# Hunting for New Physics with Unitarity Boomerangs

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Although the unitarity triangles (UTs) carry information about the Kobayashi-Maskawa (KM) quark mixing matrix, it explicitly contains just three parameters which is one short to completely fix the KM matrix. It has been shown recently, by us, that the unitarity boomerangs (UB) formed using two UTs, with a common inner angle, can completely determine the KM matrix and, therefore, better represents, quark mixing. Here, we study detailed properties of the UBs, of which there are a total 18 possible. Among them, there is only one which does not involve very small angles and is the ideal one for practical uses. Although the UBs have different areas, there is an invariant quantity, for all UBs, which is equal to a quarter of the Jarlskog parameter J squared. Hunting new physics, with a unitarity boomerang, can reveal more information, than just using a unitarity triangle.

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#### Introduction

The Kobayashi-Maskawa [1] (KM) quark mixing matrix,  $V_{KM}$ , can describe all laboratory data on quark mixing and CP violation. There are different ways of parameterizing the KM matrix. For three generations of quarks,  $V_{KM}$  is a 3 × 3 unitary mixing matrix with three rotation angles ( $\theta_1$ ,  $\theta_2$ ,  $\theta_3$ ) and one CP violating phase  $\delta$ . The magnitudes of the elements  $(V_{KM})_{ij}$  of  $V_{KM}$  are physical quantities which do not depend on parametrization. However, the value of  $\delta$  does [2–5]. Care must be exercised in quoting a value of  $\delta$ , as it depends on how the matrix is parameterized. It is therefore desirable to employ only physically-measurable quantities. To this end, it has long ago been suggested that a unitarity triangle (UT) be used[6] as a useful presentation for the quark flavor mixing, especially of CP violation [7].

Because the unitary nature of the KM matrix, the elements  $(V_{KM})_{ij}$  in the matrix satisfy

$$\sum_{i} (V_{KM})_{ij} (V_{KM})_{ik}^* = \delta_{jk} , \quad \sum_{i} (V_{KM})_{ji} (V_{KM})_{ki}^* = \delta_{jk}, \tag{1}$$

where the first and second indices of  $(V_{KM})_{ij}$  take the values u, c, t, ... and d, s, b, ...,respectively. For three generations of quarks, when  $j \neq k$ , these equations define six triangles in a plane, the UTs. All of the six UTs have the same area. A(UT), which is equal to half of the value of the Jarlskog parameter [8] J, so that A(UT) = J/2. The inner angles of a given UT are therefore closely related to the CP violating measure J. When the inner angles are measured independently, their sum, whether it turns out to be consistent with precisely 180°, provides a test for the unitarity of the KM matrix. The unitarity triangle is a popular way, to present CP violation, with three generations of quarks.

A UT, however, does not contain all the information encoded in the KM matrix,  $V_{KM}$ . Although a UT has three inner angles and three sides, it contains only three independent parameters. The three parameters can be chosen to be two of the three inner angles and the area, or the three sides, or some combination thereof. One needs an additional parameter fully to represent the physics contained in the KM matrix. This is not a surprise because the UT involved only two, of the three, rows or columns of the  $3 \times 3$  matrix,  $V_{KM}$ ,

An improved presentation is thus rendered desirable, in order better to present the KM matrix,  $V_{KM}$ . We have, in a recent paper [5], proposed a new graphic representation of the KM matrix by using the boomerang diagram, the unitarity boomerang (UB). The boomerang diagram contains information from not just one UT, but two UTs, from among the six possible different UTs. Some extensions of UT analysis to lepton section has been considered in the Ref. [9].

Out of the six possible UTs, there are 9 different ways to have a common inner angle from two UTs. In addition one can align the two longer sides or one longer and one shorter sides to form a UB. Therefore total 18 possible UBs. We find that among these UBs, there is only one which does not involve very small angles and is good for practical uses. Although the UBs have different areas, there is an invariant quantity for all UBs which is equal to a quarter of the Jarlskog parameter J squared. We also discuss how new physics can be tested by using information from UBs.

#### The Unitarity Triangles and Boomerangs

The UBs are constructed by using two UTs. Let us summarize some of the relevant information for UTs here. There are six triangles. The inner angles of three UTs involving two columns are defined by,  $\sum_{i} (V_{KM})_{ij} (V_{KM})_{ik}^* = 0$ ,  $j \neq k$ ,

$$UT(1): \quad (V_{KM})_{ub}(V_{KM})_{us}^* + (V_{KM})_{cb}(V_{KM})_{cs}^* + (V_{KM})_{tb}(V_{KM})_{ts}^* = 0, \alpha_1 = \arg\left(-\frac{(V_{KM})_{ub}(V_{KM})_{us}^*}{(V_{KM})_{cb}(V_{KM})_{cs}^*}\right), \ \beta_1 = \arg\left(-\frac{(V_{KM})_{tb}(V_{KM})_{ts}^*}{(V_{KM})_{ub}(V_{KM})_{us}^*}\right), \gamma_1 = \arg\left(-\frac{(V_{KM})_{cb}(V_{KM})_{cs}^*}{(V_{KM})_{tb}(V_{KM})_{ts}^*}\right),$$

$$UT(2): \quad (V_{KM})_{ud}(V_{KM})_{ub}^* + (V_{KM})_{cd}(V_{KM})_{cb}^* + (V_{KM})_{td}(V_{KM})_{tb}^* = 0,$$
  

$$\alpha_2 = \arg\left(-\frac{(V_{KM})_{td}(V_{KM})_{tb}^*}{(V_{KM})_{ud}(V_{KM})_{ub}^*}\right), \quad \beta_2 = \arg\left(-\frac{(V_{KM})_{cd}(V_{KM})_{cb}^*}{(V_{KM})_{td}(V_{KM})_{tb}^*}\right),$$
  

$$\gamma_2 = \arg\left(-\frac{(V_{KM})_{ud}(V_{KM})_{ub}^*}{(V_{KM})_{cd}(V_{KM})_{cb}^*}\right),$$

$$UT(3): \quad (V_{KM})_{ud}(V_{KM})_{us}^{*} + (V_{KM})_{cd}(V_{KM})_{cs}^{*} + (V_{KM})_{td}(V_{KM})_{ts}^{*} = 0,$$
  

$$\alpha_{3} = \arg\left(-\frac{(V_{KM})_{td}(V_{KM})_{ts}^{*}}{(V_{KM})_{cd}(V_{KM})_{cs}^{*}}\right), \quad \beta_{3} = \arg\left(-\frac{(V_{KM})_{cd}(V_{KM})_{cs}^{*}}{(V_{KM})_{ud}(V_{KM})_{us}^{*}}\right),$$
  

$$\gamma_{3} = \arg\left(-\frac{(V_{KM})_{ud}(V_{KM})_{us}^{*}}{(V_{KM})_{td}(V_{KM})_{ts}^{*}}\right).$$
(2)

For convenience, we have labeled the UT(i) by the missing *i*th quark in the down-quark sector.

The inner angles of the three UTs involving two rows are given by,  $\sum_{i} (V_{KM})_{ji} (V_{KM})_{ki}^* =$ 

 $0, j \neq k,$ 

$$\begin{aligned} UT(1'): & (V_{KM})_{cd}(V_{KM})_{td}^* + (V_{KM})_{cs}(V_{KM})_{ts}^* + (V_{KM})_{cb}(V_{KM})_{tb}^* = 0 , \\ & \alpha_1' = \arg\left(-\frac{(V_{KM})_{cs}(V_{KM})_{ts}^*}{(V_{KM})_{cd}(V_{KM})_{td}^*}\right) , \quad \beta_1' = \arg\left(-\frac{(V_{KM})_{cd}(V_{KM})_{td}^*}{(V_{KM})_{cb}(V_{KM})_{tb}^*}\right) , \\ & \gamma_1' = \arg\left(-\frac{(V_{KM})_{cb}(V_{KM})_{tb}^*}{(V_{KM})_{cs}(V_{KM})_{ts}^*}\right) , \end{aligned}$$

$$UT(2'): \quad (V_{KM})_{ud}(V_{KM})_{td}^* + (V_{KM})_{us}(V_{KM})_{ts}^* + (V_{KM})_{ub}(V_{KM})_{tb}^* = 0,$$
  

$$\alpha'_2 = \arg\left(-\frac{(V_{KM})_{ub}(V_{KM})_{tb}^*}{(V_{KM})_{ud}(V_{KM})_{td}^*}\right), \beta'_2 = \arg\left(-\frac{(V_{KM})_{us}(V_{KM})_{ts}^*}{(V_{KM})_{ub}(V_{KM})_{tb}^*}\right),$$
  

$$\gamma'_2 = \arg\left(-\frac{(V_{KM})_{ud}(V_{KM})_{td}^*}{(V_{KM})_{us}(V_{KM})_{ts}^*}\right),$$

$$UT(3'): \quad (V_{KM})_{ud}(V_{KM})_{cd}^{*} + (V_{KM})_{us}(V_{KM})_{cs}^{*} + (V_{KM})_{ub}(V_{KM})_{cb}^{*} = 0,$$
  

$$\alpha'_{3} = \arg\left(-\frac{(V_{KM})_{ub}(V_{KM})_{cb}^{*}}{(V_{KM})_{us}(V_{KM})_{cs}^{*}}\right), \quad \beta'_{3} = \arg\left(-\frac{(V_{KM})_{us}(V_{KM})_{cs}^{*}}{(V_{KM})_{ud}(V_{KM})_{cd}^{*}}\right),$$
  

$$\gamma'_{3} = \arg\left(-\frac{(V_{KM})_{ud}(V_{KM})_{cd}^{*}}{(V_{KM})_{ub}(V_{KM})_{cb}^{*}}\right).$$
(3)

Here the UT(i') labeled by the missing i'th quark in the up-quark sector.

It is clear from the above definitions that among the 18 inner angles of the six UTs, only 9 of them are different. Explicitly, we have

$$\begin{aligned}
\alpha_1 &= \alpha'_3 , & \beta_1 = \beta'_2 , & \gamma_1 = \gamma'_1 , \\
\alpha_2 &= \alpha'_2 , & \beta_2 = \beta'_1 , & \gamma_2 = \gamma'_3 , \\
\alpha_3 &= \alpha'_1 , & \beta_3 = \beta'_3 , & \gamma_3 = \gamma'_2 .
\end{aligned}$$
(4)

We will choose the 9 inner angles without "prime" in our later discussions.

As pointed out earlier that for a given UT, it contains only 3 independent parameters which is not enough to completely determine parameters in the KM matrix. In a previous paper [5] we have shown that using two triangles with one from UT(i) and another from UT(i') one can always form a boomerang like diagram, the unitarity boomerang and (UB)contains all information need to reconstruct the KM matrix. Let us show more details in the following. There are total 18 different ways of constructing UBs. There are 9 different ways to pair up a common angle by taking one UT from UT(i) and another from UT(i'). One can then overlap the longer side from on UT with the shorter side of the other UT, or overlap the longer sides of the two UTs. We will label the former 9 UBs as  $B_{ii'a}$  and the later 9 UBs as  $\tilde{B}_{ii'a}$ . Here the index *a* indicates the common angle. The common angles and their current central experimental values of the UBs are given in the following

$$\begin{pmatrix} B(\tilde{B})_{11\gamma_{1}} & B(\tilde{B})_{12\beta_{1}} & B(\tilde{B})_{13\alpha_{1}} \\ B(\tilde{B})_{21\beta_{2}} & B(\tilde{B})_{22\alpha_{2}} & B(\tilde{B})_{23\gamma_{2}} \\ B(\tilde{B})_{31\alpha_{3}} & B(\tilde{B})_{32\gamma_{3}} & B(\tilde{B})_{33\beta_{3}} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \gamma_{1} = 0.0207 & \beta_{1} = 1.151 & \alpha_{1} = 1.970 \\ \beta_{2} = 0.435 & \alpha_{2} = 1.535 & \gamma_{2} = 1.171 \\ \alpha_{3} = 2.686 & \gamma_{3} = 0.455 & \beta_{3} = 5.531 \times 10^{-4} \end{pmatrix} .$$

$$(5)$$

In the above when calculating the inner angles, we have used the central values [2]  $|(V_{KM})_{ud}| = 0.97419$ ,  $|(V_{KM})_{us}| = 0.2257$ ,  $|(V_{KM})_{ub}| = 0.00359$  and  $\alpha_2 = 88^\circ$ . We will use these values in our later discussions for illustrations.

We now display the UBs. In Fig. 1 we show some details for the boomerang formed by using UT(2) and UT(2'). Figs. 1. a) and 1. b) are for  $B_{22\alpha_2}$ , and  $\tilde{B}_{22\alpha_2}$ . The 9  $B_{ii'a}$ are shown in Fig. 2. One can also construct the 9 UBs of  $\tilde{B}_{ii'a}$ . But they contain similar information as those of  $B_{ii'a}$ , they can be obtained by flipping one of the UT in each of the UB, we therefore have not displayed them.

As can be seen from Fig. 2, the UBs, except the  $B_{22\alpha_2}$ , all involve a very small angle in the diagram making it difficult to construct, with high accuracy. The  $B_{22\alpha_2}$  can display the information more easily. One should therefore work with the  $B_{22\alpha_2}$ . From the  $B_{22\alpha_2}$ , one can easily obtain approximate solutions for the four physical parameters. Taking the ratio, of the two sides AC/AC' or AB/AB', one obtains,  $|(V_{KM})_{ud}/(V_{KM})_{tb}^*| \approx c_1$  since  $|(V_{KM})_{tb}|$ is very close to 1. With  $c_1$  and therefore  $s_1$  known, the length of the sides AB and AC' then provide the values for  $s_2$  and  $s_3$ . Here the angles  $c_i$  and  $s_i$  refer to the cosine and sine of the angles in the original KM matrix parametrization. One can also obtain more accurate expressions, as shown in Ref.[5]. In this parametrization, the CP violating phase  $\delta$  is, to a very good approximation, equal to [5, 10]  $\alpha_2$ .

#### Invariant quantity for CP violation

Information on CP violation are, also, fully encoded, in the UBs. The Jarlskog parameter J plays a fundamental role for CP violation with three generations. In the UT representation, it is related to the area of the UT. All UTs have the same area of J/2. One can also construct a geometric representation of CP violation invariant quantity for UB representation. Although the shapes of the UBs vary a lot, there exists a quantity related to the UB constructed areas represent CP violation in an invariant form.

Consider the areas  $A_{ABB'}$  and  $A_{ACC'}$  of the two triangles ACC' and ABB' in Fig. 1 for  $B_{22\alpha_2}$ , we find

$$A_{22} = A_{ABB'} = \frac{1}{2} \frac{|(V_{KM})_{td}|}{|(V_{KM})_{ub}|} J , \quad A'_{22} = A_{ACC'} = \frac{1}{2} \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{td}|} J ; \quad A_{22}A'_{22} = \frac{1}{4}J^2.$$
(6)

The two triangles ABB' and ACC' are similar to each other.

Similarly for  $\tilde{B}_{22\alpha_2}$ , one has

$$\tilde{A}_{22} = \tilde{A}_{\tilde{A}\tilde{B}\tilde{B}'} = \frac{1}{2} \frac{|(V_{KM})_{td}|}{|(V_{KM})_{ub}|} J , \quad \tilde{A}'_{22} = \tilde{A}_{\tilde{A}\tilde{C}\tilde{C}'} = \frac{1}{2} \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{td}|} J , \quad \tilde{A}_{22}\tilde{A}'_{22} = \frac{1}{4} J^2 .$$

$$\tag{7}$$



FIG. 1: The unitarity boomerangs  $B_{22\alpha_2}$  and  $\tilde{B}_{22\alpha_2}$ . Figure 1.a) overlaps one long and one short sides from the two UTs. The sides are:  $AB = |(V_{KM})_{td}(V_{KM})^*_{tb}|$ ,  $AC = |(V_{KM})_{ud}(V_{KM})^*_{ub}|$ ,  $BC = |(V_{KM})_{cd}(V_{KM})^*_{cb}|$ ,  $AB' = |(V_{KM})_{ud}(V_{KM})^*_{td}|$ ,  $AC' = |(V_{KM})_{ub}(V_{KM})^*_{tb}|$ , and  $B'C' = |(V_{KM})_{us}(V_{KM})^*_{ts}|$ . Figure 1.b) is for  $\tilde{B}_{22\alpha}$ . The red (left) lines and black (right) lines indicate the UT taking from UT(i) and UT(i'), respectively.

Again the two triangles are similar to each other.

This can be generalized to all UBs. The results can be collected in matrix forms. Writing  $A_{22}$ ,  $A'_{22}$ ,  $\tilde{A}_{22}$  and  $\tilde{A}'_{22}$  like areas for other UBs in similar ways, the resultant matrix forms  $A, A', \tilde{A}$  and  $\tilde{A}'$  are given by

$$\begin{aligned}
A &= \frac{1}{2} \begin{pmatrix} \frac{|(V_{KM})_{tb}|}{|(V_{KM})_{cs}|} & \frac{|(V_{KM})_{ts}|}{|(V_{KM})_{ub}|} & \frac{(V_{KM})_{cs}|}{|(V_{KM})_{ub}|} \\ \frac{|(V_{KM})_{cb}|}{|(V_{KM})_{td}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{ub}|} & \frac{(V_{KM})_{ub}|}{|(V_{KM})_{ub}|} \\ \frac{|(V_{KM})_{cs}|}{|(V_{KM})_{td}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{ub}|} & \frac{(V_{KM})_{ub}|}{|(V_{KM})_{cs}|} \\ \frac{|(V_{KM})_{cb}|}{|(V_{KM})_{cb}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{ud}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{ud}|} \\ \frac{|(V_{KM})_{cb}|}{|(V_{KM})_{cb}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{ud}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{ud}|} \\ \frac{|(V_{KM})_{cb}|}{|(V_{KM})_{cb}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{ud}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{cb}|} \\ \frac{|(V_{KM})_{cb}|}{|(V_{KM})_{cb}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{ub}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{cb}|} \\ \frac{|(V_{KM})_{cb}|}{|(V_{KM})_{cb}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{ub}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{cb}|} \\ \frac{|(V_{KM})_{cb}|}{|(V_{KM})_{cb}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{ub}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{ub}|} \\ \frac{|(V_{KM})_{cb}|}{|(V_{KM})_{ub}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{ub}|} & \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{ub}|} \\ \frac{|(V_{KM})_{ub}|}{|(V_{KM})_{ub}|} & \frac{|(V_{KM})_{ub$$

One immediately finds an invariant quantity:  $I_{UB} = A_{ij}A'_{ij} = \tilde{A}_{ij}\tilde{i}_{j} = J^2/4$ . Here no summations on *i* and *j*. If this quantity is zero, there is no CP violation. The universal nature of the Jarlskog parameter *J* is also present in the *UB* representation.

## Unitarity Boomerang and New Physics



FIG. 2: The 9 unitarity boomerangs  $B_{ii'a}$ . The red (left) lines and black (right) lines indicate the UTs taking from UT(i) and UT(i'). In the figure, the symbols double back-slash and double slash indicate the UT for i = 3 and i' = 3. For the 23, 32, 33 entries, the length in the figure for UT(3) and UT(3') should be scaled up by 5. For the 13 and 31 entries, the triangles involve UT(3,3') and UT(1,1') should be scaled up by 10 and 2, respectively.

We now discuss how possible new physics information may show up in the UB analysis. There are different ways the UBs can be used to hunt for new physics. We will discuss two ways to detect new physics beyond the three generation KM model.

One of them is to see if a UB can be formed as expected after the relevant sides, such as the sides shown in Fig.1 are measured. The construction of the UB uses the property that there is a common angle. With this constraint, if there is new physics to change the length of the sides in a fashion which is not a universal scaling, CP conserving or violating, then one cannot close the UB. Another way of presenting this situation is that, if one constructs the UT(2) and UT(2') first, then there may not be a common inner angle.

The above possibility may reveal information which cannot be obtained using only one UT. An example in which this might happen is the simple extension to four generation of quarks with the addition of t' and b'. In the case  $V_{t'd} = 0$ , the defining equation, Eq.(2), for UT(2) is still the same and one can define effective inner angles and sides. If one just checks whether the sum of inner angles is 180° from direct measurements of individual angles, and the angles determined by the sides, they are consistent. No sign of new physics will show up by this analysis. However, the UT(2') may be modified by a new term  $V_{ub'}V_{tb'}^*$ . Only information on UT(2') is also compared with that from UT(2), it is possible to decide if the type of new physics described above exists. The UB analysis contains this comparison together all in one.

Another is to use the property of the invariant quantity  $I_{UB}$  discussed earlier. It must equal to  $J^2/4$ . Taking the square root, one can check with one of the UT areas in the same UB. If more than one UB are constructed, one can also compare if their corresponding invariant quantities are equal.

### Discussion

Although the unitarity triangle carries information, about the Kobayashi-Maskawa quark mixing matrix, it explicitly contains just three parameters, which is one short to completely fix the KM matrix. The unitarity boomerangs formed using two UTs, with a common inner angle, can completely determine the KM matrix, and therefore better represents quark mixing information. Out of the six possible UTs, there are 9 different ways to have a

common inner angle from two UTs. In addition, one can align the two longer sides or one longer and one shorter side to form a UB. Therefore there are total 18 possible UBs. By studying the unitarity boomerangs, one can obtain all the information enshrined in KM matrix. We find that although the UBs have different areas, there is an invariant quantity for all UBs which is equal to a quarter of the Jarlskog parameter J squared. This is an universal representation of CP violation in the UB framework. We have also discussed how new physics can be hunted for by using information from UBs.

As far as graphic representation of the KM matrix, the proposal, to move from a single triangle to a boomerang combination, reflects, more than anything else, the impressive precision which has been attained by high-energy experiment. A unitarity boomerang contains all information of the KM matrix.

If new physics exists which modifies the unitarity nature, deviations from the expected UB can exist. The UB construction of the mixing matrix elements can also return, with information about new physics.

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- [1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
- [2] Particle Data Group, C. Amsler, et al., Phys. Lett. B667, 1 (2008).
- [3] L. L. Chau and W. Y. Keung, Phys. Rev. Lett. 53, 1802 (1984).
- [4] H. Fritzsch and Z.-Z. Xing, Phys. Rev. D57, 594 (1998); Y. Koide, Phys. Lett. B607, 123 (2005).
- [5] P.H. Frampton and X.-G. He, Phys. Lett. B688, 67 70 (2010)[arXiv:1003.0310[hep-ph]].
- [6] J.D. Bjorken, Nucl. Phys. B(Proc. Suppl.)11, 325(1989).
- [7] J.H. Christensen, J.W. Cronin, V.L Fitch and R. Turlay, Phys. Rev. Lett. 13, 138 (1964).
- [8] C. Jarlskog, Phys. Rev. Lett. 55, 1039(1985); Z. Phys. C29, 491(1986).
  P.H. Frampton and C. Jarkskog, Phys. Lett. B152, 421 (1985).

- [9] S.-W. Li and B.-Q. Ma, arXiv:1003.5854[hep-ph]
- [10] Y. Koide, Phys. Rev. **D73**, 073002 (2006).