Evidence for Three Nucleon Force Effects in p-d Elastic Scattering

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Abstract

A new measurement of the p-d differential cross section at $E_p = 1$ MeV has been performed. These new data and older data sets at energies below the deuteron breakup are compared to calculations using the two–nucleon Argonne v_{18} and the three–nucleon Urbana IX potentials. A quantitative estimate of the capability of these interactions to describe the data is given in terms of a χ^2 analysis. The χ^2 per datum drastically improves when the three-nucleon interaction is included in the Hamiltonian.

The new generation of NN potentials describes the two-nucleon (2N) observables with a χ^2 per datum ≈ 1 [1–3]. This high accuracy obtained in the description of the 2N system does not imply that a similar accuracy will be achieved in the description of larger nuclear systems, in particular the three-nucleon (3N) data. In fact, the simplest observable in the 3N system, the binding energy, is underpredicted by each of the new NN potentials. The energy deficit ranges from 0.5 to 0.9 MeV depending on the off-shell and short range parametrization of the NN interaction. This underbinding problem has not yet been solved, and a number of effects beyond the static NN interaction have been considered (a review is given in ref. [4]). For example, considerable efforts have been put into calculating relativistic corrections and three-nucleon force (3NF) contributions to the 3N binding energy.

It is common practice to look at the 3N bound state problem as the solution of the non-relativistic Schrödinger equation using phenomenological NN interactions and then to introduce a 3NF to provide supplementary binding. The models for the 3NF are usually based on two–pion exchange with intermediate Δ-isobar excitation, and the strength of the interaction is adjusted to reproduce the ^{3}H binding energy.

Once the 3N binding energy is well reproduced, the description of several other observables improves as well. For example, the $A = 3$ r.m.s radii [5], the asymptotic normalization constants η [6] and the doublet n-d scattering lengths [7] are now in much better agreement with the experimental values. These observables have the property to scale with 3N binding energy (the so-called Phillips lines) [8].

With respect to the 3N continuum, a complete quantitative analysis in terms of χ^2 of the 3N data versus theory has not yet been made for any of the new NN potentials. Therefore, there is a need to evaluate in detail the ability of those interactions to describe the 3N scattering data. In ref. [9] a detailed analysis has been performed for the total n-d cross section in which calculations solving the Faddeev equations have been compared to the data. This analysis has been recently repeated [10] by taking into account new high-precision measurements [11]. The analysis could not be extended to the differential cross section, due to lack of an adequate data set. In ref. [12] a new set of precise measurements of d-p elastic

observables at $E_d = 270$ MeV has been presented. The differential cross section as well as some polarization observables has been analyzed with Faddeev calculations using modern NN potentials including 3NF contributions. The χ^2 per datum has been studied in a limited angular range ($\theta_{c.m.} = 50° - 180°$) in order to avoid the effects of the Coulomb interaction, which has been neglected in that calculations. At this very high energy a definite sensibility to three-body forces has been observed.

Recently a rigorous solution of the p-d scattering problem has been obtained by the Pisa group [7,13] allowing for a detailed study of this reaction for which an extensive and high precision data set exists. In refs. [14,15] phase shift analyses have been performed in order to reproduce the p-d differential cross section and vector and tensor analyzing powers. From these analyses it was possible to make comparisons to the theoretical phase-shift and mixing parameters and quantitatively relate the found differences in the P–wave parameters to the so called "A_y puzzle" [15].

In the present paper we use these calculations in an attempt to analyze in a quantitative way the capability of the modern NN interactions to describe the p-d differential cross section in the low energy regime. To this aim we will present a new precise measurement of the p-d differential cross section at $E_p = 1$ MeV and its theoretical description in terms of the Argonne v_{18} potential (AV18) [2] plus the Urbana 3NF (UR) [16]. The new high precision data were taken as a part of a measurement program of p-d scattering observables including the two vector (A_y and iT_{11}) and the three tensor (T_{20}, T_{21}, and T_{22}) analyzing powers at $E_p = 1$ MeV. The complete set of data will be published elsewhere [17] and is part of a program developed at Triangle Universities Nuclear Laboratory (TUNL) to study the properties of the few body interactions at low energies.

The measurement of cross sections for p-d scattering was conducted at TUNL using the 10 MV FN tandem accelerator. The deuteron beam was accelerated to an energy of 2.0 MeV and directed by a dipole magnet to a scattering chamber. The magnet and a feedback system with the FN tandem kept the beam energy constant to within ± 5 keV. The targets were made of thin hydrogenated carbon foils containing approximately 0.5×10^{18} H/cm^2 and $1.0 \times$

10^{18} C/cm^2 [18] and were replaced often during the experiment. The elastically-scattered deuterons and recoil protons were counted in two pairs of silicon surface-barrier detectors placed 10° apart symmetrically with respect to the beam direction. In the measurement of relative cross sections the detectors covered an angular range of $\theta_{lab} = 7°$ to 64° ($\theta_{c.m.} = 21°$ to 166°). Another two pairs of silicon detectors were mounted to the chamber wall and set at 15° and 42° to normalize the yields in the rotating detectors. The statistical accuracy of the relative cross sections was less than 0.5% and the systematic error was less than 0.8%. A sample spectrum for ^{1}H(d,d) scattering at $\theta_{lab} = 26°$ is shown in Fig.1.

The absolute normalization of the cross section measurement was obtained by relating the measured differential p-d cross sections to the well-known p-p cross section [19]. The normalization procedure involved producing in sequence proton and deuteron beams of the same magnetic rigidity. This procedure assured that both beams were transported through the beamline in the same way and could be put on approximately the same spot on the target by only adjusting the dipole magnet after the ion source and changing the FN tandem terminal voltage. The scattering chamber was left with the same setup as for the relative measurement with detectors placed at two angles which provided three normalization points [^{1}H(d,d) and ^{1}H(d,p) for 25.0° and ^{1}H(d,p) for 35.0°]. Each target used in normalization runs remained in the beam for a very short period of time ($\approx 10~\mu$C) to reduce the effects of target deterioration. After the proton beam was put on target, three targets were cycled as in the case of the measurements with the deuteron beam. This process of switching from deuteron beam to proton beam back to deuteron beam was repeated four times with consistent results.

An Au target was utilized to determine systematic errors in the experimental setup and data collection process. At incident energies of $E_d = 2.0$ MeV and $E_p = 4.0$ MeV, the ^{197}Au(d,d) and ^{197}Au(p,p) cross sections should follow the Rutherford formula. For our tests, the detectors pairs were placed at 140.0° and 150.0°. The target consisted of 170 μg/cm^2 of Au evaporated on a 10 μg/cm^2 carbon foil. At the end of each cycle of runs with hydrogenated carbon targets, the Au target was placed in the beam (either deuteron or

proton). The ratio of the ^{197}Au(d, d) and ^{197}Au(p, p) scattering measurements were found to be within 0.6% of the calculated values. The overall accuracy of the present absolute cross section measurements is 0.8%.

The calculations of p-d scattering have been done using the Pair Correlated Harmonic basis [13] to expand the scattering p-d wave function. The scattering matrix has been obtained using the Kohn variational principle in its complex form [20] and, successively, the cross section has been calculated using the formula given in eq.(4) of ref. [21]. The accuracy of this method in the calculations of the phase shift and mixing parameters has been studied in ref. [20]. In ref. [22] a detailed comparison has been performed by comparing the present technique with the results obtained by solving the Faddeev equations in momentum space. The numerical accuracy of the present technique has been found to be of the order of 0.1%.

The results of the measurements for the p-d cross section at $E_{lab} = 1$ MeV are given in Fig. 2 (open circles) and compared to the theoretical predictions. The two curves shown in Fig. 2 correspond to calculations using the AV18 potential (dotted line) and including also the Urbana 3NF (solid line).

There is a good agreement between the scattering data and both calculations, though the cross section calculated using the AV18 potential is slightly higher than the one obtained with the AV18+UR model. This can be understood as arising from the additional attraction introduced by the 3NF which overall increases the binding of ^{3}He and, at low energy, reduces the cross section. It is known that the inclusion of the Urbana 3NF modifies mainly the $J = 1/2^+$ state, which is reflected in a change in the $^2S_{1/2}$ phase shift and the $\eta_{1/2+}$ mixing parameter as noted in ref. [14]. It is also evident from the figure that the calculation using the AV18+UR potential, which gives a better description of the bound system, also gives better agreement with the elastic scattering data.

In order to gain a better understanding of the quality of the agreement between theory and data, in Fig. 3 we present the values of experimental differential cross section data divided by the theoretical values calculated with the AV18+UR potential model. In this plot we also present the results of two other measurements of differential cross sections at

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the same energy [23,24]. The present data (open circles) and the theoretical predictions are in agreement within 1%. This is also the case with the data from ref. [23] (open squares) though the theoretical cross section seems to be below these data. We observe disagreement of $1-2\%$ between our data and data of ref. [24] (open triangles) at forward angles. These data are slightly below the theoretical cross section at forward angles and slightly above at backward angles. This behavior is in general present when the comparison between theory and experiment is performed at somewhat different energies.

A χ^2 per datum analysis of the theoretical calculations with respect to the experimental data is made by the evaluation of the quantity

$$\chi^2 = \frac{1}{N} \sum_i \frac{(c f_i^{exp} - f_i^{th})^2}{(\Delta f_i)^2} \ , \tag{1}$$

where f_i^{exp} is the i–th datum at angle θ_i and Δf_i its error, f_i^{th} is the theoretical value at the same angle and the total number of points is N. The parameter c is introduced to allow a variation in the absolute normalization of the data. Its value is slightly varied around $c = 1$ looking for a minimum in the value of χ^2. This is illustrated in Fig.4(a) where χ^2 has been plotted as a function of the parameter c comparing the present data ($N = 56$) to the AV18+UR theoretical cross section (solid line). A minimum has been obtained at the value $\chi^2 = 1.03$ by lowering the normalization of the data by 0.2%. A similar analysis using the data from ref. [23] ($N = 12$) gives a minimum at $\chi^2 = 0.26$ by lowering the data by 1.0%, though a change of 0.6% is enough to obtain a $\chi^2 = 1$. In both cases the change in the absolute normalization is within the limits defined by the systematic errors. Therefore we can conclude that the calculation of the differential cross sections using the AV18+UR potential model gives a χ^2 per datum ≈ 1.0. The analysis of the data from ref. [24] ($N = 20$) gives a value of $\chi^2 \approx 6$.

It is instructive to perform the χ^2 analysis of the present data with the calculation using the AV18 potential even though it underbinds ^{3}He by 0.8 MeV. In this case, with $c = 1$, the result is $\chi^2 \approx 50$. This value can be reduced to $\chi^2 \approx 8$ by changing the normalization by 3%, which is far outside systematic errors of the data and is equal to 28 when restricted to the

limits of the quoted systematic error. Therefore, a second conclusion can be reached that the differential cross section can not be correctly described using one of the new modern NN interactions, in this case the AV18 potential. Following the studies of the Bochum-Cracow group on the sensitivity of the n-d differential cross sections to the different potentials (see ref. [9] page 163), this conclusion should be valid for the other modern NN interactions as well. Thus, we have observed large three-nucleon force effects in the p-d differential cross section at low energies through a detailed χ^2 analysis between theory and data.

The same analysis can be performed at other energies. Here we will limit the analysis to energies below the deuteron breakup threshold in order to avoid the appearance of open channels. High quality measurements exist at $E_{lab} = 2.0$ and 3.0 MeV [23,25]. These data are compared to the cross sections predicted with AV18+UR potential in Fig. 5. In order to establish the quality of the agreement between theory and experiment, the calculations of χ^2 are given in Fig. 4(b,c) for the two sets of measurements at both energies. At 2 MeV, the two data points of ref. [25] at most forward angles disagree distinctly with the other data and are not included in the data base. Analysis of these data gives the value of $\chi^2 < 1$ which is obtained by changing the total normalization less than 1% (solid line) at both energies. For the data set of ref. [23] at $E_{lab} = 2$ MeV the minimum is at $\chi^2 = 2.9$ with a change in the absolute normalization of 0.3%. At $E_{lab} = 3$ MeV the value $\chi^2 = 2.4$ is obtained and it can be reduced to $\chi^2 = 1.8$ when the outlier point at 158° is removed.

In Table I we collect the χ^2–values obtained from the analysis of the cross section data at three energies. For the calculations with AV18+UR potential the χ^2 values obtained with $c = 1$ as well as the minimum χ^2 found by varying the parameter c are given. Remarkably, the present data and the high quality data of ref. [25] at $E_{lab} = 2$ and 3 MeV give a $\chi^2 \leq 1$ allowing less than 1% variation in the absolute normalization of the data. For the sake of comparison, calculations using the AV18 potential are also given in Table I. The χ^2 using the values of c previously optimized for AV18+UR potential as well as the minimum χ^2 obtained after the variation of the parameter c are shown. The minimization procedure improves the χ^2 by a factor of 5 to 10. However the change in the absolute normalization is

about 3%, considerably outside the limit due to systematic errors of the data. Considering the present data and the data of ref. [25] we observe that the χ^2 for the calculations with AV18 potential decreases as the energy increases. This trend could be a manifestation of the previously observed fact that the cross section is overpredicted at low energies and drops below the data for energies above 30 MeV [9].

In conclusion we have presented a new high-quality measurement of the p-d differential cross section at $E_p = 1$ MeV with absolute normalization to p-p elastic scattering. The new measurement allows for a detailed comparison of the data to the calculated cross section using one of the new NN forces, the AV18 interaction, with and without the inclusion of the Urbana 3NF. In addition, the present data help to resolve a significant descrepancy which existed between previous experiments performed at this energy. The use of χ^2 analysis for these comparisons provides a quantitative measure of the ability of various NN and 3N Hamiltonians to reproduce the experimental data. The calculations with the AV18+UR potential are in excellent agreement with the data with a χ^2 per datum ≈ 1. The same degree of agreement is obtained with respect to the data from ref. [25] at $E_p = 2$ and 3 MeV. However, at these two energies the data from ref. [23] show some scatter and the χ^2 does not reach unity.

The calculations using only the AV18 potential give much larger values of χ^2. This inability of AV18 potential to describe adequately the 3N scattering data confirms the evidence of a deficiency of modern NN potentials which need to be supplemented with a 3NF to avoid the underprediction of the binding energy of ^{3}He. In order to provide evidence of 3NF effects beyond those related to the correct description of the binding energy the present analysis of the differential cross section has to be extended to higher energies and to other observables such as the vector and tensor analyzing powers. Studies along these lines have already begun [26,27] and are presently being pursued vigorously.

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TABLES

TABLE I. χ^2 per datum of the AV18 and AV18+UR p-d differential cross section compared to the present data and to the data from ref.[23,25] at three different energies. The number in parenthesis corresponds to the value of the parameter c defined in eq.(1).

	$E_p = 1$ MeV		$E_p = 2$ MeV		$E_p = 3$ MeV	
	present	ref.[23]	ref.[25]	ref.[23]	ref.[25]	ref.[23]
AV18+UR	1.15 (1.)	3.43 (1.)	1.01 (1.)	3.34 (1.)	3.24 (1.)	4.52 (1.)
	1.03 (0.998)	0.26 (0.990)	0.53 (0.995)	2.97 (1.004)	0.89 (1.010)	1.80 (1.010)
AV18	50.2 (0.998)	22.7 (0.990)	16.9 (0.995)	24.5 (1.004)	15.8 (1.010)	13.8 (1.010)
	7.66 (1.030)	3.70 (1.020)	2.09 (1.026)	5.06 (1.030)	1.28 (1.038)	2.92 (1.032)

FIGURES

Figure 1. Typical spectrum of particles resulting from scattering the deuteron beam on thin hydrogenated carbon foil from relative cross section experiment.

Figure 2. Present data (open circles) for the p-d differential cross section are compared to the theoretical curves calculated with the AV18 potential (dotted line) and the AV18+UR potential (solid line).

Figure 3. Present data (open circles) and the data from ref. [23] (open squares) and from ref. [24] (open triangles) divided by the values calculated using AV18+UR potential.

Figure 4. χ^2 per datum as a function of the absolute normalization at $E_p = 1$ MeV (a), 2 MeV (b) and 3 MeV (c) obtained by comparing the cross sections calculated using AV18+UR potential to: (a) present data (solid line) and data from ref. [23] (dotted line), (b) and (c) data from ref. [25] (solid line) and ref. [23] (dotted line).

Figure 5. The p-d differential cross section calculated using AV18+UR potential (solid line) compared to the data from ref. [23] (open squares) and ref. [25] (open circles) at 2 MeV (a) and at 3 MeV (b).