# Hunting for New Physics with Unitarity Boomerangs 

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Although the unitarity triangles (UTs) carry information about the KobayashiMaskawa (KM) quark mixing matrix, it explicitly contains just three parameters which is one short to completely fix the KM matrix. It has been shown recently, by us, that the unitarity boomerangs $(U B)$ formed using two $U T s$, with a common inner angle, can completely determine the KM matrix and, therefore, better represents, quark mixing. Here, we study detailed properties of the $U B s$, of which there are a total 18 possible. Among them, there is only one which does not involve very small angles and is the ideal one for practical uses. Although the $U B s$ have different areas, there is an invariant quantity, for all $U B s$, which is equal to a quarter of the Jarlskog parameter $J$ squared. Hunting new physics, with a unitarity boomerang, can reveal more information, than just using a unitarity triangle.

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## Introduction

The Kobayashi-Maskawa [1] (KM) quark mixing matrix, $V_{K M}$, can describe all laboratory data on quark mixing and CP violation. There are different ways of parameterizing the KM matrix. For three generations of quarks, $V_{K M}$ is a $3 \times 3$ unitary mixing matrix with three rotation angles $\left(\theta_{1}, \theta_{2}, \theta_{3}\right)$ and one CP violating phase $\delta$. The magnitudes of the elements
$\left(V_{K M}\right)_{i j}$ of $V_{K M}$ are physical quantities which do not depend on parametrization. However, the value of $\delta$ does [2-5]. Care must be exercised in quoting a value of $\delta$, as it depends on how the matrix is parameterized. It is therefore desirable to employ only physically-measurable quantities. To this end, it has long ago been suggested that a unitarity triangle (UT) be used [6] as a useful presentation for the quark flavor mixing, especially of CP violation [7].

Because the unitary nature of the KM matrix, the elements $\left(V_{K M}\right)_{i j}$ in the matrix satisfy

$$
\begin{equation*}
\sum_{i}\left(V_{K M}\right)_{i j}\left(V_{K M}\right)_{i k}^{*}=\delta_{j k}, \quad \sum_{i}\left(V_{K M}\right)_{j i}\left(V_{K M}\right)_{k i}^{*}=\delta_{j k} \tag{1}
\end{equation*}
$$

where the first and second indices of $\left(V_{K M}\right)_{i j}$ take the values $u, c, t, \ldots$ and $d, s, b, \ldots$, respectively. For three generations of quarks, when $j \neq k$, these equations define six triangles in a plane, the UTs. All of the six UTs have the same area. $A(U T)$, which is equal to half of the value of the Jarlskog parameter [8] $J$, so that $A(U T)=J / 2$. The inner angles of a given $U T$ are therefore closely related to the CP violating measure $J$. When the inner angles are measured independently, their sum, whether it turns out to be consistent with precisely $180^{\circ}$, provides a test for the unitarity of the KM matrix. The unitarity triangle is a popular way, to present CP violation, with three generations of quarks.

A $U T$, however, does not contain all the information encoded in the KM matrix, $V_{K M}$. Although a $U T$ has three inner angles and three sides, it contains only three independent parameters. The three parameters can be chosen to be two of the three inner angles and the area, or the three sides, or some combination thereof. One needs an additional parameter fully to represent the physics contained in the KM matrix. This is not a surprise because the $U T$ involved only two, of the three, rows or columns of the $3 \times 3$ matrix, $V_{K M}$,

An improved presentation is thus rendered desirable, in order better to present the KM matrix, $V_{K M}$. We have, in a recent paper [5], proposed a new graphic representation of the KM matrix by using the boomerang diagram, the unitarity boomerang $(U B)$. The boomerang diagram contains information from not just one UT, but two UTs, from among the six possible different $U T s$. Some extensions of $U T$ analysis to lepton section has been considered in the Ref. [9].

Out of the six possible $U T s$, there are 9 different ways to have a common inner angle from two UTs. In addition one can align the two longer sides or one longer and one shorter
sides to form a $U B$. Therefore total 18 possible $U B s$. We find that among these $U B s$, there is only one which does not involve very small angles and is good for practical uses. Although the $U B s$ have different areas, there is an invariant quantity for all $U B s$ which is equal to a quarter of the Jarlskog parameter $J$ squared. We also discuss how new physics can be tested by using information from $U B s$.

## The Unitarity Triangles and Boomerangs

The $U B s$ are constructed by using two UTs. Let us summarize some of the relevant information for UTs here. There are six triangles. The inner angles of three UTs involving two columns are defined by, $\sum_{i}\left(V_{K M}\right)_{i j}\left(V_{K M}\right)_{i k}^{*}=0, j \neq k$,

$$
\begin{array}{ll}
U T(1): & \left(V_{K M}\right)_{u b}\left(V_{K M}\right)_{u s}^{*}+\left(V_{K M}\right)_{c b}\left(V_{K M}\right)_{c s}^{*}+\left(V_{K M}\right)_{t b}\left(V_{K M}\right)_{t s}^{*}=0, \\
& \alpha_{1}=\arg \left(-\frac{\left(V_{K M}\right)_{u b}\left(V_{K M}\right)_{u s}^{*}}{\left(V_{K M}\right)_{c b}\left(V_{K M}\right)_{c s}^{*}}\right), \beta_{1}=\arg \left(-\frac{\left(V_{K M}\right)_{t b}\left(V_{K M}\right)_{t s}^{*}}{\left(V_{K M}\right)_{u b}\left(V_{K M}\right)_{u s}^{*}}\right), \\
& \gamma_{1}=\arg \left(-\frac{\left(V_{K M}\right)_{c b}\left(V_{K M}^{*}\right)_{c s}^{*}}{\left(V_{K M}\right)_{t b}\left(V_{K M}\right)_{t s}^{*}}\right), \\
U T(2): \quad\left(V_{K M}\right)_{u d}\left(V_{K M}\right)_{u b}^{*}+\left(V_{K M}\right)_{c d}\left(V_{K M}\right)_{c b}^{*}+\left(V_{K M}\right)_{t d}\left(V_{K M}\right)_{t b}^{*}=0, \\
& \alpha_{2}=\arg \left(-\frac{\left(V_{K M}\right)_{t d}\left(V_{K M}\right)_{t b}^{*}}{\left(V_{K M}\right)_{u d}\left(V_{K M}\right)_{u b}^{*}}\right), \beta_{2}=\arg \left(-\frac{\left(V_{K M}\right)_{c d}\left(V_{K M}\right)_{c b}^{*}}{\left(V_{K M}\right)_{t d}\left(V_{K M}\right)_{t b}^{*}}\right), \\
& \gamma_{2}=\arg \left(-\frac{\left(V_{K M}\right)_{u d}\left(V_{K M}\right)_{u b}^{*}}{\left(V_{K M}\right)_{c d}\left(V_{K M}\right)_{c b}^{*}}\right), \\
U T(3): \quad & \left(V_{K M}\right)_{u d}\left(V_{K M}\right)_{u s}^{*}+\left(V_{K M}\right)_{c d}\left(V_{K M}\right)_{c s}^{*}+\left(V_{K M}\right)_{t d}\left(V_{K M}\right)_{t s}^{*}=0, \\
& \alpha_{3}=\arg \left(-\frac{\left(V_{K M}\right)_{t d}\left(V_{K M}\right)_{t s}^{*}}{\left(V_{K M}\right)_{c d}\left(V_{K M}\right)_{c s}^{*}}\right), \beta_{3}=\arg \left(-\frac{\left(V_{K M}\right)_{c d}\left(V_{K M}\right)_{c s}^{*}}{\left(V_{K M}\right)_{u d}\left(V_{K M}\right)_{u s}^{*}}\right), \\
& \gamma_{3}=\arg \left(-\frac{\left(V_{K M}\right)_{u d}\left(V_{K M}\right)_{u s}^{*}}{\left(V_{K M}\right)_{t d}\left(V_{K M}\right)_{t s}^{*}}\right) . \tag{2}
\end{array}
$$

For convenience, we have labeled the $U T(i)$ by the missing $i$ th quark in the down-quark sector.

The inner angles of the three UTs involving two rows are given by, $\sum_{i}\left(V_{K M}\right)_{j i}\left(V_{K M}\right)_{k i}^{*}=$
$0, j \neq k$,

$$
\begin{array}{ll}
U T\left(1^{\prime}\right): & \left(V_{K M}\right)_{c d}\left(V_{K M}\right)_{t d}^{*}+\left(V_{K M}\right)_{c s}\left(V_{K M}\right)_{t s}^{*}+\left(V_{K M}\right)_{c b}\left(V_{K M}\right)_{t b}^{*}=0, \\
& \alpha_{1}^{\prime}=\arg \left(-\frac{\left(V_{K M}\right)_{c s}\left(V_{K M}\right)_{t s}^{*}}{\left(V_{K M}\right)_{c d}\left(V_{K M}\right)_{t d}^{*}}\right), \beta_{1}^{\prime}=\arg \left(-\frac{\left(V_{K M}\right)_{c d}\left(V_{K M}\right)_{t d}^{*}}{\left(V_{K M}\right)_{c b}\left(V_{K M}\right)_{t b}^{*}}\right), \\
& \gamma_{1}^{\prime}=\arg \left(-\frac{\left(V_{K M}\right)_{c b}\left(V_{K M}\right)_{t b}^{*}}{\left(V_{K M}\right)_{c s}\left(V_{K M}\right)_{t s}^{*}}\right), \\
U T\left(2^{\prime}\right): \quad & \left(V_{K M}\right)_{u d}\left(V_{K M}\right)_{t d}^{*}+\left(V_{K M}\right)_{u s}\left(V_{K M}\right)_{t s}^{*}+\left(V_{K M}\right)_{u b}\left(V_{K M}\right)_{t b}^{*}=0, \\
& \alpha_{2}^{\prime}=\arg \left(-\frac{\left(V_{K M}\right)_{u b}\left(V_{K M}\right)_{t b}^{*}}{\left(V_{K M}\right)_{u d}\left(V_{K M}\right)_{t d}^{*}}\right), \beta_{2}^{\prime}=\arg \left(-\frac{\left(V_{K M}\right)_{u s}\left(V_{K M}\right)_{t s}^{*}}{\left(V_{K M}\right)_{u b}\left(V_{K M}\right)_{t b}^{*}}\right), \\
& \gamma_{2}^{\prime}=\arg \left(-\frac{\left(V_{K M}\right)_{u d}\left(V_{K M}\right)_{t d}^{*}}{\left(V_{K M}\right)_{u s}\left(V_{K M}\right)_{t s}^{*}}\right), \\
U T\left(3^{\prime}\right): \quad & \left(V_{K M}\right)_{u d}\left(V_{K M}\right)_{c d}^{*}+\left(V_{K M}\right)_{u s}\left(V_{K M}\right)_{c s}^{*}+\left(V_{K M}\right)_{u b}\left(V_{K M}\right)_{c b}^{*}=0, \\
& \alpha_{3}^{\prime}=\arg \left(-\frac{\left(V_{K M}\right)_{u b}\left(V_{K M}\right)_{c b}^{*}}{\left(V_{K M}\right)_{u s}\left(V_{K M}\right)_{c s}^{*}}\right), \beta_{3}^{\prime}=\arg \left(-\frac{\left(V_{K M}\right)_{u s}\left(V_{K M}\right)_{c s}^{*}}{\left(V_{K M}\right)_{u d}\left(V_{K M}\right)_{c d}^{*}}\right), \\
& \gamma_{3}^{\prime}=\arg \left(-\frac{\left(V_{K M}\right)_{u d}\left(V_{K M}\right)_{c d}^{*}}{\left(V_{K M}\right)_{u b}\left(V_{K M}\right)_{c b}^{*}}\right) . \tag{3}
\end{array}
$$

Here the $U T\left(i^{\prime}\right)$ labeled by the missing $i^{\prime}$ th quark in the up-quark sector.

It is clear from the above definitions that among the 18 inner angles of the six UTs, only 9 of them are different. Explicitly, we have

$$
\begin{array}{lll}
\alpha_{1}=\alpha_{3}^{\prime}, & \beta_{1}=\beta_{2}^{\prime}, & \gamma_{1}=\gamma_{1}^{\prime}, \\
\alpha_{2}=\alpha_{2}^{\prime}, & \beta_{2}=\beta_{1}^{\prime}, & \gamma_{2}=\gamma_{3}^{\prime}, \\
\alpha_{3}=\alpha_{1}^{\prime}, & \beta_{3}=\beta_{3}^{\prime}, & \gamma_{3}=\gamma_{2}^{\prime} . \tag{4}
\end{array}
$$

We will choose the 9 inner angles without "prime" in our later discussions.

As pointed out earlier that for a given $U T$, it contains only 3 independent parameters which is not enough to completely determine parameters in the KM matrix. In a previous paper [5] we have shown that using two triangles with one from $U T(i)$ and another from $U T\left(i^{\prime}\right)$ one can always form a boomerang like diagram, the unitarity boomerang and $(U B)$ contains all information need to reconstruct the KM matrix. Let us show more details in the following.

There are total 18 different ways of constructing $U B s$. There are 9 different ways to pair up a common angle by taking one $U T$ from $U T(i)$ and another from $U T\left(i^{\prime}\right)$. One can then overlap the longer side from on $U T$ with the shorter side of the other $U T$, or overlap the longer sides of the two UTs. We will label the former $9 U B s$ as $B_{i i^{\prime} a}$ and the later $9 U B s$ as $\tilde{B}_{i i^{\prime} a}$. Here the index $a$ indicates the common angle. The common angles and their current central experimental values of the $U B s$ are given in the following

$$
\begin{gather*}
\left(\begin{array}{lll}
B(\tilde{B})_{11 \gamma_{1}} & B(\tilde{B})_{12 \beta_{1}} & B(\tilde{B})_{13 \alpha_{1}} \\
B(\tilde{B})_{21 \beta_{2}} & B(\tilde{B})_{22 \alpha_{2}} & B(\tilde{B})_{23 \gamma_{2}} \\
B(\tilde{B})_{31 \alpha_{3}} & B(\tilde{B})_{32 \gamma_{3}} & B(\tilde{B})_{33 \beta_{3}}
\end{array}\right) \\
\Rightarrow\left(\begin{array}{lll}
\gamma_{1}=0.0207 & \beta_{1}=1.151 & \alpha_{1}=1.970 \\
\beta_{2}=0.435 & \alpha_{2}=1.535 & \gamma_{2}=1.171 \\
\alpha_{3}=2.686 & \gamma_{3}=0.455 & \beta_{3}=5.531 \times 10^{-4}
\end{array}\right) \tag{5}
\end{gather*}
$$

In the above when calculating the inner angles, we have used the central values [2] $\left|\left(V_{K M}\right)_{u d}\right|=0.97419,\left|\left(V_{K M}\right)_{u s}\right|=0.2257,\left|\left(V_{K M}\right)_{u b}\right|=0.00359$ and $\alpha_{2}=88^{\circ}$. We will use these values in our later discussions for illustrations.

We now display the $U B s$. In Fig. 1 we show some details for the boomerang formed by using $U T(2)$ and $U T\left(2^{\prime}\right)$. Figs. 1. a) and 1. b) are for $B_{22 \alpha_{2}}$, and $\tilde{B}_{22 \alpha_{2}}$. The $9 B_{i i^{\prime} a}$ are shown in Fig. 2. One can also construct the $9 U B s$ of $\tilde{B}_{i i^{\prime} a}$. But they contain similar information as those of $B_{i i^{\prime} a}$, they can be obtained by flipping one of the $U T$ in each of the $U B$, we therefore have not displayed them.

As can be seen from Fig. 2, the $U B s$, except the $B_{22 \alpha_{2}}$, all involve a very small angle in the diagram making it difficult to construct, with high accuracy. The $B_{22 \alpha_{2}}$ can display the information more easily. One should therefore work with the $B_{22 \alpha_{2}}$. From the $B_{22 \alpha_{2}}$, one can easily obtain approximate solutions for the four physical parameters. Taking the ratio, of the two sides $A C / A C^{\prime}$ or $A B / A B^{\prime}$, one obtains, $\left|\left(V_{K M}\right)_{u d} /\left(V_{K M}\right)_{t b}^{*}\right| \approx c_{1}$ since $\left|\left(V_{K M}\right)_{t b}\right|$ is very close to 1 . With $c_{1}$ and therefore $s_{1}$ known, the length of the sides AB and AC ' then provide the values for $s_{2}$ and $s_{3}$. Here the angles $c_{i}$ and $s_{i}$ refer to the cosine and sine of the angles in the original KM matrix parametrization. One can also obtain more accurate expressions, as shown in Ref.[5]. In this parametrization, the CP violating phase $\delta$ is, to a very good approximation, equal to $[5,10] \alpha_{2}$.

## Invariant quantity for CP violation

Information on CP violation are, also, fully encoded, in the $U B s$. The Jarlskog parameter $J$ plays a fundamental role for CP violation with three generations. In the $U T$ representation, it is related to the area of the $U T$. All UTs have the same area of $J / 2$. One can also construct a geometric representation of CP violation invariant quantity for $U B$ representation. Although the shapes of the $U B s$ vary a lot, there exists a quantity related to the $U B$ constructed areas represent CP violation in an invariant form.

Consider the areas $A_{A B B^{\prime}}$ and $A_{A C C^{\prime}}$ of the two triangles $A C C^{\prime}$ and $A B B^{\prime}$ in Fig. 1 for $B_{22 \alpha_{2}}$, we find

$$
\begin{equation*}
A_{22}=A_{A B B^{\prime}}=\frac{1}{2} \frac{\left|\left(V_{K M}\right)_{t d}\right|}{\left|\left(V_{K M}\right)_{u b}\right|} J, \quad A_{22}^{\prime}=A_{A C C^{\prime}}=\frac{1}{2} \frac{\left|\left(V_{K M}\right)_{u b}\right|}{\left|\left(V_{K M}\right)_{t d}\right|} J ; \quad A_{22} A_{22}^{\prime}=\frac{1}{4} J^{2} . \tag{6}
\end{equation*}
$$

The two triangles $A B B^{\prime}$ and $A C C^{\prime}$ are similar to each other.

Similarly for $\tilde{B}_{22 \alpha_{2}}$, one has

$$
\begin{equation*}
\tilde{A}_{22}=\tilde{A}_{\tilde{A} \tilde{B} \tilde{B}^{\prime}}=\frac{1}{2} \frac{\left|\left(V_{K M}\right)_{t d}\right|}{\left|\left(V_{K M}\right)_{u b}\right|} J, \quad \tilde{A}_{22}^{\prime}=\tilde{A}_{\tilde{A} \tilde{C} \tilde{C}^{\prime}}=\frac{1}{2} \frac{\left|\left(V_{K M}\right)_{u b}\right|}{\left|\left(V_{K M}\right)_{t d}\right|} J, \quad \tilde{A}_{22} \tilde{A}_{22}^{\prime}=\frac{1}{4} J^{2} . \tag{7}
\end{equation*}
$$


a

$B^{\prime}$
b

FIG. 1: The unitarity boomerangs $B_{22 \alpha_{2}}$ and $\tilde{B}_{22 \alpha_{2}}$. Figure 1.a) overlaps one long and one short sides from the two UTs. The sides are: $A B=\left|\left(V_{K M}\right)_{t d}\left(V_{K M}\right)_{t b}^{*}\right|, A C=\left|\left(V_{K M}\right)_{u d}\left(V_{K M}\right)_{u b}^{*}\right|$, $B C=\left|\left(V_{K M}\right)_{c d}\left(V_{K M}\right)_{c b}^{*}\right|, A B^{\prime}=\left|\left(V_{K M}\right)_{u d}\left(V_{K M}\right)_{t d}^{*}\right|, A C^{\prime}=\left|\left(V_{K M}\right)_{u b}\left(V_{K M}\right)_{t b}^{*}\right|$, and $B^{\prime} C^{\prime}=$ $\left|\left(V_{K M}\right)_{u s}\left(V_{K M}\right)_{t s}^{*}\right|$. Figure 1.b) is for $\tilde{B}_{22 \alpha}$. The red (left) lines and black (right) lines indicate the $U T$ taking from $U T(i)$ and $U T\left(i^{\prime}\right)$, respectively.

Again the two triangles are similar to each other.

This can be generalized to all $U B s$. The results can be collected in matrix forms. Writing $A_{22}, A_{22}^{\prime}, \tilde{A}_{22}$ and $\tilde{A}_{22}^{\prime}$ like areas for other $U B s$ in similar ways, the resultant matrix forms $A, A^{\prime}, \tilde{A}$ and $\tilde{A}^{\prime}$ are given by

One immediately finds an invariant quantity: $I_{U B}=A_{i j} A_{i j}^{\prime}=\tilde{A}_{i j} \tilde{}^{\prime}{ }_{i j}=J^{2} / 4$. Here no summations on $i$ and $j$. If this quantity is zero, there is no CP violation. The universal nature of the Jarlskog parameter $J$ is also present in the $U B$ representation.

## Unitarity Boomerang and New Physics



FIG. 2: The 9 unitarity boomerangs $B_{i i^{\prime} a}$. The red (left) lines and black (right) lines indicate the $U T s$ taking from $U T(i)$ and $U T\left(i^{\prime}\right)$. In the figure, the symbols double back-slash and double slash indicate the $U T$ for $i=3$ and $i^{\prime}=3$. For the 23, 32, 33 entries, the length in the figure for $U T(3)$ and $U T\left(3^{\prime}\right)$ should be scaled up by 5 . For the 13 and 31 entries, the triangles involve $U T\left(3,3^{\prime}\right)$ and $U T\left(1,1^{\prime}\right)$ should be scaled up by 10 and 2 , respectively.

We now discuss how possible new physics information may show up in the $U B$ analysis. There are different ways the $U B s$ can be used to hunt for new physics. We will discuss two ways to detect new physics beyond the three generation KM model.

One of them is to see if a $U B$ can be formed as expected after the relevant sides, such as the sides shown in Fig. 1 are measured. The construction of the $U B$ uses the property that there is a common angle. With this constraint, if there is new physics to change the length of the sides in a fashion which is not a universal scaling, CP conserving or violating, then one cannot close the $U B$. Another way of presenting this situation is that, if one constructs the $U T(2)$ and $U T\left(2^{\prime}\right)$ first, then there may not be a common inner angle.

The above possibility may reveal information which cannot be obtained using only one $U T$. An example in which this might happen is the simple extension to four generation of quarks with the addition of $t^{\prime}$ and $b^{\prime}$. In the case $V_{t^{\prime} d}=0$, the defining equation, Eq.(2), for $U T(2)$ is still the same and one can define effective inner angles and sides. If one just checks whether the sum of inner angles is $180^{\circ}$ from direct measurements of individual angles, and the angles determined by the sides, they are consistent. No sign of new physics will show up by this analysis. However, the $U T\left(2^{\prime}\right)$ may be modified by a new term $V_{u b^{\prime}} V_{t b^{\prime}}^{*}$. Only information on $U T\left(2^{\prime}\right)$ is also compared with that from $U T(2)$, it is possible to decide if the type of new physics described above exists. The $U B$ analysis contains this comparison together all in one.

Another is to use the property of the invariant quantity $I_{U B}$ discussed earlier. It must equal to $J^{2} / 4$. Taking the square root, one can check with one of the $U T$ areas in the same $U B$. If more than one $U B$ are constructed, one can also compare if their corresponding invariant quantities are equal.

## Discussion

Although the unitarity triangle carries information, about the Kobayashi-Maskawa quark mixing matrix, it explicitly contains just three parameters, which is one short to completely fix the KM matrix. The unitarity boomerangs formed using two UTs, with a common inner angle, can completely determine the KM matrix, and therefore better represents quark mixing information. Out of the six possible UTs, there are 9 different ways to have a
common inner angle from two UTs. In addition, one can align the two longer sides or one longer and one shorter side to form a $U B$. Therefore there are total 18 possible $U B s$. By studying the unitarity boomerangs, one can obtain all the information enshrined in KM matrix. We find that although the $U B s$ have different areas, there is an invariant quantity for all $U B s$ which is equal to a quarter of the Jarlskog parameter $J$ squared. This is an universal representation of CP violation in the $U B$ framework. We have also discussed how new physics can be hunted for by using information from $U B s$.

As far as graphic representation of the KM matrix, the proposal, to move from a single triangle to a boomerang combination, reflects, more than anything else, the impressive precision which has been attained by high-energy experiment. A unitarity boomerang contains all information of the KM matrix.

If new physics exists which modifies the unitarity nature, deviations from the expected $U B$ can exist. The $U B$ construction of the mixing matrix elements can also return, with information about new physics.

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