Incipient nodal pairing in planar d-wave superconductors

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The possibility of a second pairing transition $d \to d + is$ $(d + id')$ in planar d-wave superconductors which occurs in the absence of external magnetic field, magnetic impurities or boundaries is established in the framework of the non-perturbative phenomenon of dynamical chiral symmetry breaking in the system of 2+1-dimensional Dirac-like nodal quasiparticles. We determine the critical exponents and quasiparticle spectral functions that characterize the corresponding quantum critical behavior and discuss some of its potentially observable spectral and transport features.

The recent ARPES [1] and optical conductivity [2] studies of the superconducting high-T_c compound $Bi_2Sr_2CaCu_2O_{8+\delta}$ have challenged the physical picture of what was once thought to be an essentially mundane BCS-like pairing state with only weakly interacting quasiparticles near the nodes of the $d_{x^2-y^2}$ order parameter. Most notably, in a striking contrast with the well-known theoretical prediction of the cubic in temperature quasiparticle damping rate obtained in the conventional d-wave picture [3], both of the above probes [1,2] revealed a much weaker, approximately linear, temperature dependence.

Whether the above findings indicated a behavior generic for all cuprates or not, their observation has prompted the idea of a quantum-critical behavior associated with a zero temperature quantum phase transition. Should such a transition occur in a sufficient proximity to the physical slice (the standard "temperature-doping" plane) of the, conceivably, multi-dimensional phase diagram of the cuprates, the fluctuations of a new order parameter could provide for a strong enhancement of quasiparticle scattering resulting in the sought-after linear quasiparticle damping rate.

Recently, M. Vojta, Y. Zang, and S. Sachdev proposed a phenomenological theory of the nodal quasiparticles coupled to the fluctuations of a secondary (is or id_{xy}) pairing order parameter. By applying a one-loop ϵ-expansion about $D = 3 + 1$ space-time dimensions, they found an evidence of a quantum critical point residing inside the superconducting phase [4].

In the present Letter, we develop a direct 2 + 1-dimensional approach that will allow us to establish the existence of the critical point in question beyond perturbation theory and make quantitative predictions for the corresponding quantum critical behavior.

In order to facilitate the following discussion we make use of the rotationally-covariant Nambu representation where the d-wave quasiparticles are described in terms of the two species ($i = 1, 2$), each having $N = 2$ spin components labeled by α, of the two-component Nambu spinors $\psi_{i\alpha} = (c_\alpha(\mathbf{k}_i), \epsilon_{\alpha\beta}c_\beta^\dagger(-\mathbf{k}_i))$ which are constructed from the quasiparticle states near the pairs of the opposite nodes located at $\pm\mathbf{k}_i = \pm(k_F, \pm k_F)/\sqrt{2}$.

Upon rotating the axes of the coordinate system by $\pi/4$, we cast the linear in momentum kinetic energy of the nodal quasiparticles in the form of the (anisotropic) Dirac Lagrangian

$$\mathcal{L}_\psi = i \sum_{\alpha=1}^N \overline{\psi}_{1\alpha}(\gamma_0\partial_0 - v_F\gamma_1\partial_x - v_\Delta\gamma_2\partial_y)\psi_{1\alpha}$$
$$+(\psi_{1\alpha} \leftrightarrow \psi_{2\alpha}, \quad x \leftrightarrow y) \qquad (1)$$

where v_F and v_Δ are related to the momentum derivatives of the normal state quasiparticle dispersion and the parent $d_{x^2-y^2}$-symmetrical gap function, respectively. In Eq.(1), we use the irreducible 2×2 representation for the γ-matrices in $2 + 1$ dimensions $\gamma_\mu = (\sigma_2, i\sigma_1, i\sigma_3)$ and the notation $\overline{\psi}_i = \psi_i^\dagger\gamma_0$. Also, in order to make Eq.(1) more symmetrical we perform a rotation in the Nambu space on one of the two fermion species: $\psi_{2\alpha} \to \tilde{\psi}_{2\alpha} = (\sigma_1 + \sigma_3)\psi_{2\alpha}/\sqrt{2}$.

Next, we combine the spinors $\psi_{1\alpha}$ and $\tilde{\psi}_{2\alpha}$ into one four-component Dirac fermion $\Psi_\alpha = (\psi_{1\alpha}, \tilde{\psi}_{2\alpha})$ and introduce a (reducible) 4-dimensional representation for the γ-matrices $\Gamma_\mu = \gamma_\mu \otimes \sigma_3$ where the second factor in the tensor product acts in the subspace of the two species.

In the phenomenological secondary pairing scenario, the nodal Dirac fermions couple to a real boson field ϕ corresponding to the imaginary part of the total gap function. Its own critical dynamics is governed by a generic ϕ^4-theory

$$\mathcal{L}_\phi = \frac{1}{2}(\frac{1}{c^2}(\partial_0\phi)^2 - (\nabla\phi)^2 - m^2\phi^2) - \frac{\lambda}{24}\phi^4 \qquad (2)$$

At $T = 0$ the theory (2) undergoes the conventional $D = 3$ Ising transition driven by the quartic self-interaction of the field ϕ that results in spontaneous breaking of the reflection (Z_2) symmetry $\phi \to -\phi$ as the parameter m^2 is tuned into its critical value m_c^2.

Alternatively, the Z_2 symmetry breaking can also be driven by a linear coupling between ϕ and the nodal fermions

$$\mathcal{L}_g = g\phi \sum_{\alpha=1}^N \overline{\Psi}_\alpha \mathbf{\Lambda} \Psi_\alpha \qquad (3)$$

provided that both (1) and (3) remain invariant under the discrete chiral transformation

$$\phi \to -\phi, \qquad \Psi_\alpha \to \Gamma_5 \Psi_\alpha, \qquad \overline{\Psi}_\alpha \to -\overline{\Psi}_\alpha \Gamma_5 \qquad (4)$$

where $\Gamma_5 = -\mathbf{1} \otimes \sigma_2$ anticommutes with all Γ_μ.

By choosing $\mathbf{\Lambda} = \mathbf{1} \otimes \mathbf{1}$ in (3), one couples ϕ to a Z_2-(and, accordingly, time reversal-) odd fermion mass operator. In terms of the original nodal quasiparticles, the latter reads as $\psi_1^\dagger \sigma_2 \psi_1 + \psi_2^\dagger \sigma_2 \psi_2$ which appears to coincide with the order parameter of the is pairing between the opposite nodes, while the chiral reflection (4) corresponds to a permutation of the two fermion species: $\psi_{1\alpha} \leftrightarrow \psi_{2\alpha}$.

With all the three velocities in (1) and (2) set equal ($v_F = v_\Delta = c = 1$) the theory described by the Lagrangian $\mathcal{L}_\psi + \mathcal{L}_\phi + \mathcal{L}_g$ becomes manifestly Lorentz- and Z_2-invariant (we will return to the spatially anisotropic case later).

In fact, the above Lagrangian can be readily recognized as that of the Higgs-Yukawa (HY) model where the phenomenon of spontaneous chiral symmetry breaking (CSB) has long been discussed as an alternative to the Higgs mechanism [5]. Unlike its $1 + 1$- and $3 + 1$-dimensional counterparts which appear to be asymptotically free and trivial (Gaussian), respectively, the $2 + 1$ dimensional HY model possesses a non-Gaussian infrared fixed point at a finite coupling g_c where CSB occurs via spontaneous generation of a non-zero fermion mass $M = g\langle\phi\rangle$.

The above correspondence allows one to identify the CSB phenomenon with the conjectured second pairing transition ($d \to d + is$) below which the nodal quasiparticles become fully gapped. Albeit being absent in any finite order of perturbation theory, spontaneous CSB occurs at the level of the non-perturbative mean field "gap equation" for the fermion mass

$$1 = \frac{4Ng^2}{m^2} T \sum_{\omega_n} \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{1}{\omega_n^2 + \mathbf{k}^2 + M^2(g,T)} \qquad (5)$$

which accounts for the tadpole ("Hartree") contribution to the fermion propagator [5].

At $T = 0$ Eq.(5) yields a critical coupling $g_c^2 = \pi m^2/\Omega N$ as a function of the ultraviolet cutoff Ω set by the amplitude of the parent d-wave gap. At strong coupling ($g > g_c$), the chiral symmetry (4) is spontaneously broken, and the fermion mass scales alongside with the order parameter $M(g,0) \propto\, <\phi> \propto (g - g_c)^\beta$.

At finite T the chiral symmetry gets restored above a transition line in the $g - T$ plane which terminates at the quantum critical point. In the symmetry broken (strong coupling, low-T) phase, the mean field equation (5) yields the fermion mass $M(g,T) = M(g,0) + 2T \ln[(1 + \sqrt{1 - 4\exp(-M(g,0)/T)})/2]$ which vanishes along a critical line, resulting in the non-BCS-like relation $M(g,0)/T_c^* = 2\ln 2$ between the maximum gap and critical temperature T_c^* of the nodal pairing.

The quantum critical point associated with the CSB transition manifests itself via anomalous operator dimensions that can be found from the solution of the coupled Dyson equations for the fermion propagator $\langle\Psi\overline{\Psi}\rangle = (Z_p \slashed{p} + \Delta_p)^{-1}$ with all the tadpoles absorbed into the definition of the bare fermion mass (hereafter, $\slashed{p} = \Gamma_\mu p_\mu$ and $p^2 = p^\mu p_\mu$):

$$(Z_p - 1)\, \slashed{p} + \Delta_p =$$
$$\int \frac{d^3k}{(2\pi)^3} \frac{Z_{p-k}(\slashed{p} - \slashed{k}) + \Delta_{p-k}}{Z_{p-k}^2(p-k)^2 + \Delta_{p-k}^2} \frac{\Lambda_{p,k}^2}{k^2 + m^2 + \Pi_k}, \qquad (6)$$

and the fermion polarization operator

$$\Pi_k = N \mathrm{Tr} \int \frac{d^3p}{(2\pi)^3} \Lambda_{p,k}^2 \frac{Z_p\, \slashed{p} + \Delta_p}{Z_p^2 p^2 + \Delta_p^2} \frac{Z_{k+p}(\slashed{k} + \slashed{p}) + \Delta_{k+p}}{Z_{k+p}^2(k+p)^2 + \Delta_{k+p}^2} \qquad (7)$$

which modifies the boson propagator $\langle\phi\phi\rangle = (k^2 + m^2 + \Pi_k)^{-1}$. In particular, the (ultraviolet divergent) momentum-independent part of (7) contributes to the renormalized boson mass $\delta m^2 = m^2 + \Pi_0$ which vanishes at the critical point $g = g_c$, consistent with Eq.(5).

In the lowest $1/N$ order, $Z_p = 1$, $\Delta_p = M$, $\Lambda_{p,k} = g$, and the polarization operator (7) assumes the form

$$\Pi_k = \Pi_0 + \frac{g^2 N}{\pi} \left(\frac{4M^2 - k^2}{2\sqrt{-k^2}} \tan^{-1} \left(\frac{\sqrt{-k^2}}{2M} \right) - M \right)$$

In the critical regime ($\delta m = M = 0$), the infrared behavior of the boson propagator is governed solely by the fermion polarization $\Pi_k - \Pi_0 \propto \sqrt{-k^2}$ (hereafter, $\propto k$), while the bare k^2 term can be completely neglected. This implies that at the CSB fixed point the mean field dimension of ϕ changes from its bare value $1/2$ to 1, thereby rendering the kinetic and quartic terms in Eq.(2) totally irrelevant in the renormalization group sense.

We mention, in passing, that besides indicating the existence of a fixed point at a critical coupling $g_c^2 \propto \epsilon = 4 - D$, the one-loop ϵ-expansion used in Ref. [4] also predicts that the (formally relevant for any $\epsilon > 0$) quartic term in (2) scales towards strong coupling as well: $\lambda \to \lambda_c \propto \epsilon$, which appears to be an artifact of the corresponding one-loop renormalization group equations.

With the only relevant term $m^2\phi^2$ in Eq.(2) remaining, the critical behavior at the CSB infrared fixed point turns out to be identical to the ultraviolet asymptotical regime of the $2 + 1$-dimensional Gross-Neveu model which is renormalizable in the $1/N$-expansion, N being the number of Dirac fermion species [5].

Namely, the mean field (or $N = \infty$) scaling behavior of the boson propagator $< \phi\phi > \propto 1/k$ gives rise to the logarithmic divergence of the momentum integrals in Eqs.(6,7) describing the first order $1/N$ corrections

to the fermion and boson wave functions as well as the gap and vertex functions. By differentiating these logarithmic corrections with respect to the external momenta, one arrives at the usual renormalization group equations whose solution yields the infrared asymptotics of the renormalization factors

$$Z_p = \left(\frac{p}{\Omega}\right)^{-\frac{2}{3\pi^2 N}} \quad , \quad \Delta_p = M\left(\frac{p}{\Omega}\right)^{\frac{2}{\pi^2 N}} \qquad (8)$$

$$\Pi_k - \Pi_0 = k\left(\frac{k}{\Omega}\right)^{\frac{16}{3\pi^2 N}} \quad , \quad \Lambda_{p,k} = g\left(\frac{max(p,k)}{\Omega}\right)^{\frac{2}{\pi^2 N}}$$

from which the dimensions of the fermion $[\Psi]$ and boson $[\phi]$ fields can be readily read off. Notably, the anomalous dimension of the fermion operator $[\Psi] - 1 \approx 1/3\pi^2 N$ remains rather small even for $N = 2$.

As regards the Lorentz-non-invariance of the bare fermion Lagrangian (1), in the quantum-critical regime the momentum integrals in Eqs.(6,7) are dominated by the momenta parallel to the external ones, and, therefore, the anisotropic velocity factors can be scaled away without affecting the above power counting.

The critical exponents characterizing the CSB transition satisfy the hyperscaling relations which allow one to express all of them in terms of only two independent ones, e.g., the anomalous dimension exponent $\eta = 2 - D + 2[\phi] \approx 1 - (16/3\pi^2 N)$ and the exponent $\nu = \beta/[\phi] \approx 1 + (8/3\pi^2 N)$ controlling the correlation length: $1/L_\xi = \delta m^2/(g^2 N) \propto |g - g_c|^\nu$. The latter remains finite in the symmetry broken (ordered) phase, as breaking discrete chiral symmetry does not result in the appearance of a Goldstone mode below T_c^*.

The fact that the critical exponents demonstrate an explicit N-dependence implies that the universality class of the CSB transition is different for different N, the $D = 3$ Ising transition ($\nu = 0.63, \quad \eta = 0.025$) being recovered only in the limit $N \to 0$.

In the physical case of $N = 2$, the above first order estimates for the critical exponents ($\nu \approx 1.14, \quad \eta \approx 0.73$) compare favorably with the available Monte Carlo results ($\nu = 1.00, \quad \eta = 0.75$) [6], since the actual (inverse) parameter of this expansion $N\text{Tr}(\mathbf{1}\otimes\mathbf{1}) = 4N$ remains fairly large even for $N = 2$. For comparison, the first-order estimate for the anomalous dimension exponent given by the ϵ-expansion of Ref. [4] is $\eta \approx 4\epsilon/7 = 0.57$, although the agreement can be improved in the higher orders [5].

We emphasize that the above discussion pertains to the bulk properties of the layered d-wave superconductors and is, therefore, unrelated to the previously proposed scenario of a surface-induced is pairing [7].

The alternate case of id_{xy} pairing corresponds to a different choice of the coupling matrix $\mathbf{\Lambda} = \mathbf{1} \otimes \sigma_3$ in Eq.(3). This yields a chiral invariant order parameter $\overline{\Psi}_\alpha \mathbf{1} \otimes \sigma_3 \Psi_\alpha = \psi_{1\alpha}^\dagger \sigma_2 \psi_{1\alpha} - \psi_{2\alpha}^\dagger \sigma_2 \psi_{2\alpha}$ which causes the term (3) to explicitly break the chiral symmetry (4).

However, contrary to the is order parameter, the id_{xy} pairing and the corresponding boson field ϕ change their signs under parity transformation ($x \to -x, y \to y$) which acts upon the four-component spinors as $\Psi_\alpha \to \sigma_1 \otimes \sigma_1 \Psi_\alpha$. It is this very property that allows for the existence of a zero-field spin or thermal Hall current in the id_{xy}-paired state [8].

Therefore, by substituting parity for the chiral transformation (4), one finds yet another Z_2 symmetry which gets spontaneously broken upon the onset of the id_{xy} order described by the same gap equation (5). The resulting critical behavior then appears to be identical to that found in the case of the is pairing. It will be, however, different from the previously discussed second order $d \to d + id'$ transition induced by magnetic impurities [9] as well as the first order one driven by external magnetic field [10] (both these scenarios have recently been revisited and critically assessed in Ref. [11]).

After having fully identified the nature of the second pairing transition, we now proceed with computing the experimentally measurable quasiparticle damping. Whether attainable by varying a single physical parameter such as doping or not, a zero temperature quantum critical point affects the finite temperature dynamics of the system in a whole domain of the $g - T$ plane where the correlation length L_ξ exceeds the thermal one $\propto 1/T$ (the dynamical exponent equals unity due to the Lorentz invariance).

In this quantum-critical regime, the renormalization cut-off is set by the temperature, and, therefore, the small energy- and momentum-dependencies of the fermion and boson propagators are no longer governed by the anomalous operator dimensions from Eq.(8). Instead, the latter appear in the residues $\propto T^{2([\Psi,\phi]-1)}$ of these (now, pole-like) propagators.

After factoring out the fermion wave function renormalization factor, the damping of the nodal quasiparticles with $\mathbf{p} = 0$ is given by the expression: $\Sigma(\varepsilon) = \text{Im}\{\text{Tr}\,[\Gamma_0(Z_p\langle\Psi\overline{\Psi}\rangle)^{-1}]\}$.

The universal function $\Sigma/T = \Phi(x, y)$ incorporates, alongside with the low- versus high-energy asymptotics for $x = \varepsilon/T < 1$ (> 1), the crossovers from the quantum-critical to the other, renormalized classical and quantum disordered, regimes for $y = \Omega(|g - g_c|/g_c)^\nu/T < 1$ (> 1), respectively.

The non-linear equation determining the function $\Phi(x, y)$ results from including $\Sigma(\varepsilon)$ into the fermion propagator in Eq.(6)

$$\Sigma(\varepsilon) = \int \frac{d\omega}{2\pi} \int \frac{d\mathbf{q}}{(2\pi)^2} \left\{\tanh\frac{\varepsilon+\omega}{2T} - \coth\frac{\omega}{2T}\right\}$$

$$\text{Im}\left[\frac{\varepsilon+\omega+i\Sigma(\varepsilon+\omega)}{(\varepsilon+\omega+i\Sigma(\varepsilon+\omega))^2 - \mathbf{q}^2}\right]\text{Im}\left[\frac{g^2}{\Pi(\omega,\mathbf{q})+m^2}\right] \quad (9)$$

and, correspondingly, into Eq.(7) for the spatially isotropic, albeit manifestly non-Lorentz-invariant, finite

temperature polarization operator

$$\Pi(\omega,\mathbf{q}) = g^2 N \int \frac{d\varepsilon}{2\pi} \int \frac{d\mathbf{p}}{(2\pi)^2} \left\{ \tanh\frac{\varepsilon+\omega}{2T} - \tanh\frac{\varepsilon}{2T} \right\}$$

$$\frac{(\varepsilon+i\Sigma(\varepsilon))(\varepsilon+\omega+i\Sigma(\varepsilon+\omega)) - \mathbf{p}(\mathbf{p}+\mathbf{q})}{[(\varepsilon+i\Sigma(\varepsilon))^2 - \mathbf{p}^2][(\varepsilon+\omega+i\Sigma(\varepsilon+\omega))^2 - (\mathbf{p}+\mathbf{q})^2]}$$

(10)

Notably, in the quantum-critical regime ($y < 1$) the energy-independent renormalization factors drop out from Eqs.(9,10), thanks to the underlying Ward identities of the HY model [5].

Both analytical and numerical, analyses of the coupled equations (9,10) show that the fermion damping behaves as $\Sigma \propto T$ and $\propto \varepsilon$ at high and low temperatures, respectively, in agreement with experiment [1] and the results of Ref. [4].

Below the crossover ($y > 1$) to the quantum disordered (weak coupling, low-T) regime, the infrared cutoff is provided by the (inverse) correlation length L_ξ^{-1}, and the damping of the Lorentz-invariant nodal quasiparticles appears to retain its energy dependence even for $\varepsilon < T$, unlike the case of fermions with extended Fermi surface. In the energy intervals $x \geq 1$, $1/y^4 < x < 1$, and $x \leq 1/y^4$, the self-consistent solution of Eqs.(9,10) behaves as $\Phi(x,y) \propto x^3/y^2$, $\propto x^{1/2}/y^2$, and $\propto 1/y^4$, respectively. For $\varepsilon \sim T$ it reproduces the perturbative second order result $\Sigma \propto T^3$ (see [3]) valid for a generic short-ranged coupling, while for $\varepsilon \to 0$ we obtain $\Sigma \propto T^5$, one possible experimental implication being a significant narrowing of the nodal ARPES linewidth below the crossover.

Likewise, in the renormalized classical (strong coupling, low-T) regime one can expect a suppression of the nodal quasiparticle density of states that can manifest itself in tunneling and specific heat data as well as a sharp decrease of both, thermal and optical, conductivities accompanied by an increasing thermal Hall angle [12].

Furthermore, according to experiment, the linear temperature/energy dependence of the quasiparticle damping extends well into the normal phase [1]. This might be suggestive of a possibility of applying the HY model to the pseudogap state, as the above discussion relies on the presence of the $d_{x^2-y^2}$-symmetrical (pseudo)gap in the local quasiparticle spectrum rather than the onset of the superconducting coherence across the entire system. To this end, it should be interesting to contrast the quantum critical scenario described in this paper against the results of the previous approaches to the pseudogap problem which have focussed on the couplings between the nodal Dirac fermions and magnetic order parameters [13] as well as classical [14] and quantum [15] superconducting phase fluctuations.

To summarize, we carried out a non-perturbative analysis of the secondary pairing transition in planar d-wave superconductors. By identifying the transition in question with the HY model of the $2+1$-dimensional Dirac fermions we succeeded in finding all the critical exponents and determining the behavior of the quasiparticle propagators. At $\varepsilon > T$ the quantum-critical fermion propagator was found to have a numerically small anomalous dimension, while in the opposite limit it exhibits linear damping, as observed in experiment.

Should a secondary pairing occur in the bulk d-wave superconductors upon tuning the phenomenological parameter g in a (yet, unspecified) way, it can manifest itself as a marked change in quasiparticle spectral and transport properties even in the absence of boundaries (cf. [7]), magnetic impurities (cf. [9]), and/or external magnetic field (cf. [10]). Also, the two-dimensional spatial parity's remaining intact or being spontaneously broken may allow for an experimental discrimination between the two different kinds (is versus id_{xy}) of incipient ordering with broken time-reversal symmetry.

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