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Modified Dispersion Relations from Closed Strings in Toroidal Cosmology

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Abstract

A long-standing problem of theoretical physics is the exceptionally small value of the cosmological constant $\Lambda \sim 10^{-120}$ measured in natural Planckian units. Here we derive this tiny number from a toroidal string cosmology based on closed strings. In this picture the dark energy arises from the correlation between momentum and winding modes that for short distances has an exponential fall-off with increasing values of the momenta.The freeze-out by the expansion of the background universe for these transplanckian modes may be interpreted as a frozen condensate of the closed-string modes in the three non-compactified spatial dimensions.

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1 Introduction

In this work we will attempt to make a quantitative argument about the origin of dark energy from string theory. The transition from string theory to conventional cosmology is of importance not only to theoretical physics in general but to inflationary cosmology in particular. Corrections to short distance physics due to the nonlocal nature of strings contribute to dark energy. The possibility to detect their signature observationally is very intriguing. In Ref. [1] it was shown that a nonlinear dispersion function modifying the frequency of the transplanckian perturbation modes [2] can produce the right contribution to the dark energy of the universe [3]. The physics mechanism that gave rise to dark energy was the freeze-out of these ultralow frequency modes by the expansion of the background universe. Superstring duality [4] was invoked to justify the dispersion function. This work attempts to carry out this derivation.

In Section 2 we review some preliminaries of the Friedman-Robertson-Walker (FRW) cosmological solutions found for string theory in a D-dimensional torus [5, 6, 7, 8]. The quantum hamiltonian from closed string theory obtained in [9] by using the correspondence principle between string and quantum operators, is reviewed in Section 3. Although the background is an FRW universe, it is globally nontrivial in [9], thus it allows two types of quantum string field configurations, twisted and untwisted fields.

Based on the equivalence between Euclidean path integral and statistical partition functions, we perform in Section 4 the calculation of a coarse-grained effective action [10, 11] for the momentum and winding modes of the system described in Section 2 for the case of 3 expanding spatial dimensions R in the T^D toroidal topology. The string scale is taken as the natural UV lattice cutoff scale of the theory. The renormalization group equations (RGE) of the coupling constants for the winding and momentum modes describe the evolution from early to late times of their entanglement. Based on T-duality the whole spectrum is obtained by exchanging momentum to winding modes and $R \to R^{-1}$. Their coupling is strong when the radius of the torus R is of the same order as the string scale $\sqrt{\alpha'}$, i.e. during the phase transition from a winding dominated universe to a momentum mode dominated universe. Due to the expanding background, we have a non-equilibrium dynamics and calculate the effective action by splitting our modes into the open system degrees of freedom (low energy modes, mainly momentum modes) and the environment degrees of freedom (high energy modes, mainly winding modes). The coarse-graining is performed by integrating out the environmental degrees of freedom. The scale factor $a(t) = R(t)$ serves as the collective coordinate that describes the order parameter for the environment degrees of freedom. The effective action calculated in this way contains the influence of the environment at all times in a systematic way and the coarse graining process encodes the dispersion function and corrections to short distance physics due to the correlation between the two types of modes in the system and environment. This procedure results in the RGEs for the coupling constants that offers information about their running to trivial and nontrivial fixed points at early and late times, therefore the flow of one family of lagrangians (string theory phase) to another family of lagrangians at late times (conventional 3+1 quantum theory). Results of this non-equilibrium phase transition are summarised in Section 5 with a discussion about the possibility of their observational signatures through the equation of state of the frozen short distance modes. In this section we also briefly touch upon the issue of the two field configurations in a globally nontrivial topology and the instabilities in the theory arising from their interaction. A detailed summary of the main coarse-graining formulas and procedure [10] needed in Section 4, are attached in the Appendix. In essence, the dark energy arises from the study of the UV behavior of the correlations with environmental modes.

2 Toroidal String Cosmology.

We consider the string cosmological scenario proposed by Brandenberger and Vafa[1] [5, 7, 12]. Strings propagate in compact space, a box with D spatial dimensions and periodic boundary conditions, the T^D torus. It was argued that [5] a thermodynamic description of the strings with positive specific heat, is well defined only when all the spatial dimensions are compact.

Let us begin with the Universe placed in a T^D box with a size of the order of the string scale, that we are taking to be the Planck scale. In such a space, string states also contain winding modes, which are characteristic of having an extended object like a string, "winding" around the compact spatial dimension, besides the usual momentum modes, and oscillator modes with energy independent of the size of the box. The energy of the winding modes increases with the size of the box as wR, while the energy of the momentum modes decreases as m/R. The spectrum is symmetric under the exchange $R \leftrightarrow 1/R$ and $m \leftrightarrow w$. This symmetry known as T-duality [4] is not only a symmetry of the spectrum but of the theory.

The BV model [5] argues that if the Universe expands adiabatically in more than 3 spatial dimensions, it would not be possible to maintain the winding modes in thermal equilibrium. As their energy density grows with the radius, their number would have to decrease, for example through annihilation processes. But typically strings do not meet in more than 3 spatial dimensions and do not interact with each other; therefore the winding modes fall out of equilibrium [13]. In summary, their growing energy density will tend to slow down the expansion of the universe and eventually stop it. But if the Universe starts to contract, the dual scenario of the momentum modes opposing contraction would take place and the Universe may oscillate between expanding/contracting eras. In what follows we use this argument of [5] to justify the assumption that only $D = 3$ dimensions of the T^D torus will expand to create an FRW universe.

Cosmological solutions for an arbitrary number of anisotropic toroidal spatial dimensions T^D were found by Mueller in [6]. He studied the cosmology of bosonic strings propagating in the background defined by a time-dependent dilaton field, $\Phi(t)$, and space-time metric

$$ds_d^2 = G_{\mu\nu}(X)dX^\mu dX^\nu = -dt^2 + \sum_{i=1}^{D} 4\pi R_i^2(t) dX_i^2 \, , \tag{1}$$

The radii of the torus, $R_i(t)$, become the time-dependent scale-factors, and the spacetime

[1] Herein referred to as the BV model.

dimensions is $d = 1 + D$. The equations of motion of the bosonic string in background fields are obtained from the following action[2] [14]

$$I = \frac{1}{4\pi\alpha'} \int d^2\sigma \sqrt{g} \left[g^{mn} G_{\mu\nu}(X) \partial_m X^\mu \partial_n X^\nu + \frac{1}{2}\alpha' \Phi R^{(2)} \right] , \tag{2}$$

where g_{mn} is the two-dimensional world-sheet metric, and $R^{(2)}$ the world-sheet scalar curvature. The background field equations are obtained by imposing the condition that the theory be free from Weyl anomalies. To lowest order in perturbation theory this leads to the equations:

$$\beta^G_{\mu\nu} = R_{\mu\nu} + \nabla_\mu \nabla_\nu \Phi = 0 \,, \tag{3}$$

$$\beta^\Phi = \frac{d-26}{3\alpha'} - R + (\nabla\Phi)^2 - 2\nabla^2\Phi = 0 \,, \tag{4}$$

Using the metric given in Eq. (1), they reduce to:

$$\ddot{\Phi} - \sum i \frac{\ddot{R}_i}{R_i} = 0 \,, \tag{5}$$

$$\frac{\ddot{R}_i}{R_i} + \sum_{j \neq i} \frac{\dot{R}_i \dot{R}_j}{R_i R_j} - \dot{\Phi} \frac{\dot{R}_i}{R_i} = 0 \,, \tag{6}$$

$$\ddot{\Phi} - \frac{1}{2}\dot{\Phi}^2 + \sum_{i<j} \frac{\dot{R}_i \dot{R}_j}{R_i R_j} = \frac{d-26}{3\alpha'} \,. \tag{7}$$

When $D = 25$, the solutions obtained in [6] are:

$$e^{-\Phi(t)} \propto t^p \,, \tag{8}$$

$$R_i(t) \propto t^{p_i} \,, \tag{9}$$

with the constraints,

$$\sum_{i=1}^{D} p_i^2 = 1 \,, \qquad \sum_{i=1}^{D} p_i = 1 - p \,. \tag{10}$$

Note that these solutions are found in the absence of matter sources. In general the backreaction of the matter action of the strings in T^D alters the solutions for the background geometry[3]. It is clear that we can have an arbitrary number of compact spatial dimensions D_c with $p_i < 0$, that are decreasing with time [4], and $D - D_c$ expanding spatial dimensions with $p_i > 0$. Among the many solutions found in [6] we select the solution $D - D_C = 3$ that although it is not unique it is justified by the BV argument. The assumption that our Universe is expanding in only 3 spatial dimensions, with the remaining $D - 3$ being small

[2] The antisymmetric tensor field is taken to be zero.

[3] See [7, 8] and references therein for the geometry solutions in the presence of a matter action. Inclusion of matter sources alters the solutions of [6] due to the backreaction of the winding modes, such that the scale factor approaches asymptotically a constant value at late times .

[4] We do not address the concern that the time dependence of the compactified R_i endangers the constancy of the dimensionless parameters in the $D = 3$ theory.

and compact, as well as considering a constant dilaton field[5] ($p = 0$), are consistent with Mueller's solutions Eqs. (10). The issue of stabilising the dilaton is beyond the scope of this paper, and we assume that the dilaton has acquired a mass and become stable at some fixed value. It is also assumed that the backreaction of the matter string sources on the backround geometry is small enough such that the deviations from the FRW metric, Eq. (10), can be neglected.

Due to the toroidal string cosmology, the three expanding dimensions contain both types of modes: *momentum* and *winding*, propagating in the 3+1 FRW space-time. The number of winding modes at each stage of the evolution of the Universe is determined by the dynamics of the background. In the next section, we touch base with quantum field theory through correspondence principle between string and quantum operators, in order to use coarse graining techniques for studying the influence of the winding modes on the momentum modes as the Universe expands.

3 Quantum Hamiltonian from Closed String Theory.

Let us consider BV model [5] of a D-dimensional anisotropic torus with radius $\bar{R}_i$, by including the dynamics of both modes: momentum modes, $p_{1,i} = m/\bar{R}_i$ (where m is the wavenumber), and winding modes with momenta $p_{2,i} = w\bar{R}_i/\alpha'$. The dimensionless quantity for the radius is $R_i = \bar{R}_i/\sqrt{\alpha'}$, where α' is the string scale. Based on the arguments reviewed in Section 2, we *choose* a cosmology with three toroidal radii equal and large $R \gg 1$ in units of the string or Planckian scale, with the other $(D-3)$ toroidal radii equal and small $R_C \ll 1$. Here the subscript C refers to compactified dimensions. Then, $R(t)$ becomes the scale factor for the 3+1 metric in conventional FRW (Friedman Robertson Walker) cosmology $R(t) = a(t)$, while R_C corresponds to the radius, in this factorizable metric, of the $D-3$ compact dimensions z_j that decrease with time,

$$ds_D^2 = -dt^2 + 4\pi R^2(t)dx_i^2 + 4\pi R_C^2(t)dz_j^2 = a(\eta)^2[-d\eta^2 + dy^2] + ds_{D-3}^2 \,. \tag{11}$$

Using the string toroidal solution of [6] the time-dependence of these radii is:

$$R(t) = \alpha_U t^{p_U} \tag{12}$$

$$R_C(t) = \alpha_C t^{p_C} \tag{13}$$

The solutions in Ref. [6] show that p_U and p_C depend on the dimensionality D in an interesting way. There is a plethora of possible solutions but if we assume, for example, that the dilaton is time-independent and the compactification is isotropic we find that for $4 \leq D < \infty$, then $0.5 \leq p_U < 1/\sqrt{3} \simeq 0.577$. Let us take $D = 4$ where the scale factor behaves as a radiation-dominated universe; if, in fact, $D \geq 5$ we can assume that the $D-4$ additional dimensions have $p_C^{'} \ll p_C$ to achieve the same result. In this case, $p_C = -0.5$. Here we do not, however, need to specialise to a particular solution.

[5] The authors of [7] argued that a constant dilaton background may not be consistent with a *high* temperature phase of strings thermodynamics.

What we have in mind for the dark energy is the correlation of momentum to *winding* string modes. The question is, given the well-known form for the kinetic energy of these strings, *e.g.* [15], how to describe best the interaction between the winding and momentum modes. Some aspects are addressed in [15] who focuses on the smallness of temperature (T/T_H). For temperature T very much below the Hagedorn or string temperature T_H we expect that only very small winding numbers $w_i = 0$ or 1 in the compact dimensions are of any significance [15]. Similar arguments apply to the momentum modes m_i for the time-reversed case.

Let us consider the small parameter $\delta(t)$, taken to be:

$$\delta = \frac{R_C}{R} \sim t^{p_C - p_U} \tag{14}$$

For the case $D = 3$ (d=4), for example $\delta \sim t^{-1} \sim (T/T_H)^2$ and is an extremely small number ($\sim 10^{-60}$) at present. The point is that in the $\delta \to 0$ limit these modes are in separate spaces and for very small δ are therefore expected to be highly restricted. The compactified dimensions can be integrated out, and we are left with the momentum and winding modes in the remaining $D = 3$ spatial dimensions.

The partition function for this system was calculated, from first principles, by summing up over their momenta in [15]:

$$Z = \sum_\sigma e^{-n_\sigma \epsilon_\sigma}\,, \tag{15}$$

where n_σ is the number of strings in state σ with energy ϵ_σ

$$\epsilon_\sigma = p_0 = \sqrt{\left(\frac{m}{R}\right)^2 + (wR)^2 + N + \tilde{N} - 2}\,, \tag{16}$$

and σ counts over $(m,\, w)$, with the constraint $N - \tilde{N} = mw$ for closed strings where N and $\tilde{N}$ are the sums over the left- and right- mover string excitations, respectively. By now, in Eq.(16), we are considering only the large 3 spatial dimensions. The string state can also be described by its left and right momenta, $k_L = p_1 + p_2$, $k_R = p_1 - p_2$. The string state for left and right modes can be expanded in terms of the creation and annihilation operators α_m, $\tilde{\alpha}_n$, with higher excitation string states given by $N = \sum_{n=1}^{\infty} \alpha_{-n}\alpha_n$ (similarly for $\tilde{N}$), and string energy $L_0 + \tilde{L}_0 = p_1^2 + p_2^2 + (N + \tilde{N} - 2)/\alpha'$.

We would like to write the path integral for this configuration in terms of quantum fields[6]. The path integral is calculated from the hamiltonian density. In order to use the correspondence between the Euclidean path integral of the persistence vacuum amplitude $|\langle in|out\rangle|^2$ and the partition function Z, we need to write a hamiltonian density over the fields in configuration space in such a way that its Fourier transform in k-space corresponds to the string energy expression Eq. (16).

Thus in writing a Coarse-Grained Effective Action (CGEA), the kinetic terms are unambiguous while for the interaction terms we must appeal to simplicity and the requirement of T-duality. Closed-string field theory provides guidance, since in *e.g.* [16] truncation at a

[6] Below we use quantum string equations under the assumption that the dilaton is massive and stable.

quartic coupling can be sensible, and this will lead to a CGEA which is renormalisable and satisfies useful RG equations.

Generally, closed string field theory contains couplings of all non-polynomial orders. In a semi-classical approximation we may restrict to genus $g = 0$ since the genus g contribution is proportional to $\hbar^g$ [17].

The quantum hamiltonian is in any case known for the classical string in axi-symmetric or toroidal backgrounds [9]. They explicitly calculated the quantum hamiltonian and demonstrated the correspondence principle between the string operators L_0, $\widetilde{L}_0$ and quantum field operators in the form (in the notation of [9])

$$\begin{aligned} \hat{H} &= \hat{L}_0 + \hat{\widetilde{L}}_0 = \frac{1}{2}\alpha'\left(-E^2 + p_a^2 + \frac{1}{2}(Q_+^2 + Q_-^2)\right) + N + \widetilde{N} - 2c_0 \\ & -\alpha'\left[(q+\beta)Q_+ + \beta E\right] - \alpha'\left[(q-\alpha)Q_- + \alpha E\right] J_L \\ & \frac{1}{2}\alpha' q\left[(q+2\beta)J_R^2 + (q-2\alpha)J_L^2 + 2(q+\beta-\alpha)J_R J_L\right] \end{aligned} \tag{17}$$

$$\hat{L}_0 - \hat{\widetilde{L}}_0 = N - \widetilde{N} - mw \tag{18}$$

where $J_{R,L}$ are bilinear quadratic operators in terms of creation and annihilation operators and the higher string oscillators $N, \tilde{N}$ contribute the string mass. Therefore the J_R^2 term would be a quartic interaction in terms of creation and annihilation operators.

This particular solution is for a cylindrical topology (Melvin model) where the uncompactified x_1 and x_2 are written in polar coordinates $x_1 + ix_2 = \rho e^{i\phi}$ and x_3 is also uncompactified (but could be compactified along with additional similar coordinates), together with time and one additional compactified dimension $y \subset (0, 2\pi R)$. Although an exact solution for the hamiltonian of the string matter in a toroidal background is not yet known, a quartic potential energy was advocated and found in [8] by arguments similar to those of Eq. (17), for the classical string and the three string coupling level. We take this as an indication, in the subsequent section (if the exact solution were known to all orders), that an quantum hamiltonian analogous to Eq.(17) for closed strings on a torus, similarly containing only quartic terms as suggested by [9], exists for our present case of $(T_3) \times (T_{D-3}) \times (time)$ and focus on the uncompactified 3 spatial dimensions.

The hamiltonian depicted in Eq.(17) is for a static background,i.e a constant scale factor $R(t)$. In the next section, we base our calculation in the coarse-grained effective action (CGEA) formalism where the dynamics of an expanding background is replaced by scaling on a static background.

Thus Eq.(17) which applies to a static background (as in Eq.(16)) can be generalized to a cosmologically-expanding background as in Eq.(11) by using this technique of re-scaling, as we shall discuss in the next Section. This strategy is necessitated by the absence of an exact string solution in the time-dependent background.

4 Coarse Grained Effective Action (CGEA) and RGE's

4.1 General case of the d=D+1 Universe

Our system of winding and momentum modes is described by nonequilibrium dynamics due to the expanding background spacetime. All the information about the evolution of these modes will be contained in the effective action. Therefore we need to write the path integral in the configuration space of the quantum fields in order to obtain the effective action. This information must be extracted from the torus analogs of the quantum mechanics hamiltonian of Eq.(17) such that its Fourier transform in momentum k-space recovers the string energy spectrum Eq. (16). Correlation functions are obtained by using the correspondence between the Euclidean path integral of the persistence vacuum amplitude $|\langle in|out\rangle|^2$ and the partition function Z. All the string quantum operators below are promoted to quantum field operators with the corresponding hamiltonian density $\mathcal{H}(t,x)$ in configuration space derived from the quantum string hamiltonian $\mathcal{H}(t)$.

The following calculations are done in the conformally flat background Eq. (11)[7], through the scaling of the fields and operators with the conformal factor $a(\eta)$. The momentum field $\phi_1(R,x)$ and the winding field $\phi_2(R,x)$ are defined by the relation:

$$\phi_i(x) = \int e^{ip_i x}\phi_i(p_i)d^3p_i\,, \quad \int |\nabla\phi_i|^2 d^3x = \int d^3p_i p_i^2\phi_i(p_i)\,, \tag{19}$$

where

$$\nabla = R\partial/\partial x \ = \partial/\partial y\,, \tag{20}$$

and $p_i = p_1,\, p_2$. Let us also define two new fields, $\psi_L(R,x)$ and $\psi_R(R,x)$, with momenta $k_L,\, k_R$ that are the left and right combinations of the Kaluza Klein momentum and winding modes

$$\psi_L(R,x) = \phi_1(R,x) + \phi_2(R,x)\,, \tag{21}$$
$$\psi_R(R,x) = \phi_1(R,x) - \phi_2(R,x)\,. \tag{22}$$

These fields live in the expanding (3+1) spacetime dimensions. Similarly there is another set of fields $\Psi_{c,a}$ that are functions of the compact dimensions z_a. Their energy contribution to the total hamiltonian density is $\mathcal{H}_C(p_a) = A_a p_a^2$ where A_a is a constant parameter with dimensions of inverse volume of the compact space, and p_a are the momenta of these fields in the extra compact dimensions, with a running over the $D-3$ dimensions.

The Hamiltonian density ansatz that would describe the energy of our two string states in the $D=3$ expanding dimensions with energy $H = L_0 + \widetilde{L}_0$, including the oscillators from string's higher excitations $(N + \widetilde{N} - 2)/\alpha'$, is similar to the hamiltonian of spin waves in a periodic lattice[8]. The Ginzburg-Landau hamiltonian for a Heisenberg magnet obtained in [9] by means of CFT bears similarity with $\lambda\phi^4$ quantum field theory in a well-known manner.Our lattice spacing is given by the string scale $\sqrt{\alpha'}$. Therefore the hamiltonian density can be written for this dual lattice in terms of wave functional "spin" fields $\psi_L(R,x), \psi_R(R,x)$ of Eqs. (21), (22) as follows

$$\mathcal{H} = \mathcal{H}_3 + \mathcal{H}_C\,, \tag{23}$$

[7] We do not address issues of the matter backreaction on the geometry. They are treated in [7, 8].

[8] Torus is obtained by identifying the first and the last lattice sites, thus the periodicity.

with

$$\mathcal{H}_3 = |\nabla\psi_L|^2 + |\nabla\psi_R|^2| + |\nabla\psi_L||\nabla\psi_R| + m_0^2(|\psi_L|^2 + |\psi_R|^2) + g_1(|\psi_L|^4 + |\psi_R|^4) + g_2|\psi_L|^2|\psi_R|^2\,, \tag{24}$$

where the fields ψ_L, ψ_R are expanded in terms of the mode functions $u_n, \tilde{u}_n$,

$$\psi_L = \Sigma u_n b_n + u_n^* b_n^+\,, \quad \psi_R = \Sigma \tilde{u}_n \tilde{b}_n + \tilde{u}_n^* \tilde{b}_n^+\,, \tag{25}$$

and $b_n, \tilde{b}_n$ are the normalised quantum creation and annihilation operators of $\alpha_n, \tilde{\alpha}_n$. The commutation relation for the unnormalised operators are such that $[\alpha_n, \alpha_m^+] = \omega_\pm \delta_{nm}$ with $\omega_\pm$ the frequency of left and right moving modes.

The periodic lattice condition $N - \widetilde{N} = mw$ introduces an interaction term in the hamiltonian $\mathcal{H}_3$ of the form $\nabla\psi_L\nabla\psi_R$. In terms of the 2-component state $\Psi_{EN} = (\psi_L, \psi_R)$, the hamiltonian reads,

$$\mathcal{H}_3 = |\nabla\Psi_{EN}|^2 + \nabla\Psi_{EN}\widehat{X}\nabla\Psi_{EN} + m_0^2|\Psi_{EN}|^2 + g_1|\Psi_{EN}|^4 + (g_2 - 2g_1)|\Psi_{EN}\widehat{X}\Psi_{EN}|^2\,, \tag{26}$$

with

$$\widehat{X} = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix} \tag{27}$$

The system is known as the dual momentum-space lattice, and for $g_2 = 2g_1$ reduces to the XYZ model of condensed matter. Let us for simplicity limit to the XYZ model case, $g_2 = 2g_1$, for the rest of this paper.

These periodic lattice systems studied in 3+1 dimensions in terms of Bloch wavefunctions have a solution which respect to lattice translation invariance, $exp(-pl)$, with the lattice spacing "l" equal to the string scale $\sqrt{\alpha'}$. The interaction term, in the tight-binding approximation, lifts the degeneracy between the energy eigenstates due to the leakage/tunnelling of the wavefunction from one lattice site to the neighbour site. As a result the gap energy produced between the ground (bound) state and higher excitation states is

$$p^2\Delta_p = p^2|\cos(2\theta)| = p^2\ |2cos^2(\theta) - 1|\,, \tag{28}$$

in which

$$pl = p\sqrt{\alpha'} = \sqrt{\alpha'(p_1^2 + p_2^2)} = \sqrt{(\frac{m}{R})^2 + (wR)^2}\,, \tag{29}$$

and $\theta \to \theta + ipl$. Therefore,

$$\Delta_p \leq 2\cosh^2(pl) - 1\,. \tag{30}$$

The first term in θ is a pure phase of rotation of the "spin-wave" in the dual lattice, but the second term describes the tunnelling of the wavefunction to the nearest neighbour[9]. The gap energy of Eq. (28) introduces a correction to the kinetic energy, such that in momentum space the hamiltonian reads

$$\mathcal{H}_3 = z_p p^2|\Psi_{EN}|^2 + m_0^2|\Psi_{EN}|^2 + g_1|\Psi_{EN}|^4\,, \tag{31}$$

[9] In condensed matter this is known as Coulomb dipole type of vortex interaction.

with $z_p = 1+\Delta_p$. This correction contributes to the wavefunction renormalization constant of the field Ψ_{EN}. We can then make a partial (finite) renormalization of the hamiltonian in order to recover the canonically normalised kinetic term, such that

$$\Psi_{EN} \;\rightarrow\; \widetilde{\Psi}_{EN} = z_p^{1/2}\Psi_{EN}\,, \tag{32}$$

$$m_0^2 \;\rightarrow\; \tilde{m}_0^2 = z_p^{-1} m_0^2 = \frac{m_0^2}{1+\Delta_p}\,, \tag{33}$$

$$g_1 \;\rightarrow\; \tilde{g}_1 = z_p^{-2} g_1\,. \tag{34}$$

The hamiltonian density $\mathcal{H}_3$ finally reads,

$$\mathcal{H}_3 = |\nabla\widetilde{\Psi}_{EN}|^2 + \tilde{m}_0^2|\widetilde{\Psi}_{EN}|^2 + \tilde{g}_1|\widetilde{\Psi}_{EN}|^4\,. \tag{35}$$

The action in D-dimension therefore is

$$S_D = \int dt\; R_C d^{(D-3)} z_a\; a(t) d^3x \left(\mathcal{H}_3[\widetilde{\Psi}_{EN}(x)] + \mathcal{H}_C[\Psi_{c,a}(z)]\right) \tag{36}$$

$$= V_C \int a(t)dt\; d^3x \mathcal{H}_3[\widetilde{\Psi}_{EN}(x)] + V_U \int R_C dt\; d^{D-3}z \mathcal{H}_C[\Psi_{c,a}(z)]\,, \tag{37}$$

where V_C (V_U) is the volume factor obtained from integrating out the contribution from the compact $(D-3)$ (uncompact) dimensions. The partition function Z is then,

$$Z = Z_c Z_3\,, \tag{38}$$

$$Z_c = \int D\Psi_{c,a} e^{-V_U \int R_C dt\; d^{D-3}z \mathcal{H}_C[\Psi_{c,a}]}\,, \tag{39}$$

$$Z_3 = \int D\widetilde{\Psi}_{EN} e^{-V_C \int a(t)dt\; d^3x \mathcal{H}_3[\widetilde{\Psi}_{EN}]}\,. \tag{40}$$

The contribution Z_c to the path integral is easy to calculate since the integral over the compact dimension fields is a simple gaussian,

$$Z_c = \int D\Psi_{c,a} e^{-V_U \int d^{D-3}p_a A_a \Psi_{c,a} p_a^2 \Psi_{c,a}} = \prod_a \sqrt{\frac{\pi}{A_a V_U}}\,. \tag{41}$$

The contribution of these fields to the path integral is proportional to the volume of the compact space R_C^a they live in, thus their contribution is relevant only around the string scale because at late times the volume of the compact metric decreases rapidly with time. In either case their contribution rescales the normalisation constant of the path integral, which we will allow for the moment to be arbitrary[10], such that

$$Z = N' \int D\widetilde{\Psi}_{EN} e^{-V_C \int a(t)dt\; d^3x \mathcal{H}_3[\widetilde{\Psi}_{EN}]} \tag{42}$$

$$= N' Z_3\,. \tag{43}$$

The volume of the compact dimensions, V_C, is roughly of order unity in terms of string units, and it can be reabsorbed into the parameters of $\mathcal{H}_3$.

[10]Varying the action with respect to the metric care should be taken to account for the effect of the compact metric volume on the Newtons constant of the reduced (3+1) metric.

We would like to find a simplified description for the dynamics of our non-equilibrium system, consisting of both winding and momentum modes, while incorporating the back-reaction of the short wavelength modes to it, in the reduced $(3+1)$ dimensions. This is done by carrying out the necessary steps of coarse graining which are the following [10]: 1) distinguish the system from the environment, 2) coarse grain the environment, 3) measure how the coarse grained environment influences the system in providing an effective dynamics for our reduced system. As we will see below the environment, consisting of the short wavelength modes, has a time dependent order parameter due to the expanding background universe, thus the need for using non-equilibrium dynamics methods[11].

4.2 The D=3 Universe

At this point, in evaluating the reduced 3 dimensional partition function Z, we want to separate our modes into system (S) + environment (E) degrees of freedom, and coarse grain by integrating out the degrees of freedom for the environment. This amounts to finding out the backreaction of the coarse grained environment on the system, and eventually leads to the RGE's[12]. We use many of the results and the approach of [10] in what follows. We will consider as environment all the short wavelength modes with momenta

$$(E): \qquad \frac{\Lambda}{b} < p^E = \frac{1}{\sqrt{\alpha'}}[(m/R)^2 + (wR)^2]^{1/2} < \Lambda\,, \tag{44}$$

where the cutoff $\Lambda = (\alpha')^{-1/2}$ is the string scale because $(\alpha')^{1/2}$ is identified with the lattice spacing l, and $b = a(t)/a(t_0)$ is the coarse grain scaling parameter, where t_0 is the initial time. The scale factor $a(t)$ plays the role of the collective coordinate describing the environmental degrees of freedom. Time in this procedure is playing the role of a scaling parameter and dynamics is being replaced by scaling [10]. This is an artificial procedure (known as Kadanoff-Migdal transform [18]) that relates the microscopic and macroscopic properties of a system based on the existence of scaling properties of the system in the infrared limit. Thus $a(t)$ is treated simply as a parameter while carrying out the coarse graining in this "static limit", 3-dimensional Minkowski field theory of the foliation $a(t)$=constant hypersurface at each $t_n = t_0 + n\Delta t$.

The system modes are the ones with:

$$(S): \qquad p^S < \frac{\Lambda}{b}\,, \tag{45}$$

¿From the above definitions of system and environment, Eqs. (45) and (44), at initial times when $b \approx 1$ we have $p \leq \Lambda/b$ thus all our modes, momentum and winding, are in the system; but at later times when $b \geq 1$ more and more winding modes systematically transfer to

[11]A whole program with a detailed treatment of the conceptual and formal techniques of coarse-graining has been pioneered and developed by Hu and collaborators in [10]. They showed that for a special class of expansion, the dynamics of spacetime can be equivalently replaced by a scaling transformation with time playing the role of the scaling parameter.

[12]The running of the coupling constants with time depends on how one selects the environment and the system.

the environment because the condition of Eq. (45), $p = \frac{1}{\sqrt{\alpha'}}[(m/R)^2 + (wR)^2]^{1/2} \leq \Lambda/b$ is satisfied only for vanishingly small winding numbers $w \to 0$. As t becomes large, the system contains $m \leq R\Lambda$, $w = 0$, i.e. all the modes except $m \leq R\Lambda, w = 0$ have transfered to the environment. The Euclidean path-integral of this 2 field system $\widetilde{\Psi}_{EN}$ with hamiltonian density given in Eq. (35), is[13]

$$Z = |\langle R < 1|R > 1\rangle|^2 = \int \prod_{\Lambda/b \leq p \leq \Lambda} D\tilde{\Psi}_E \prod_{0 \leq p \leq \Lambda/b} D\tilde{\Psi}_S \; e^{-S[\widetilde{\Psi}_{EN}]} \,, \tag{46}$$

where the field is split into high and low energy as follows: $\widetilde{\Psi}_{EN} = \tilde{\Psi}_S + \tilde{\Psi}_E$, e.g. $\tilde{\Psi}_E$ denotes the modes with 'environment' momenta p^E given by Eq. (44). After this splitting of the modes into (System+ Environment), we can separate the terms in the action $S[\widetilde{\Psi}_{EN}]$ into:

$$S[\widetilde{\Psi}_{EN}] = S_S[\tilde{\Psi}_S] + S_0[\tilde{\Psi}_E] + S_I[\tilde{\Psi}_E, \tilde{\Psi}_S] \,, \tag{47}$$

where S_S, S_0 are the action depending on system, environment variables and S_I is the piece that depends on the interaction of system variables to the environment variables.

$$S_S[\tilde{\Psi}_S] = \int a(t)dt \int d^3x (\tilde{\Psi}_S G_S^{-1} \tilde{\Psi}_S + g_1 \tilde{\Psi}_S^4) \,, \tag{48}$$

$$S_0[\tilde{\Psi}_E] = \int a(t)dt \int d^3x \tilde{\Psi}_E G_E^{-1} \tilde{\Psi}_E \,, \tag{49}$$

$$S_I[\tilde{\Psi}_E, \tilde{\Psi}_S] = \int a(t)dt \int d^3x g_1 \left[4\tilde{\Psi}_S^3 \tilde{\Psi}_E + 6\tilde{\Psi}_S^2 \tilde{\Psi}_E^2 + 4\tilde{\Psi}_S \tilde{\Psi}_E^3 + \tilde{\Psi}_E^4\right] . \tag{50}$$

$G_{S,E}$ are Green's functions for open system (S) and environment (E) given by:

$$G_S^{<\Lambda/b}[p^S < \Lambda/b] = [(p^S)^2 + \tilde{m}_0^2]^{-1} \,, \tag{51}$$

$$G_E^{\Lambda/b}[p^E \leq \Lambda/b] = [(p^E)^2 + \tilde{m}_0^2]^{-1} \,, \tag{52}$$

The Green function for the whole (closed system, S+E) system $G[p]$ satisfies:

$$G[p] = G_S^{<\Lambda/b}[p^S] + G_E^{\Lambda/b}[p^E] \,. \tag{53}$$

After integrating out the high energy p^E modes Eq. (44) in the action, we are left with an effective action that depends only on the system variables $p^S < \Lambda/b$, such that:

$$S_{eff}[\tilde{\Psi}_S] = S_S[\tilde{\Psi}_S] + \Delta S[\tilde{\Psi}_S] \,. \tag{54}$$

$S_S[\tilde{\Psi}_S]$ is the portion of the action that all along depends only on the system variables. The term ΔS results from the interaction of the system with the environment, but it depends only on the system variables after the coarse-graining. It gives rise to corrections $\delta\tilde{m}_0^2$ and $\delta\tilde{g}_1$ to the mass and coupling parameters in the action (see [10], Appendix for details). Therefore, the effective action S_{eff} will have the same form as $S_S[\tilde{\Psi}_S]$ with parameters $\tilde{m}^2$ and $\tilde{g}$ defined as

$$\tilde{m}^2 = \tilde{m}_0^2 + \delta\tilde{m}_0^2 \tag{55}$$

$$\tilde{g} = \tilde{g}_1 + \delta\tilde{g}_1 \,. \tag{56}$$

[13] From now on, we drop the subindex "3" from the the action.

We assumed that $\cosh^2(p\sqrt{\alpha'})$ in the expression of Δ_p that enters in the mass term $\tilde{m}_0^2$ Eq. (33), is a slowly varying function of momenta p and consider it to be a constant while carrying out the procedure of coarse graining[14].

Let us rescale our variables in the effective action Eq. (54), such that

$$p' = bp, \quad \tilde{\Psi}'(p') = b^{-(D+2)/2}\tilde{\Psi}_{EN}(p'/b)\,. \tag{57}$$

Clearly, the original cutoff Λ and range of momenta are restored after rescaling. Dynamics has been replaced by scaling of parameters in a *static spacetime* [10]. This procedure can be repeated n times, for very small time increments $\Delta t = (t_f - t_0)/n$ between the initial and final times,

$$S_{eff}(\tilde{\Psi}') = b^{-D}\int d^D p'\tilde{\Psi}(p'/b)\left[\left(\frac{p'}{b}\right)^2 + \tilde{m}^2 + \tilde{g}\langle\tilde{\Psi}^2\rangle\right]\tilde{\Psi}(p'/b) \tag{58}$$

$$= \int d^D p'\tilde{\Psi}'(p')\left[(p')^2 + b^2\tilde{m}^2 + b^{4-D}\tilde{g}\langle\tilde{\Psi}^2\rangle\right]\tilde{\Psi}'(p')\,. \tag{59}$$

$S_{eff}[\tilde{\Psi}']$ will have the same form as the original one in Eq. (54) provided that we identify the mass term and coupling constant

$$\tilde{m}'^2 = b^2\tilde{m}^2\,, \quad \tilde{g}' = b^{4-D}\tilde{g}\,. \tag{60}$$

Although we are formally keeping the dimensionality to be an arbitrary D in discussing the RGEs below, in fact our reduced system has $D = 3$ and we take that limit at the end. Repeating this procedure n times (with $n \to \infty$), results in the RGEs for the coupling constants.

The canonical two-point correlation function at high energy for system-environment interaction is calculated from the path integral of the canonical fields $\tilde{\Psi}_{S,E}$ in momentum space (Fourier transform of $G^{\Lambda/b}$). It is related to the correlation function of the original fields Ψ_E (which decreases at high energy) as follows

$$\langle\Psi_E\Psi_E\rangle = \frac{\langle\tilde{\Psi}_E\tilde{\Psi}_E\rangle}{z_p}\,. \tag{61}$$

where $z_p = 1 + \Delta_p$ and Δ_p is given in Eq.(30). This is the crucial result for the interpretation of the cosmological dark energy. Because of the mass gap, the correlation function is suppressed exponentially in p-space. It is very familiar that a mass gap leads to an exponential fall off in x-space, but here for the dual lattice the exponential fall off is in momentum space. This may be traced to the T-duality of the closed strings and the resultant interchange of the IR/UV limits. As we will show, for the rescaled fields $\tilde{\Psi}_E$ the correlation function increases at high energy leading to an exponential decrease in the dispersion $\omega(p)$.

The two-point correlation function at low energies, $p^S \leq \Lambda/b$ is related to the canonical one in the same way. The canonical two-point function is the Fourier transform of $G^{<\Lambda/b}$,

[14] We are keeping only first order corrections to the mass term due to the Δ_p. Contributions from higher order terms to the mass correction $\delta\tilde{m}_0^2$, like Δ'_p, Δ''_p,...have been ignored.

and at low momenta it goes like a polynomial:

$$\langle \tilde{\Psi}_S \tilde{\Psi}_S \rangle = \frac{1}{(p^S)^2 + \tilde{m}'^2} \,. \tag{62}$$

The reason for this different behaviour of the correlation function is because before splitting Ψ into high + low energy modes, the total canonical two-point correlation function is:

$$\langle \tilde{\Psi}' \tilde{\Psi}' \rangle = G[p'] = \frac{1}{p'^2 + \tilde{m}'^2} \,, \tag{63}$$

When writing it in terms of the two point function of the original fields (by dividing with the z_p normalisations factor), it goes as an exponential for large momentum p, and as a polynomial for low momenta.

The correlation length is given by

$$\xi \simeq (\tilde{m}')^{-1} \,. \tag{64}$$

Clearly, ξ diverges at very high momenta (early times) that indicate that the correlation length is very large and signals a phase transition. Alternatively, the correlation length goes to zero at low momenta (late times) indicating that the theory becomes local at late times.

Let us denote $\tau = \ln b$, and $\epsilon = 4 - D$. The RGEs for this system at hand are known from the analogy of our partition function,

$$Z = N \prod_{p' < \Lambda} \int D\tilde{\Psi}' e^{-S_{eff}[\tilde{\Psi}']} \,, \tag{65}$$

to the dual lattice Ising model,

$$\frac{dg_1}{d\tau} = \epsilon g_1 - A(36 g_1^2 + g_2^2) \,, \tag{66}$$

$$\frac{dg_2}{d\tau} = \epsilon g_2 - A(24 g_1 g_2 + 8 g_2^2) \,, \tag{67}$$

$$\frac{dx}{d\tau} = 2x + 12(1 - x)(1 + \frac{g_2}{6 g_1}) \frac{g_1}{\Lambda^\epsilon} \,, \tag{68}$$

with $x = \tilde{m}'^2 \Lambda^{-2}$, and A a numerical constant.

For the case we considered, $g_2 = 2 g_1$ ($g_1 \to \tilde{g}$) thus Eq. (66) reduces and becomes identical to Eq. (67). The solution to the RGEs will tell us the running of the couplings constant $\tilde{m}'^2$ and $\tilde{g}'$ to their nontrivial fixed points with time, $\tilde{m}'^2 = f_m(b)$, $\tilde{g}' = f_g(b)$. These relation $f_i(b)$ are replaced in the expression for the correlation function and length, Eqs. (63) and (64). Note that $G^{-1}[p]$ is the dispersed frequency with short distance modifications contained in the $\tilde{m}_0^2$ term:

$$\tilde{m}_0{}^2 \overset{p \to \infty}{\rightarrow} \frac{m_0^2}{2 \cosh^2 p\sqrt{\alpha'}} \simeq \frac{1}{2} m_0^2 e^{-2\sqrt{\alpha'} p} \,. \tag{69}$$

where $b_f = a(t_f)/a(t_0)$ and t_f is the final time that can be taken to be future infinity. It is clear from the expression for the modified canonical mass of Eq.(69), which originated from the interaction between the system and the environment at short distances, that the correlation length of Eq.(64) diverges exponentially at high energies and the correlation function between the original fields falls off exponentially, Eq.(61). From the RGEs, the correlation length ξ and the quartic coupling constant g_1 vanish with time, $\xi \rightarrow_{t\rightarrow\infty} 0$, and the system is dominated only by $free$ momentum modes (m/R) $(w \rightarrow 0)$. The RGEs in combination with the CGEA thus describe the dynamics evolution of our entangled system of winding and momentum modes at early times to a free gas of momentum modes at late times, due to the backreaction of the high energy environmental (mainly winding) modes. Also notice that by using the Tolman relation for the temperature in an expanding background, $T = T_c/a(t)$, we can express the RGEs and the partition function Z as functions of temperature rather than the scale factor $a(t)$ (or equivalently b), i.e.:

$$Z = Z[\frac{T - T_c}{T_c}] \, , \tag{70}$$

where T_c corresponds to $a(t_0)$, i.e. $b = 1$. The correlation length ξ diverges around $b = 1, T = T_c$, $\xi \overset{t\rightarrow 0}{\rightarrow} \infty$. Thus our system breaks down due to strong correlations at early times or high temperatures, but this simply signals the Hagedorn phase-transition at around the critical temperature T_c.

5 Dark Energy from Closed String Theory. Discussion

We argue that closed strings on a toroidal cosmology lead to a plausible explanation of the dark energy phenomenon. Although bosonic strings have been used, it is expected that superstrings will lead to a similar conclusion. Certainly it is crucial that closed strings are involved because open strings do not have the same aspect of winding around the torus.

The scale factor of the universe $a(\eta)$ has been used as a collective coordinate for the environment degrees of freedom, and as the fundamental scaling parameter in the coarse-graining. The choice of a $D = 3$ expanding cosmology was chosen phenomenologically. An argument for this choice in the BV model was presented in [5], and we believe this argument does provide a possible justification. It is encouraging that inclusion of branes gives a similar result [12]. It has further been assumed that the mass gap Δ_p can be safely assumed to be slowly-varying during our coarse-graining procedure.

We would like to make the reader aware of another subtlety related to the torus topology of our background. A globally nontrivial topology like $T^3 \times R^1$ admits two types of quantum field configurations, twisted and untwisted fields, due to the periodic and anti-periodic boundary conditions imposed on the fibre bundle of the manifold. This is a long-standing problem [19] that does not have a definite remedy. The problem is the following: twisted fields can have a negative two-point function. These fields interact with each other while preserving the symmetries of the hamiltonian. Their interaction thus contributes a negative mass squared term to the effective mass of the untwisted field due to the negative two-point function of the twisted field and render the untwisted field unstable. It is often assumed that

Nature simple chose to preserve the untwisted configuration only or forbids their interaction due to some, as yet unknown, symmetry [20].

String theory preserves Lorentz invariance. This symmetry has been broken for the open system of our low energy string modes due to the backreaction from the coarse grained environment. Their correlation results in our dispersion relation. If a specific frame must be chosen, it could be *e.g.* the rest frame of the CMB. The formalism needed for the calculation of the stress-energy tensor and the equation of state of the non-linearly dispersed short-distance modes in the presence of Lorentz non-invariance and an expanding background while lacking an effective lagrangian for this short-distance physics requires further development. The initial condition for our model is a vacuum state conformally equivalent to the Minkowki spacetime - the so-called Bunch-Davis vacuum[21]. Finally, before summarising we should note that if there are other modes without the exponential suppression at high k, all that we need is one such mode to lead to the frozen tail comprising the dark energy.

The high wave number behaviour e^{-ak} of the dispersion relation $\omega(k)$ leads again to the correct estimate for the dark energy as a fraction $\sim 10^{-120}$ of the total energy during inflation. This dark energy is certainly completely stringy because our derivation depends on the existence of winding modes, as seen by the role of the generalised level-matching condition

$$N - \widetilde{N} = \Sigma_i m_i w_i$$

This correlation between momentum and winding modes leads to the quantum hamiltonian [15] and hence to the interpretation of the dark energy as the weak correlation with the winding mode energy at short distances. [16] The excitation modes of these correlations with energy less than the current expansion rate are currently frozen by the expanding background.

Within string cosmology there has always been the question of the fate of the winding modes in the uncompactified three spatial dimensions, whether they combine to a single string per horizon which wraps around the universe. Our remedy is intuitively appealing that while the momentum modes are in evidence as quarks, leptons, gauge bosons, etc. the winding modes are now condensed uniformly in the environmental background, hence with a weak correlation at short distances to the momentum modes, frozen by the expansion of the FRW universe in the form of the dark energy.

The observed small value $\Lambda \sim 10^{-120}$ in natural units has an explanation in the toroidal cosmology of closed strings and thus the dark energy provides an exciting opportunity to connect string theory to precision cosmology. We may argue that numerically the size of the cosmological constant in the present approach is a combination of the string scale and the Hubble expansion rate in the sense that $\Lambda/M_{Planck}^4 \simeq 10^{-120} \simeq (H_0/M_{Planck})^2$. Therefore

[15] We would like to remind the reader of the approximation made in Sec.3 for obtaining the toroidal string quantum hamiltonian since an exact solution for this class of backgrounds does not exist as yet.

[16] The correlations in the transplanckian regime contribute to the total energy density of the universe with two types of modes: The excitation modes of these correlations with energy less than the current Hubble expansion rate, H, are currently frozen by the expanding background therefore their kinetic energy is nearly zero. All the other modes in the transplanckian regime whose frequency is higher than H oscillate and redshift away at a rate that will be determined by their equation of state.

the correct amount of dark energy obtained by this frequency dispersion function does not require any fine tuning and relies, besides a physical mechanism (such as freeze-out), only on the string scale as the parameter of the theory. However, our approach does not solve the second puzzle about the dark energy namely, the coincidence problem for the following reason: The expansion rate of the universe is determined by the *total energy density* in the universe by the relation given in the Friedman equation. As can be seen from our dispersion function which approaches conventional cosmology in the subplanckian regime ($k \leq M_{pl}$), most of the other contributions to the energy density are not frozen modes. Therefore the Hubble rate H^2 is not always proportional to the dark energy of the frozen modes due to the contributions in H^2 from other forms of energy densitites.H^2 is dominated by frozen modes (and thus proportional to the dark energy ρ_{DE}) only at some late times $t \geq t_E$ when all other energy contributions ρ_{other} have diluted enough below ρ_{DE} due to their redshift.

The quantitative effort we have made in this work suggests that an interpretation of the dark energy in terms of string theory is more convincing than either a simple cosmological constant or the use of a slowly- varying scalar field with fine tuned parameters.

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Notes Added.

(i) We have not addressed the equation of state $w = p/\rho$ for dark energy in this work. A recent paper by Lemoine *et al.* (`hep-th/0109128`) attempts to calculate w from a dispersion relation similar to that discussed in the present paper, arrives at a bizarre result $w = -186$ and concludes correctly that this value appears in contradiction with data. However, this result for w depends sensitively on the assumed initial conditions (*e.g.* Eq. (34) of that work). We agree with Lemoine *et al.* that an acceptable equation of state is not automatic but depends on initial conditions. Our conclusion would be that an acceptable equation of state requires initial conditions dictated by string theory. We hope to return to this question in a future publication.

(ii). Starobinsky (`astro-ph/0104043`) suggests that trans-Planckian physics is irrelevant to inflationary cosmology; however, his result is based on a discussion using the WKB approximation and as he himself emphasizes does not apply to a dispersion relation which falls to zero, $\omega(k) \to 0, k \to \infty$, at large k as we have assumed in the present article.

Appendix

In this appendix we summarise the derivation of the coarse-grained action $S_{eff}[\Psi_S]$, Eq. (54). For details we refer the reader to the original papers in [10].

We start with the euclidean action for the quartic interaction scalar model under consideration, Eq. (35), in a spatially-flat FRW universe,

$$S[\Psi] = \int a(t)dt \int d^3x \left[|\nabla\Psi|^2 + \tilde{m}_0^2|\Psi|^2 + \tilde{g}_1|\Psi|^4 \right] . \tag{71}$$

Time in this approach is considered as a parameter, and the scale factor $a(t)$ is regarded as a constant instead of a dynamical function. In this way, different values of $a(t)$=constant labels different spatial sections in 3-dimensional Minkowsky space, related to each other by "scaling" transformations, with scaling parameter

$$b = a(t)/a(t_0) . \tag{72}$$

In this sense, *dynamics* is replaced by *scaling*.

In (3-dimensional) momentum space, the action is given by:

$$\begin{aligned} S[\Psi] \;\; = \;\; & \int a(t)dt \left[\int d^3p\Psi(p) \left(p^2 + \tilde{m}_0^2\right) \Psi(-p) \right. \\ & \left. + \tilde{g}_1 \int \prod_i^4 d^3p_i \Psi[p_1]\Psi[p_2]\Psi[p_3]\Psi[p_4]\delta^3(p_1+p_2+p_3+p_4) \right] . \end{aligned} \tag{73}$$

We now separate the field modes into system and environment, $\Psi = \Psi_S + \Psi_E$, with the choice:

$$\Psi_S : \; p^S \leq \Lambda/b , \qquad \Psi_E : \; \Lambda/b \leq p^E \leq \Lambda . \tag{74}$$

We can then also write the action as system+environment, plus the interaction between system and environment:

$$S[\Psi] = S[\Psi_S] + S_0[\Psi_E] + S_I[\Psi_E, \Psi_S] , \tag{75}$$

where:

$$S_S[\Psi_S] \;=\; \int a(t)dt \left[\int d^3p^S \Psi_S(p^S) G_S^{-1} \Psi_S(-p^S) + \tilde{g}_1 \int \prod_{i=1}^4 d^3p_i^S \Psi_S^4(p_i^S)\delta^3(\sum p_i^S) \right], \tag{76}$$

$$S_0[\Psi_E] \;=\; \int a(t)dt \int d^3p^E \Psi_E(p^E) G_E^{-1} \Psi_E(-p^E) , \tag{77}$$

$$\begin{aligned} S_I[\Psi_E,\Psi_S] \;=\; & \tilde{g}_1 \int a(t)dt \left[\int \prod_{i=1}^4 d^3p_i^E \Psi_E(p_1^E)\Psi_E(p_2^E)\Psi_E(p_3^E)\Psi_E(p_4^E)\delta^3(\sum p_i^E) \right. \\ & + 4 \int \prod_{i=1}^3 d^3p_i^E \; d^3p^S \Psi_E(p_1^E)\Psi_E(p_2^E)\Psi_E(p_3^E)\Psi_S(p^S)\delta^3(p_1^E+p_2^E+p_3^E+p^S) \\ & + 6 \int \prod_{i=1}^2 d^3p_i^E \; \prod_{j=1}^2 d^3p_j^S \Psi_E(p_1^E)\Psi_E(p_2^E)\Psi_S(p_1^S)\Psi_S(p_2^S)\delta^3(p_1^E+p_2^E+p_1^S+p_2^S) \\ & \left. + 4 \int d^3p^E \; \prod_{j=1}^3 d^3p_j^S \Psi_E(p^E)\Psi_S(p_1^S)\Psi_S(p_2^S)\Psi_S(p_3^S)\delta^3(p^E+p_1^S+p_2^S+p_3^S) \right] \end{aligned} \tag{78}$$

(79)

where $G_{S,E}$ are Green's functions given by:

$$G_S^{<\Lambda/b}[p^S < \Lambda/b] = [(p^S)^2 + \tilde{m}_0^2]^{-1}, \quad (80)$$

$$G_E^{\Lambda/b}[p^E > \Lambda/b] = [(p^E)^2 + \tilde{m}_0^2]^{-1}, \quad (81)$$

and the Green's function for the whole system is defined as

$$G[p] = G_S^{<\Lambda/b}[p^S] + G_E^{\Lambda/b}[p^E]. \quad (82)$$

Now, the "environment" fields can be integrated out from the partition function, such that:

$$Z[\Psi] = N \int D\Psi e^{-S[\Psi]} = N \int D\Psi_S \int D\Psi_E e^{-(S[\Psi_S]+S_0[\Psi_E]+S_I[\Psi_E,\Psi_S])} \quad (83)$$

$$= N' \int D\Psi_S e^{-S[\Psi_S]} \langle e^{-S_I[\Psi_E,\Psi_S]} \rangle_{\Psi_E}. \quad (84)$$

The average $\langle \cdots \rangle_{\Psi_E}$ is defined with respect to the free action for the environment fields $S_0[\Psi_E]$,

$$\langle e^{-S_I[\Psi_E,\Psi_S]} \rangle_{\Psi_E} = \int D\Psi_E e^{-(S_0[\Psi_E]+S_I[\Psi_E,\Psi_S])} / \int D\Psi_E e^{-S_0[\Psi_E]} \quad (85)$$

$$= e^{-\Delta S[\Psi_S]}. \quad (86)$$

The coarse-grained effective action is then

$$S_{eff}[\Psi_S] = S[\Psi_S] + \Delta S[\Psi_S], \quad (87)$$

with

$$\Delta S[\Psi_S] = -\ln \langle exp\ S_I[\Psi_E, \Psi_S] \rangle. \quad (88)$$

When the interaction between system and environment modes is small ($\tilde{g}_1 \ll 1$), the contribution from Eq. (88) can be expanded in a Dyson-Feynman series, which contains only even powers of the system fields (odd powers of the environment fields average to zero). The first terms in the series give rise then to corrections to the mass ($\delta \tilde{m}_0^2(b)$) and coupling constant ($\delta \tilde{g}_1(b)$) parameters, which can be absorbed by redefining the original mass and coupling parameters in the action $S_S[\Psi_S]$. Therefore, the effective action will be given by:

$$S_{eff}[\Psi] = \int a(t)dt \left[\int d^3p^S \Psi_S(p^S) \left((p^S)^2 + \tilde{m}^2(b) \right) \Psi_S(-p^S) \right.$$
$$\left. + \int \prod_i^4 d^3p_i^S \tilde{g}(b) \Psi_S[p_1^S] \Psi_S[p_2^S] \Psi_S[p_3^S] \Psi_S[p_4^S] \delta^3(p_1^S + p_2^S + p_3^S + p_4^S) \right], \quad (89)$$

where

$$\tilde{m}^2(b) = \tilde{m}_0^2 + \delta \tilde{m}_0^2(b), \quad (90)$$

$$\tilde{g}(b) = = \tilde{g}_1 + \delta \tilde{g}_1(b). \quad (91)$$

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