

A Solution to the Strong CP Problem with Gauge-Mediated Supersymmetry Breaking

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Abstract

We demonstrate that a certain class of low scale supersymmetric “Nelson-Barr” type models can solve the strong and supersymmetric CP problems while at the same time generating sufficient weak CP violation in the $K^0 - \bar{K}^0$ system. In order to prevent one-loop corrections to $\bar{\theta}$ which violate bounds coming from the neutron electric dipole moment (EDM), one needs a scheme for the soft supersymmetry breaking parameters which can naturally give sufficient squark degeneracies and proportionality of trilinear soft supersymmetry-breaking parameters to Yukawa couplings. We show that a gauge-mediated supersymmetry breaking sector can provide the needed degeneracy and proportionality, though that proves to be a problem for generic Nelson-Barr mod-

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els. The workable model we consider here has the Nelson-Barr mass texture enforced by a gauge symmetry; one also expects a new U(1) gauge superfield with mass in the TeV range. The resulting model is predictive. We predict a measurable neutron EDM and the existence of extra vector-like quark superfields which can be discovered at the CERN Large Hadron Collider. Because the 3×3 Cabbibo-Kobayashi-Maskawa matrix is approximately real, the model also predicts a flat unitarity triangle and the absence of substantial CP violation in the B system at future B factories. We discuss the general issues pertaining to the construction of such a workable model and how they lead to the successful strategy. A detailed renormalization group study is then used to establish the feasibility of the model considered.

I. INTRODUCTION

The strong CP problem is without question one of the most important problems faced by the Standard Model (SM). Its origin lies in the necessity of adding the so-called θ term to the effective QCD Lagrangian due to the contribution of instantons present in the topologically nontrivial QCD vacuum [1]:

$$\mathcal{L}_{\text{eff}} = \frac{\theta\alpha_s}{8\pi} F_{\mu\nu}^A \tilde{F}^{A\mu\nu} , \quad (1.1)$$

where the dual field strength is given by $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$. Through the anomaly in the axial U(1) current of QCD, chiral U(1) transformations lead to shifts in θ , leaving the physical combination $\bar{\theta} = \theta - \arg \det M_q$, where M_q is the quark mass matrix. Since \mathcal{L}_{eff} clearly violates CP, it gives a strong interaction contribution to the neutron electric dipole moment [2] and leads to the experimental constraint

$$\bar{\theta} < 10^{-9} . \quad (1.2)$$

The real problem therefore is one of naturalness or fine-tuning: Why is $\bar{\theta}$ so incredibly small?

There are currently three notable classes of possible solutions to this problem: (1) vanishing up quark mass, (2) the axion [3,4] or (3) CP conservation and subsequent spontaneous breaking. The first and simplest possibility appears to be disfavored by current algebra relations between pseudoscalar meson masses [5], but is still controversial (see *e.g.* Ref. [6]). Of these, the most popular is the invisible axion alternative [4]. Here one introduces a global chiral U(1)_{PQ} (Peccei-Quinn [7]) symmetry which is spontaneously broken at a high energy scale f and explicitly broken by instantons. The θ parameter is replaced by a dynamical field — a pseudo Goldstone boson of the U(1)_{PQ} — whose potential dynamically relaxes $\bar{\theta}$ to zero. The advantage of this scheme is that it is simple, generic and has observable consequences both in terrestrial experiments and in astrophysics and cosmology. Astrophysical constraints from axion-induced cooling during stellar evolution [8] and effects on the neutrino signal from supernova 1987A [9] give a lower bound, $f \gtrsim 10^{10}$ GeV, while a cosmological

upper bound of $\sim 10^{12}$ GeV is given by the contribution to the universal energy density of the vacuum energy associated with $U(1)_{PQ}$ breaking as the axion vacuum expectation value relaxes to zero [10]. On the aesthetic side, one may complain that we are merely replacing the $\bar{\theta}$ fine-tuning problem with another: the smallness of the ratio of the weak scale to the $U(1)_{PQ}$ breaking scale $\sim 10^{-(8-10)}$. Another possible problem is the dependence of the solution on a global symmetry, generally not preserved by gravity, so not likely to appear from a more fundamental theory. This appears to be a significant problem [11], at least in Einstein gravity. However, it has been argued that gravitational violations of global symmetries may be suppressed in certain extensions, including string theories [12], where at least the universal dilaton-axion is always present. The axion alternative also does not provide an explanation of weak CP violation. Here one must assume the Kobayashi-Maskawa (KM) origin of CP violating phases. Of course the ultimate test is to detect actually an axion [13].

In this paper we will focus on the third alternative. That is, we will assume that the fundamental theory of nature preserves CP and that at sub-Planck energies it is spontaneously broken. Indeed there is evidence that CP has its origin as a gauge symmetry remnant of superstring theories [14]. In this manner the smallness of $\bar{\theta}$ reflects the existence of an underlying symmetry. Such models were first constructed in the context of Grand Unified theories (GUTs) by Nelson and refined by Barr [15] and incorporate extra heavy quarks which mix with the observed quarks. Then, relying on specific symmetries, one can obtain a texture of the full quark mass matrices which guarantee the tree-level vanishing of $\bar{\theta}$ after the spontaneous CP violation (SCPV). After integrating out the heavy fields, the low energy quark mass matrices contain the usual KM phase. The generic difficulty with these models comes from the need to ensure that large contributions to $\bar{\theta}$ do not arise at higher loops, while at the same time having sufficient weak CP violation from the KM phase. Thus while the SCPV approach is conceptually rather simple, it is not so generic and requires careful model building.

Given some of the tantalizing hints of low energy supersymmetry and the plausible gauge origin of CP symmetry, it is worthwhile to attempt to construct a supersymmetric

(SUSY) model with a Nelson-Barr type mechanism for solving the strong CP problem. The SCPV feature then also resolves the so-called SUSY phases problem. The latter problem was originally described in the context of a minimal supergravity origin of the soft SUSY breaking terms [16], and is usually worse in a general SUSY breaking scenario. There are two extra phases in the universal soft mass parameters, beyond δ_{KM} and $\bar{\theta}$, which give CP violating effects in the low energy effective theory. These can be written as effective phases in the coefficients A and B of the trilinear and bilinear soft SUSY breaking scalar terms, respectively, given by

$$\phi_A = \arg(AM_{1/2}^*) \quad \phi_B = \arg(BM_{1/2}^*) , \quad (1.3)$$

where $M_{1/2}$ is the universal gaugino mass. The problem is that from 1-loop diagrams involving squarks, these phases must be fine-tuned to order $10^{-2} - 10^{-3}$ to satisfy the limit on the neutron electric dipole moment unless all the superpartners are “heavy”, ~ 1 TeV. With CP spontaneously broken in a sector independent of SUSY breaking, these phases would be naturally zero at first order.

Attempts to realize the Nelson-Barr mechanism in SUSY models [17] have, however, run up against a formidable difficulty: There generically exist potentially large 1-loop contributions to $\bar{\theta}$ in these models [18]. The dangerous diagrams are shown in Fig.1, where now in the supersymmetric case $\bar{\theta}$ also gets contributions from the argument of the gluino mass (Fig.1b):

$$\bar{\theta} = \theta - \arg \det M_q - 3 \arg M_g . \quad (1.4)$$

As discussed at length in Ref. [18], one requires an exceptionally high degree of proportionality of the soft SUSY breaking trilinear scalar couplings to their associated Yukawa couplings as well as degeneracy among the soft squark mass terms for each charge and color sector, if these contributions are to be sufficiently suppressed. This is equivalent to the statement that when the quark and squark mass matrices are diagonalized by the same set of unitary matrices, no phase can appear in the diagrams of Fig.1. The degree of proportionality and

degeneracy required among the soft SUSY breaking parameters is very difficult to maintain due to the effects of renormalization.

So, is SCPV doomed to be disfavored as a solution to the strong CP problem in supersymmetric models? We will argue that models with a specifically modified Nelson-Barr mechanism together with the recently popular gauge-mediated SUSY breaking (GMSB) scenario [19–23] can overcome the difficulty. The GMSB scenario ensures that the soft masses at the intrinsic SUSY breaking scale M_{mess} are proportional and degenerate, while renormalization effects that violate these conditions are reduced by having to run soft masses and couplings from the messenger scale $M_{mess} \simeq 10 - 100$ TeV instead of the reduced Planck mass $\simeq 2 \times 10^{18}$ GeV. In the minimal version of such GMSB models [20,22], the A - and B -terms are zero at M_{mess} . This tends to give additional suppression of the dangerous contributions to $\bar{\theta}$. However, large third generation Yukawa couplings can still lead to significant violations of proportionality and degeneracy and a detailed numerical analysis of the situation is necessary to determine if the supersymmetric Nelson-Barr type models are viable solutions to the strong CP problem. We will answer this in the affirmative with what to our knowledge is the only full renormalization group (RG) analysis of such models in the literature.

The models to be discussed here have CP spontaneously broken at low energies (of order a TeV), and a source of weak CP violation distinct from that in the Standard Model, namely the exchange of a new U(1) gauge boson, the symmetry of which enforces the Nelson-Barr texture. The KM phase is very small. This makes it possible to account for the smallness of $\bar{\theta}$ without making weak CP violation inconsistent with observations. In the SUSY-GUT Nelson-Barr models, even with gauge-mediated SUSY breaking, one predicts a too large $\bar{\theta}$ if the experimental requirement that $\delta_{KM} \sim \mathcal{O}(1)$ is imposed. Moreover, one has a richer and quite distinct phenomenology. The extra quarks and other fields needed to construct the Nelson-Barr texture and break CP will be within the reach of future accelerators. A non-supersymmetric version of the type of model is the aspon model [24–26]. The situation with SUSY incorporated is first discussed in Ref. [27] where its advantage over the generic

SUSY Nelson-Barr type models is highlighted. Another possible advantage for such low scale models would arise if it were somehow possible to imbed the sector responsible for the Nelson-Barr texture in the SUSY breaking and messenger sectors. An immediate disadvantage of this approach is that by breaking CP at low energies, one introduces a serious domain wall problem [28]. However, this can be solved via a period of inflation just above the weak scale. Indeed, in the SUSY context, some authors have argued that this type of inflation can be natural [29] and desirable for other reasons (*e.g.* as a solution to the cosmological moduli problem).

This article is organized as follows: In Section 2, the considerations leading to viable models for solving the strong and supersymmetric CP problems are discussed. We also note some intriguing alternatives worthy of further consideration. We give in Section 3 a detailed summary of the dangerous 1-loop contributions to $\bar{\theta}$ and their dependence on proportionality and squark-degeneracy violating mass insertions. The question of the detailed structure of the spontaneous CP breaking part of the superpotential is taken up in Section 4. We emphasize its importance in determining certain dangerous contributions to $\bar{\theta}$ and present a minimal example we use in further analysis. The renormalization group analysis used to estimate the $\bar{\theta}$ contributions is described in Section 5 and the full numerical results presented. Some remarks on related questions of interest are presented in the conclusion.

II. MODEL-BUILDING CONSIDERATIONS

We assume full CP symmetry in the visible sector, including the soft SUSY breaking terms, down to energies where spontaneous CP breaking occurs. From Eq.(1.4) we see that there should be no tree-level phases in the quark mass determinant, nor in the SUSY breaking gluino mass. The latter holds by assumption and the former is obtained through a Nelson-Barr texture. To obtain the texture, we introduce an extra heavy right-handed down quark superfield \bar{D} together with its mirror D coupling to the ordinary down quarks

via the superpotential¹

$$W_d = Y_d^{ij} Q_j \bar{d}_i H_d + \mu_D D \bar{D} + \gamma^{ia} D \chi_a \bar{d}_i, \quad (2.1)$$

where the VEVs of the scalar components of χ_a contain a relative phase, thus breaking CP. The details of the superpotential accomplishing this are postponed to Section 4, and here it is sufficient to note that at least two χ fields are necessary, since one phase can always be absorbed by a field redefinition of the extra quarks. After CP and $SU(2)_W \times U(1)_Y$ breaking, we have the down sector fermion mass matrix:

$$m_q = \begin{pmatrix} m_d & x\mu_D \mathbf{a} \\ 0 & \mu_D \end{pmatrix}, \quad (2.2)$$

where m_d is the usual 3×3 down sector mass matrix and \mathbf{a} is a complex 3-vector with components $a^i = \frac{1}{x\mu_D} \gamma^{ia} \langle \chi_a \rangle$, and the real parameter x is defined such that \mathbf{a} is normalized to 1, *i.e.* $\mathbf{a}^\dagger \mathbf{a} = 1$. The magnitude of mixing between the ordinary quarks and the extra singlet is characterized by x . Clearly the determinant of m_q is real and at energies below μ_D the low energy effective theory has a KM phase of at most order x . Without some additional source for weak CP violation, this must be $\mathcal{O}(1)$ and as we shall see this in turn makes the suppression of 1-loop contributions to $\bar{\theta}$ problematic.² For this reason, we shall assume that CP is broken at relatively low scales with a nonstandard mechanism for weak CP violation.

The specific form of the mass matrix in Eq.(2.2) can be enforced by a variety of symmetries, though by the non-renormalization theorems, the Nelson-Barr texture is not upset by renormalization of terms in the superpotential. In the aspon scenario, as discussed in Ref. [25], the D , \bar{D} and χ_a can be given charges 1, -1 and 1, respectively, under a new gauged $U(1)$ symmetry. The major source of weak CP violation, in the $K - \bar{K}$ system for instance,

¹ Other type of phenomenological features from an extra vector-like quark singlet have been studied by various authors in a different context. See for example Ref. [30], and references therein.

²This is exactly the reason why the small x scenario is discarded in the analysis of Ref. [18].

then comes from exchange of the new U(1) gauge boson (aspon) which becomes massive at the scale where CP is spontaneously broken. This places an upper bound on the mass scale of CP breaking of $\mathcal{O}(\text{TeV})$. More important in our SUSY version, this allows the parameter x to be small, *e.g.* $x^2 \sim 10^{-5}$, which contributes significantly in suppression $\bar{\theta}$ from loop corrections. Note that we will need at least some extra mirror partners for χ_a superfields to cancel the gauge anomaly introduced by their fermionic components.

If the Nelson-Barr texture is obtained from a discrete/global symmetry, one must rely on superbox diagrams involving gluino and chargino exchange to generate ϵ_K ; a scenario recently re-analyzed in the context of the minimal supersymmetric Standard Model (MSSM) [31]. However, in present setting, this proves very difficult to do. This is unfortunate since it is easier to construct unifiable models in this latter case.³

When SUSY is broken we generate nonzero gaugino masses and soft scalar bilinear and trilinear couplings,⁴ including the following terms relevant for the down-sector analysis [a complete description is given in Eqs.(A2,A3)]:

$$\begin{aligned}
V_{\text{soft}}^d = & \hat{Q}^\dagger \tilde{m}_Q^2 \hat{Q} + \hat{d} \tilde{m}_d^2 \hat{d}^\dagger + \hat{D}^\dagger \tilde{m}_D^2 \hat{D} + \hat{D} \tilde{m}_D^2 \hat{D}^\dagger + \chi_a \tilde{m}_{\chi ab}^2 \chi_b^\dagger \\
& + \hat{d} h_d \hat{Q} H_d + \hat{d}_i h_\gamma^{ia} \hat{D} \chi_a + B_D \mu_D \hat{D} \hat{D} + \text{h.c.}
\end{aligned} \tag{2.3}$$

The general form of the down squark mass matrix can be written as,

$$\mathcal{M}_d^2 = \begin{pmatrix} \mathcal{M}_{RR}^2 & \mathcal{M}_{RL}^2 \\ \mathcal{M}_{RL}^{2\dagger} & \mathcal{M}_{LL}^2 \end{pmatrix}, \tag{2.4}$$

³To construct a unifiable model in the discrete symmetry case, we need to add a pair of heavy lepton doublets coupling in a superpotential analogous to Eq.(2.1). The same thing can be done to make the aspon model GUT-compatible, but unifying the extra U(1) with the other gauge interactions is not possible.

⁴Note that the trilinear couplings, h_d and h_γ^{ia} , are also commonly written as products of the A -parameters and the corresponding Yukawa couplings, *e.g.* elements of the h_d matrix correspond to $A_d^{ij} Y_d^{ij}$ (no sum).

where

$$\mathcal{M}_{LL}^2 = \begin{pmatrix} \tilde{m}_d^2 + m_d^\dagger m_d & x\mu_D m_d^\dagger \mathbf{a} \\ x\mu_D \mathbf{a}^\dagger m_d & \mu_D^2(1+x^2) + \tilde{m}_D^2 \end{pmatrix}, \quad (2.5)$$

$$\mathcal{M}_{RR}^2 = \begin{pmatrix} \tilde{m}_d^2 + m_d m_d^\dagger + x^2 \mu_D^2 \mathbf{a} \mathbf{a}^\dagger & x\mu_D^2 \mathbf{a} \\ x\mu_D^2 \mathbf{a}^\dagger & \mu_D^2 + \tilde{m}_D^2 \end{pmatrix}, \quad (2.6)$$

$$\mathcal{M}_{RL}^2 = \begin{pmatrix} h_d v_d + m_d \mu_H \tan\beta & M_5^2 \mathbf{b} \\ 0 & B_D \mu_D \end{pmatrix}. \quad (2.7)$$

In the expression for \mathcal{M}_{RL}^2 we have used

$$M_5^2 b^i = h_\gamma^{ia} \langle \chi_a \rangle - \gamma^{ia} \langle F_{\chi_a} \rangle, \quad (2.8)$$

where \mathbf{b} is normalized to 1 and F_{χ_a} is the F -term for the χ_a field, which depends on the specific form of the soft SUSY breaking mass terms related to the spontaneous CP breaking part of the superpotential. The form of these squark mass matrices will be critical in the calculation of $\bar{\theta}$ in the next section. In particular, the $\langle F_{\chi_a} \rangle$'s bear complex phases independent of those in the $\langle \chi_a \rangle$'s in a generic setting, and hence constitute a major source of trouble.

We have implicitly assumed in the above that the sectors responsible for the Nelson-Barr texture and CP breaking are disjoint from those involved in the intrinsic breaking of supersymmetry. Since the successful example model we focus on has gauge-mediated SUSY breaking at a relatively low scale M_{mess} , it is *a priori* possible that all or part of the extra field content and symmetries required for the Nelson-Barr mechanism is contained in the SUSY breaking hidden and messenger sectors. Although this is an intriguing possibility, we have not been able to construct viable models of this type thus far. Actually, there appears to be an intrinsic incompatibility between the role a field takes in CP violation and the one it takes in SUSY breaking, as far as constraining $\bar{\theta}$ is concerned. Recall that in GMSB, the scalar particles get their soft SUSY breaking masses from the gauge interactions they share with the messenger sector particles which see SUSY breaking directly. The squark masses in

each sector, for instance, would then be degenerate, as the process is flavor blind. The extra singlet \bar{D} introduced here may easily upset the situation. Naively, the best strategy is to make the GMSB also blind to the aspon U(1). We will see below that this happens to have an even more important merit — it guarantees the suppression of the very dangerous $\langle F_{\chi_a} \rangle$'s. In other words, hiding the CP-breaking sector from SUSY breaking helps to suppress the SUSY loop contributions to $\bar{\theta}$. This is the less ambitious strategy we have taken in the model analyzed in detail below.

III. CALCULATION OF 1-LOOP $\bar{\theta}$ CONSTRAINTS

Here we review and extend the work in Refs. [18,27] to compute the 1-loop contributions to $\bar{\theta}$ of Figs.1a and 1b, using the mass-insertion approximation [32]. These results are generally valid for any type of low energy SCPV model, with or without the U(1)_A. The scale, μ_D , and hence the characteristic scale of the SCPV, here is chosen near or below \tilde{m}_{sq} , the average squark mass. This is more or less dictated by the aspon scenario of weak CP [25]. The D and \bar{D} superfields are handled on the same footing as the other quark superfields. The analysis is basically the same as that given in Ref. [27] except here we pay full attention to the explicit phase factors and family indices, and also treat the M_5^2 term, as given by Eq.(2.8), in full detail. These turn out to be very important in understanding how the scenario can provide a feasible solution. In taking the large μ_D limit, which corresponds to the situation discussed in Ref. [18], one has to be careful in handling the loop momentum integrals properly. The latter are however not explicitly given in this paper, though they are included in our numerical computations.

The 1-loop contribution is given by

$$\begin{aligned} \delta\bar{\theta} &= \text{Im Tr } m_F^{-1} \delta m_F + 3M_g^{-1} \delta M_g \\ &= \frac{\alpha_s}{4\pi} \sum_{i,I} \text{Im}[Z^{iI*} Z^{(i+4)I}] \mathcal{M}_{dI}^2 \left(\frac{M_g}{m_{F_i}} \frac{8/3}{M_g^2 - \mathcal{M}_{dI}^2} \ln \frac{\mathcal{M}_{dI}^2}{M_g^2} + \frac{m_{F_i}}{M_g} \frac{3}{\mathcal{M}_{dI}^2 - m_{F_i}^2} \ln \frac{\mathcal{M}_{dI}^2}{m_{F_i}^2} \right), \quad (3.1) \end{aligned}$$

where m_{F_i} runs over the four eigenvalues of the quark mass matrix [cf. Eq.(2.2)] and \mathcal{M}_{dI}^2

over the eight eigenvalues for the squarks [cf. Eq.(2.4)], all in the down-sector; Z^{IJ} is the unitary rotation that diagonalizes the squark mass matrix in the quark mass eigenstate basis. This full formula, while it can be used in the numerical calculations once all the quantities involved are known, hides its physics content behind the Z -matrix elements. In the limit of exact degeneracy and proportionality, the latter is just the identity matrix and $\bar{\theta}$ is zero. Otherwise, the mass-insertion approximation, as discussed below, is more illustrative.

We first assume an approximate degeneracy and that the diagonal blocks in \mathcal{M}_d^2 dominate over the off-diagonal block \mathcal{M}_{RL}^2 and write

$$\begin{aligned}\tilde{m}_d^2 &= \bar{m}_d^2 \times \mathbf{1} + \delta\tilde{m}_d^2, & \tilde{m}_D^2 &= \bar{m}_d^2 + \delta\tilde{m}_D^2, \\ \tilde{m}_d^2 &= \bar{m}_d^2 \times \mathbf{1} + \delta\tilde{m}_d^2, & \tilde{m}_D^2 &= \bar{m}_d^2 + \delta\tilde{m}_D^2.\end{aligned}\tag{3.2}$$

The squarks are then treated as scalars of masses \bar{m}_d^2 and \bar{m}_D^2 with the $\delta\tilde{m}_d^2$ and \mathcal{M}_{RL}^2 treated as admissible mass-insertions in the loop-diagrams Figs.1a and 1b. Explicit forms of the matrices needed to diagonalize m_F are useful. Expressions up to order x^2 are available in the literature [33]. To parametrize the effect of proportionality violation among the three families, we write

$$h_d = \bar{A}_d Y_d + \delta A_d.\tag{3.3}$$

The situation for the related parameter in the d - D mixings is more complicated. Recall that $M_5^2 b^i = h_\gamma^{ia} \langle \chi_a \rangle - \gamma^{ia} \langle F_{\chi_a} \rangle$ [Eq.(2.8)]. It has been emphasized in Ref. [27] that the F -terms being small is paramount to the success of any model of the Nelson-Barr type. These terms are dangerous because in general one has no reason to expect these F -terms to obey even an approximate proportionality (to the $x\mu_D a^i$ terms). On the contrary, contributions of the other part to $\bar{\theta}$ can be interpreted as a proportionality violation among the γ^{ia} 's by writing

$$h_\gamma^{ia} = \bar{A}_\gamma \gamma^{ia} + \delta A_\gamma^{ia};\tag{3.4}$$

the term proportional to \bar{A}_γ does not contribute. We further introduce the simplified notation:

$$\delta A_\gamma c^i = \frac{1}{x\mu_D} h_\gamma^{ia} \langle \chi_a \rangle - \bar{A}_\gamma a^i, \quad (3.5)$$

where complex vector \mathbf{c} is normalized to 1. Hence, we have

$$M_5^2 b^i = \bar{A}_\gamma (x\mu_D a^i) + \delta A_\gamma (x\mu_D c^i) - \gamma^{ia} \langle F_{\chi_a} \rangle. \quad (3.6)$$

In terms of the above notation, the list of major contributions to $\bar{\theta}$ is given in Tables 1a and 1b. The $\bar{\theta}$ contributions involving M_5^2 are complicated. To make it easier to see the effects of the different parts, we list some of those terms in tables before and after the above mentioned splitting. For example, entry 1 in the Table 1a is split into two parts: the first part is a proportionality violation effect involving δA_γ and $\text{Im}(a_i^* c^i)$ (both are suppressed in our model), the second is the F -term contribution ($\gamma^{ia} \langle F_{\chi_a} \rangle$), where the relevant complex phase is taken to be $\mathcal{O}(1)$. One other notable feature among the $\bar{\theta}$ contributions is the combination $M_5^2 b^i - x\mu_D B_D a^i$, as shown in entry 9 of Table 1b. When the $M_5^2 b^i$ term is split as above, the second term actually can be combined with the first term in Eq.(3.6) to give $x\mu_D (\bar{A}_\gamma - B_D) a^i$, which can be interpreted as a proportionality violation among the corresponding trilinear and bilinear terms. The other parts involve δA_γ and $\gamma^{ia} \langle F_{\chi_a} \rangle$, as explicitly shown in the table. All other entries with a M_5^2 can be split and interpreted in the same way. We will see in the final result that the F -term contribution *is the most dangerous*.

IV. THE SPONTANEOUS CP VIOLATION SECTOR

Spontaneous breaking of the $U(1)_A$ symmetry is the only source of CP violation in our model. This CP violation effect feeds directly into the $x\mu_D$ and M_5^2 terms in the quark and squark mass matrices, with complex phase vectors \mathbf{a} and \mathbf{b} , respectively. To implement the mechanism, we need a sector of $U(1)_A$ -charged SM singlet superfield with a superpotential that not only gives rise to the complex $\langle \chi_a \rangle$'s, but also gives us a good control on the dangerous $\langle F_{\chi_a} \rangle$'s. Soft SUSY breaking terms should also be taken into consideration, when determining the true scalar potential. The F -terms, of course, characterize SUSY breaking. We consider the scenario in which the messengers communicating SUSY breaking to the

visible sector are $U(1)_A$ -blind, *i.e.* they do not carry any $U(1)_A$ charges; furthermore, they are not directly coupled to the CP-breaking sector. The superfields of the latter are then hidden from SUSY breaking.

We have to consider at least five superfields, two $\bar{\chi}$'s of conjugate $U(1)_A$ charges to the χ_a 's and a singlet \aleph , in order to have both gauge anomaly cancellation and a possible CP violating vacuum solution [27,34]. We consider the superpotential⁵

$$W_\chi = \bar{\chi}_a \mu_\chi^{ab} \chi_b + \aleph \bar{\chi}_a \lambda^{ab} \chi_b + \lambda_\aleph \aleph^3 + \mu_\aleph \aleph^2. \quad (4.1)$$

The five F -flat conditions yield four independent equations, which, together with the D -flat condition, give a unique vacuum solution. The solution is CP violating for most of the parameter space. Hence, neglecting the soft SUSY breaking terms, we have a SUSY preserving vacuum that breaks CP.

The GMSB scenario we considered allows the unwanted soft SUSY breaking terms of the sector to be zero at M_{mess} . They are, however, generated through RG evolution, as discussed in the next section. With their nonvanishing values taken into consideration, the scalar potential is then given by

$$V_\chi = D_\chi^2 + F_{\chi_a} F_{\chi_a}^* + F_{\bar{\chi}_a} F_{\bar{\chi}_a}^* + F_\aleph F_\aleph^* + V_{s\chi} \quad (4.2)$$

where

$$\begin{aligned} V_{s\chi} = & \bar{\chi}_a B_\chi^{ab} \chi_b + \bar{\chi}_a h_\chi^{ab} \chi_b \aleph + h_\aleph \aleph^3 + B_\aleph \aleph^2 \\ & + \bar{\chi}_a^\dagger \tilde{m}_{\bar{\chi}ab}^2 \bar{\chi}_b + \chi_a \tilde{m}_{\chi ab}^2 \chi_b^\dagger + \aleph^\dagger \tilde{m}_\aleph^2 \aleph. \end{aligned} \quad (4.3)$$

Solving for the potential minimum to determine the $\langle F_{\chi_a} \rangle$ values is not tractable, as W_χ and $V_{s\chi}$ involves a large number of parameters which are not otherwise constrained, apart

⁵In Ref. [27], a W_χ without the μ_χ -terms is suggested. While that could have a CP violating vacuum with SUSY preserved, the situation is not as general and natural as the one considered here, and would have to rely on a linear \aleph term to fix the symmetry breaking scale.

from yielding a CP violating solution. However, one can easily obtain a reasonable order of magnitude estimate of the shifts in the $\langle F_{\chi_a} \rangle$'s as a result of including the small $V_{s\chi}$ terms.

For example, the equation

$$\frac{\partial V_\chi}{\partial \bar{\chi}_a} = 2D_\chi \frac{\partial D_\chi}{\partial \bar{\chi}_a} - (\mu_\chi^{ab} + \aleph \lambda^{ab}) F_{\chi_b} + F_{\aleph} \frac{\partial F_{\aleph}^*}{\partial \bar{\chi}_a} + B_\chi^{ab} \chi_b + h_\lambda^{ab} \chi_b \aleph + \bar{\chi}_b^\dagger \tilde{m}_{\bar{\chi}ba}^2 = 0 \quad (4.4)$$

suggests that $\langle F_\chi \rangle$ (here we drop all indices and phases) is given by the magnitude of

$$B_\chi \quad \text{or} \quad \tilde{m}_{\bar{\chi}}^2 \quad \text{or} \quad h_\lambda \langle \chi \rangle / \lambda. \quad (4.5)$$

An alternative way to estimate the $\langle F_{\chi_a} \rangle$'s is given by the SUSY breaking diagrams shown in Fig.2. Here Figs.2a and 2b give exactly the same results as the first two terms listed above. Figure 2c, however, gives the estimate $\langle F_\chi \rangle \sim h_\lambda \langle \chi \rangle \lambda / 16\pi^2$. For perturbative values of the λ coupling, this is of course smaller than the third estimate in Eq.(4.5), hence we neglect it. A similar diagram, Fig.2d, also suggests a contribution $\sim h_\gamma \langle \chi \rangle \gamma / 16\pi^2$, though the γ dependence of the $\langle F_\chi \rangle$'s is implicitly incorporated into the generation of the $V_{s\chi}$ terms through RG running. We will use all these in our numerical estimates to determine whether the F -term is sufficiently small that its contributions to $\bar{\theta}$, listed in Tables 1 and 2, are under control. Finally, we emphasize again that the complex phases in the $\langle F_{\chi_a} \rangle$'s are not related to those of the $\langle \chi_a \rangle$'s directly.

V. RENORMALIZATION GROUP ANALYSIS

As pointed out in the introduction, we need a full theory for the soft SUSY breaking parameters to see if the $\bar{\theta}$ constraints can be satisfied and the GMSB scenario may provide the only viable possibility. In particular, we use here only the minimal version of such a theory [20]. This version has a few special merits: it provides practically a one-parameter model of soft SUSY breaking, radiative breaking of electroweak symmetry is naturally implemented and, within the MSSM framework, it has been studied with extensive renormalization group analysis and shown to be compatible with all known experimental constraints [22]. From our

perspective of solving the strong CP problem by augmenting the $U(1)_A$ sector, it actually represents a relatively demanding setting among GMSB models, where a large $\tan\beta$ allows all the Yukawa couplings of the third family to have substantial effects on the RG-runnings. A smaller $\tan\beta$ in general would only make it easier to satisfy the $\bar{\theta}$ constraints.

We will refrain from elaborating extensively on the details of the GMSB model or the RG-analysis itself. For more specific details on finding the correct electroweak symmetry breaking vacuum and meeting other experimental constraints in the minimal GMSB model, readers are referred to Ref. [22]. *Our interest here is in adapting the machinery to our extended model at a level of sufficient sophistication to calculate $\bar{\theta}$ to 1-loop and establish our solution to the strong CP problem.*

We use 1-loop renormalization group equations (RGE's) with naive step thresholds between $M_Z = M_{SUSY}$ and M_{mess} . The RG-improved tree level Higgs potential is considered in finding the electroweak-symmetry breaking solution. The RGE's for the extra content of the model are also implemented at the 1-loop level; relevant formulae are in Appendix A. The $U(1)_A$, and hence CP symmetry, breaking is imposed by hand. The idea is to study the general situation independent of the details of the SCPV sector, as the latter is to a certain extent more flexible and less constrained. It is important to note that the extra superfield content in the model is partially decoupled from the MSSM part, with the only direct coupling being gauge couplings of D and \bar{D} , and the small Yukawa couplings γ^{ia} . The computation concerning the SCPV sector, as well as the RG analysis can certainly be made more sophisticated, however, we consider our treatment sufficient for our purpose. In the sample analysis for which numerical results are presented in this paper in detail (Appendix B and the last column of Table 1), the values of the various γ^{ia} Yukawa couplings are generated randomly in the range 0.005 – 0.01. The latter is chosen to target an x -value of around 0.01. The value of μ_D is fixed at 500 GeV; $M_{mess} \sim \Lambda$ at 50 TeV. For the soft SUSY breaking parameters from GMSB, all A - and B - terms are taken to be zero at $M_{mess} \equiv X$. The scalar soft masses from GMSB are given by

$$\tilde{m}^2(X) = \frac{\Lambda^2}{8\pi} \left\{ C_3 \alpha_3^2(X) + C_2 \alpha_2^2(X) + \frac{3}{5} Y^2 \alpha_1^2(X) \right\} f(y), \quad (5.1)$$

where $C_3 = 4/3, 0$ for triplets and singlets of $SU(3)_C$, $C_2 = 3/4, 0$ for doublets and singlets of $SU(2)_L$; $Y = Q - T_3$ is the hypercharge. The function $f(y)$, derived in Ref. [21], is simply set to 1. Note that the above formula is independent of the $U(1)_A$ charge; a SM singlet with or without $U(1)_A$ charge, such as the χ_a and \aleph scalars, has no initial soft mass. Gaugino masses are likewise given by the MSSM formula, omitted here. The new $U(1)_A$ gaugino (aspino) has no tree-level SUSY breaking mass. The gauge coupling g_A is taken to be around g_{em} . The SUSY-breaking aspino mass M_A then remains vanishingly small even after finite loop effects and RG evolution are taken into account.

After the electroweak symmetry breaking solution is obtained, various $\bar{\theta}$ contributions are calculated through the mass-insertion approximation to order x^2 . To impose the $U(1)_A$ symmetry breaking, we set, in the sample analysis, $|\langle\chi_1\rangle|^2 + |\langle\chi_2\rangle|^2 \simeq \mu_D^2$ and choose random values for the VEVs and their complex phases within the constraint. Effects of higher order in x are checked to be insignificant.

Values of parameters in W_χ are needed for the RG-runnings of particularly the SCPV sector soft SUSY breaking parameters, discussed in the previous section. To simplify the situation, we input all these mass parameters as μ_D and all dimensionless couplings as random numbers in the range $0.1 - 0.8$, for the sample calculation. This oversimplification certainly begs the question of consistency of the vacuum solution for this sector, or the whole model. However, in the small x domain of interest, the influence of the extra ingredients on the values of the other MSSM parameters is insignificant, as to be expected. The only practical effect of those parameters is in the RG evolution of the related soft terms which we needed to estimate the $\langle F_\chi \rangle$'s. We have checked, for instance, that the particular input values used in the sample run reported here does lead to generic magnitudes of the latter.

Appendix B contains a collection of some of the numerical results, while those for the $\bar{\theta}$ contributions, *without* the $\langle F_\chi \rangle$'s are listed in the last column of Table 1. Estimates of the $\langle F_\chi \rangle$, following the discussion in the previous section, and their contribution to $\bar{\theta}$ are

given in Table 2 (second column). The latter can be easily checked using the $\langle F_\chi \rangle$ value and the listing of M_5^2 -terms in Table 1. We also list in Table 2 results from a number of different runs with different values of the γ 's (reflected by the x -value obtained) and λ 's. The former, which can have a significant effect on the various MSSM parameters, are restricted by $x^2 \sim 10^{-3} - 10^{-5}$ from the weak CP considerations. Our results indicate that only a relatively large value of x can upset the strong CP solution, by first driving $\langle F_\chi \rangle$ too large (see column 4 of Table 2). One should be cautious in using this result quantitatively, as our $\langle F_\chi \rangle$ estimates are meant to be conservative upper bounds. However, the result is certainly illustrative of the importance of the $\langle F_\chi \rangle$ in estimating $\bar{\theta}$. With x restricted to the workable range, the basic features of the RGE solutions are quite stable. This is true even with a relatively large variation of the λ 's, as illustrated by column 5 and 6 of Table 2. Note that though the $h_\lambda \langle \chi \rangle / \lambda$ term may have an explicit dependence on λ , its numerical value does not have a large variation with λ as one might naively expect. This is, like the approximate proportionality of a general A -term, a natural result of the RG equations.

All in all, the F -term contributions to $\bar{\theta}$ dominate, and the overall $\bar{\theta}$ value is comfortably within the required bound for the major region of the parameter space of our model under consideration, hence solving the strong CP problem.

VI. CONCLUSIONS

To recapitulate, we discussed a complete spontaneous CP violation model with gauge-mediated supersymmetry breaking and why this type of model is particularly favored over a generic supersymmetric SCPV model in solving the strong CP problem. Results from numerical RGE studies are used to explicitly establish the feasibility of the approach. The treatment of parameters in the SCPV sector is admittedly oversimplified. The correlations between x and μ_D , and between the various mass parameters at the μ_D scale and the values of the various λ coupling, for instance, are neglected. However, it is easy to see from our discussion that such details are not going to change the essential features of our results,

though they would determine explicitly the specific “large- x ” region of the parameter space that could be ruled out. The model has a rich spectrum of new particles at the μ_D or SCPV scale. Until such experimental data become available, a detailed study of the parameter space may not be feasible.

While the weak CP aspects of this model have been analyzed in the non-supersymmetric setting as in Ref. [25], SUSY particles could lead to new contributions through super-box and penguin diagrams. These contributions are in general subdominant, as are the Standard Model box diagrams.

Our model predicts a measureable neutron EDM, which could be close to the present experimental bound for $x > .01$. The model also has an extra pair of vector-like quark superfields, a new gauge boson, and a number of neutral fermions and scalars with no direct couplings to the Standard Model gauge bosons, all with masses around the TeV scale. This scale is dictated by the weak CP phenomenology. Hence it offers a rich spectrum of new particles to be discovered at the CERN Large Hadron Collider. Because the 3×3 Cabbibo-Kobayashi-Maskawa matrix is approximately real, the model also predicts a flat unitarity triangle and the absence of substantial CP violation in the B system at future B factories [26]. Moreover, there will be a lack of any substantial CP violating effects in the up-quark sector.

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APPENDIX A: RENORMALIZATION GROUP EQUATIONS

Here we collect the modified one loop MSSM renormalization group equations (RGE's) to account for the extra vector-like chiral superfields and the new Yukawa couplings in our model. In many cases we give only the extra contributions and refer the interested reader to Ref. [35] with whom we share conventions. The complete two-loop renormalization group equations for a softly broken supersymmetric theory can be found in Refs. [35,36].

The complete superpotential can be written as

$$\begin{aligned}
 W = & \bar{u}Y_uQH_u + \bar{d}Y_dQH_d + \bar{e}Y_eLH_d + \mu_H H_u H_d + \bar{d}_i \gamma^{ia} D\chi_a + \mu_D \bar{D}D \\
 & + \bar{\chi}_a \mu_\chi^{ab} \chi_b + \aleph \bar{\chi}_a \lambda^{ab} \chi_b + \lambda_\aleph \aleph^3 + \mu_\aleph \aleph^2
 \end{aligned} \tag{A1}$$

where family indices are implicit except in the new Yukawa coupling and $a, b = 1, 2$. The soft supersymmetry breaking Lagrangian can be written as $\mathcal{L}_{\text{soft}} = \mathcal{L}_{\text{soft}}^{\text{MSSM}} + \mathcal{L}_{\text{soft}}^{\text{extra}}$, where

$$\begin{aligned}
 -\mathcal{L}_{\text{soft}}^{\text{MSSM}} = & \hat{u}h_u \hat{Q}H_u + \hat{d}h_d \hat{Q}H_d + \hat{e}h_e \hat{L}H_d + B_H \mu_H H_u H_d + \text{h.c.} \\
 & + \hat{Q}^\dagger \tilde{m}_Q^2 \hat{Q} + \hat{L}^\dagger \tilde{m}_L^2 \hat{L} + \hat{u} \tilde{m}_u^2 \hat{u}^\dagger + \hat{d} \tilde{m}_d^2 \hat{d}^\dagger + \hat{e} \tilde{m}_e^2 \hat{e}^\dagger \\
 & + m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d,
 \end{aligned} \tag{A2}$$

and

$$\begin{aligned}
 -\mathcal{L}_{\text{soft}}^{\text{extra}} = & \hat{d}_i h_\gamma^{ia} \hat{D}\chi_a + B_D \mu_D \hat{D}\hat{D} + \bar{\chi} B_\chi^{ab} \chi + \bar{\chi}_a h_\chi^{ab} \chi_b \aleph + h_\aleph \aleph^3 + B_\aleph \aleph^2 + \text{h.c.} \\
 & + \hat{D} \tilde{m}_D^2 \hat{D}^\dagger + \hat{D}^\dagger \tilde{m}_D^2 \hat{D} + \bar{\chi}_a^\dagger \tilde{m}_{\chi ab}^2 \bar{\chi}_b + \chi_a \tilde{m}_{\chi ab}^2 \chi_b^\dagger + \aleph^\dagger \tilde{m}_\aleph^2 \aleph.
 \end{aligned} \tag{A3}$$

Note that B_χ and B_\aleph are defined in a different way from B_D and B_H ; the former have dimension (mass)² and are analogs of $B_D \mu_D$ and $B_H \mu_H$. Also, B_χ is a 2×2 matrix. Finally we have supersymmetry breaking gaugino masses $M_a (a = 1, 2, 3)$ for the MSSM and a possible gaugino mass M_A under $U(1)_A$.

We give the MSSM one loop RGE's for these interactions below. The gauge couplings are computed to two loops and are given by

$$\frac{dg_a}{dt} = \frac{g_a^3}{16\pi^2} B_a^{(1)} + \frac{g_a^3}{(16\pi^2)^2} \left(\sum_{b=1}^3 B_{ab}^{(2)} g_b^2 - \sum_{x=u,d,e,\gamma} C_a^x \text{Tr}(Y_x^\dagger Y_x) \right), \tag{A4}$$

where $B^{(1)} = (\frac{33}{5} + \frac{2}{5}N_D + \frac{3}{5}N_L, 1 + N_L, -3 + N_D)$,

$$B^{(2)} = \begin{pmatrix} \frac{199}{25} + \frac{8}{75}N_D + \frac{9}{25}N_L & \frac{27}{5} + \frac{9}{5}N_L & \frac{88}{5} + \frac{32}{15}N_D \\ \frac{9}{5} + \frac{3}{5}N_L & 25 + 7N_L & 24 \\ \frac{11}{5} + \frac{4}{15}N_D & 9 & 14 + \frac{34}{3}N_D \end{pmatrix}, \quad (\text{A5})$$

and

$$C^{u,d,e,\gamma} = \begin{pmatrix} \frac{26}{5} & \frac{14}{5} & \frac{18}{5} & \frac{4}{5} \\ 6 & 6 & 2 & 0 \\ 4 & 4 & 0 & 2 \end{pmatrix}. \quad (\text{A6})$$

In the above we have allowed for the possibility of N_D heavy vector-like pairs of charge $\frac{1}{3}$ color triplets [SU(2) singlets] and N_L such pairs of hypercharge -1 SU(2) doublets (color singlets). These include both the extra mirror pairs $D + \bar{D}$ and $L + \bar{L}$ which can interact directly with MSSM fields, but also extra mirror pairs originating in the messenger sector at higher scales. We have for simplicity omitted the possible Yukawa interactions of the extra mirror lepton doublets and the two loop $U(1)_A$ contributions. In the actual computations, we use $N_D = 1$, $N_L = 0$. At one loop we also have

$$16\pi^2 \frac{dg_A}{dt} = 10g_A^3, \quad (\text{A7})$$

when the $U(1)_A$ is present.

Using the above definitions, the two loop gaugino mass equations are

$$\begin{aligned} \frac{dM_a}{dt} = & \frac{2g_a^2}{16\pi^2} B_a^{(1)} M_a + \frac{2g_a^3}{(16\pi^2)^2} \left[\sum_{b=1}^3 B_{ab}^{(2)} g_b^2 (M_a + M_b) \right. \\ & \left. + \sum_{u,d,e,\gamma} C_a^x \left(\text{Tr}(Y_x^\dagger h_x) - M_a \text{Tr}(Y_x^\dagger Y_x) \right) \right]. \end{aligned} \quad (\text{A8})$$

At one loop we have

$$16\pi^2 \frac{dM_A}{dt} = 20g_A^2 M_A. \quad (\text{A9})$$

The running down quark Yukawa matrix is modified by the presence of the $D-\bar{d}$ couplings γ^{ia} which are treated as 3×2 matrices below. We have for the up and down one loop Yukawas:

$$16\pi^2 \frac{dY_d}{dt} = Y_d \left(\text{Tr} \left(3Y_d^\dagger Y_d + Y_e^\dagger Y_e \right) + 3Y_d^\dagger Y_d + Y_u^\dagger Y_u - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2 \right) + \gamma\gamma^\dagger Y_d, \quad (\text{A10})$$

$$16\pi^2 \frac{dY_u}{dt} = Y_u \left(3\text{Tr} \left(Y_u^\dagger Y_u \right) + 3Y_u^\dagger Y_u + Y_d^\dagger Y_d - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right), \quad (\text{A11})$$

and for γ :

$$16\pi^2 \frac{d\gamma}{dt} = \gamma \left(\text{Tr} \left(\gamma^\dagger \gamma \right) + 2\gamma^\dagger \gamma + \lambda^\dagger \lambda - \frac{16}{3}g_3^2 - \frac{4}{15}g_1^2 - 4g_A^2 \right) + Y_d Y_d^\dagger \gamma, \quad (\text{A12})$$

where for γ we have included the effect of a possible extra $U(1)_A$ as described in the text.

The running supersymmetric μ_D parameter is given by

$$16\pi^2 \frac{d\mu_D}{dt} = \mu_D \left(\text{Tr} \left(\gamma^\dagger \gamma \right) - \frac{16}{3}g_3^2 - \frac{4}{15}g_1^2 - 4g_A^2 \right). \quad (\text{A13})$$

The equation for the corresponding soft mass parameter is

$$16\pi^2 \frac{dB_D}{dt} = 2\text{Tr} \left(\gamma^\dagger h_\gamma \right) + \frac{32}{3}g_3^2 M_3 + \frac{8}{15}g_1^2 M_1 + 8g_A^2 M_A. \quad (\text{A14})$$

Similar parameters for the SCPV sector have

$$16\pi^2 \frac{d\mu_\chi}{dt} = \mu_\chi \lambda^\dagger \lambda + \lambda \lambda^\dagger \mu_\chi + 3\mu_\chi \gamma^\dagger \gamma - 4g_A^2 \mu_\chi, \quad (\text{A15})$$

$$\begin{aligned} 16\pi^2 \frac{dB_\chi}{dt} &= B_\chi \left(\lambda^\dagger \lambda + 3\gamma^\dagger \gamma \right) + \lambda \lambda^\dagger B_\chi + \lambda \left[\text{Tr} \left(\lambda^\dagger B_\chi \right) + \lambda_{\mathbb{N}} B_{\mathbb{N}} \right] \\ &\quad + 2\mu_\chi \lambda^\dagger h_\lambda + 2h_\lambda \lambda^\dagger \mu_\chi + 6\mu_\chi \gamma^\dagger h_\gamma - 4 \left(B_\chi - 2\mu_\chi M_A \right) g_A^2, \end{aligned} \quad (\text{A16})$$

and

$$16\pi^2 \frac{d\mu_{\mathbb{N}}}{dt} = \mu_{\mathbb{N}} \left[\lambda_{\mathbb{N}}^2 + 2\text{Tr} \left(\lambda^\dagger \lambda \right) \right], \quad (\text{A17})$$

$$\begin{aligned} 16\pi^2 \frac{dB_{\mathbb{N}}}{dt} &= B_{\mathbb{N}} \left[\lambda_{\mathbb{N}}^2 + 2\text{Tr} \left(\lambda^\dagger \lambda \right) \right] + \lambda_{\mathbb{N}} \left[\lambda_{\mathbb{N}} B_{\mathbb{N}} + 2\text{Tr} \left(\lambda^\dagger B_\chi \right) \right] \\ &\quad + 2\mu_{\mathbb{N}} \left[\lambda_{\mathbb{N}} h_{\mathbb{N}} + 2\text{Tr} \left(\lambda^\dagger h_\lambda \right) \right]. \end{aligned} \quad (\text{A18})$$

There are also RGE's for the extra Yukawa couplings:

$$16\pi^2 \frac{d\lambda}{dt} = \lambda \left(\text{Tr} \left(\lambda^\dagger \lambda \right) + \frac{1}{2}\lambda_{\mathbb{N}}^2 + 2\lambda^\dagger \lambda + 3\gamma^\dagger \gamma - 4g_A^2 \right), \quad (\text{A19})$$

$$16\pi^2 \frac{d\lambda_{\mathbb{N}}}{dt} = \lambda_{\mathbb{N}} \left[\lambda_{\mathbb{N}}^2 + 2\text{Tr} \left(\lambda^\dagger \lambda \right) \right]. \quad (\text{A20})$$

Note that μ_x , B_x , λ and h_λ are all 2×2 matrices.

The relevant soft supersymmetry breaking trilinear coupling RGE's are given by

$$\begin{aligned}
16\pi^2 \frac{dh_d}{dt} = & h_d \left(\text{Tr} \left(3Y_d^\dagger Y_d + Y_e^\dagger Y_e \right) + 5Y_d^\dagger Y_d + Y_u^\dagger Y_u - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2 \right) + \gamma\gamma^\dagger h_d \\
& + Y_d \left(\text{Tr} \left(6Y_d^\dagger h_d + 2Y_e^\dagger h_e \right) + 4Y_d^\dagger h_d + 2Y_u^\dagger h_u \right. \\
& \left. + \frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{14}{15}g_1^2 M_1 \right) + 2h_\gamma \gamma^\dagger Y_d, \tag{A21}
\end{aligned}$$

$$\begin{aligned}
16\pi^2 \frac{dh_u}{dt} = & h_u \left(3\text{Tr} \left(Y_u^\dagger Y_u \right) + 5Y_u^\dagger Y_u + Y_d^\dagger Y_d - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right) \\
& + Y_u \left(6\text{Tr} \left(Y_u^\dagger h_u \right) + 4Y_u^\dagger h_u + 2Y_d^\dagger h_d \right. \\
& \left. + \frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{26}{15}g_1^2 M_1 \right), \tag{A22}
\end{aligned}$$

$$\begin{aligned}
16\pi^2 \frac{dh_\gamma}{dt} = & h_\gamma \left(\text{Tr} \left(\gamma^\dagger \gamma \right) + 3\gamma^\dagger \gamma + \lambda^\dagger \lambda - \frac{16}{3}g_3^2 - \frac{4}{15}g_1^2 - 4g_A^2 \right) + Y_d Y_d^\dagger h_\gamma \\
& + \gamma \left(2\text{Tr} \left(\gamma^\dagger h_\gamma \right) + 3\gamma^\dagger h_\gamma + \lambda^\dagger h_\lambda + \frac{32}{3}g_3^2 M_3 + \frac{8}{15}g_1^2 M_1 + 8g_A^2 M_A \right) \\
& + 2h_d Y_d^\dagger \gamma, \tag{A23}
\end{aligned}$$

$$\begin{aligned}
16\pi^2 \frac{dh_\lambda}{dt} = & h_\lambda \left(\text{Tr} \left(\lambda^\dagger \lambda \right) + 3\gamma^\dagger \gamma + 3\lambda^\dagger \lambda + \frac{1}{2}\lambda_\mathbb{N}^2 - 4g_A^2 \right) \\
& + \lambda \left(2\text{Tr} \left(\lambda^\dagger h_\lambda \right) + 6\gamma^\dagger h_\gamma + 3\lambda^\dagger h_\lambda + \lambda_\mathbb{N} h_\mathbb{N} + 8g_A^2 M_A \right), \tag{A24}
\end{aligned}$$

$$16\pi^2 \frac{dh_\mathbb{N}}{dt} = h_\mathbb{N} \left(\frac{9}{2}\lambda_\mathbb{N}^2 + 3\text{Tr} \left(\lambda^\dagger \lambda \right) \right) + 6\lambda_\mathbb{N} \text{Tr} \left(\lambda^\dagger h_\lambda \right). \tag{A25}$$

Finally we give the modifications of the MSSM soft hermitian quadratic mass parameters (see Ref. [35] for a complete description). All of the MSSM RGE's have the following change in a D-term contribution: the factor \mathcal{S} defined in Eq.(4.27) of [35] is now given by

$$\mathcal{S} = m_{H_u}^2 - m_{H_d}^2 - \tilde{m}_D^2 + \tilde{m}_D^2 + \text{Tr} \left(\tilde{m}_Q^2 - \tilde{m}_L^2 - 2\tilde{m}_u^2 + \tilde{m}_d^2 + \tilde{m}_e^2 \right). \tag{A26}$$

Besides this modification, the equation for \tilde{m}_d^2 has the only nontrivial change due to the coupling γ :

$$\begin{aligned}
16\pi^2 \frac{d\tilde{m}_d^2}{dt} = & \left(2\tilde{m}_d^2 + 4m_{H_d}^2 \right) Y_d Y_d^\dagger + 4Y_d \tilde{m}_Q^2 Y_d^\dagger + 2Y_d Y_d^\dagger \tilde{m}_d^2 + \gamma\gamma^\dagger \tilde{m}_d^2 + \tilde{m}_d^2 \gamma\gamma^\dagger \\
& + 2\tilde{m}_D^2 \gamma\gamma^\dagger + 2\gamma \tilde{m}_\chi^2 \gamma^\dagger + 4h_d h_d^\dagger + 2h_\gamma h_\gamma^\dagger \\
& - \frac{32}{3}g_3^2 |M_3|^2 - \frac{8}{15}g_1^2 |M_1|^2 + \frac{2}{5}g_1^2 \mathcal{S}, \tag{A27}
\end{aligned}$$

where \tilde{m}_χ^2 is a 2×2 matrix. The equations for the new soft squark masses are

$$16\pi^2 \frac{d\tilde{m}_D^2}{dt} = 2\text{Tr} \left(\tilde{m}_D^2 \gamma^\dagger \gamma + \gamma \tilde{m}_\chi^2 \gamma^\dagger + \gamma^\dagger \tilde{m}_d^2 \gamma + h_\gamma^\dagger h_\gamma \right) - \frac{32}{3} g_3^2 |M_3|^2 - \frac{8}{15} g_1^2 |M_1|^2 - 8g_A^2 |M_A|^2 - \frac{2}{5} g_1^2 \mathcal{S} + 2g_A^2 \mathcal{S}_A, \quad (\text{A28})$$

$$16\pi^2 \frac{d\tilde{m}_D^2}{dt} = -\frac{32}{3} g_3^2 |M_3|^2 - \frac{8}{15} g_1^2 |M_1|^2 - 8g_A^2 |M_A|^2 + \frac{2}{5} g_1^2 \mathcal{S} - 2g_A^2 \mathcal{S}_A, \quad (\text{A29})$$

$$16\pi^2 \frac{d\tilde{m}_\chi^2}{dt} = 3\gamma^\dagger \gamma \tilde{m}_\chi^2 + 3\tilde{m}_\chi^2 \gamma^\dagger \gamma + 6\gamma^\dagger \gamma \tilde{m}_D^2 + 6\gamma^\dagger \tilde{m}_d^2 \gamma + 6h_\gamma^\dagger h_\gamma + \lambda^\dagger \lambda \tilde{m}_\chi^2 + \tilde{m}_\chi^2 \lambda^\dagger \lambda + 2\lambda^\dagger \lambda \tilde{m}_\chi^2 + 2\lambda^\dagger \tilde{m}_\chi^2 \lambda + 2h_\lambda^\dagger h_\lambda - 8g_A^2 |M_A|^2 - 2g_A^2 \mathcal{S}_A, \quad (\text{A30})$$

$$16\pi^2 \frac{d\tilde{m}_\chi^2}{dt} = \tilde{m}_\chi^2 \lambda \lambda^\dagger + \lambda \lambda^\dagger \tilde{m}_\chi^2 + 2\lambda \lambda^\dagger \tilde{m}_\chi^2 + 2\lambda \tilde{m}_\chi^2 \lambda^\dagger + 2h_\lambda h_\lambda^\dagger - 8g_A^2 |M_A|^2 + 2g_A^2 \mathcal{S}_A, \quad (\text{A31})$$

$$16\pi^2 \frac{d\tilde{m}_\chi^2}{dt} = (3\lambda_\chi^2 + 2\text{Tr} \lambda^\dagger \lambda) \tilde{m}_\chi^2 + 2\text{Tr} (\lambda \tilde{m}_\chi^2 \lambda^\dagger) + 2\text{Tr} (\lambda^\dagger \tilde{m}_\chi^2 \lambda) + h_\chi^2 + 2\text{Tr} h_\lambda^\dagger h_\lambda, \quad (\text{A32})$$

where $\mathcal{S}_A = 3\tilde{m}_D^2 - 3\tilde{m}_D^2 + \text{Tr}(\tilde{m}_\chi^2 - \tilde{m}_\chi^2)$.

APPENDIX B: SOME NUMERICAL RESULTS

We collect here some of the numerical results in our sample calculation. Recall that we use $M_{mess} = 50$ TeV, $\mu_D = \mu_\chi = 500$ GeV, and random values of γ 's and λ 's in the ranges .005 – .01 and .1 – .8 respectively. As inputs we also used the values $M_t = 175$ GeV and $\alpha_s = 0.12$.

The electroweak-symmetry breaking solution is obtained with

$$\tan\beta = 43.18, \quad \mu_H = -370.5 \text{ GeV}, \quad B_H = 3.938 \text{ GeV}. \quad (\text{B1})$$

In the following, we concentrate on the down-sector as it is the only one of relevance to the understanding the $\bar{\theta}$ value. Average values of the squared left- and right-handed squark masses are $5.337 \times 10^5 \text{ GeV}^2$ and $5.031 \times 10^5 \text{ GeV}^2$, respectively, and \bar{A}_d is -243.5 GeV . The lack of proportionality of the A_d -terms is given by

$$\delta A / \tilde{m}_{sq} = \begin{pmatrix} 0. & 2.12 \times 10^{-8} & 8.45 \times 10^{-7} \\ 1.18 \times 10^{-9} & 6.39 \times 10^{-7} & 4.02 \times 10^{-7} \\ -2.37 \times 10^{-7} & 1.73 \times 10^{-6} & 1.63 \times 10^{-2} \end{pmatrix}, \quad (\text{B2})$$

while degeneracy violations are given by

$$\delta\tilde{m}_d^2/\tilde{m}_{sq}^2 = \begin{pmatrix} 0. & 2.54 \times 10^{-5} & -6.13 \times 10^{-4} \\ 2.54 \times 10^{-5} & -2.53 \times 10^{-4} & 4.42 \times 10^{-3} \\ -6.13 \times 10^{-4} & 4.42 \times 10^{-3} & -0.172 \end{pmatrix}, \quad (\text{B3})$$

and

$$\delta\tilde{m}_{\bar{d}}^2/\tilde{m}_{sq}^2 = \begin{pmatrix} 0. & -8.22 \times 10^{-6} & -1.12 \times 10^{-5} \\ -8.22 \times 10^{-6} & -1.59 \times 10^{-4} & -1.17 \times 10^{-5} \\ -1.12 \times 10^{-5} & -1.17 \times 10^{-5} & -0.135 \end{pmatrix}, \quad (\text{B4})$$

where \tilde{m}_{sq}^2 is the average squark mass. In addition, we have

$$\delta\tilde{m}_D^2/\tilde{m}_{sq}^2 = 1.65 \times 10^{-5}, \quad (\text{B5})$$

$$\delta\tilde{m}_{\bar{D}}^2/\tilde{m}_{sq}^2 = -5.64 \times 10^{-2}. \quad (\text{B6})$$

The proportional part of the A -terms for the γ Yukawas is very close to the B_D value, given by

$$\bar{A}_\gamma \sim B_D = -248.1 \text{ GeV}, \quad (\text{B7})$$

with a difference of only 4.27×10^{-2} GeV. Some other quantities of interest are :

$$M_g(\equiv M_3) = 478.1 \text{ GeV}, \quad x = 1.213 \times 10^{-2};$$

and, as defined by Eq.(3.5),

$$\delta A_\gamma = 4.782 \times 10^{-4} \text{ GeV}, \quad \text{Im}(\mathbf{a}^\dagger \mathbf{c}) = 5.531 \times 10^{-3}. \quad (\text{B8})$$

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Table Captions.

Table 1: Analysis of the major 1-loop $\bar{\theta}$ contributions and numerical results from the sample run. Table 1a contains contributions from gluino mass corrections; Table 1b contains those from quark mass corrections. Entry 1 of Table 1a and entries 9, and 10 of Table 1b are shown together with explicit splittings of M_5^2 according to Eq.(3.6) below the first lines. Numerical results given in the last column do not include the $\langle F_\chi \rangle$ term contributions, but otherwise are complete, *i.e.* they include all other numerical factors from color indices summation, momentum loop integrals, and full summation over family indices (i, j, k) so that the full $\bar{\theta}$ value without the F -term contributions, apart from some unlisted subdominating terms, is given by the sum of all the entries.

Table 2: Estimates of the $\langle F_\chi \rangle$ term and its contribution to $\bar{\theta}$, for our sample run and a few runs with different γ and λ inputs (μ_D and μ_χ 's are all set at 500 GeV, M_{mess} at 50 TeV). Note that the entries B_χ , $\tilde{m}_{\bar{\chi}}^2$, $h_\lambda \langle \chi \rangle / \lambda$, and $h_\gamma \langle \chi \rangle \gamma / 16\pi^2$ are our $\langle F_\chi \rangle$ estimates, as discussed; all these are quantities of dimension (mass)² in units of GeV² (not shown explicitly). The $\langle F_\chi \rangle$ estimates and its contributions to $\bar{\theta}$ are meant to be upper bounds. Overall $\bar{\theta}$ contributions from gluino and quark mass corrections without the F -term are also listed.

Figure Captions.

Fig.1 : 1-loop mass-correction diagrams leading to $\bar{\theta}$ contributions. (a) 1-loop quark mass; (b) 1-loop gluino mass.

Fig.2 : Diagrams giving estimates of $\langle F_\chi \rangle$ magnitudes. Note that $\langle \chi \rangle$, $\langle \bar{\chi} \rangle$, μ_χ and all propagator masses in the diagrams can be taken as around the same scale, namely μ_D ; a SUSY breaking vertex or mass insertion is required in each case, as shown.

FIGURES

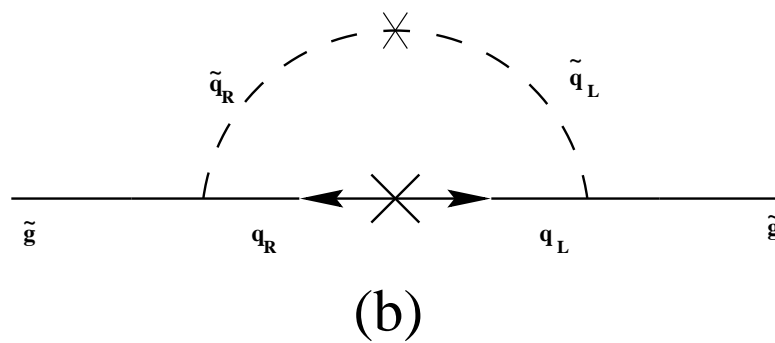
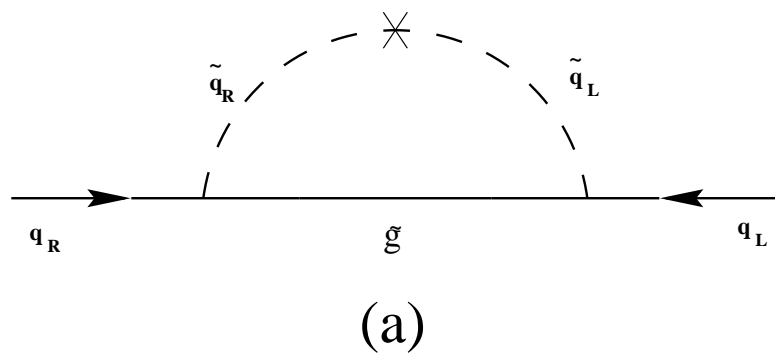


FIG. 1. 1-loop mass-correction diagrams leading to $\bar{\theta}$ contributions. (a) 1-loop quark mass; (b) 1-loop gluino mass.

FIG. 2. Diagrammatic estimates of $\langle F_\chi \rangle$ magnitudes. Note that $\langle \chi_a \rangle$ and all the mass insertions in the diagrams are around the same scale, namely μ_D ; a SUSY breaking vertex or mass insertion is required in each case, as shown.

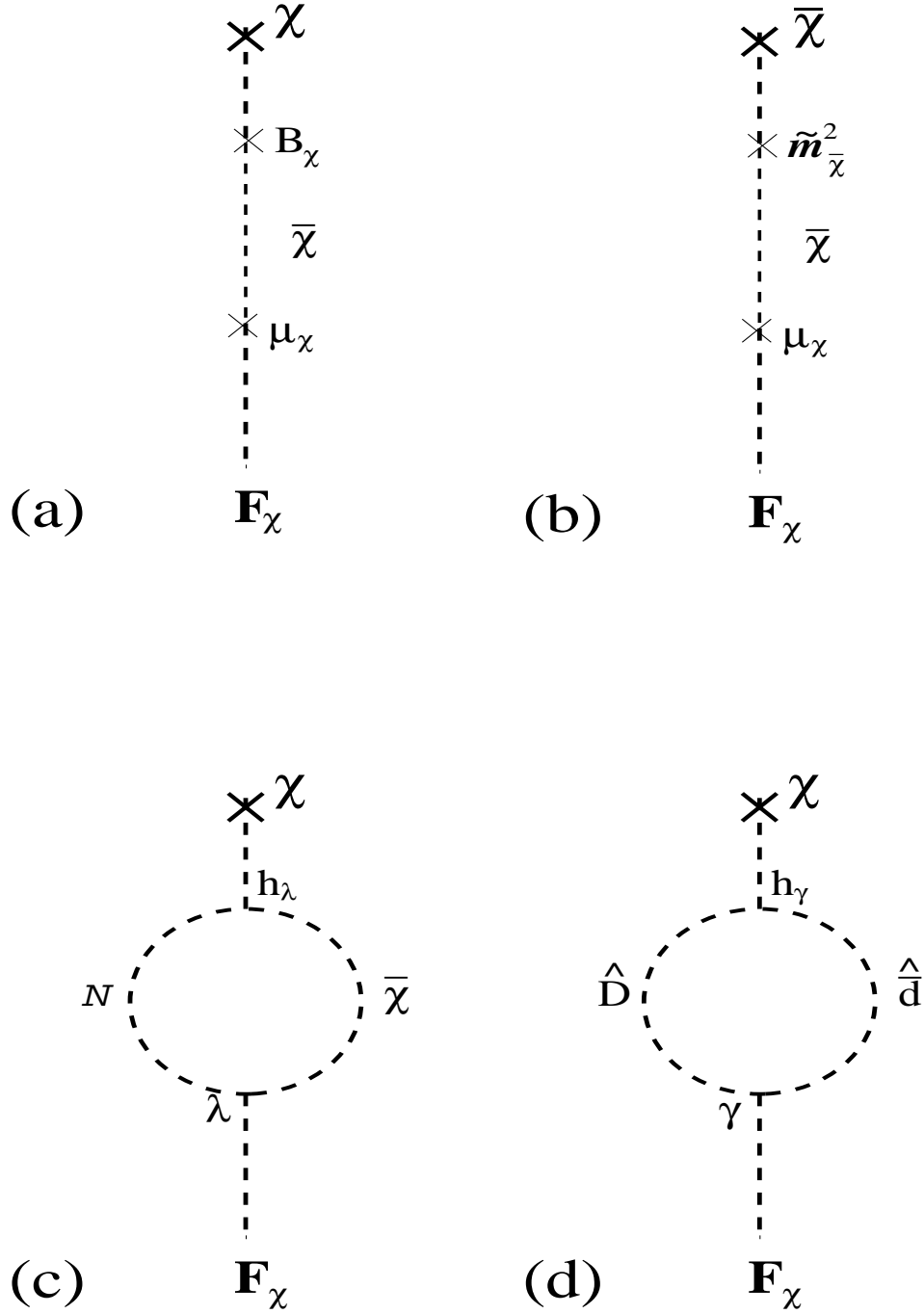


Table 1: Analysis of the major 1-loop $\bar{\theta}$ contributions and numerical results from the sample run. Table 1a contains contributions from gluino mass corrections; Table 1b contains those from quark mass corrections. Entry 1 of Table 1a and entries 9, and 10 of Table 1b are shown together with explicit splittings of M_5^2 according to Eq.(3.6) below the first lines. Numerical results given in the last column do not include the $\langle F_\chi \rangle$ term contributions, but otherwise are complete, *i.e.* they include all other numerical factors from color indices summation, momentum loop integrals, and full summation over family indices (i, j, k) so that the full $\bar{\theta}$ value without the F -term contributions, apart from some unlisted subdominating terms, is given by the sum of all the entries.

Table 1a: Gluino mass correction contributions

No.	factors of $\bar{\theta}$ contribution								
(1)	$\frac{\alpha_s}{4\pi} x$	$\frac{\mu_D}{\tilde{m}_{sq}}$	$\frac{\tilde{m}_{sq}}{M_g}$	$\frac{M_5^2}{\tilde{m}_{sq}^2}$	$\text{Im}(a_i^* b^i)$	-5.17×10^{-15}			
–	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{\tilde{m}_{sq}}{M_g}$	$\frac{\delta A_\gamma}{\tilde{m}_{sq}}$	$\text{Im}(a_i^* c^i)$	–			
–	$\frac{\alpha_s}{4\pi} x$	$\frac{\mu_D}{\tilde{m}_{sq}}$	$\frac{\tilde{m}_{sq}}{M_g}$	$\frac{\gamma^{ia} F_{\chi a} }{\tilde{m}_{sq}^2}$		(cf. table 2)			
(2)	$\frac{\alpha_s}{4\pi} x$	$\frac{\mu_D}{\tilde{m}_{sq}}$	$\frac{\tilde{m}_A^2 m_i^2}{\tilde{m}_{sq}^4}$	$\frac{\tilde{m}_{sq}}{M_g}$	$\frac{M_5^2}{\tilde{m}_{sq}^2}$	$\text{Im}(a_i^* b^i)$	-1.53×10^{-14}		
(3)	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{v_d m_i}{\tilde{m}_{sq}^2}$	$\frac{\tilde{m}_{sq}}{M_g}$	$\frac{\delta A^{ji}}{\tilde{m}_{sq}}$	$\text{Im}(a_j^* a^i)$	1.53×10^{-18}		
(4)	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{\tilde{m}_A m_i^2}{\tilde{m}_{sq}^3}$	$\frac{\tilde{m}_{sq}}{M_g}$	$\frac{\delta \tilde{m}_d^{2jk}}{\tilde{m}_{sq}^2}$	$\text{Im}(a_j^* a^i)$	1.09×10^{-16}		
(5)	$\frac{\alpha_s}{4\pi} x$	$\frac{\mu_D}{\tilde{m}_{sq}}$	$\frac{\tilde{m}_{sq}}{M_g}$	$\frac{\delta \tilde{m}_d^{2ij}}{\tilde{m}_{sq}^2}$	$\frac{M_5^2}{\tilde{m}_{sq}^2}$	$\text{Im}(a_j^* b^i)$	1.27×10^{-14}		
(6)	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{v_d m_i}{\tilde{m}_{sq}^2}$	$\frac{\tilde{m}_{sq}}{M_g}$	$\frac{\delta \tilde{m}_d^{2jk}}{\tilde{m}_{sq}^2}$	$\frac{\delta A^{ki}}{\tilde{m}_{sq}}$	$\text{Im}(a_j^* a^i)$	2.39×10^{-19}	
(7)	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{v_d m_i}{\tilde{m}_{sq}^2}$	$\frac{\tilde{m}_{sq}}{M_g}$	$\frac{\delta \tilde{m}_d^{2ki}}{\tilde{m}_{sq}^2}$	$\frac{\delta A^{jk}}{\tilde{m}_{sq}}$	$\text{Im}(a_j^* a^i)$	-5.99×10^{-18}	
(8)	$\frac{\alpha_s}{4\pi}$		$\frac{v_d m_i}{\tilde{m}_{sq}^2}$	$\frac{\tilde{m}_{sq}}{M_g}$	$\frac{\delta A^{ji}}{\tilde{m}_{sq}}$	$\frac{(M_5^2)^2}{\tilde{m}_{sq}^4}$	$\text{Im}(b_j^* b^i)$	8.43×10^{-20}	
(9)	$\frac{\alpha_s}{4\pi} x$	$\frac{\mu_D}{\tilde{m}_{sq}}$	$\frac{v_d m_i \tilde{m}_A}{\tilde{m}_{sq}^3}$	$\frac{\tilde{m}_{sq}}{M_g}$	$\frac{\delta A^{ji}}{\tilde{m}_{sq}}$	$\frac{M_5^2}{\tilde{m}_{sq}^2}$	$\text{Im}(a_j^* a^i)$	1.14×10^{-18}	
(10)	$\frac{\alpha_s}{4\pi} x$	$\frac{\mu_D}{\tilde{m}_{sq}}$	$\frac{v_d^2}{\tilde{m}_{sq}^2}$	$\frac{\tilde{m}_{sq}}{M_g}$	$\frac{\delta A^{jk}}{\tilde{m}_{sq}}$	$\frac{\delta A^{ik}}{\tilde{m}_{sq}}$	$\frac{M_5^2}{\tilde{m}_{sq}^2}$	$\text{Im}(a_j^* b^i)$	5.26×10^{-22}

Table 1b: Quark mass correction contributions

No.	factors of $\bar{\theta}$ contribution							
(1)	$\frac{\alpha_s}{4\pi} x$	$\frac{\mu_D}{\tilde{m}_{sq}}$	$\frac{M_q}{\tilde{m}_{sq}}$		$\frac{M_5^2}{\tilde{m}_{sq}^2}$	$\text{Im}(a_i^* b^i)$	-1.52×10^{-15}	
(2)	$\frac{\alpha_s}{4\pi} x$	$\frac{\mu_D}{\tilde{m}_{sq}}$	$\frac{\tilde{m}_A B_D}{\tilde{m}_{sq}^2}$	$\frac{M_q}{\tilde{m}_{sq}}$		$\text{Im}(a_i^* b^i)$	3.45×10^{-15}	
(3)	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{v_d m_i}{\tilde{m}_{sq}^2}$	$\frac{M_q}{\tilde{m}_{sq}}$		$\frac{\delta A^{ji}}{\tilde{m}_{sq}}$	$\text{Im}(a_i^* a^i)$	3.10×10^{-19}
(4)	$\frac{\alpha_s}{4\pi} x$	$\frac{\mu_D}{\tilde{m}_{sq}}$	$\frac{m_i}{m_j}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta \tilde{m}_d^{2ij}}{\tilde{m}_{sq}^2}$		$\text{Im}(a_i^* b^j)$	2.02×10^{-14}
(5)	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{v_d}{m_k}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta \tilde{m}_d^{2jk}}{\tilde{m}_{sq}^2}$	$\frac{\delta A^{ik}}{\tilde{m}_{sq}}$	$\text{Im}(a_i^* a^j)$	2.00×10^{-15}
(6)	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{v_d}{m_j}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta \tilde{m}_D^2}{\tilde{m}_{sq}^2}$	$\frac{\delta A^{ij}}{\tilde{m}_{sq}}$	$\text{Im}(a_i^* a^j)$	8.00×10^{-18}
(7)	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{m_i \tilde{m}_A}{m_k \tilde{m}_{sq}}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta \tilde{m}_d^{2kj}}{\tilde{m}_{sq}^2}$	$\frac{\delta \tilde{m}_d^{2ik}}{\tilde{m}_{sq}^2}$	$\text{Im}(a_i^* a^j)$	-7.49×10^{-13}
(8)	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{m_j \tilde{m}_A}{m_i \tilde{m}_{sq}}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta \tilde{m}_D^2}{\tilde{m}_{sq}^2}$	$\frac{\delta \tilde{m}_d^{2ji}}{\tilde{m}_{sq}^2}$	$\text{Im}(a_i^* a^j)$	2.26×10^{-13}
(9)	$\frac{\alpha_s}{4\pi} x$	$\frac{\mu_D}{\tilde{m}_{sq}}$	$\frac{m_j}{m_i}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta \tilde{m}_d^{2ji}}{\tilde{m}_{sq}^2}$	$\text{Im}(a_j^* (\frac{M_5^2}{\tilde{m}_{sq}^2} b^i - x \mu_D \frac{B_D}{\tilde{m}_{sq}^2} a^i))$	-5.74×10^{-15}	
-	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{m_j}{m_i}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta \tilde{m}_d^{2ji}}{\tilde{m}_{sq}^2}$	$\frac{\bar{A}_\gamma - B_D}{\tilde{m}_{sq}}$	$\text{Im}(a_j^* a^i)$	--
-	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{m_j}{m_i}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta \tilde{m}_d^{2ji}}{\tilde{m}_{sq}^2}$	$\frac{\delta A_\gamma}{\tilde{m}_{sq}}$	$\text{Im}(a_j^* c^i)$	--
-	$\frac{\alpha_s}{4\pi} x$	$\frac{\mu_D}{\tilde{m}_{sq}}$	$\frac{m_j}{m_i}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta \tilde{m}_d^{2ji}}{\tilde{m}_{sq}^2}$	$\frac{\gamma^{ia} F_{\chi a} }{\tilde{m}_{sq}^2}$		(cf. table 2)
(10)	$\frac{\alpha_s}{4\pi}$		$\frac{v_d}{m_i}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta A^{ji}}{\tilde{m}_{sq}}$	$\frac{(M_5^2)^2}{\tilde{m}_{sq}^4}$	$\text{Im}(b_j^* b^i)$	1.78×10^{-16}
-	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{v_d \bar{A}_\gamma^2}{m_i \tilde{m}_{sq}^2}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta A^{ji}}{\tilde{m}_{sq}}$		$\text{Im}(a_j^* a^i)$	--
-	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{v_d}{m_i}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta A^{ji}}{\tilde{m}_{sq}}$	$\frac{\delta A_\gamma^2}{\tilde{m}_{sq}^2}$	$\text{Im}(c_j^* c^i)$	--
-	$\frac{\alpha_s}{4\pi} x^2$	$\frac{\mu_D^2}{\tilde{m}_{sq}^2}$	$\frac{v_d \bar{A}_\gamma}{m_i \tilde{m}_{sq}}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta A^{ji}}{\tilde{m}_{sq}}$	$\frac{\delta A_\gamma}{\tilde{m}_{sq}}$	$\text{Im}(c_j^* a^i + a_j^* c^i)$	--
-	$\frac{\alpha_s}{4\pi} x$	$\frac{\mu_D}{\tilde{m}_{sq}}$	$\frac{v_d \bar{A}_\gamma}{m_i \tilde{m}_{sq}}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta A^{ji}}{\tilde{m}_{sq}}$	$\frac{\gamma^{ia} F_{\chi a} }{\tilde{m}_{sq}^2}$		(cf. table 2)
-	$\frac{\alpha_s}{4\pi} x$	$\frac{\mu_D}{\tilde{m}_{sq}}$	$\frac{v_d}{m_i}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta A^{ji}}{\tilde{m}_{sq}}$	$\frac{\delta A_\gamma}{\tilde{m}_{sq}}$	$\frac{\gamma^{ia} F_{\chi a} }{\tilde{m}_{sq}^2}$	(cf. table 2)
-	$\frac{\alpha_s}{4\pi}$		$\frac{v_d}{m_i}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta A^{ji}}{\tilde{m}_{sq}}$	$\frac{\gamma^{ja} F_{\chi a} }{\tilde{m}_{sq}^2}$	$\frac{\gamma^{ia} F_{\chi a} }{\tilde{m}_{sq}^2}$	(cf. table 2)
(11)	$\frac{\alpha_s}{4\pi} x$	$\frac{\mu_D}{\tilde{m}_{sq}}$	$\frac{v_d B_D}{m_i \tilde{m}_{sq}}$	$\frac{M_q}{\tilde{m}_{sq}}$	$\frac{\delta A^{ji}}{\tilde{m}_{sq}}$	$\frac{M_5^2}{\tilde{m}_{sq}^2}$	$\text{Im}(b_j^* a^i)$	3.08×10^{-21}

Table 2: Estimates of the $\langle F_\chi \rangle$ term and its contribution to $\bar{\theta}$, for our sample run and a few runs with different γ and λ inputs (μ_D and μ_χ 's are all set at 500 GeV, M_{mess} at 50 TeV). Note that the entries B_χ , \tilde{m}_χ^2 , $h_\lambda \langle \chi \rangle / \lambda$, and $h_\gamma \langle \chi \rangle \gamma / 16\pi^2$ are our $\langle F_\chi \rangle$ estimates, as discussed; all these are quantities of dimension (mass)² in units of GeV² (not shown explicitly). The $\langle F_\chi \rangle$ estimates and its contributions to $\bar{\theta}$ are meant to be upper bounds. Overall $\bar{\theta}$ contributions from gluino and quark mass corrections without the F -term are also listed.

No.	1	2 (sample)	3	4	5	6
x	.0081	.012	.02	.068	.0086	.0077
λ	.23 – .44	.18 – .68	.53 – .77	.27 – .61	.0038 – .0094	.75 – 1.3
B_χ	1.5	3.4	9.5	101	1.9	1.5
\tilde{m}_χ^2	3.9	3.5	.24	18	4.7	2.5
$h_\lambda \langle \chi \rangle / \lambda$	2.3	1.0	11	7.9	4.6	1.8
$\frac{h_\gamma \langle \chi \rangle \gamma}{16\pi^2}$.033	.054	.16	1.9	.036	.032
$\langle F_\chi \rangle$ estimate	10	10	25	130	10	10
$\frac{\gamma^{ia} F_{\chi a} }{\tilde{m}_{sq}^2}$ estimate	10^{-7}	10^{-7}	10^{-7}	10^{-5}	10^{-7}	10^{-7}
$\longrightarrow \bar{\theta}$	10^{-11}	10^{-11}	10^{-11}	10^{-8}	10^{-11}	10^{-11}
$\bar{\theta}$ (quark)	10^{-14}	10^{-13}	10^{-11}	10^{-9}	10^{-13}	10^{-13}
$\bar{\theta}$ (gluino)	10^{-15}	10^{-15}	10^{-14}	10^{-11}	10^{-15}	10^{-15}