Energy-momentum uncertainties as possible origin of threshold anomalies in UHECR and TeV-γ events

Y. Jack Ng[(a)*], D.-S. Lee[(b)], M. C. Oh[(a)], and H. van Dam[(a)]

(a) Institute of Field Physics, Department of Physics and Astronomy,

University of North Carolina, Chapel Hill, NC 27599-3255

(b) Department of Physics, National Dong Hwa University,

Shoufeng, Hualien 974, Taiwan, Republic of China

Abstract

A threshold anomaly refers to a theoretically expected energy threshold that is not observed experimentally. Here we offer an explanation of the threshold anomalies encountered in the ultra-high energy cosmic ray events and the TeV-γ events, by arguing that energy-momentum uncertainties due to quantum gravity, too small to be detected in low-energy regime, can affect particle kinematics so as to raise or even eliminate the energy thresholds. A possible modification of the energy-momentum dispersion relation, giving rise to time-of-flight differences between photons of different energies from gamma ray bursts, is also discussed.

PACS: 98.70.Sa; 98.70.Rz; 04.60.-m; 11.30.-j

---

*Corresponding author. E-mail: yjng@physics.unc.edu

I. INTRODUCTION

The observation of ultra-high energy cosmic rays [1] with energy exceeding the Greisen-Zatsepin-Kuz'min cutoff [2] at $\sim 5 \times 10^{19}$ eV has presented the physics and astrophysics community quite a conundrum. The GZK cutoff is based on pion photo-production by inelastic collisions of cosmic-ray nucleons with the cosmic microwave background

$$p + \gamma(CMB) \longrightarrow p + \pi. \tag{1}$$

(Actually, the dominant contribution to the GZK cutoff comes from the $\Delta(1232)$-resonance. But the difference between m_Δ and $m_p + m_\pi$ would modify our results below only slightly. Moreover, if the Δ formation is not possible, a weakened version of the GZK cutoff may result from non-resonant pion photo-production. Also we should add that the exact composition of the cosmic rays is not known. But even if they are heavy nuclei like Fe rather than nucleons, they would still be photo-disintegrated, and the GZK cutoff remains more or less intact.) In the CMB frame, such a collision requires a threshold energy of the cosmic ray proton given by

$$E_{th} = \frac{(m_p + m_\pi)^2 - m_p^2}{4\omega} \simeq 5 \times 10^{19} \;\; eV, \tag{2}$$

for an average CMB photon energy $\omega \sim 1.4 \times 10^{-3}$ eV (and $m_p \simeq 9.4 \times 10^8$ eV for the proton mass, $m_\pi \simeq 1.4 \times 10^8$ eV for the pion mass). For protons above this energy, the pion photo-production from CMB will dominate beyond the mean free path. And for pion photo-production cross-section of $\sim 200\mu b$ and density ~ 550 photons/cc for the CMB, this mean free path is of order 1 Mpc, much smaller than the intergalactic distances. This would imply that the protons would need to originate within our galaxy. But the known maximal galactic magnetic fields are too weak to accelerate the protons to such ultra-high energies. Furthermore, such high energy $\sim 10^{20}$ eV protons are hardly deflected by the interstellar magnetic fields and hence should have a direction identifiable with some source. But the observed UHECR events are oriented along the extragalactic plane and have no known correlation with any identifiable sources. Thus we are forced to conclude that nature has found a way to evade the GZK cutoff, an energy threshold that, as we have just seen, is well established theoretically. This phenomenon of a theoretically expected energy threshold not observed experimentally has come to be known as a threshold anomaly.

There has not been a lack of attempts [3] to explain these extraordinary cosmic rays. They include protons originated from nearby topological defects/monopolium [4], magnetic monopoles [5], and solutions like the decay of massive supersymmetric hadrons [6]. Other explanations include "Z-boson bursts" [7] and decay products of hypothetical super-heavy relic particles [8]. Exotic origins have also been suggested, such as: gamma-ray bursts [9], spinning supermassive black holes associated with presently inactive quasar remnants [10], and colliding galaxy systems [11].

The recent observation of 20 TeV γ-rays [12] from Mk 501 is also puzzling. [13] Theoretically such events are not expected since a high energy photon propagating in the intergalactic medium can suffer inelastic impacts with photons in the Infra-Red background resulting in the production of an electron-positron pair

$$\gamma + \gamma(IR) \longrightarrow e^{+} + e^{-}. \tag{3}$$

For such a collision, the threshold energy of the high energy photon is given by

$$E_{th} = \frac{m_e^2}{\omega} \simeq 10 \;\; TeV, \tag{4}$$

for an average photon energy of $\omega \sim 0.025$ eV in the IR background (and $m_e \simeq 0.5 \times 10^6$ eV for the electron mass). Thus γ-rays above 10 TeV lose energy drastically during their propagation from their source to the Earth. It is very unlikely that they can survive their long trip from distant Mk 501 with any significant flux. Here then is another threshold anomaly. Compared to the UHECR events, the TeV-γ events have elicited only a few explanations, such as: there may be a possible upturn in the intrinsic spectrum emitted by Mk 501; the distance to Mk 501 or the background IR intensity may have been overestimated; and multiple TeV-γ emitted coherently by Mk 501 may have been mistaken to be a single photon event with higher energy [14].

There is one solution to the UHECR paradox and recently used also to explain the TeV-γ events that deserves special mention. Numerous authors [15,11,17] have suggested that these events are a signal of violation of ordinary Lorentz invariance at the energies in question. These violations are too small to have been detected at the available accelerator energies. But at the highest observed energy region they can affect particle kinematics so as to suppress or even forbid the inelastic collisions (Eq. (1) and (3)), thereby evading the two cutoffs.

In this paper we will adopt a proposal [18], which bears some similarity to the one just mentioned, to solve the UHECR and TeV-γ puzzles. It is based on the observation that, due to quantum gravitational effect, energy and momentum, like distances and time intervals, cannot be measured with infinite accuracies. The energy-momentum uncertainties of the form (with positive a)

$$\delta E \gtrsim E \left(\frac{E}{E_P}\right)^a, \qquad \delta p \gtrsim p \left(\frac{p}{m_P c}\right)^a, \tag{5}$$

a natural consequence of quantum gravitational effects, [19] upon inserted into the energy-momentum conservation equations or the energy-momentum dispersion relation, can mimic the effects of violation of ordinary Lorentz invariance in a particular way. (Here E_P denotes the Planck energy, m_P denotes the Planck mass, and we have restored the factor of c.) They can be interpreted as the *physical* origin of the threshold anomalies. We have little to say about the origins of UHECR and TeV-γ per se. We simply want to point out that there is a *natural* mechanism that can potentially raise or even eliminate the two energy thresholds under consideration.

The outline of the paper is as follows: In the next Section, we review the argument used by two of us (Ng and van Dam) [19] years ago leading to energy-momentum uncertainties of the form given by Eq. (5). In Section III, we use the energy-momentum uncertainties to explain the threshold anomalies encountered in the UHECR and TeV-γ events. We also give another plausible interpretation of energy-momentum uncertainties and apply it to future time-of-flight measurements of photons of different energies from gamma ray bursts. The concluding section is devoted for discussion.

II. ENERGY-MOMENTUM UNCERTAINTIES

Just as there are uncertainties in distance and time interval measurements, there are uncertainties in energy-momentum measurements. Both types of uncertainties [19] come from the same source, viz., quantum fluctuations of space-time metrics [20] giving rise to space-time foam. We will consider two leading models of space-time foam. In the first model, the fluctuations of the metric are given by [21]

$$\delta g_{\mu\nu} \gtrsim \frac{l_P}{l}, \tag{6}$$

for a measurement in a space-time region of volume l^4. Here $l_P \equiv (\hbar G/c^3)^{1/2}$ is the Planck length. Since $\delta l^2 = l^2 \delta g$, this translates into an uncertainty in distance measurements given by $\delta l \gtrsim l_P$. We can calculate the minimum uncertainty in momentum for a particle with momentum p by regarding δp as the uncertainty of the momentum operator $p = -i\hbar\partial/\partial x$, associated with $\delta x = l_P$. For any function $f(x)$, $(\delta p)f$ is given by

$$(\delta p)f = \frac{\hbar}{i}\left(\delta x \frac{\partial^2 f}{\partial x^2} + \frac{\partial f}{\partial x}\frac{\partial \delta x}{\partial x}\right). \tag{7}$$

Taking the function $f(x)$ to be a momentum eigenstate $f = exp(ipx/\hbar)$, we get

$$(\delta p)e^{ipx/\hbar} = i\frac{p^2 l_P}{\hbar}e^{ipx/\hbar}. \tag{8}$$

This yields

$$|\delta p| \sim p\left(\frac{p}{m_P c}\right), \tag{9}$$

where $m_P \equiv (\hbar c/G)^{1/2}$ is the Planck mass.

An alternative derivation of Eq. (9) goes as follows: Imagine sending a particle of momentum p to probe a structure of spatial extent l so that

$$p \sim \frac{\hbar}{l}. \tag{10}$$

Consider the coupling of the metric to the energy-momentum tensor of the particle,

$$(g_{\mu\nu} + \delta g_{\mu\nu})t^{\mu\nu} = g_{\mu\nu}(t^{\mu\nu} + \delta t^{\mu\nu}), \tag{11}$$

where we have noted that the uncertainty in $g_{\mu\nu}$ can be translated into an uncertainty in $t_{\mu\nu}$. Eqs. (6) and (11) can now be used to give

$$\delta p \gtrsim p\left(\frac{l_P}{l}\right), \tag{12}$$

which, with the aid of Eq. (10), yields Eq. (9). We can also mention that the momentum uncertainty is actually fixed by dimensional analysis, once the uncertainty in the metric is given by Eq. (6). The corresponding statement for energy uncertainties is

$$\delta E \sim E\left(\frac{E}{E_P}\right). \tag{13}$$

Next let us consider the second space-time foam model [19,22] (which we actually favor over the first model for reasons we have given in Ref. [18,23], including its natural connection to the holographic principle and black hole physics). It is given by

$$\delta g_{\mu\nu} \gtrsim \left(\frac{l_P}{l}\right)^{2/3}, \tag{14}$$

corresponding to $\delta l \gtrsim (l l_P^2)^{1/3}$. Repeating the above procedure we get

$$\delta E \gtrsim E\left(\frac{E}{E_P}\right)^{2/3}, \qquad \delta p \gtrsim p\left(\frac{p}{m_P c}\right)^{2/3}. \tag{15}$$

Note that, for both space-time foam models, the energy-momentum uncertainties are negligible except when we consider processes involving very energetic particles. We should also mention that we have not found the proper (presumably nonlinear) transformations of the energy-momentum uncertainties between different reference frames. Therefore we will apply the results only in the frame in which we do the observations. In the following, we will write the energy-momentum uncertainties in the form given by Eq. (5) with $a = 1, 2/3$ for the space-time foam models given by Eq. (6) and Eq. (14) respectively.

III. SOLVING THE THRESHOLD ANOMALIES

Now that we know the energy-momentum uncertainty expressions, we have to figure out how and where we should apply them. It all comes down to the question of correctly interpreting the physics. Relevant to the discussion of the UHECR events and the TeV-γ events is the scattering process in which an energetic particle of energy E_1 and momentum $\mathbf{p_1}$ collides head-on with a soft photon of energy ω in the production of two energetic particles with energy E_2, E_3 and momentum $\mathbf{p_2}$, $\mathbf{p_3}$. Henceforth let us adopt $c = 1$. At threshold, (ordinary) energy-momentum conservation demands

$$E_1 + \omega = E_2 + E_3, \qquad p_1 - \omega = p_2 + p_3, \tag{16}$$

and the (ordinary) energy-momentum dispersion relation takes the form

$$E_i = (p_i^2 + m_i^2)^{1/2}, \tag{17}$$

where $i = 1, 2, 3$ refers to the particle with energy E_i, momentum p_i, and mass m_i. For the UHECR and TeV-γ events, these two equations yield the threshold energies given by Eqs. (2) and (4) respectively. But for the problem of threshold anomalies at hand, we believe Eqs. (16) and (17) can receive crucial modifications from energy-momentum uncertainties. [18] Let us, therefore, consider (separately) modifying (i) the conservation expressions and (ii) the dispersion relation. (The suggestion that the dispersion relation may be modified by quantum gravity first appeared in Ref. [24].)

(i) Modifying energy-momentum conservation relations

While the energy-momentum dispersion relation is the conventional one given by Eq. (17),

$$E_i \simeq p_i + \frac{m_i^2}{2p_i}, \tag{18}$$

where we have used the fact that p_i is very large compared to m_i, the energy-momentum conservation is modified to read

$$E_1 + \delta E_1 + \omega = E_2 + \delta E_2 + E_3 + \delta E_3, \tag{19}$$

and

$$p_1 + \delta p_1 - \omega = p_2 + \delta p_2 + p_3 + \delta p_3. \tag{20}$$

Thus, in this scheme, energy-momentum is conserved up to energy-momentum uncertainties, while the dispersion relation is still dictated by Lorentz invariance. We have omitted $\delta\omega$, the contribution coming from the uncertainty of ω because ω is small. Substituting Eq. (18) into Eq. (19) and making use of Eq. (20), we can rewrite Eq. (19) as

$$4\omega \simeq \frac{m_2^2}{p_2} + \frac{m_3^2}{p_3} - \frac{m_1^2}{p_1} + \varepsilon\frac{1}{E_P^a}(p_1^{1+a} - p_2^{1+a} - p_3^{1+a}). \tag{21}$$

Here we have used Eq. (5) and the fact that $E_i \simeq p_i$ for energetic particles to put

$$\delta p_i - \delta E_i \simeq \varepsilon\frac{p_i^{1+a}}{2E_P^a}, \tag{22}$$

thereby defining the parameter ε. We do not know how to calculate ε; but since $\delta E_i \simeq \delta p_i$ for high energy, we expect that it can be fairly *small* compared to one.

The solution to Eq. (21) for the threshold energy $E_{th} \simeq p_1$ of the incoming energetic particle can be easily worked out for the case of TeV-γ for which $m_1 = 0$, $m_2 = m_3 = m_e$, the electron mass, and $p_2 = p_3 \simeq p_1/2$. It satisfies the following equation

$$E_{th}\omega \simeq m_e^2 + \varepsilon\frac{2^a - 1}{2^{2+a}}\frac{E_{th}^{2+a}}{E_P^a}. \tag{23}$$

For the general case, the threshold energy E_{th} is given by [17]

$$4E_{th}\omega \simeq (m_2 + m_3)^2 - m_1^2 + \varepsilon\frac{E_{th}^{2+a}}{E_P^a}\left(1 - \frac{m_2^{1+a} + m_3^{1+a}}{(m_2 + m_3)^{1+a}}\right), \tag{24}$$

with $m_1 = m_2 = m_p$, the proton mass and $m_3 = m_\pi$, the pion mass for UHECR. (One can easily check that Eq. (24) contains Eq. (23) as a special case.)

To explain the TeV-γ events, we need to raise the threshold energy to $E_{th} \simeq 20TeV$. With $E_P \simeq 10^{28}$ eV for the Planck energy, Eq. (23) gives $\varepsilon \simeq 4.2 \times 10^{-5}$ for $a = 2/3$ and $\varepsilon \simeq 2.5$ for $a = 1$. To explain the UHECR events, we need the threshold shift from 5×10^{19} eV to $E_{th} = 3 \times 10^{20}$ eV; Eq. (24) yields $\varepsilon \sim 10^{-17}, 10^{-15}$ for $a = 2/3, 1$ respectively. Indeed, as expected, ε is small compared to one in general. (But the smallness of ε for the UHECR case suggests that there may be a fine-tuning problem. More on the allowed values of ε later.) It is amazing that such a small modification coming from energy-momentum uncertainties can have such a large effect in shifting the threshold energies by a factor of 2 and 6 for the TeV-γ and UHECR events respectively. To repeat, energy-momentum uncertainties from quantum gravity effects can potentially be the physical origin of the two threshold anomalies.

The following comment is now in order. Effects of energy-momentum uncertainties yield a negative ε as likely as a positive ε. Then what happens to the negative ε case? The answer is that negative values of ε would shift the energy thresholds in the opposite ("wrong") direction. They correspond to events not seen; therefore, there is nothing that needs to be explained in the first place.

(ii) Modifying the energy-momentum dispersion relation

Consider energy-momentum conservation given by Eq. (16) but the energy-momentum dispersion relation modified to read

$$(E_i + \delta E_i)^2 = (p_i + \delta p_i)^2 + m_i^2, \tag{25}$$

which, for high energy ($E_i \simeq p_i$), becomes

$$E_i \simeq \frac{1}{2} p_i \left(2 + \frac{m_i^2}{p_i^2} + \varepsilon \frac{p_i^a}{E_P^a} \right), \tag{26}$$

where ε is defined by Eq. (22) as in scheme (i). Eq. (26) is the starting point of the approach adopted by many of the Lorentz invariance violation advocates [15,17]. Here it is the result of energy-momentum uncertainties (due to quantum gravity) in the dispersion relation. Using Eq. (26) and Eq. (16), we recover Eq. (21) for the threshold energy except for a sign change for ε. But as we have argued above, the sign of ε is irrelevant. To raise the threshold energy, all we need this time is to pick negative values for ε. It follows that, as far as the UHECR events and TeV-γ events are concerned, the threshold anomalies are explained in the same way as in (i).

In passing we mention that we have used the same ε parameter for all different particle species. If we have used different ε parameters for different particle species, we will get a scheme which bears some resemblance to that advocated by Coleman and Glashow [16]. (Such dependence of ε on particle species is natural if, e.g., δp_i and δE_i cancel so completely in Eq. (22) that its right hand side is reduced by a factor of m_i^2/p_i^2. But in that case, the effect from energy-momentum uncertainties is so small that we recover the ordinary threshold equation; in other words, we will need another way to solve the threshold anomalies.)

Are the two schemes (i) and (ii) equivalent? No, not entirely. Consider the modified energy-momentum dispersion relation for photon given by Eq. (26)

$$E^2 \simeq c^2 k^2 + \varepsilon E^2 \left(\frac{E}{E_P}\right)^a , \tag{27}$$

where we have restored c in writing $p = ck$. The speed of (massless) photon

$$v = \frac{\partial E}{\partial k} \simeq c\left(1 + \varepsilon \frac{1+a}{2}\frac{E^a}{E_P^a}\right) , \tag{28}$$

becomes energy-dependent! Thus modified energy-momentum dispersion relation (scheme (ii)), unlike modified energy-momentum conservation relations (scheme (i)), predicts time-of-flight differences between simultaneously-emitted photons of different energies, E_1 and E_2, given by

$$\delta t \simeq \varepsilon t \frac{1+a}{2}\frac{E_1^a - E_2^a}{E_P^a}, \tag{29}$$

where t is the average overall time of travel from the photon source. An upper bound [25,17] on the absolute value of ε can be obtained from the observation [26] of simultaneous (within experimental uncertainty of $\delta t \le 200$ sec) arrival of 1-TeV and 2-TeV γ-rays from Mk 421 which is believed to be $\sim$ 143 Mpc away from the Earth. Using Eq. (29) we obtain $|\varepsilon| \le 1.3 \times 10^{-3}, 1.4 \times 10^2$ for $a = 2/3, 1$ respectively. Note that these bounds for ε are consistent with those values from UHECR and TeV-γ events. For an analysis of the time-lag signature see Ref. [27].

IV. DISCUSSION

In the preceding section, we have obtained the various values of ε corresponding to the two observed threshold energies. But an examination of Eqs. (23) and (24) shows that, with those values of ε, the two equations can each be solved by *two* different real and positive E_{th}'s the larger of which being the observed threshold energy. Now the following question arises: given ε, which of the two solutions for E_{th} would nature pick? Perhaps neither. The point is that, for real and positive ε and E_{th}, there is a maximum value of ε above which there is *no* solution to the two equations. In that case, the threshold cutoffs are completely removed (i.e., the threshold anomalies are trivially solved). This consideration leads us to the following bounds on the (magnitude of the) ε parameter: $\varepsilon \gtrsim 4.6 \times 10^{-5}, 3.8 \times 10^{-17}$ for the TeV-γ and UHECR respectively for the case of $a = 2/3$, and $\varepsilon \gtrsim 3.0, 1.5 \times 10^{-14}$ respectively for the case $a = 1$. These values of ε are still consistent with the bounds from photon time-of-flight delay measurements given above.

So far we have considered the effects from either modified energy-momentum conservation relations or a modified energy-momentum dispersion relation. Let us now consider scheme (iii), the case with both the conservation relations and the dispersion relation modified. As for scheme (ii), time-of-flight differences between simultaneously-emitted photons of different energies are predicted. But as far as the threshold anomalies are concerned, one can check that this scheme offers no explanation as the effects of energy-momentum uncertainties cancel out in the threshold equation, yielding

$$E_{th} = \frac{(m_2 + m_3)^2 - m_1^2}{4\omega}, \tag{30}$$

the ordinary threshold condition which we try to explain away for the UHECR events and the TeV-γ events. (It is not surprising that one gets back the ordinary threshold condition for this case because one can redefine E_i and p_i by absorbing δE_i and δp_i so that all energy-momentum uncertainty effects disappear from the threshold equation.)

So, which of the three schemes is the correct one? Frankly we cannot decide. However, the three schemes give different experimental predictions (or "post-dictions"). So in principle they can be subject to further experimental checks. (But at least we may have provided advocates of Lorentz invariance violation [15–17,25] some physical justification coming from energy-momentum uncertainties due to quantum gravitational fluctuations.) Our attitude is that we should proceed in such a way as to preserve as much as possible the framework which has been so productive in describing the various physical interactions. Thus we would like, on the large scale of experimental equipment, to preserve time translation-, space translation-, and Lorentz- invariance. This would support the familiar conservation laws to a sufficient extent. But it does not necessarily mean that space-time in the small is Minkowskian. Following Einstein and Wigner we could presumably blame small scale oscillations of the metric (which, as we have argued in Sec. II, lead to energy-momentum uncertainties) for possible deformations of Minkowskian invariance. Once this fact is accepted, we would expect some effects in the energy-momentum dispersion relation for the individual particles participating in a collision as well as in the conservation laws of energy and linear momentum in such a collision. At the very least, we should not accept strict Lorentz invariance and energy-momentum conservation on faith but rather regard them as plausible hypotheses subject to experimental tests! Nature may have kindly provided us with the UHECR and TeV-γ puzzles for such tests.

Acknowledgments

One of us (YJN) thanks G. Amelino-Camelia for a useful correspondence and John Ellis for calling to his attention some relevant references. This work was supported in part by the US Department of Energy under #DE-FG05-85ER-40219, the Bahnson Fund at the University of North Carolina at Chapel Hill, and the Republic of China National Science Council under NSC89-2112-M-259-001 and NSC89-2112-M-259-012-Y.

REFERENCES

[1] M.A. Lawrence et al., J. Phys. **G17** (1991) 733; N. N. Efimov et al., in 22nd Intl. Cosmic Pay Conf. (1991) Dublin; D.J. Bird et al., Astrophys. J. **441** (1995) 144; M. Takeda et al., Phys. Rev. Lett. **81** (1998) 1163; A. Watson, in Proc. Snowmass Workshop (1996) 126.

[2] K. Greisen, Phys. Rev. Lett. **16** (1966) 748; G. T. Zatsepin and V.A. Kuz'min, JETP Lett. **41** (1966) 78.

[3] A.V. Olinto, Phys. Rept. **333-334** (2000) 329-348.

[4] P. Bhattacharjee, C.T. Hill, and D.N. Schramm, Phys. Rev. Lett. **69** (1992) 567; G. Sigl, astro-ph/9611190; V. Berezinsky and A. Vilenkin, Phys. Rev. Lett. **79** (1997) 5202.

[5] T. W. Kephart and T.J. Weiler, Astropart. Phys. **4** (1996) 271; I.F.M. Albuquerque, G. R. Farrar, and E.W. Kolb, hep-ph/9805288.

[6] G.R. Farrar and P.L. Biermann, Phys. Rev. Lett. **81** (1998) 3579.

[7] D. Fargion, B. Mele, and A. Salis, Ap. J. **517** (1999) 725; T.J. Weiler, Astropart. Phys. **11** (1999) 303; J.L. Crooks, J.O. Dunn, and P.H. Frampton, astro-ph/0002089.

[8] P.H. Frampton, B. Keszthelyi, and Y. J. Ng, Intl. J. Mod. Phys. **D8** (1999) 117.

[9] E. Waxman, Phys. Rev. Lett. **75** (1995) 386; M. Vietri, Astrophys. J. **453** (1995) 883.

[10] E. Boldt and P. Ghosh, Mon. Not. R. Astron. Soc. **307** (1999) 491.

[11] C.J. Cesarsky, Nucl. Phys. (Proc. Suppl.) **B28** (1992) 51.

[12] F.A. Aharonian et al., Astronomy and Astrophysics **349** (1999) 11A.

[13] R. J. Protheroe and H. Meyer, astro-ph/0005349, to appear in Phys. Lett. B.

[14] M. Harwit, P.J. Protheroe, and P.L. Biermann, Ap. J. **524** (1999) L91.

[15] L. Gonzalez-Mestres, physics/9704017; R. Aloisio, P. Blasi, P.L. Ghia, and A.F. Grillo,

astro-ph/0001258; O. Bertolami and C.S. Carvalho, Phys. Rev. **D61** (2000) 103002; H. Sato, astro-ph/0005218; T. Kifune, Astrophys. J. Lett. **518** (1999) L21; W. Kluzniak, astro-ph/9905308.

[16] S. Coleman and S.L. Glashow, Phys. Rev. **D59** (1999) 116008.

[17] G. Amelino-Camelia and T. Piran, hep-ph/0006210; astro-ph/0008107.

[18] Y. J. Ng, gr-qc/0006105.

[19] Y. J. Ng and H. van Dam, Mod. Phys. Lett. **A9** (1994) 335; ibid. **A10** (1995) 2801; in Proc. of Fundamental Problems in Quantum Theory, eds. D.M. Greenberger and A. Zeilinger, Ann. New York Acad. Sci. **755** (1995) 579.

[20] For lightcone fluctuations, see L. H. Ford, Phys. Rev. **D51** (1995) 1692.

[21] C. W. Misner, K.S. Thorne, and J.A. Wheeler, Gravitation (W.H. Freeman, 1973), pp. 1190-1194.

[22] F. Karolyhazy, Il Nuovo Cimento **A42** (1966) 390; N. Sasakura, Prog. Theor. Phys. **102** (1999) 169, and hep-th/0001161.

[23] Y.J. Ng and H. van Dam, Found. Phys. **30** (2000) 795; Phys. Lett. **B477** (2000) 429.

[24] G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, and D.V. Nanopoulos, Int. J. Mod. Phys. **A12** (1997) 607.

[25] G. Amelino-Camelia, J. Ellis, N.E. Mavromatos, D.V. Nanopoulos, and S. Sarkar, Nature **393** (1998) 763. The idea of bounding ε originated with this paper.

[26] B. E. Schaefer, Phys. Rev. Lett. **82** (1999) 4964; S.D. Biller et al., Phys. Rev. Lett. **83** (1999) 2108.

[27] J. Ellis, K. Farakos, N.E. Mavromatos, V.A. Mitsou, and D.V. Nanopoulos, Astrophys. J. **535** (2000) 139.