IFP-472-UNC

PRL-TH-93/13

hep-ph/9308348

August 1993

Common Origin for the Solar and Atmospheric Neutrino Deficits

Anjan S. Joshipura,

Theory Group, Physical Research Laboratory, Navrangpura, Ahmedabad 380009, India

P.I. Krastev*

Institute of Field Physics, Department of Physics and Astronomy, The University of North Carolina at Chapel Hill, CB – 3255, Phillips Hall, Chapel Hill, NC 27599-3255

Abstract

Large mixing induced $\nu_\mu \leftrightarrow \nu_e$ transitions can explain the deficit in the flux of the low energy atmospheric ν_μ whereas the solar neutrino deficit can be simultaneously explained through MSW transitions $\nu_e \to \nu_\tau$. A combined analysis of both these effects is presented. The large $\nu_e - \nu_\mu$ mixing affects the MSW transition between ν_e and ν_τ significantly. As a consequence, a large region of parameters ruled out by experiment in the two generation case is now allowed. The mass hierarchy as well as the mixing pattern required arise naturally in a model for neutrino masses proposed by Zee.

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*Permanent address: Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, BG–1784 Sofia, Bulgaria.

Two independent sets of experiments point towards the existence of neutrino oscillations. The flux of the solar neutrinos is found to be depleted by varied amounts compared to the expectations based on the standard solar model in four different experiments namely, Homestake [1], Kamioka [2], GALLEX [4] and SAGE [3]. Similarly, the flux ratio of the low energy ($\leq$ GeV) ν_μ to ν_e of atmospheric origin is seen to be smaller than the theoretically expected one in experiments by the Kamioka II [5] and IMB [6] group. Independent measurements by the Soudan* [7] and the Frejus collaborations [8] are not inconsistent with the above but the statistical significance of these results is much less than the corresponding one in cases of Kamioka and IMB results. Theoretically, both these deficits could originate from the same common source, namely neutrino oscillations. However the range of the relevant parameters ((mass)^2 difference Δ and mixing angle θ) required by the solar and atmospheric neutrino data are quite different.

The solar neutrino deficit can be explained through neutrino oscillations occurring either in vacuum [9] or in the presence of solar matter [10]. The former requires $\Delta \approx 10^{-10}$ (eV)^2, $\sin^2 2\theta \approx 0.75-1.0$ [9] while the latter becomes important if either $\Delta \approx (5-10)\times 10^{-6}$ (eV)^2, $\sin^2 2\theta \approx (0.2-0.9)$ or $\Delta \sin^2 2\theta \approx (3-4)\times 10^{-8}$ (eV)^2 corresponding respectively to the large angle adiabatic and non adiabatic Mikheyev Smirnov Wolfenstein (MSW) solutions [11]. In contrast, the deficit in the atmospheric muon neutrinos can be explained by means of neutrino oscillations [12] when parameters assume typical values $\Delta \approx 10^{-3} - 10^{-2}$ (eV)^2 and $\sin^2 2\theta \approx 0.5$.

It is quite clear that a simple two generation picture used in deriving above restrictions is inadequate to simultaneously explain the solar and the atmospheric neutrino deficits. A natural possibility is to consider more realistic three generation picture. When all three neutrino flavors are involved, the vacuum oscillations can account [13] for both the deficits

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*The latest, still preliminary result from the SOUDAN–II detector seems to confirm the results of Kamiokande and IMB, albeit with larger statistical errors.

if $|\Delta_{ij}| \equiv |m_i^2 - m_j^2|$ (i, j=1,2,3) $\simeq (0.5 - 1.2) \times 10^{-2}$ (eV)^2 and at least two of the mixing angles are large. On the other hand if $|\Delta| \leq 10^{-5}$ eV^2, then the matter effects inside the Sun become important and they should be taken into account. There are two independent possibilities in this case.

(i) The oscillations between ν_e and ν_μ generate the solar deficit while the reduction in the low energy atmospheric neutrino flux could be due to the ν_μ oscillating to ν_τ. This would require a large θ_{23} and $|\Delta_{23}| \approx 10^{-2} - 10^{-3}$ (eV)^2. In this case, the high energy muon neutrinos originating in the atmosphere also undergo oscillations as they travel through the Earth. Such oscillations have not been found [14,15]. This negative evidence combined with the positive evidence for oscillations in case of the low energy neutrinos [2,6] strongly constrain the relevant parameters and a very narrow range $2 \times 10^{-3} \leq |\Delta_{23}| \leq 0.4$ (eV)^2, $0.4 \leq \sin^2 2\theta_{23} \leq 0.7$ is consistent with all the available information† [17].

(ii) The other possibility is to have a large mixing between ν_e and ν_μ and relatively large $|\Delta_{21}|$ to generate the deficit in the atmospheric flux. The MSW solution for the solar neutrino can be obtained in this case if Δ_{31} falls in the appropriate range. The upcoming high energy muons should oscillate in this case as well but these oscillations will be affected by the presence of matter inside the Earth. The values of parameters which are consistent with the observations of the low and high energy atmospheric ν_μ flux as well with the laboratory search for $\nu_e - \nu_\mu$ oscillations were determined to be [17]: $4 \times 10^{-3} \leq |\Delta_{12}| \leq 0.02$ (eV)^2, $0.35 \leq \sin^2 2\theta_{12} \leq 0.7$.

In case (i), the solar and the atmospheric neutrino deficits are explained through two independent sets of parameters and the third generation does not play any more significant role than to provide this set. In contrast, in case (ii), the presence of a large mixing between

†Allowed regions of neutrino oscillation parameters solving the solar neutrino problem in scenario (i) have been found in ref. [16].

the first two generations can influence the values of Δ_{31} and θ_{13} allowed by the MSW solution. The depletion of the solar neutrino is caused in this case not only by the resonant conversion of the neutrinos inside the Sun but also by their oscillations in vacuum arising due to large mixing between the first two generations. Thus one expects non-trivial departure compared to the case involving only two generations. In this letter we study quantitatively the case (*ii*).

The MSW mechanism in the case of three generations is more involved because of the presence of the two independent (mass)^2 differences and two additional angles as compared with two generations. However in a number of situations [18], one can effectively reduce the problem to that of two generations. Case (*ii*) which requires vastly different values for $|\Delta_{31}|$ and $|\Delta_{21}|$ falls under this category. One of the mass eigenstates is expected to remain unaffected by matter and the other two to undergo a resonant transition. This can be seen analytically by means of an approximation [19]. Let us assume that mixing between the second and the third generation is very small and neglect it altogether. The evolution of neutrino states inside the Sun is governed by a (mass)^2 matrix which can be conveniently expressed in flavor basis as follows:

$$M_A^2 = R_{12}(\theta_{12})R_{13}(\theta_{13})\begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta_{21} & 0 \\ 0 & 0 & \Delta_{31} \end{pmatrix} R_{13}^T(\theta_{13})R_{12}^T(\theta_{12}) + \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{1}$$

where we have subtracted a term $m_1^2 I$ proportional to the identity matrix from the actual (mass)^2 matrix. R_{ij} is rotation in the (ij) plane described by the mixing angle θ_{ij}. $A = 2\sqrt{2}G_F nE$ with n and E denoting the electron density inside the Sun and the neutrino energy respectively. One could exactly diagonalize the upper 2×2 block of this matrix by a rotation in the 12 plane by an angle θ_{12A}.

$$(M_A')^2 \equiv R_{12}^T(\theta_{12A})M_A^2 R_{12}(\theta_{12A}); \tag{2}$$

$$\tan\theta_{12A} \equiv -\frac{2(M_A^2)_{12}}{(M_A^2)_{11} - (M_A^2)_{22}}. \tag{3}$$

We are interested in the limit $|\Delta_{21}| \gg |\Delta_{31}|$ and A. In this case, the θ_{12A} depends very mildly on A:

$$\theta_{12A} \approx \theta_{12} + \epsilon; \qquad \text{with} \quad \epsilon \equiv \sin 2\theta_{12} \frac{A}{2\Delta_{21}}. \tag{4}$$

In addition, the matrix $(M'_A)^2$ also assumes a simple form when non leading terms of O(ϵ) are neglected:

$$(M'_A)^2 \approx \begin{bmatrix} s_{13}^2\Delta_{31} + Ac_{12}^2 & 0 & s_{13}c_{13}\Delta_{31} \\ 0 & \Delta_{21} & 0 \\ s_{13}c_{13}\Delta_{31} & 0 & \Delta_{31}c_{13}^2 \end{bmatrix} + O(\epsilon). \tag{5}$$

where $c_{ij} \equiv \cos\theta_{ij}$; $s_{ij} \equiv \sin\theta_{ij}$. It is seen from the above equation, that in the limit ϵ going to zero, the second generation decouples and MSW resonance could occur between the first and the third generation. The resonance condition however differ in this case from the two generation case. As follows from eq.(5), the resonance occurs if

$$A\ \cos^2(\theta_{12}) = \Delta_{31}\ \cos(2\theta_{13}). \tag{6}$$

For the range of Δ_{21} that is of interest to us from the point of view of the atmospheric neutrino deficit, $\epsilon \leq 10^{-2}$ and to a good approximation a resonance occurs between only two of the neutrinos. The third mass eigenstate remains more or less independent of A. This argument is confirmed by the exact calculation of the variation of the effective neutrino masses in matter with the solar density displayed in fig. 1 for typical values of the parameters.

We shall work with eq.(5) with O(ϵ) terms neglected and comment upon validity of such an assumption later on. The probability $P_{\nu_e\nu_e}$ for the ν_e produced in the Sun to remain ν_e at the detector follows immediately using eq.(5) and the standard MSW picture [11]. Initially, the ν_e would be a mixture of three mass eigenstates determined by the form of the mixing matrix R at the point of production. One of the mass eigenstates remains independent of A while the other two change and undergo resonant transition. Let X denote the probability of non-adiabatic transition between the two transforming states. The survival probability is then given by

$$P_{\nu_e\nu_e} = \frac{1}{2}\left(c_{12}^4(1+(1-2X)\cos(2\theta_{13})\cos(2\theta_{13A_0})) + 2s_{12}^4\right). \qquad (7)$$

As follows from eq.(4), the angle θ_{12A} is mildly dependent upon A and we have used its vacuum value in the above equation. The θ_{13A_0} is the value of θ_{13A} at the production point. This can be determined from eq.(5)

$$\tan\theta_{13A_0} \equiv -\frac{\Delta_{31}\sin(2\theta_{13})}{\Delta_{31}\cos(2\theta_{13}) - A_0 c_{12}^2}. \qquad (8)$$

The exact expression for X depends upon the assumed variation of the density in the vicinity of the resonance layer. We shall use the following form which is valid when this variation is linear but which provides a good approximation to the actual situation in most cases [11]

$$X = \exp\left[-\frac{\pi\Delta_{31}\sin^2 2\theta_{13}}{4\,E\cos(2\theta_{13})\left|\frac{d}{d\,x}ln\;n\right|_R}\right]. \qquad (9)$$

The above expression follows from a straightforward generalization of the method used in case of purely two generation case [11]. Unlike the resonance condition, eq.(6), X turns out to be independent of the large mixing θ_{12} between the first two generations and retains its form given in purely two generation case [11].

We have performed several numerical tests of the accuracy of the analytical expressions for the electron–neutrino survival probability given by eqs.(8). In particular, we have checked the accuracy of this expression for several values of the parameters Δ_{ij} and θ_{ij}. This has been done by comparing the results of explicit numerical calculation of the survival probability with the corresponding values given by eq.(8). One potential source of discrepancy between the values to be compared is the nonequivalence of the two approaches. The analytical formula presumes an averaging over all vacuum oscillation lengths. On the other hand, the numerically obtained survival probability at the surface of the Sun has still to be averaged over two oscillation lengths L_{12} and L_{31}. It is not clear a priori in which order this averaging should be performed numerically. We have obtained good agreement between the analytical and numerical results when we averaged first over the shorter oscillation length L_{12} and then over the longer one L_{31}. In this case typically the relative error of the analytical formula does not exceed several percent.

Having checked numerically the accuracy of the analytical formula we have proceeded with the analysis of the results from the solar neutrino experiments in terms of three neutrino oscillations in matter. We have used the latest available data from the four solar neutrino experiments taking data at present. For the ratio of the experimentally measured to the theoretically predicted event rate in the chlorine experiment [1] we have used the value

$$R_{Cl} = 0.28 \pm 0.03. \tag{10}$$

The relevant ratio determined from the published results of the $Ga - Ge$ experiments SAGE [3] and GALLEX [4] is

$$R_{\mathrm{SAGE}} = 0.44 \pm 0.21. \tag{11}$$

$$R_{\mathrm{GALLEX}} = 0.66 \pm 0.12. \tag{12}$$

We have used also the latest result from the $\nu_e e$ scattering experiment conducted by the Kamiokande collaboration [2]

$$R_{Kamioka} = 0.49 \pm 0.08. \tag{13}$$

In our calculations we have used the solar model [20] which takes into account the diffusion of hellium and is in excellent agreement with the data from helioseizmology. The restrictions on the parameters Δ_{31} and θ_{13} implied by the solar neutrino experiments depend upon the value of θ_{12}. We have chosen values of θ_{12} allowed by the combined search for the ν_μ oscillations in atmospheric as well as accelerator neutrino experiments. These values, as determined in ref. [17], fall in the range $0.35 \leq \sin^2 2\theta_{12} \leq 0.7$. They were determined for the case of two generations. But the presence of the third generation is not expected to significantly influence this determination as long as the $\nu_\tau - \nu_\mu$ and $\nu_e - \nu_\mu$ mixings are small as assumed here. We shall use two representative values of θ_{12} which fall in the allowed band in the following. We also do not expect the results of the analysis of the solar neutrino deficit with the effect of the Earth on the oscillations of two neutrinos leading to unobserved seasonal and day–night variations [21] to change considerably with the inclusion of the third neutrino. The mixing angle θ_{13} relevant for the solar neutrino oscillations is too small to affect the neutrino oscillations in the Earth because of the much smaller dimensions of the Earth as compared with that of the Sun. Also the oscillation length corresponding to Δ_{31} is much larger then the diameter of the Earth.

The neutrino survival probability for each set of neutrino oscillation parameters has been computed by numerically finding the resonant point and substituting in eq.9 the corresponding value of the logarithmic derivative of the electron number density. With the neutrino survival probabilities so obtained we have calculated the corresponding event rates in each detector and have compared them with the ones predicted in the standard solar model. The allowed regions of parameters for which the neutrino conversion hypothesis under consideration cannot be rejected at certain c.l. are determined by minimizing the function:

$$\chi^2 = \sum_{i=Cl,Ga,H_2O} \left(\frac{R_i^{exp} - R_i^{th}}{\sigma_i} \right)^2 . \tag{14}$$

They are shown in fig.2 for two different values of the large mixing angle θ_{12}. Note that with diminishing the mixing angle θ_{12} the allowed region of parameters $\Delta_{31} - \sin^2 2\theta_{13}$ converges to the one determined in the two–neutrino oscillation scenario [22]. As a less trivial result from the above calculation it follows that a relatively large part of the adiabatic region of parameters, i.e. the horizontal branch of the MSW triangle between $\sin^2 2\theta_{13} = 0.07$ and $\sin^2 2\theta_{13} = 0.9$ is allowed at the 95 % c.l. for $\sin^2 2\theta_{13} = 0.65$. The mixed solution, i.e. the upper part of the hypotenuse of the MSW triangle, is also allowed at the 90 % c.l. for $\sin^2 2\theta_{12} = 0.65$ and at the 95 % c.l. for $\sin^2 2\theta_{12} = 0.45$. This result is characteristic of the three–neutrino mixing scheme we are considering here whereas in the "standard" two–neutrino oscillation scenario the adiabatic and mixed solutions are ruled out at the 95 % c.l. from the combined analysis of the results of the four solar neutrino experiments [22].

These results change considerably when we include in our analysis the recoil electron energy spectrum data obtained by the Kamiokande–II collaboration. Instead of comparing only the mean value of the suppression factor R_{H_2O} with the predicted ones for each set of neutrino oscillation parameters, we have deleted the corresponding summand from eq.(13) and have added instead the χ^2 function for the 12 recoil electron energy bins as given in [23]. We end up with the plot shown in fig.3. In this case only a part of the so called "mixed" solution survives but most of the adiabatic region is ruled out at the 95 % c.l.. However, the allowed region of parameters is still different from the corresponding one in the two–neutrino oscillation case.

Solar neutrino experiments that are in operation, as well as proposed ones that are being developed will make it possible to test the proposed three–neutrino oscillation solution of the solar neutrino problem. For $\sin^2 2\theta_{12} = 0.65$ and $\sin^2 2\theta_{13} = 5 \times 10^{-4}$ and $\Delta_{31} = 8 \times 10^{-5}$ eV^2 the signals in Ga-Ge detectors should be close to 82 SNU, in chlorine detectors about 2.52 SNU and the signal in water Cherenkov-detectors should be about 0.33 of the predicted one within the standard solar model. For $\sin^2 2\theta_{13} = 6 \times 10^{-3}$ and $\Delta_{31} = 5 \times 10^{-6}$ eV^2 the signals in Ga-Ge and chlorine detectors should be correspondingly 53 SNU and 2.20 SNU and the suppression of the signal in $\nu_e e$ scattering experiments should be 0.45. Finally, for $\sin^2 2\theta_{13} = 0.7$ and $\Delta_{31} = 10^{-4}$ eV^2 the signals in these detectors should be 55 SNU, 2.28 SNU with a suppression of 0.34 of the signal in neutrino–electron scattering experiments. The pp neutrino signals alone should be suppressed by factors of 0.66, 0.63 and 0.44 correspondingly, whereas 0.862 MeV ^7Be neutrino signals should be suppressed by factors 0.67, 0.47 and 0.44 respectively. Thus by comparing the signals in Borexino [24], SNO [25], ICARUS [26] and Superkamiokande [27] it will be possible to pinpoint the allowed region of parameters.

The proposed hellium detector of solar neutrinos HELLAZ [28] is expected to measure the fluxes of both pp and ^7Be neutrinos. Moreover, the spectrum of pp neutrinos will possibly be measured too. As shown in [29] this spectrum does not depend on details of the solar model and its shape can be predicted with a great precision. Any significant deviation from the standard shape will be an unequivocal evidence in favour of neutrino conversion taking place either in the Sun, between the Sun and the Earth and/or in the Earth.

The scenario we have considered requires a hierarchy $|\Delta_{21}| \gg |\Delta_{31}|$. Such hierarchy does not occur in seesaw models based on grand unified groups such as $SO(10)$ if the right handed neutrino masses are assumed to be flavor independent. However there exist other mechanisms for neutrino mass generation which could lead to the required values for the Δ_{ij}. A concrete example is provided by the model of Zee [30] which in fact comes very close to the present scenario. This model contains two hierarchical Δ_{ij}. The larger one describes the $\nu_e - \nu_\mu$ oscillations while the $\nu_e - \nu_\tau$ oscillations are of longer wavelength and are controlled by the other (mass)^2 difference [31]. The model contains only left handed neutrinos whose majorana masses are radiatively generated and are described by the following mass matrix

$$M_\nu = m_0 \begin{bmatrix} 0 & \sigma & \cos\alpha \\ \sigma & 0 & \sin\alpha \\ \cos\alpha & \sin\alpha & 0 \end{bmatrix}, \tag{15}$$

where m_0, σ and α are parameters defined by Wolfenstein [31]. In the limit σ going to zero, one has a massless state and two with masses $\pm m_0$. We need to identify the massless state with "ν_μ". To first order in σ, one has [32]

$$m_1 = m_0(1 + 1/2\sigma \sin 2\alpha) \tag{16}$$

$$m_2 = -\sigma \sin 2\alpha \tag{17}$$

$$m_3 = -m_0(1 - 1/2\sigma \sin 2\alpha) \tag{18}$$

Hence, $\Delta_{21} \approx m_0^2$; $\Delta_{31} \approx 2m_0^2\sigma \sin 2\alpha$. Thus our scenario gets realized by choosing $m_0^2 \approx 10^{-3}$ (eV)^2 and $2\sigma \sin 2\alpha \approx -10^{-2}$.

The mixing pattern predicted in the model also comes close to the one required here. The mixing matrix is given in the limit σ going to zero by:

$$U M_\nu U^T = diag.(m_1, m_2, m_3) \tag{19}$$

$$U^T = \begin{bmatrix} 1/\sqrt{2}\cos\alpha & \sin\alpha & 1/\sqrt{2}\cos\alpha \\ 1/\sqrt{2}\sin\alpha & -\cos\alpha & 1/\sqrt{2}\sin\alpha \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{bmatrix}. \tag{20}$$

This coincides with the mixing matrix that we have used namely,

$$U^T = R_{12}(\theta_{12})R_{13}(\theta_{13}), \tag{21}$$

if one identifies $\theta_{12} = \pi - \alpha$ and $\theta_{13} = \pi + \pi/4$. α and hence θ_{12} is not fixed within the model but would be required to lie in the range appropriate for solving the atmospheric neutrino problem. In contrast, $\sin^2\theta_{13}$ is already fixed around [33] its maximal value. This value is seen to be allowed by the restrictions coming from the solar neutrino experiments as displayed in figs.(2, 3). Thus, in addition to having the required hierarchy in masses, the mixing angles in the model are also consistent with the solar and atmospheric neutrino data.

Conclusions. We have attempted to describe the observed deficits in the solar and atmospheric neutrinos in terms of the neutrino oscillations involving three generations. The deficits in the flux of the low energy atmospheric ν_μ is not as clearly established [34] as the one observed in case of the solar neutrinos. However if such a deficit gets firmly established then the three generation scenario considered here would provide an interesting mechanism for understanding both deficits in a coherent manner. As discussed above, the conventionally employed two generation MSW picture is still approximately applicable in the present case but the large mixing θ_{12} changes the allowed region of parameters in a significant manner. Moreover, $\nu_e - \nu_\mu$ oscillations with relatively large $(\text{mass})^2$ difference envisaged here can be studied in laboratory as well. Future experiments may confirm or rule out the present scenario.

Acknowledgements. A part of this work was done when A.S.J. was visiting the Theory Division of CERN and the II. Inst. for Theoretical Physics, Hamburg. He thanks G. Kramer and S. Pakvasa and S. Rindani for discussions related to this work. The work of P.K. has been partially supported by grant No. DE-FG05-85ER-40219 of the U.S. Department of Energy. He thanks John Bahcall for a number of very informative and fruitful discussions while visiting the Institute for Advanced Studies, Y. Totsuka and Y. Takeuchi for the detailed explanation of the procedure adopted by the Kamiokande collaboration for the analysis of the recoil electron energy spectrum and J.T. Liu for numerous discussions and invaluable help concerning computer graphics.

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Figure Captions

Fig.1 Masses of three flavors of neutrinos as a function of A expressed in units of 10^{-5} eV^2 for the values of the vacuum parameters: $m_1^2 = 0$; $m_2^2 = 500$; $m_3^2 = 1$ in units of 10^{-5} eV^2; $\theta_{12} = 40^0$; $\theta_{13} = 5^0$ and $\theta_{23} = 0$.

Fig.2 Allowed region of parameters $\sin^2(2\theta_{13})$ and Δm_{31} at the 90 % c.l. (solid line) and 95 % c.l. (dashed line) from the analysis of the results of the four solar neutrino experiments. The values of the mixing angle θ_{12} have been chosen from the region allowed by the analysis of the deficit of atmospheric ν_μ in terms of oscillations of two neutrino flavours in the Earth [5], $\sin^2(2\theta_{12}) = 0.65$ (fig.2a) and $\sin^2(2\theta_{12}) = 0.45$ (fig. 2b).

Fig.3 The same as in fig.2 with the recoil electron energy spectrum as measured by the Kamiokande-II collaboration taken into account.