Seeking Evolution of Dark Energy

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Abstract

We study how observationally to distinguish between a cosmological constant (CC) and an evolving dark energy with equation of state $\omega(Z)$. We focus on the value of redshift Z^* at which the cosmic late time acceleration begins and $\ddot{a}(Z^*) = 0$. Four $\omega(Z)$ are studied, including the well-known CPL model and a new model that has advantages when describing the entire expansion era. If dark energy is represented by a CC model with $\omega \equiv -1$, the present ranges for $\Omega_\Lambda(t_0)$ and $\Omega_m(t_0)$ imply that $Z^* = 0.743$ with 4% error. We discuss the possible implications of a model-independent measurement of Z^* with better accuracy.

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Introduction   The interface between astrophysics and particle physics has never been stronger than now because our knowledge of gravity comes in large part from observational astronomy and cosmology. At particle colliders, seeking the constituent of dark matter is an important target of opportunity.

Dark energy is widely regarded as the most important issue in all of physics and astronomy. Other than the Cosmological Constant (CC) model, and the very interesting, if not yet fully satisfying, usage of string theory, there is no compelling theory. So we are motivated to pursue a purely phenomenological approach to attempt to make progress towards the understanding of dark energy.

The discovery of cosmic acceration [1, 2] in 1998 has revolutionized theoretical cosmology. The simplest theoretical interpretation is as a CC with constant density and equation of state (EoS) $\omega \equiv -1$. We shall refer to alternatives to the CC model, with $\omega(Z)$ redshift-dependent, as evolutionary dark energy.

The equations which govern cosmic history, which assume Einstein's equations, isotropy, homogeneity (FLRW metric [3–6]), and flatness as expected from inflation, are (with $c = 1$):

$$H(t)^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)\rho \tag{1}$$

and

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{4\pi G}{3}(\rho + 3p), \tag{2}$$

together with the continuity equation:

$$a\frac{d\rho}{da} = -3(\rho + p). \tag{3}$$

In these equations, p is pressure and ρ is the density with components $\rho = \rho_\Lambda + \rho_m + \rho_\gamma$. Although, for small redshifts, the radiation term is by far the smallest, we still include it.

Using Eq. (2), the CC model with $\omega \equiv -1$, and the WMAP7 values[1] [7]

$$\Omega_\Lambda(t_0) = 0.725 \pm 0.016 \quad \text{and} \quad \Omega_m(t_0) = 0.274 \pm 0.013, \tag{4}$$

we find that [8]

$$Z^* = \left(\frac{2\Omega_\Lambda(t_0)}{\Omega_m(t_0)}\right)^{\frac{1}{3}} - 1 \;\; = \;\; 0.743 \pm 0.030. \tag{5}$$

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[1] Note that we use the WMAP7 [7] value for $\Omega_m(t_0)$, unless explicitly stated otherwise. We use $\Omega_\gamma(t_0) = 5 \times 10^{-5}$.

With evolution, Eq. (5) is modified. It is worth mentioning that Z^* is a constant of Nature, like Hubble's constant, which can in principle be measured precisely, without reference to any theoretical model. We shall introduce various evolutionary models, including the very popular CPL model [9, 10] $\omega^{(CPL)}(Z)$ and a new proposal $\omega^{(new)}(Z)$ that is more physically motivated with respect to the whole expansion history[2]. We present figures which make predictions for Z^*, and we suggest

slowly varying criteria which support the use of $\omega^{(new)}(Z)$ over $\omega^{(CPL)}(Z)$.

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[2] References [9, 10] do, however, specify that the CPL model is to be used only for $0 \leq Z \lesssim 2$.

Evolutionary Dark Energy Models   When we consider dark energy models, we must specify an equation of state $\omega(Z)$. There is an infinite number of choices for $\omega(Z)$: our objective in the present article is to suggest a sensible choice for the functional form of $\omega(Z)$, which can be valid for the entire extent of the present expansion era. Since there exists no compelling evolutionary theory, we choose to consider models for $\omega(Z)$ each containing two free parameters, which we designate as ω_0 and ω_1, each ornamented by a superscript which denotes the model. To add more parameters would be premature. The first three are aleady in the literature, while the fourth is, to our knowledge, new.

(i) <u>Linear model</u>   (lin)

The evolutionary equation of state (EEoS) is:

$$\omega^{(lin)}(Z) = \omega_0^{(lin)} + \omega_1^{(lin)} Z \tag{6}$$

(ii) <u>Chevallier-Polarski-Linder model</u> [9, 10]   (CPL)

The EEoS for CPL is:

$$\omega^{(CPL)}(Z) = \omega_0^{(CPL)} + \omega_1^{(CPL)} \frac{Z}{1+Z} \tag{7}$$

(iii) <u>Shafieloo-Sahni-Starobinsky model</u> [11]   (SSS)

The SSS version of the EEoS is:

$$\omega^{(SSS)}(Z) = -\frac{1 + \tanh[(Z - \omega_0^{(SSS)})\omega_1^{(SSS)}]}{2} \tag{8}$$

(iv) <u>New proposal</u>   (new)

Here we consider, as a novel EEoS, a simple modification of the CPL EEoS:

$$\omega^{(new)}(Z) = \omega_0^{(new)} + \omega_1^{(new)} \frac{Z}{2+Z} \tag{9}$$

Analysis of the Models   We begin with the model where the EEoS is linear in redshift, $\omega^{(lin)}(Z)$. In Fig. 1 are shown $\omega_0^{(lin)} - \omega_1^{(lin)}$ curves for Z^* in the range $0.4 \le Z^* \le 0.9$. Several interesting features of Fig. 1 deserve discussion. First, the dot at (0, -1) confirms $Z^* = 0.743 \pm 0.030$ for the CC model. If Z^* is measured to be $Z^* > 0.75$, it is necessary that $\omega_1^{(lin)} < 0$. If Z^* is meaured to be $Z^* > 0.831$, we find that $\omega_0^{(lin)} > -1$. As Z^* increases, the requisite $\omega^{(lin)}(Z)$ becomes more and more distinct from the CC model. In Fig. 1, we note that for any measured value of Z^*, there are two possible values of $\omega_0^{(lin)}$ for each value of $\omega_1^{(lin)}$.

The EEoS for $\omega^{(lin)}(Z)$ possesses singular behavior for $Z \to \infty$ because $\omega^{(lin)}(Z) \to \pm\infty$ for $\omega_1^{(lin)}$ being positive or negative, respectively.

The reader will remark the confluence of the Z^*-orbits in Fig. 1, which is an artifact of the restriction to $0.4 \le Z^* \le 0.9$. For values of Z^* near to but outside this range, the confluence desists. A similar phenomenon appears in later plots.

We next discuss the model $\omega^{(CPL)}(Z)$ [9, 10] in which the EEoS is linear in $(1-a)$, where a is the scale factor. In Fig. 2 are shown $\omega_0^{(CPL)} - \omega_1^{(CPL)}$ curves for Z^* in the range $0.6 \le Z^* \le 1.0$. There are several features of Fig. 2 to note. The dot at (0, -1) confirms $Z^* = 0.8$, which is the resulting Z^* value from Eq.(5) when the value $\Omega_m(t_0) = 0.255$ is used, as determined by [11] as the best fit value for the CPL model. If Z^* is measured to be $Z^* > 0.810$, we find that $\omega_1^{(CPL)} < 0$. As Z^* increases, the necessary $\omega^{(CPL)}(Z)$ becomes more and more distinct from the CC model. In Fig. 2, we see that for any measured value of Z^*, there are two possible values of $\omega_0^{(CPL)}$ for each value of $\omega_1^{(CPL)}$.

Now we examine Fig. 3 for the SSS model. We see that for a certain range of $Z^* > 0.8$, which includes the best fit value of Z^* which we discuss later, both $\omega_1^{(SSS)}$ and $\omega_0^{(SSS)}$ must be greater than zero. Note also the degeneracy in $\omega_0^{(SSS)}$ for $Z^* = 0.85, 0.9$.
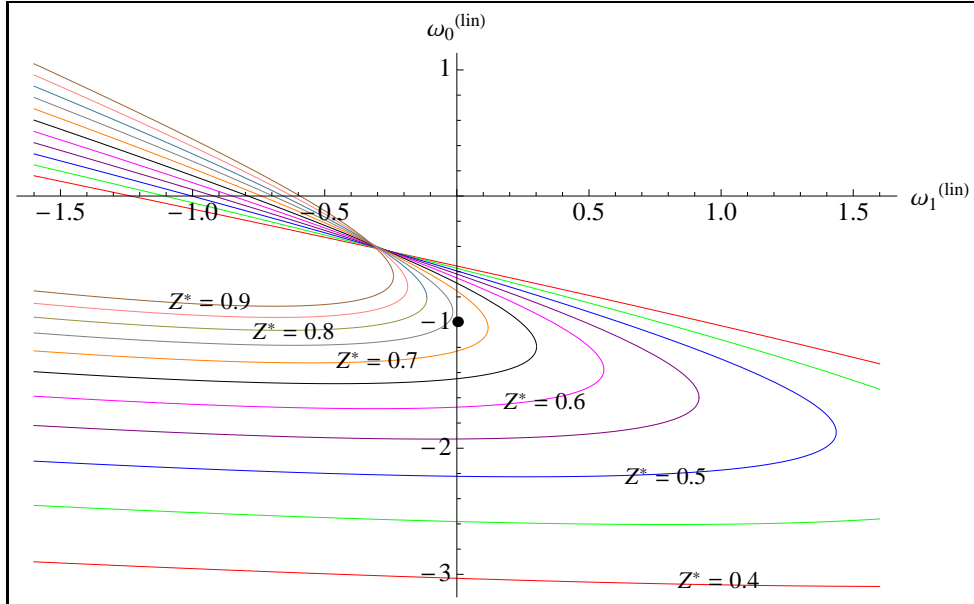

FIG. 1. $\omega_0^{(lin)}$ is plotted against $\omega_1^{(lin)}$ for $0.4 \le Z^* \le 0.9$ in increments of 0.05. The dot at (0,-1) represents the CC model. Here we use $\Omega_m(t_0) = 0.275$, which is consistent with the result of [12] for the best fit for this model.
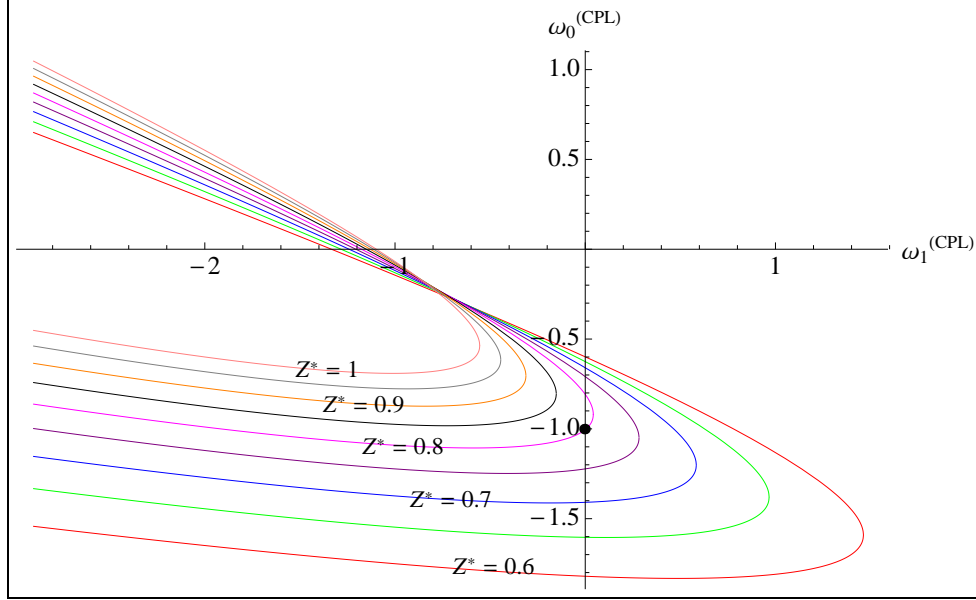
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FIG. 2. $\omega_0^{(CPL)}$ is plotted against $\omega_1^{(CPL)}$ for $0.6 \leq Z^* \leq 1.0$ in increments of 0.05. The dot at (0,-1) represents the CC model. We use $\Omega_m(t_0) = 0.255$, which is consistent with the analysis in [11] for the best fit for this model.

Fig. 4 displays the new model. Once again we see that the CC model has $Z^* = 0.743$. For $Z^* \geq 0.81$, $\omega_0^{(new)} > -1.0$. We note that for $Z^* > 0.75$, $\omega_1^{(new)}$ must be negative.

In Fig. 5, we show $\omega^{(lin)}(Z)$, $\omega^{(CPL)}(Z)$, and $\omega^{(SSS)}(Z)$ as a function of Z (including the future, $-1 \leq Z < 0$) using the best-fit parameters of [12] for $\omega^{(lin)}(Z)$ and those of [11] for the other 2 models. $\omega^{(new)}(Z)$ is also plotted using the choice of $\omega_0^{(new)} = -1$ and $\omega_1^{(new)} = 0.1$.

Unlike the CC model, where the future of the universe is infinite exponential expansion, the best fit for $\omega^{(lin)}(Z)$ necessarily leads to a big rip, at a finite time in the future [13]. The EEoS for $\omega^{(CPL)}(Z)$ possesses singular behavior for $Z \to -1$ because $\omega^{(CPL)}(Z) \to \pm\infty$ for $\omega_1^{(CPL)}$ being negative or positive, respectively. For the SSS model, $\omega^{(SSS)}(Z)$ varies in the range $0 > \omega^{(SSS)}(Z) > -1$.

As for our fourth and last model $\omega^{(new)}(Z)$, from Eq.(9), we note that this choice has the advantage that for all Z this EEoS lies between $(\omega_0^{(new)} - \omega_1^{(new)})$ and $(\omega_0^{(new)} + \omega_1^{(new)})$. This is illustrated in Fig 5 where, for the choices $\omega_0^{(new)} = -1.0$ and $\omega_1^{(new)} = +0.1$, the EEoS falls smoothly from -0.9 at the big bang to -1.1 at the big rip.

Slowly Varying Criteria   We now discuss which $\omega(Z)$ for dark energy is the best for observers to employ. Regretfully, there is no theoretical guidance about evolution. Nevertheless, we here propose slowly varying criteria which is based purely on grounds of aesthetics and, especially, conservatism. All the present data are consistent with the CC model $\omega^{(CC)} \equiv -1$. We choose to remain proximate to it, as supported by [14, 15]. One consideration is that we prefer any global $\omega(Z)$ to have analytic, non-singular behavior for $Z \to -1$ and $Z \to \infty$. Finally, we propose to impose the inequality representing conservatism,

$$|\omega(Z) + 1| \ll 1 \quad \text{for all} \quad -1 \leq Z < \infty. \tag{10}$$

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Next, we consider application of our slow variation criteria to the four specific models we have discussed in the present article. For this task, Fig. 5 will be used. To be fair to their inventors, these EEoS were intended to apply for only a limited range of redshift.

(i) <u>Linear model</u>

By studying the EEoS in Eq. (6), we notice that for $Z \to +\infty$, $\omega^{(lin)}(Z)$ approaches $\pm\infty$ depending on the sign of $\omega_1^{(lin)}$. Also, $\omega^{(lin)}(Z)$ violates the criterion of Eq. (10). We conclude that this linear model is disfavored according to our slow variation criteria.

(ii) <u>CPL model</u>

By examining the EEoS in Eq. (7), we note that as $Z \to -1$, $\omega^{(CPL)}(Z) \to \mp\infty$ for a $(\pm)$ sign for $\omega_1^{(CPL)}$. Therefore, according to our criteria, this model is disfavored.

(iii) <u>SSS model</u>

Looking at Eq. (8), we see that $\omega^{(SSS)}(Z)$ varies from 0 to -1 for the best fit given in [11], so it is non-singular. By this token, however, it does not satisfy Eq. (10). This model, then, is also disfavored by our criteria. It should be noted, though, that [11] studied this EEoS as a toy model to illustrate the importance of an EEoS that fits data well for small and large positive Z.

(iv) <u>New model</u>

This model is non-singular for all $-1 \leq Z < \infty$ because $[Z/(2+Z)]$ varies smoothly from -1 to $+1$, as can be seen from examining Eq. (9). It also satisfies Eq. (10) if we choose appropriate values for $\omega_0^{(new)}$ and $\omega_0^{(new)}$.
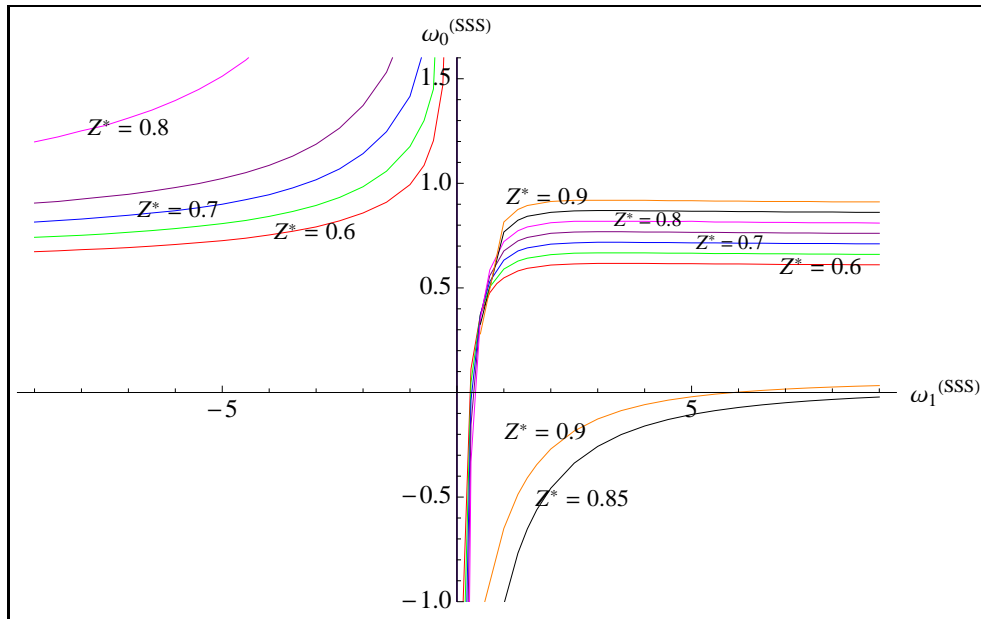

FIG. 3. $\omega_0^{(SSS)}$ is plotted against $\omega_1^{(SSS)}$ for $0.6 \leq Z^* \leq 0.9$ in increments of 0.05. We use $\Omega_m(t_0) = 0.255$, which is consistent with the analysis in [11] for the best fit for this model.
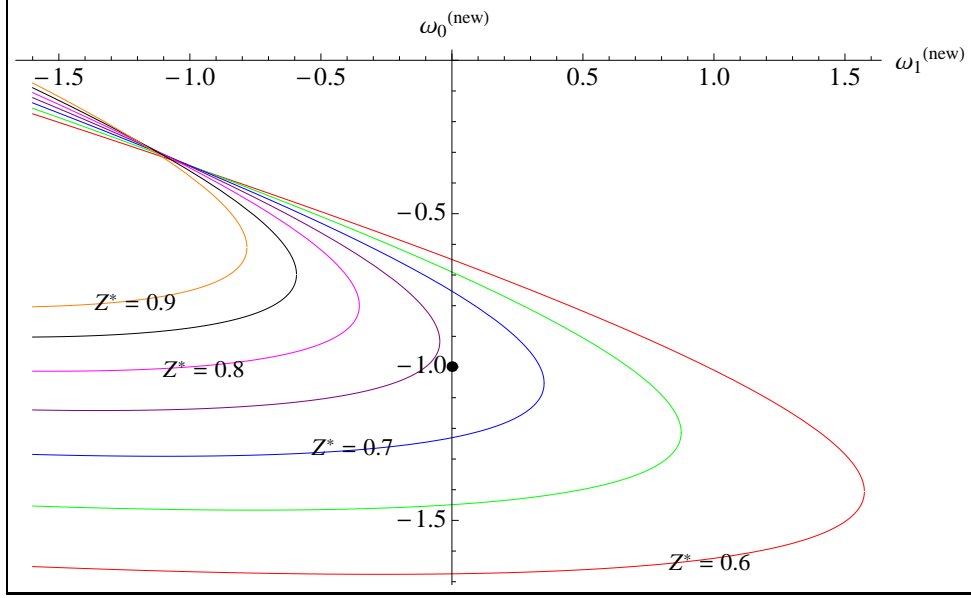
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FIG. 4. $\omega_0^{(new)}$ is plotted against $\omega_1^{(new)}$ for $0.6 \leq Z^* \leq 0.9$ in increments of 0.05. The dot at $(0,-1)$ represents the CC model. We use $\Omega_m(t_0) = 0.275$.
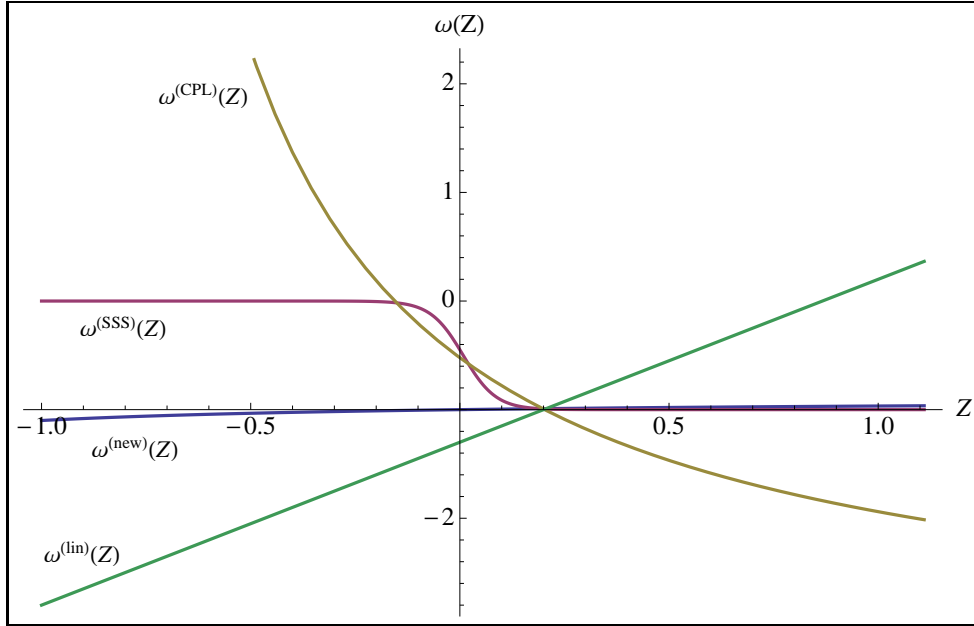

FIG. 5. $\omega(Z)$ for all the models is plotted against Z from the highest Z^* value to $Z = -1$ (when the scale factor a is infinite). The horizontal axis at $\omega(Z) = -1$ is the line representing the CC model. We use $\omega_0^{(lin)} = -1.3$, $\omega_1^{(lin)} = 1.5$, and $Z^* = 0.4067$ (from the best fit for this model given in [12]). We use $\omega_0^{(CPL)} = -0.522$, $\omega_1^{(CPL)} = -2.835$, and $Z^* = 1.11$ for the CPL model and $\omega_0^{(SSS)} = 0.008$, $\omega_1^{(SSS)} = 12.8$, and $Z^* = 0.855$ for the SSS model (both from the respective best fits given in [11]). $\omega^{(new)}(Z)$ is plotted using $\omega_0^{(new)} = -1$ and $\omega_1^{(new)} = 0.1$, which gives $Z^* = 0.729$.

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Discussion and Conclusions The outstanding observational question about dark energy is whether it is a CC model with $\omega(Z) \equiv -1$ or an evolutionary model with a non-trivial EEoS. The model-independent observational measurement of Z^* is very useful for making this distinction.

The theoretical prediction of Z^* is, however, dependent on the EoS that is assumed. The value for the CC model using the WMAP7 values of $\Omega_m(t_0)$ and $\Omega_\Lambda(t_0)$ is $Z^* = 0.743 \pm 0.030$. As the data become even more precise, the error on Z^* will diminish, making it observationally easier to detect deviation, if any, from the CC model.

In the four EEoS models listed earlier, the possible values of Z^* for different values of the parameters ω_0 and ω_1 can be read off from our plots. These plots show that degeneracies appear. For a given Z^* and a specific type of EEoS, there is an allowed curve in the ω_1-ω_0 plane. For all $0.4 \leq Z^* \leq 1.0$, there exist disallowed regions in the ω_1-ω_0 plane.

To go further, we have to give criteria for selecting one EEoS. The most popular choice in the last few years has been the CPL model [9, 10] because it approximates the linear model at low redshift. Also, it has a simple interpretation in terms of the scale factor:

$$\omega^{(CPL)}(Z) = \omega_0^{(CPL)} + \omega_1^{(CPL)}(1 - a(Z)) \tag{11}$$

However, as $a(Z) \to \infty$ ($Z \to -1$), the CPL model diverges. Of course, the authors of [9, 10] intended their model to be applicable only for a limited range of Z. However, it seems preferable for $\omega(Z)$ to cover the entire range of cosmic history, both the past and future.

Our novel EEoS also approximates the linear model at low redshift. It has, like the CPL model, a straightforward physical interpretation in terms of the scale factor:

$$\omega^{(new)}(Z) = \omega_0^{(new)} + \omega_1^{(new)} \left(\frac{1 - a(Z)}{1 + a(Z)} \right) \tag{12}$$

We think one advantage of this new model over the CPL model is that it is non-singular for $-1 \leq Z < \infty$. A second advantage is that it can satisfy the slow variation criterion of Eq. (10).

In conclusion, the choice of an evolutionary alternative to the CC model depends theoretically on constraining the EEoS, and we have proposed a new EEoS which not only has a simple physical interpretation but also is well-behaved for all possible redshifts. The model-independent extraction of Z^* from observational data is a familiar process [16]. A more accurate model-independent estimate of Z^* by global fits to all relevant data is worthwhile. It is an interesting issue how the present considerations of evolutionary dark energy are related to the possible occurence of a big rip [13]. This requires ultra-negative pressures of dark energy, so it is interesting that situations involving phantom energy have appeared in the context of extra spatial dimensions in string theory [17].

It has been argued [18] that dark energy effects can be detected only by studying physical systems as large as galaxies. Thus, it is unlikely that any terrestrial experiment can be sensitive to dark energy.

Understanding dark energy may, or may not (in the CC model), require a gravitational theory more complete than general relativity, which has been accurately confirmed [19] only at the scale of the solar system, say $\sim 10^{12}$ meters, while dark energy operates above the galactic size, say $\sim 10^{20}$ meters. Thus, it is likely, even probable, that study of dark energy will inform us, in the near future, how to go beyond Einstein, which is the most important direction both for particle physics and astrophysics.

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