appenzeller, and Ph. Avouris, Nano Lett. 1, 453 (2001).
[159] X. Liu, C. Lee, C. Zhou, and J. Han, Appl. Phys. Lett. 79, 3329 (2001).
[160] M. Bockrath, J. Hone, A. Zettl, P.L. McEuen, A.G. Rinzler, and R.E. Smalley, Phys. Rev. B 61, 10606 (2000).
[161] C. Joachim, J. K. Gimzewski, and A. Aviram, Nature (London) 408, 541 (2000).

