Three Generations in Minimally Extended Standard Models

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Abstract

We present a class of minimally extended standard models with the gauge group $SU(3)_C \times SU(N)_L \times U(1)_X$ where for all $N \geq 3$, anomaly cancelation requires three generations. At low energy, we recover the Standard Model (SM), while at higher energies, there must exist quarks, leptons and gauge bosons with electric charges shifted from their SM values by integer multiples of the electron charge up to $\pm [N/2] \, e$. Since the value $N = 5$ is the highest N consistent with QCD asymptotic freedom, we elaborate on the 3–5–1 model.

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I.   INTRODUCTION

With expected availability of data in *pp*-scattering up to 14 TeV center-of-mass energy in the next few years at the Large Hadron Collider (LHC), it is reasonable to anticipate additional gauge bosons as well as more chiral fermions beyond the Standard Model (SM). We expect the Standard Model to be the low energy limit of any new physics seen at the LHC. Hence, whatever the new physics, it should be an extension of the Standard Model. In order to explore the parameter space of such models and at the same time make testable predictions for the LHC, it is best to consider minimal extensions of the Standard Model, since these have the best hope of being testable and they are likely to be contained in the ultimate model.

Merely adding sequential quarks and leptons beyond the first three generations, while keeping the Standard Model $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group, is the simplest possibility, but perhaps not the most attractive scenario based on purely aesthetic grounds. For instance, eight families are the most allowed by QCD asymptotic freedom. But the symmetry requirement for baryons [1] seems to suggest that the number of colors should remain three. Thus, a somewhat more elegant model would have the overall gauge group grow in rank at higher energy while the color gauge group remains fixed. In this approach, only the chiral electroweak gauge group is extended from rank 2 to rank N for $N \geq 3$, namely we consider $SU(2)_L \longrightarrow SU(N)_L$. There will remain a $U(1)_X$ factor in the non-simple gauge group. For the Standard Model, we have $X = Y$ (the weak hypercharge) which cancels all triangle anomalies, generation by generation. For $N \geq 3$, however, X, though still unpredicted, becomes notably simpler than Y.

Following the above strategy, it is shown here how to generalize the SM gauge group to a minimally extended standard model (MESM) with gauge group $SU(3)_C \times SU(N)_L \times U(1)_X$ such that for all integers $N \geq 3$, unlike the SM case, anomaly cancelation requires three generations. At low energy, the models reduce to the Standard Model. At higher energies, there must exist quarks and leptons with electric charges up to $Q = q \pm [N/2]\, e$ where q is the corresponding SM charge and the symbol $[x]$ denotes the integer part of x, together with gauge bosons which have electric charges up to $Q = \pm [N/2 + 1]\, e$.

II. MINIMALLY EXTENDED STANDARD MODELS

Our construction is inspired at the first stage by the 331-model [2, 3] but involves generalization to give an infinite sequence of three-generation anomaly-free extensions. (For reviews and related work, see [4–7].)

It is easier to start with odd $N = (2n+1)$, then describe even $N = 2n$. For $N = (2n+1)$, the first and second generations of fermions are assigned under $(SU(3)_C, SU(N)_L)_X$ as

$$(3,\, N\,)_{-\frac{1}{3}} + (\bar{3},\, 1\,)_{+\frac{1}{3}} + (\bar{3},\, 1\,)_{+\frac{1}{3}\pm 1} + ... + (\bar{3},\, 1\,)_{+\frac{1}{3}\pm n} + (1,\, \bar{N}\,)_0 + (1,\, 1\,)_0. \qquad (1)$$

More explicitly, the first two generations of quarks are assigned in (1) as the $\mathbf{N}$ of $SU(N)_L$:

$$Q_{iL} = \begin{pmatrix} U_i^{n-1} \\ U_i^{n-2} \\ . \\ . \\ U_i^1 \\ u_i \\ d_i \\ D_i^{-1} \\ . \\ . \\ D_i^{-(n-1)} \\ D_i^{-n} \end{pmatrix}_L , \quad i = 1, 2 ; \qquad X = -\frac{1}{3}, \qquad (2)$$

where the notations U_i^{n-1} and D^{-n} mean that the charge is shifted up by $(n-1)\,e$ units and down by $-n\,e$ units respectively. The first two generations of leptons are assigned in (1) as

the $\bar{\mathbf{N}}$ of $SU(N)_L$:

$$f_{lL} = \begin{pmatrix} E_l^{-n} \\ E_l^{-(n-1)} \\ . \\ . \\ E_l^{-2} \\ l^- \\ \nu_l \\ l^+ \\ E_l^2 \\ . \\ . \\ E_l^{n-1} \\ E_l^n \end{pmatrix}_L , \quad l = e, \mu ; \qquad X = 0 . \tag{3}$$

We shall write triangle anomalies in the order

$$A_g = [\, SU(3)_C^3, \; SU(3)_C^2 \, X, \; SU(N)_L^3, \; SU(N)_L^2 \, X, \; X^3, \; X \,] , \tag{4}$$

where $g = 1, 2, 3$ is the generation number. The sixth and last anomaly is a mixed gravity-gauge triangle diagram. For $N \geq 3$, normalizing the defining representation of $SU(N)$ to have anomaly $+1$ and normalizing X as defined above, we find that the first two generations of fermions have anomalies

$$A_1 = A_2 = [\, 0, \; 0, \; +2, \; -1, \; n\,(n+1)\,(2n+1), \; 0\,] . \tag{5}$$

The third generation of fermions is assigned under $(\, SU(3)_C, \; SU(N)_L \,)_X$ as

$$(\, 3, \, \bar{N} \,)_{+\frac{2}{3}} + (\, \bar{3}, \, 1 \,)_{-\frac{2}{3}} + (\, \bar{3}, \, 1 \,)_{-\frac{2}{3}\pm1} + \ldots + (\, \bar{3}, \, 1 \,)_{-\frac{2}{3}\pm n} + (\, 1, \, \bar{N} \,)_0 + (\, 1, \, 1 \,)_0 . \tag{6}$$

4

More explicitly, the third generation of quarks is assigned in (6) as the $\bar{\mathbf{N}}$ of $SU(N)_L$:

$$Q_{3L} = \begin{pmatrix} B^{-(n-1)} \\ B^{-(n-2)} \\ . \\ . \\ B^{-1} \\ b \\ t \\ T^1 \\ . \\ . \\ T^{n-1} \\ T^n \end{pmatrix}_L , \qquad X = \frac{2}{3} . \tag{7}$$

Similar to the first two generations of leptons, the third generation of leptons is also assigned in (6) as the $\bar{\mathbf{N}}$ of $SU(N)_L$. Thus, the index l in (3) can actually be e, μ or τ. The anomalies due to the third generation of fermions are given by

$$A_3 = \left[\, 0,\ 0,\ -4,\ +2,\ -\,2\,n\,(n+1)\,(2n+1),\ 0 \,\right]. \tag{8}$$

Combining (5) and (8) leads to

$$A_1 + A_2 + A_3 = \left[\, 0,\ 0,\ 0,\ 0,\ 0,\ 0 \,\right]. \tag{9}$$

Thus, the anomalies from the first two generations of fermions exactly cancel those from the third generation of fermions. There are many ways to express the cancelation physically, perhaps the simplest is to say that the number of generations must equal to the number of colors.

For even $N = 2n$ [8], the simplest possibility is to imitate the Standard Model with sequential generations where the required extension is most easily seen by the breaking of $SU(2n + 1)$ to $SU(2n)$ which cannot generate new anomalies. In the $SU(2n)$ models, the leptons develop non-zero X charges but, as expected, the six anomalies listed in (4) all cancel for three generations. It is straightforward to check, that unlike the minimal odd N models, anomalies cancel generation by generation for even N as in the Standard Model.

In breaking $SU(2n + 1)$ to $SU(2n)$, there are two possibilities for $SU(2n)$. The reason is that we can assign the singlet to the fundamental irrepresentation in the natural embedding $\mathbf{N} = (\mathbf{N} - \mathbf{1}) + \mathbf{1}$ to either the highest weight or the lowest weight of the $\mathbf{N}$.

As a final remark, for a given N, the models with three generations have $3N$ quark flavors, and so we require $N \leq 5$ for QCD asymptotic freedom, so we will turn our focus to this 3–5–1 model. While the $N = 5$ model is close to losing asymptotic freedom, it is also close to being conformal at one loop, an interesting feature that we will not explore here [9–11].

III. HIGGS SECTOR AND SYMMETRY BREAKING

We expect the effective nonabelian part of the electroweak gauge group to be N = 3, 4, ... as the energy increases. As shown in [2], the $SU(3)_L$ breaking must be below ~ 4 TeV for reasons associated with the running of $SU(2)_L$ coupling constant and the embedding of $SU(2)_L$ in $SU(3)_L$. The corresponding new gauge bosons $(Y^{++}, Y^+), (Y^{--}, Y^-)$ and Z', as well as the exotic quarks $D\,(Q = -4/3\,)$ and $T\,(Q = +5/3\,)$ are predicted to have masses $\lesssim 4$ TeV. For higher $N \geq 4$, the scale of symmetry breaking for $SU(N)_L \to SU(N-1)_L$ will be successively higher.

As an illustrative example, we consider a model with the gauge group $SU(3)_C \times SU(5)_L \times U(1)_X$. As mentioned earlier, this 3-5-1 model is the maximal model that still preserves the QCD asymptotic freedom. It has the following fermion contents:

$$
f_{lL} = \begin{pmatrix} E_l^{--} \\ l^- \\ \nu_l \\ l^+ \\ E_l^{++} \end{pmatrix}, \quad l = e, \mu, \tau,
\tag{10}
$$

$$
Q_{iL} = \begin{pmatrix} U_i^1 \\ u_i \\ d_i \\ D_i^{-1} \\ D_i^{-2} \end{pmatrix}, \quad i = 1, 2 \qquad Q_{3L} = \begin{pmatrix} B^{-1} \\ b \\ t \\ T^1 \\ T^2 \end{pmatrix},
\tag{11}
$$

where we have denoted E_l^{-2} and E_l^2 by E_l^{--} and E_l^{++} respectively.

The symmetry breaking from $SU(3)_C \times SU(5)_L \times U(1)_X$ to $SU(3)_C \times SU(2)_L \times U(1)_Y$ (where Y is the hypercharge) can be achieved with three fundamental Higgs fields ϕ, χ and Δ acquiring the following vacuum expectation values (VEVs):

$$
\langle\phi\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v \end{pmatrix}, \quad \langle\chi\rangle = \begin{pmatrix} v' \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \langle\Delta\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ v'' \\ 0 \end{pmatrix}. \tag{12}
$$

These VEVs can be obtained by constructing the following Higgs potential:

$$
\begin{aligned}
V(\phi, \chi, \Delta) = {} & \alpha_1 \left(\phi^\dagger\phi - v^2\right)^2 + \alpha_2 \left(\chi^\dagger\chi - v'^2\right)^2 + \alpha_3 \left(\Delta^\dagger\Delta - v''^2\right)^2 \\
& + \alpha_4 \left[\left(\phi^\dagger\phi - v^2\right) + \left(\chi^\dagger\chi - v'^2\right)\right]^2 + \alpha_5 \left[\left(\phi^\dagger\phi - v^2\right) + \left(\Delta^\dagger\Delta - v''^2\right)\right]^2 \\
& + \alpha_6 \left[\left(\chi^\dagger\chi - v'^2\right) + \left(\Delta^\dagger\Delta - v''^2\right)\right]^2,
\end{aligned} \tag{13}
$$

where all the coefficients $\alpha_1, ..., \alpha_6$ are non-negative. Obviously, these VEVs minimize the Higgs potential.

After the symmetry breaking, we can find the expression for the hypercharge Y. With the convention $\mathrm{Tr}(\lambda_a \lambda_b) = 2\delta_{ab}$, we adopt the following representation for the generators (note our λ matrices differ from the standard notation by a reordering of rows and columns):

$$
T_3 = \frac{1}{2}\,\mathrm{diag}\,(0, 1, -1, 0, 0)\,, \tag{14}
$$

$$
\lambda_8 = \frac{1}{\sqrt{3}}\,\mathrm{diag}\,(0, 1, 1, -2, 0)\,, \tag{15}
$$

$$
\lambda_{15} = \frac{1}{\sqrt{6}}\,\mathrm{diag}\,(0, 1, 1, 1, -3)\,, \tag{16}
$$

$$
\lambda_{24} = \frac{1}{\sqrt{10}}\,\mathrm{diag}\,(-4, 1, 1, 1, 1)\,. \tag{17}
$$

It is then straightforward to show that the hypercharge Y is given by

$$
Y = 2\,X + \sqrt{3}\,\lambda_8 + \sqrt{6}\,\lambda_{15} - \sqrt{10}\,\lambda_{24}\,, \tag{18}
$$

with $X = 0$, $X = -1/3$ and $X = 2/3$ for the leptons and quarks respectively. This expression for hypercharge Y holds given that the Higgs fields ϕ, χ and Δ carry the $U(1)$ charges $X = 2$, $X = -2$ and $X = 1$ respectively. One can readily verify that (18) precisely

reproduces the SM hypercharges for those leptons and quarks. The electric charge Q is given by

$$Q = T_3 + \frac{Y}{2},\tag{19}$$

which again reproduces the SM electric charges for the leptons and quarks of the standard model sector. Moreover, this expression for Q also generates the required electric charges for $E^{--}, E^{++}, U_i^1, D_i^{-1,-2}, B^{-1}$ and $T^{1,2}$.

IV. DISCUSSIONS

Preliminary data from the LHC give no encouragement to the popular ideas of super-symmetry and extra dimensions. So it is timely to ask what are other extensions of the Standard Model which stay in four bosonic space dimensions.

We have presented minimally extended standard models (MESMs). They can be designated as *minimal* because the extension of $SU(2)_L$ to $SU(N)_L$ with some simplification of the $U(1)_X$ assignments compared to $U(1)_Y$, as we have noted, is surely the infinite sequence which most simply extends the Standard Model.

The phenomenological consequences of the MESMs are spectacular. It predicts that fermions and gauge bosons of ever increasing electric charge will appear as the available energy increases. It is unnecessary for $N \to \infty$ because of the intervention of gravity. Also, for electric charges near $\alpha^{-1} e$, the Schwinger mechanism [12] destabilizes the vacuum, which provides a finite bound on N. (The interplay between the Schwinger effect and the implied large N expansion [13–15] of the electroweak sector could be an interesting avenue to explore.) However, there is no theoretical reason not to have gauge bosons, hadrons, leptons etc., with charges $\pm 2\,e, \pm 3\,e, ...$; or quarks with large fractional charges. At the LHC, we expect to see at least the quarks with charges $Q = -4/3, +5/3$ and the doubly-charged bileptons $Y^{\pm\pm}$, both corresponding to the new level $N = 3$. It is possible that the higher levels $N \geq 4$ will be in evidence too. For example, in the case $N = 5$, the highest N allowed by QCD asymptotic freedom, there are gauge bosons with charges up to $\pm 4\,e$ coupling to channels like $\mu^-\mu^-\mu^-\mu^-$ which can provide striking signatures at the LHC [16].

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