Composite Dirac fermions in graphene

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Generalizing the notion of composite fermions to the case of "pseudo-relativistic" Quantum Hall phenomena in graphene, we discuss a possible emergence of compressible states at the filling factors $\nu = \pm 1/2, \pm 3/2$. This analysis is further extended to the nearby incompressible states viewed as Integer Quantum Hall Effect of composite Dirac fermions, as well as those that might occur at $\nu = 0, \pm 1$ as a result of (pseudo)spin-singlet pairing between the latter.

The traditional interest in Quantum Hall Effect has been rekindled by the recent experiments on mono- and double-layers of graphene where the interplay between unscreened Coulomb interactions and pseudo-relativistic kinematics of the Dirac quasiparticles has long been expected to harbor a host of novel phenomena[1].

In graphene mono-layers, the Integer Quantum Hall Effect (IQHE) plateaus were found at the integer values $\sigma_{xy} = \nu = (4n+2)$, $n = 0, \pm 1, \pm 2, \ldots$[2] (hereafter, we put $\hbar = e = c = 1$, and measure all the conductivities in units of $e^2/h \equiv 1/2\pi$). Elaborating on the earlier insight of Refs.[1], this observation was readily explained[3] by treating the low-energy excitations in graphene as (pseudo)relativistic Dirac fermions with linear dispersion and speed $v_F \sim 10^6 m/s$. These quasiparticles carry a physical spin $s = 1/2$ and possess an additional orbital ("pseudo-spin" or "valley") quantum number (hereafter referred to as R and L) corresponding to the double degeneracy of the electronic Bloch states in graphene.

The resulting $SU(4)$ symmetry of the non-interacting Hamiltonian survives the long-range Coulomb interactions, although it gets broken in the presence of the Zeeman and various additional short-range (Hubbard-like) interaction terms. A number of implications of this symmetry which generalizes independent rotations in the spin and valley subspaces have been explored in several recent papers[4].

For one, by drawing a parallel with the previous studies of spin-unpolarized double-layer Quantum Hall systems, it was argued that, apart from the Dirac kinematics, the situation in graphene is similar to that occurring in the double-layer systems in the limit of vanishing interlayer tunneling. Thus, a graphene analog of the Quantum Hall Ferromagnet was predicted to occur, which type of strongly correlated states would manifest itself as additional (interaction-induced) plateaus at *all* the integer filling factors[4].

In a recent experiment, additional plateaus were indeed observed at $\sigma_{xy} = 0, \pm 1, \ldots$[5], thus suggesting a complete lifting of the spin and valley degeneracies at the lowest ($n = 0$) relativistic Landau level (LL). As an alternative interpretation, it was pointed out[6] that the behavior reported in Ref.[5] can also be explained by invoking the "magnetic catalysis" scenario of Ref.[7] where, contrary to the predictions of Refs.[4], the valley degeneracy gets lifted *only* at the 0^{th} LL, in agreement with the data of Ref.[5].

Still awaiting its observation, however, is a graphene counterpart of the Fractional Quantum Hall Effect (FQHE). By analogy with FQHE in the conventional ("non-relativistic") two-dimensional electron gas (2DEG) with parabolic quasiparticle dispersion, one might expect that its graphene analog can also be studied by adapting the idea of statistical flux attachement to the case of the Dirac fermions.

In the present Letter, we discuss such a procedure, thereby setting the stage for a systematic analysis of the (potentially, much richer than in the case of the conventional 2DEG) realm of FQHE phenomena in graphene.

The flux attachment recipe would usually be applied to a half-(or, more generally, $1/2p$-, where p is an integer) filled uppermost LL, while the rest of the system would be treated as an inert incompressible background. In a generic $SU(N)$-invariant system, the Chern-Simons Lagrangian implementing a transformation from the original electrons to N-component composite Dirac fermions (CDFs) takes the form

$$L = \sum_{\alpha}^{N} \int_{\mathbf{r}} \Psi^{\dagger}_{\alpha} \hat{\gamma}_{\alpha} (i\partial^i + a^i_{\alpha} + A^i) \Psi_{\alpha} + \frac{1}{4\pi} \sum_{\alpha,\beta}^{N} \int_{\mathbf{r}} K^{-1}_{\alpha\beta} \epsilon_{ijk} a^i_{\alpha} \partial^j a^k_{\beta}$$

$$+ \frac{v_F}{4\pi} \sum_{\alpha,\beta}^{N} \int_{\mathbf{r}} \int_{\mathbf{r}'} {\Psi^{\dagger}}_{\alpha}(\mathbf{r}') \Psi_{\alpha}(\mathbf{r}') \frac{g}{|\mathbf{r} - \mathbf{r}'|} {\Psi^{\dagger}}_{\beta}(\mathbf{r}) \Psi_{\beta}(\mathbf{r}) \quad (1)$$

Here $g_0 = 2\pi e^2/\epsilon_0 v_F \sim 3$ is the bare Coulomb coupling, the vector potential $A^i = (0, -By/2, Bx/2)$ represents an external magnetic field, and the matrices $\hat{\gamma}_{\alpha} = (\mathbf{1}, \hat{\sigma}_x, (-1)^{\alpha} \hat{\sigma}_y)$ act in the space of spinors Ψ_{α} composed of the values of the CDF wave functions on the two sublattices of the bi-partite lattice of graphene.

In a multi-component system, the statistical flux provided by the Chern-Simons fields a^i_{α} can be attached in a number of different ways, the choice between which should ultimately be determined by the nature of the ground state in question. Accordingly, there exist different choices of the integer-valued matrix $\hat{K}$, the only condition imposed upon which is that the transformed CDFs retain their (mutual) fermionic statistics. This requirement can be readily satisfied, provided that all the matrix elements of $\hat{K}$ are even integers, though.

By varying Eq.(1) with respect to the Lagrange multipliers a^0_α, one obtains a set of constraints

$$\rho_\alpha =<\Psi^\dagger_\alpha \Psi_\alpha>=\frac{1}{2\pi}\sum_\beta^N K^{-1}_{\alpha\beta}<\nabla\times \mathbf{a}_\beta> \qquad (2)$$

which determine the average values of the effective fields $b_\alpha = B- <\nabla\times \mathbf{a}_\alpha>$ experienced by the CDF α-species.

In the FQHE states viewed as the CDF IQHE, each of the CDF species occupies an integer number $\nu_\alpha = 2\pi\rho_\alpha/b_\alpha = m_\alpha$ of the effective LLs. The total electronic filling factor is then given by the expression

$$\nu=\sum_\alpha^N \frac{2\pi\rho_\alpha}{B} = Tr(\mathbf{1}+\hat{K}\hat{m})^{-1}\hat{m} \qquad (3)$$

where $\hat{m} = diag[m_1,\dots,m_N]$.

Integrating the CDFs out in the standard manner, one obtains a quadratic Lagrangian for the vector fields

$$L_{eff}[a_\alpha, A]=\frac{1}{2}\sum_\alpha^N (a^i_\alpha+A^i)\Pi^\alpha_{ij}(a^j_\alpha+A^j) + \frac{1}{4\pi}\sum_{\alpha,\beta}^N \epsilon_{ijk}K^{-1}_{\alpha\beta}a^i_\alpha\partial^j a^k_\beta \qquad (4)$$

where $\Pi^\alpha_{ij}(\omega,\mathbf{q})$ is the CDF polarization operator.

Next, by eliminating all the statistical fields, one derives the RPA-like formula for the physical electromagnetic response function $\hat{\chi}^{-1}_{ij}(q)=\hat{\Pi}^{-1}_{ij}(q)+2\pi\hat{K}\epsilon_{ijk}q^k/q^2$ where $q^k=(\omega,\mathbf{q})$. Quantized values of the Hall conductivity corresponding to the putative FQHE plateaus are given by the formula

$$\sigma_{xy}=\sum_\alpha^N \sigma^H_\alpha - \sum_{\alpha,\beta}^N \sigma^H_\alpha(\hat{\sigma}^H+\hat{K}^{-1})^{-1}_{\alpha\beta}\sigma^H_\beta \qquad (5)$$

where $\hat{\sigma}^H=(2\pi/\omega)Im\hat{\Pi}_{xy}|_{\omega,\mathbf{q}\to 0}$ is a tensor of the CDF Hall conductivities.

As one important example of this general construction, attaching two units of the α-type flux ($\int_\mathbf{r} <\nabla\times\mathbf{a}_\alpha>=\pm 4\pi$) to the CDFs of the same type is equivalent to choosing $\hat{K}=\pm diag[2,\dots,2]$, in which case Eq.(3) yields

$$\sigma_{xy}=\sum_\alpha^N\frac{\nu_\alpha}{2\nu_\alpha\pm 1} \qquad (6)$$

Notably, the overall Hall conductivity (6) is given by a "parallel" combination of the conductivities of the individual species (each of which is, in turn, given by a "series" combination of the responses to the physical electromagnetic $\mathbf{A}$ and the corresponding statistical field $\mathbf{a}_\alpha$). This composition rule should be contrasted against the naive one, $\sigma_{xy}=\sum^N_\alpha \nu_\alpha/(2\sum^N_\alpha\nu_\alpha+1)$ (see, e.g., the last reference in Ref.[3]), which would have resulted from a series connection between the response to $\mathbf{A}$ and a parallel combination of all the statistical fields (or a single field that couples symmetrically to all the fermion species).

A more general case of the diagonal matrix $\hat{K} = diag[2p_1,\dots,2p_N]$ gives rise to a formula similar to Eq.(6) (with the factor of 2 replaced by $2p_\alpha$ in the denominator of the α-term). Furthermore, any "entangled" way of attaching the fluxes described by a non-diagonal $\hat{K}$-matrix yields an expression different from Eq.(6). Such alternative choices of the $\hat{K}$-matrix (which we do not consider in this work) would be physically appropriate if different CDF species formed mutually coherent states. In the spin-polarized ($N=2$) case, one example of this sort is provided by the matrix $\hat{K}=\begin{pmatrix}0 & 2\\ 2 & 0\end{pmatrix}$ which has been previously discussed in the context of the double-layer $\nu=1/2$ system.

The $SU(4)$-symmetry of Eq.(1) gets lowered in the presence of various symmetry-breaking terms of the form $\delta L=\sum^N_{\alpha,\beta}\int_\mathbf{r}\Psi^\dagger_\alpha\Lambda_{\alpha\beta}\Psi_\beta$ which can be of both, single-particle and many-body, nature (observe that these terms retain their form after the statistical transformation, if the matrix $\hat{\Lambda}$ is diagonal).

At the mean-field (Hartree-Fock) level, the list of such terms includes the (exchange-enhanced) Zeeman term ($\hat{\Lambda}_Z = E_Z\hat{\mathbf{1}}\otimes\hat{\mathbf{1}}\otimes\hat{\sigma}_z$) and a parity-odd mass term ($\hat{\Lambda}_M=\Delta\hat{\sigma}_z\otimes\hat{\mathbf{1}}\otimes\hat{\mathbf{1}}$ or $\Delta\hat{\sigma}_z\otimes\hat{\mathbf{1}}\otimes\hat{\sigma}_z$), where the first, second, and third factors refer to the sublattice, valley, and spin subspaces, correspondingly. These two terms lift, respectively, the spin and valley degeneracies of the 0^{th} LL, thereby splitting it into four individual sub-levels. Notice that the parity-even mass terms ($\sim\hat{\sigma}_z\otimes\hat{\sigma}_z\otimes\hat{\mathbf{1}}$ or $\hat{\sigma}_z\otimes\hat{\sigma}_z\otimes\hat{\sigma}_z$) which have been previously discussed in the context of spin-orbit coupling and Spin Hall Effect in graphene can only split the 0^{th} LL into *two* sub-levels (with the Zeeman term present).

The symmetry-breaking terms appear to be instrumental for describing, e.g., the plateau transitions $0\to\pm 1$, in which case all the four sub-levels of the 0^{th} LL are fully spin- and valley-resolved, as suggested by the strong-field ($B\gtrsim 20T$) data of Ref.[5].

The nearby FQHE states at $|\nu|<1$ can then be constructed with the use of single-component ($N=1$) CDFs which occupy an integer number of the effective LLs. Naturally, these states fall into the standard Jain's series converging towards $\nu=\pm 1/2$

$$\sigma^{N=1}_{xy}=\pm\nu^\pm_m=\pm\frac{m}{2m\pm 1} \qquad (7)$$

where $m=1,2,\dots$ and the overall $\pm$ sign is not correlated with that in the definition of $\nu^\pm_m$. Similar fractions can occur near $\nu=\pm 3/2$, thereby giving rise to the FQHE plateaus at $\sigma_{xy}=\pm(1+\nu^\pm_m)$. Since the single-particle CDF states are non-degenerate, a relative stability of the even vs odd-numerator fractions (7) is not an issue (cf. with the discussion in the last of Ref.[9]).

In the case of a residual $SU(2)$ degeneracy of either spin or valley origin, the number of relevant CDF species becomes $N = 2$. Conceivably, such a situation can occur at the $0 \to \pm 2$ plateau transitions (where, say, $\nu^{\uparrow}_{L,R} = 1/2$ or $\nu^{\uparrow,\downarrow}_{L} = 1/2$, depending on the relative magnitude of E_Z and Δ).

The data of Ref.[5] suggest that at moderately strong fields ($10T \lesssim B \lesssim 20T$) the spin degeneracy gets lifted first (at least, at the $n = \pm 1$ LLs). In this scenario, the residual valley degeneracy gives rise to a series of valley-unpolarized IQHE states of the $N = 2$ CDFs which converge towards $\nu = \pm 1$ and correspond to the plateaus

$$\sigma^{N=2}_{xy} = \pm(1 - \nu^{\mp}_{m\pm 1} + \nu^{\pm}_{m}) = \pm\frac{2m}{2m \pm 1} \qquad (8)$$

where $\nu^{\pm}_{m}$ is defined in Eq.(7).

In Eq.(8), we took into account the fact that the numbers of occupied (spin-polarized) effective LLs for the $L-$ and $R-$type CDFs differ by *one* as a result of the spectral anomaly at the 0^{th} CDF LL (the R-type states reside at the energy $E = \Delta$, whereas the L-type ones are at $E = -\Delta$). As a result, the partial Hall conductivities of the R- and L-species as functions of chemical potential obey the relation $\sigma^{R}_{xy}(-\mu) = -\sigma^{L}_{xy}(\mu)$, although their sum $\sigma^{R}_{xy} + \sigma^{L}_{xy}$ is, of course, an odd function of μ. It is worth noting that, from a formal standpoint, the anomalous IQHE observed in Refs.[2] has the very same origin.

Lastly, $SU(4)$-invariant spin- and valley-unpolarized states would be described in terms of $N = 4$ CDFs which provide a mean-field picture of the $-2 \to 2$ plateau transition in terms of the half-filled 0^{th} LL ($\nu^{\uparrow,\downarrow}_{L,R} = 1/2$) which is appropriate at relatively weak fields ($B \lesssim 10T$), according to the data of Ref.[5].

Incompressible spin- and valley-unpolarized $N = 4$ CDF states would then correspond to the plateaus

$$\sigma^{N=4}_{xy} = 2(\nu^{\pm}_{m} - \nu^{\mp}_{m\pm 1}) = \pm\frac{2}{2m \pm 1} \qquad (9)$$

Notably, the series (9) includes (pseudo)spin-singlet states at $\nu = 2/3$ and $2/5$, thus providing a possible CDF picture of the exact ground states found at these filling factors in the recent numerical studies[9].

By analogy with the conventional 2DEG[8], we conjecture that the parent CDF states at $\nu^{(N=1)} = k - 1/2$, ($k = -1, 0, 1, 2$), $\nu^{(N=2)} = \pm 1$ and $\nu^{(N=4)} = 0$ for $N = 1, 2$, and 4, respectively, behave as compressible "CDF metals" characterized by the presence of a Fermi surface of radius $k^{*}_{F} = (2\nu B/N)^{1/2}$.

The mean-field CDF dispersion relation remains linear and the effective CDF velocity determined by the strength of the Coulomb interaction, $v^{*}_{F} \sim g v_F N^{1/2}$, is comparable to v_F for g and N of order one. Due to their inherited Dirac kinematics, the Subnikov-de-Haas oscillations of the CDF resistivity at small deviations from the compressible fractions $\nu^{(N)}$ are expected to show the same Berry phase of π as that of the original Dirac quasiparticles in weak fields[2].

The CDF Fermi energy $E^{*}_{F} = v^{*}_{F} k^{*}_{F} \sim g v_F B^{1/2}$ appears to be of the same order as the distance $E_1 - E_0 = v_F(2B)^{1/2}$ between the 0^{th} and $\pm 1^{th}$ LLs, suggesting that in graphene the LL mixing effects are potentially more important than in the conventional 2DEG where they get suppressed with increasing field. Moreover, the LL mixing becomes stronger with an increasing number n of the occupied electronic LLs, as the distance between the adjacent levels decreases as $\sim |n|^{-1/2}$. It is, therefore, likely that most favorable for the formation of the parent CDF metals and their incompressible descendants is the 0^{th} LL (cf. with the conclusions drawn in Refs.[9] where the LL mixing was neglected from the outset).

In the CDF IQHE states (7,8,9), the energy gaps for well separated particle-hole excitations

$$\Delta_m \approx E^{*}_{m} - E^{*}_{m-1} = v^{*}_{F}(2B)^{1/2}\frac{m^{1/2} - (m-1)^{1/2}}{(2m-1)^{1/2}} \qquad (10)$$

scale as $\sim \sqrt{B}/m$ for large m, which dependence is similar to that found in the conventional case of "non-relativistic" composite fermions where the effective mass varies as $\sim B^{1/2}$ (see Ref.[8]). In contrast, for small m the true lowest energy excitations are likely to be represented by pairs of spin/valley (anti)skyrmions[4,9].

Despite the general possibility for the compressible states to emerge at any of the aforementioned fractions $\nu^{(N)}$, a relative stability of these states is strongly dependent on the number N of the CDF species involved. In order to proceed with the stability analysis one has to go beyond the mean-field picture by including fluctuations of the statistical fields $\mathbf{a}_{\alpha}$ controlled by the CDF polarization operator.

In the regime where typical CDF energies and momenta are small compared to k^{*}_{F} and E^{*}_{F}, respectively, $\hat{\Pi}_{ij}(q)$ is similar to that of a "non-relativistic" system with the same k_F and v_F. In particular, its transverse (with respect to the transferred momentum $\mathbf{q}$) component $\Pi^{\perp}(\omega, \mathbf{q}) = \mathbf{q} \times \bar{\mathbf{\Pi}} \times \mathbf{q}/\mathbf{q}^2 = aq^2 + ib\omega/q$, where $a \sim v^{*}_{F}/k^{*}_{F}$ and $b \sim k^{*}_{F}$, accounts for the Landau diamagnetism and damping in the CDF metal.

For $N > 1$ the CDF interactions are dominated by $N - 1$ linear combinations of the transverse components of the statistical fields which are orthogonal to the "in-phase" mode $\sum^{N}_{\alpha} a^{i}_{\alpha}$. Unlike the latter, these combinations are not affected by the unscreened Coulomb interactions, and the effective coupling between different CDF species (here $V_q = g/q$, $(\mathbf{v}_{\perp}\mathbf{v}'_{\perp}) = (\mathbf{v}\mathbf{v}') - (\mathbf{v}\mathbf{q})(\mathbf{v}'\mathbf{q})/\mathbf{q}^2$)

$$U_{\alpha\beta} = (\mathbf{v}_{\perp}\mathbf{v}'_{\perp})\frac{q^2 V_q}{(Nq^2 V_q + \Pi^{\perp})\Pi^{\perp}} \approx (\mathbf{v}_{\perp}\mathbf{v}'_{\perp})\frac{1}{N\Pi^{\perp}} \qquad (11)$$

is always attractive in the Cooper channel ($\mathbf{v} = -\mathbf{v}'$). For $N = 2$ this interaction can facilitate the onset of s-wave valley-singlet pairing[10]. Moreover, for $N = 4$ there exists a possibility of more exotic (spin-valley coupled) patterns of the $SU(4)$-symmetry breaking.

By contrast, for $N = 1$ the effective interaction is repulsive in the Cooper channel, as it is between any CDF species of the same kind for $N > 1$,

$$U_{\alpha\alpha} = -(\mathbf{v}_\perp \mathbf{v}'_\perp) \frac{(N-1)q^2 V_q + \Pi^\perp}{(Nq^2 V_q + \Pi^\perp)\Pi^\perp} \approx -(\mathbf{v}_\perp \mathbf{v}'_\perp) \frac{N-1}{N\Pi^\perp} \tag{12}$$

Although there is still a possibility of p-wave pairing between the like CDFs, this potential instability (which is also present for the $\nu = 1/2$ state in the conventional 2DEG) tends to be much weaker[11].

Since the inherent pairing instabilities make the $N > 1$ CDF metals prone to becoming incompressible paired states, it is conceivable that the compressible states at the filling factors $\nu^{(2,4)}$ should, in general, be less robust than those at $\nu^{(1)}$. Likewise, the chances of observing the novel series (8) and (9) might be rather limited, as compared to the standard one given by Eq.(7).

The CDF metals would also be highly sensitive to disorder. In the presence of potential (short-range) impurities of density ρ_i, the CDFs experience elastic scattering off of an effective random magnetic field whose vector potential is described by the Gaussian variance $< A_i(\mathbf{q}) A_j(-\mathbf{q}) >= 16\pi^2 \rho_i (\delta_{ij} - q_i q_j/\mathbf{q}^2)/\mathbf{q}^{28}$.

A transport rate for the CDF α-species can be estimated as $\Gamma^* \sim E_F^* \rho_i/|\rho_\alpha|$, and the above analysis (including the role of the symmetry-breaking terms) pertains to the regime where $E_Z, \Delta, v_F^* B^{1/2}/|m| \gtrsim \Gamma^*$. By the same token, disorder makes it more difficult to resolve metallic states at fractions $\nu \sim 1/2p$ with $p > 1$.

Evaluating the longitudinal conductivity of the CDF α-species as $\sigma^*_{xx} \approx max[|\rho_\alpha|/\rho_i, 1]$, one obtains a rough estimate for the physical conductivity of the CDF metals

$$\sigma_{xx} \approx \frac{1}{4} min[\sum_\alpha^N \frac{\rho_i}{|\rho_\alpha|}, N] \tag{13}$$

which dependence should be contrasted with that at zero field (in the case of Coulomb impurities, the latter is *proportional* to the total electron density $\sum_\alpha^N \rho_\alpha{}^2$). Interestingly enough, in the experiment of Ref.[5] the conductivity at the $\nu = 0$ plateau was found to be of order $\sigma_{xx} \approx 0.6$, possibly suggesting a precursor of the formation of the $N = 4$ CDF metal at weak fields.

Also, by analogy with the situation in the conventional 2DEG[12], we predict that the conductivity of the CDF metals is going to be temperature dependent due to quantum interference corrections which dominate over weak-(anti)localization ones and behave as

$$\delta\sigma_{xx} \propto -\ln \sigma^*_{xx} \ln \frac{\Gamma}{T} \quad or \quad -\ln^2 \frac{\Gamma}{T} \tag{14}$$

for $N = 1$ and $N > 1$, respectively, thus allowing one to discriminate (in principal) between the single- and multi-component CDF metals.

Furthermore, despite the ostensibly Fermi-liquid-like properties of the CDF metals, the electron spectral function $ImG(\mathbf{p}, \epsilon)$ exhibits a distinctly non-Fermi-liquid behavior. Repeating the calculations carried out in the case of the conventional 2DEG[13], we find a tunneling $I-V$ characteristics of the CDF metal

$$I(V) \propto \exp[-Const(E_F^*/V)^\eta] \tag{15}$$

where $\eta = 1$ for $N = 1$ and $1/2$ for $N > 1$.

To conclude, in addition to a wealth of its other remarkable properties, graphene provides a natural venue for the merger between the notions of pseudo-relativistic quasiparticles and statistical transformation from electrons to CDFs. We predict that compressible CDF states are most likely to be observed at the $|\Delta\nu| = 1$ plateau transitions (e.g., $0 \to 1$) between the fully resolved sublevels of the 0^{th} LL, since a stronger LL mixing makes it more difficult for such states to form at the $|n| \neq 0$ LLs.

In contrast, the would-be CDF metals associated with the $|\Delta\nu| = 2, 4$ transitions are generally more fragile due to their propensity towards pairing, which can drive these states incompressible. Moreover, we predict that the incompressible CDF IQHE states can occur at both, the standard (7) as well as novel (8,9), fractions.

As far as the practical possibility of testing these predictions is concerned, the detrimental effect of disorder calls for performing experiments of Refs.[2,5] in samples of substantially higher mobility. The anticipated experimental signatures of the CDF metals can then be probed by such well-established techniques as bulk tunneling, acoustic wave propagation, magnetic focusing, and other geometric resonances[8].

For one, if a compressible $\nu = 0$ state were indeed to occur at $B \lesssim 10T$, its CDF excitations would be amenable to conventional electrostatic gating. This prediction should be contrasted with such a hallmark of the zero-field Dirac kinematics as the celebrated Klein's paradox that would hinder any possibility of electrostatic confinement of the electronic Dirac excitations with vanishing Fermi momentum[2].

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