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Simplified Renormalizable $T^{'}$ Model for Tribimaximal Mixing and Cabibbo Angle

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Abstract

In a simplified renormalizable model where the neutrinos have PMNS (Pontecorvo-Maki-Nakagawa-Sakata) mixings $\tan^2\theta_{12} = \frac{1}{2}, \theta_{13} = 0, \theta_{23} = \pi/4$ and with flavor symmetry $T^{'}$ there is a corresponding prediction where the quarks have CKM (Cabibbo-Kobayashi-Maskawa) mixings $\tan 2\Theta_{12} = \frac{\sqrt{2}}{3}, \Theta_{13} = 0, \Theta_{23} = 0$.

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The standard model of particle physics is very predictive, and well tested. Given coupling constants and masses, we can calculate electroweak processes like scattering cross-sections and atomic energy levels to remarkable accuracy. We can in principle also calculate hadronic processes except for our lack of technical skill, but not for our lack of a good theory. The main reason for the precision (or potential precision) in these calculations is the symmetries satisfied by the theory–Lorentz invariance plus the electroweak gauge symmetry $SU(2) \times U(1)$ and the color sector gauge symmetry $SU(3)$ in addition to the discrete spacetime symmetries like P and CP, and the controlled way some of these symmetries are broken, e. g., gauge symmetry breaking via Higgs vacuum expectation value in the electroweak sector. However, the standard model has its limitations. There are still approximately twenty parameters needed as input for the model, including gauge coupling constants, quark and lepton mixing angles and phases, etc.

Recently, a considerable effort has been made to reduce the number of standard model input parameters. This can be done by introducing new symmetries that relate the various parameters, while eventually breaking or at least partially breaking the new symmetry. Some requirements and constraints needed for a viable theory are: the new symmetries cannot be gauged at low energy since there are no corresponding light gauge bosons in the spectrum. Broken continuous global symmetries must also be avoided since they lead to Goldstone bosons, also unseen in experiments. This leads us to the only natural choice–discrete symmetries.

To date, models of this type have usually focused on reducing the number of parameters in either the lepton or the quark sector. A notable exception is provided by models based on the binary tetrahedral group T', which is capable of providing calculability to both sectors. To show the power of this additional symmetry, we will provide a T' model that leads to the celebrated tribimaximal neutrino mixing and at the same time allows us to calculate quark mixings. As an example, we will show how the quark mixing matrix can give a purely numerical value for the Cabibbo angle that is only a few percent away from its experimental value.

The first use of the binary tetrahedral group T' in particle physics was by Case, Karplus and Yang [1] who were motivated to consider gauging a finite T' subgroup of $SU(2)$ in Yang-Mills theory. This led Fairbairn, Fulton and Klink (FFK) [2] to make an analysis of T' Clebsch-Gordan coefficients #4. As a flavor symmetry, T' first appeared in [6] motivated by the idea of representing the three quark families with the third treated differently from the first two. Since T' is the double cover of A_4, it was natural to suggest [7] that T' be employed to accommodate quarks and simultaneously the established A_4 model building for tribimaximal neutrino mixing.

In the present article we shall build such a T' model with simplifications to emphasize the largest quark mixing, the Cabibbo angle, for which we shall derive an entirely new formula as an exact angle.

#4 Other analysis of T' Clebsch-Gordan coefficients appears in [3–5]. We use FFK.

This work is a major extension of that in [8] where the constraint of renormalizability was first applied to an A_4 model and led not only to the usual tribimaximal mixing[#5]

$$\tan\theta_{12} = 1/\sqrt{2}, \qquad \theta_{23} = \pi/4, \qquad \theta_{13} = 0, \tag{1}$$

but to the simplified normal hierarchy

$$m_3 \neq 0, \qquad m_{1,2} = 0. \tag{2}$$

We review briefly this A_4 model. The leptons are assigned under $(A_4 \times Z_2)$ as

$$\left.\begin{matrix} \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \\ \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \\ \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \end{matrix}\right\} L_L(3,+1) \quad \begin{matrix} \tau_R^- \ (1_1,-1) & N_R^{(1)} \ (1_1,+1) \\ \mu_R^- \ (1_2,-1) & N_R^{(2)} \ (1_2,+1) \\ e_R^- \ (1_3,-1) & N_R^{(3)} \ (1_3,+1), \end{matrix} \tag{3}$$

which is typical of A_4 model building [9]. Imposing strict renormalizability on the lepton lagrangian allows as nontrivial terms only Majorana mass terms and Yukawa couplings to A_4 scalars [#6] $H_3(3,+1)$ and $H_3^{'}(3,-1)$

$$\begin{aligned} \mathcal{L}_Y^{(leptons)} &= \frac{1}{2} M_1 N_R^{(1)} N_R^{(1)} + M_{23} N_R^{(2)} N_R^{(3)} \\ &+ \Bigg\{ Y_1 \left(L_L N_R^{(1)} H_3\right) + Y_2 \left(L_L N_R^{(2)} H_3\right) + Y_3 \left(L_L N_R^{(3)} H_3\right) \\ &+ Y_\tau \left(L_L \tau_R H_3'\right) + Y_\mu \left(L_L \mu_R H_3'\right) + Y_e \left(L_L e_R H_3'\right) \Bigg\} + \text{h.c.}. \end{aligned} \tag{4}$$

Charged lepton masses arise from the vacuum expectation value (hereafter VEV)

$$< H_3^{'} >= \left(\frac{m_\tau}{Y_\tau}, \frac{m_\mu}{Y_\mu}, \frac{m_e}{Y_e}\right) = (M_\tau, M_\mu, M_e), \tag{5}$$

where $M_i \equiv m_i/Y_i$ $(i = \tau, \mu, e)$. Neutrino masses and mixings satisfying Eqs.(1,2) come from the see-saw mechanism [10] and the VEV [#7]

$$< H_3 >= V(1, -2, 1). \tag{6}$$

We promote A_4 to $T^{'}$ keeping renormalizability and including quarks. The left-handed quark doublets $(t,b)_L, (c,d)_L, (u,d)_L$ are assigned under $(T^{'} \times Z_2)$ to

[#5] Throughout we ignore CP violation.
[#6] All scalars are doublets under electroweak $SU(2)$.
[#7] Use [11] of $< H_3 >= V(1,1,1)$ gives Eqs.(1,2) with $2 \leftrightarrow 3$ interchanged in Eq.(2).

$$\begin{array}{lll}
\begin{pmatrix} t \\ b \end{pmatrix}_L & \mathcal{Q}_L & (\mathbf{1_1}, +1) \\
\left.\begin{array}{l} \begin{pmatrix} c \\ s \end{pmatrix}_L \\ \begin{pmatrix} u \\ d \end{pmatrix}_L \end{array}\right\} & Q_L & (\mathbf{2_1}, +1),
\end{array} \tag{7}$$

and the six right-handed quarks as

$$\begin{array}{lll}
t_R & & (\mathbf{1_1}, +1) \\
b_R & & (\mathbf{1_2}, -1) \\
\left.\begin{array}{l} c_R \\ u_R \end{array}\right\} & \mathcal{C}_R & (\mathbf{2_3}, -1) \\
\left.\begin{array}{l} s_R \\ d_R \end{array}\right\} & \mathcal{S}_R & (\mathbf{2_2}, +1).
\end{array} \tag{8}$$

We add only two new scalars $H_{1_1}(1_1, +1)$ and $H_{1_3}(1_3, -1)$ whose VEVs

$$< H_{1_1} >= m_t/Y_t, \qquad < H_{1_3} >= m_b/Y_b, \tag{9}$$

provide the (t, b) masses. In particular, no $T^{'}$ doublet $(2_1, 2_2, 2_3)$ scalars have been added. This allows a non-zero value only for Θ_{12}. The other angles vanish making the third family stable [#8]. The allowed quark Yukawa and mass terms are

$$\begin{aligned}
\mathcal{L}_Y^{(quarks)} = & \; Y_t(\{\mathcal{Q}_L\}_{\mathbf{1_1}}\{t_R\}_{\mathbf{1_1}}H_{\mathbf{1_1}}) \\
& + Y_b(\{\mathcal{Q}_L\}_{\mathbf{1_1}}\{b_R\}_{\mathbf{1_2}}H_{\mathbf{1_3}}) \\
& + Y_{\mathcal{C}}(\{Q_L\}_{\mathbf{2_1}}\{\mathcal{C}_R\}_{\mathbf{2_3}}H_{\mathbf{3}}^{'}) \\
& + Y_{\mathcal{S}}(\{Q_L\}_{\mathbf{2_1}}\{\mathcal{S}_R\}_{\mathbf{2_2}}H_{\mathbf{3}}) \\
& + \text{h.c.}.
\end{aligned} \tag{10}$$

The use of $T^{'}$ singlets and doublets [#9] for quark families in Eqs.(7,8) permits the third family to differ from the first two and thus make plausible the mass hierarchies $m_t \gg m_b$, $m_b > m_{c,u}$ and $m_b > m_{s,d}$ as outlined in [6].

The nontrivial (2×2) quark mass matrices (c, u) and (s, d) will be respectively denoted by $U^{'}$ and $D^{'}$ and calculated using the $T^{'}$ Clebsch-Gordan coefficients of FFK [2]. Dividing out $Y_{\mathcal{C}}$ and $Y_{\mathcal{S}}$ in Eq.(10) gives U and D matrices ($\omega = e^{i\pi/3}$)

$$U \equiv \left(\frac{1}{Y_{\mathcal{C}}}\right) U^{'} = \begin{pmatrix} \sqrt{\frac{2}{3}}\omega^2 M_\tau & \frac{1}{\sqrt{3}} M_e \\ -\frac{1}{\sqrt{3}}\omega^2 M_e & \sqrt{\frac{2}{3}} M_\mu \end{pmatrix}, \tag{11}$$

[#8]At the end of this paper non-vanishing Θ_{23}, Θ_{13} is related to (d, s) masses.

[#9]It is discrete anomaly free cf. [12, 13]. We thank the UF-Gainesville group for discussions.

$$D \equiv \left(\frac{1}{VY_{\mathcal{S}}}\right) D' = \begin{pmatrix} \frac{1}{\sqrt{3}} & -2\sqrt{\frac{2}{3}}\omega \\ \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}}\omega \end{pmatrix}. \tag{12}$$

Let us first consider U of Eq.(11). Noting that $m_\tau > m_\mu \gg m_e$ we may simplify U by setting the electron mass to zero, $M_e = 0$. This renders U diagonal leaving free the c, u, τ and μ masses. This leaves only the matrix D in Eq.(12) which predicts both Θ_{12} and (m_d^2/m_s^2). The hermitian square $\mathcal{D} \equiv DD^\dagger$ is

$$\mathcal{D} \equiv DD^\dagger = \left(\frac{1}{3}\right) \begin{pmatrix} 9 & -\sqrt{2} \\ -\sqrt{2} & 3 \end{pmatrix}, \tag{13}$$

which leads by diagonalization to a formula for the Cabibbo angle

$$\tan 2\Theta_{12} = \left(\frac{\sqrt{2}}{3}\right), \tag{14}$$

or equivalently[#10] $\sin\Theta_{12} = 0.218...$ close to the experimental value [#11] $\sin\Theta_{12} \simeq 0.227$.

Our result of an exact angle for Θ_{12} can be regarded as on a footing with the tribimaximal values for neutrino angles θ_{ij}, quoted in Eq.(1). Note that the tribimaximal θ_{12} presently agrees with experiment within one standard deviation (1σ). On the other hand, our analagous exact angle for Θ_{12} differs from experiment already by 9σ which is probably a reflection of the fact that the experimental accuracy for Θ_{12} is $\sim 0.5\%$ while that for θ_{12} is $\sim 6\%$. It is thus very important to acquire better experimental data on θ_{12}, θ_{23} and θ_{13} to detect their similar deviation from the exact angles predicted by Eq.(1). Our result for (m_d^2/m_s^2) from Eq.(13) is exactly 0.288... compared to the central experimental value $\simeq 0.003$ in a simplified model whose generalization to an extended scalar sector including T' doublets can avoid $\Theta_{23} = \Theta_{13} = 0$ and thereby change (m_d^2/m_s^2) due to mixing of (d, s) with the b quark.

We believe our $T' \times Z_2$ extension of the standard model is an important stride in tying the quark and lepton sectors together, providing calculability, and at the same time reducing the number of standard model parameters. The ultimate goal would be to understand the origin of this discrete symmetry. Since gauge symmetries can break to discrete symmetries, and gauge symmetries arise naturally from strings, perhaps there is a clever construction of our model with its fundamental origin in string theory.

[#10] Ellipsis ... denotes exactitude.

[#11] Experimental results are from [14]; see references therein.

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