INVESTIGATING THE IMPACTS OF A MATHEMATICS WORD PROBLEM INTERVENTION ON STUDENT PERSEVERANCE, SOLVING ACCURACY, AND SELF-EFFICACY

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ABSTRACT

Hana Van Name Baskin: Investigating the Impacts of a Mathematics Word Problem Intervention on Student Perseverance, Solving Accuracy, and Self-Efficacy
(Under the direction of Janice Anderson and Katie Baker)

Mathematical literacy and numeracy are critical for students during school and post-graduation, however, many U.S. students do not develop the mathematical skillset needed for college or the workplace, despite curricular importance placed on solving problems encountered in mathematics classrooms and spaces. Part of this skillset for critical thinking and analysis is the ability to successfully interpret and solve word problems. In an effort to increase proficiency in mathematics, through improving word problem solving ability, a schema- and cognitive-based intervention, *Solve It*!, was implemented with students, who historically performed below average on state assessments, enrolled in year-long Math I in a diverse, large, urban high school. The purpose of this study was to measure the effectiveness of the *Solve It*! instructional approach and in turn the usefulness of a schema- and cognitive-based mathematics word problem solving intervention for improvement in ability through accuracy, perseverance, and self-efficacy.

This study employed a nonequivalent control group quasi-experimental design. Correlation and ANCOVA were used to assess effectiveness of *Solve It!*. The data collected were quantitative and included: student pre- and post-intervention test scores and maintenance test scores three months post-intervention to measure solution accuracy, teacher-monitored checklists to measure student problem-solving perseverance, and student scale survey results reporting self-efficacy. The student participants were compared to peers who were also in year-long Math I classes but not introduced to *Solve It!*.

Data analysis showed that students who received the *Solve It!* intervention did not have statistically significant gains in accuracy as compared to students without the intervention. Significant correlation was not found between student perseverance and solution accuracy nor between student self-efficacy and solution accuracy while using the intervention. However, a significant correlation was found between perseverance and self-efficacy in both the control and intervention classes. Additionally, students who received the intervention had higher gains in accuracy, perseverance, and self-efficacy than those who did not receive the intervention. While these gains were not statistically significant the findings offer insight into why a schema- and cognitive-based word problem instructional methods may be employed in the mathematics classroom.

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LIST OF ABBREVIATIONS

ACT	American College Testing
CBM	Curriculum-Based Measures
CCSSM	Common Core State Standards-Mathematics
DSM	Dynamic Strategic Math
ELA	English Language Arts
FERPA	Family Educational Rights and Privacy Act
IS	Instruction Scaffolding
M	mean
MD	mathematics disabilities
MDRD	mathematical and reading disabilities
NCTM	National Council of Teachers of Mathematics
NGA	National Governors Association
p	p value
PD	professional development
PISA	Program for International Student Assessment
RPV-HECC	Read, Paraphrase, Visualize, Hypothesize, Estimate, Compute, Check
RUN	Read, Underline, Name
SBI	Standards Based Instruction
SD	standard deviation
SNC	Story Number Competencies
SQRQCQ	Survey, Question, Read, Question, Compute, and Question
t	t-test statistic

CHAPTER 1 – INTRODUCTION

Today's job market requires high school graduates to receive post-secondary education or training equivalent to skills and competencies learned in a first year of college in order to be able to support a family of four above the poverty line (ACT, 2012). However, American high school graduates are entering the workforce and college without the mathematical competency sought for entry-level jobs by employers and institutions of higher education (ACT, 2012; Castner-Lotto & Barrington, 2006). In a survey of 431 employers, 53.3% reported that their high school graduate entrants were "deficient" in mathematics (Castner-Lotto & Barrington, 2006). Of the same employers, 30.4% indicated that mathematics is "very important" to this group of entrants' successful job performance. This mathematical deficiency means that new employees lack critical thinking and problem-solving skills. Critical thinking and problem-solving skills are defined as using sound reasoning and analytic thinking; applying knowledge, facts, and data to solve workplace problems; and integrating mathematics and science concepts in complex settings (Castner-Lotto & Barrington, 2006). Competitive companies expect employees to be creative, innovative, and adaptable problem solvers and exercise the part-whole critical thinking needed to evaluate relevant details from a big picture (Bates & Phelan, 2002; Soulé & Warrick, 2015; Steinke, 2015). Employees with strong critical thinking and problem-solving skills are essential to meet the needs of a rapidly changing American workplace.

The shortage of potential employees with requisite skills is impeding companies' ability to achieve production levels, increase productivity, and meet customer demand (National Association of Manufacturers & Deloitte Development LLC, 2005). According to the National

Association of Manufacturers and Deloitte Development LLC (2005), 84% of manufacturing companies say that K-12 schools are not doing an adequate job preparing students for the workplace. It is estimated that only 50% of working Americans have a literacy level that would qualify them as competitive in their workplace (Bates & Phelan, 2002). The country is now competing on the basis of innovation; companies are changing what is produced, how it is produced, and the organization of production (Harris, 2008). Employees considered literate are now expected to possess strong communication, judgment, problem-solving, and decision-making skills (Ozgen & Bindaka, 2011). A change in workforce requirements has influenced the aims of schooling (Ravitch, 1988). A commonly accepted mission of public high schools is to prepare all students for postsecondary education, training, and the workforce without the need for remediation (Ali & Jenkins, 2002).

Schools have now aligned their standards with real-world expectations of college and the workplace by upgrading graduation requirements and coursework (e.g. completion of Algebra II as a graduation requirement) (Attewell & Domina, 2008). The Common Core State Standards (NGA, 2010) shifted emphasis from memorization and performing algorithmic procedures to demonstrating understanding and application with high levels of cognitive demand (Porter, McMaken, Hwang, & Yang, 2011). This shift is also mirrored in the standards for English Language Arts (ELA) with an emphasis on analyzing text as well as making inferences, drawing conclusions, and predicting probable consequences (Porter et al., 2011). The change in mathematics and English Language Arts standards together highlight the need for critical thinking skills in post-secondary school. High school tests are also starting to measure college- and work-ready skills instead of basic and intermediate algebra skills from the 8^{th}, 9^{th}, and 10^{th} grades (Conklin et al., 2005). In 2015, the U.S. ranked lower than 36 education systems in the mathematics achievement of 15-year olds. Average U.S. mathematics literacy scores on the Program for International Student Assessment (PISA) in 2015 were comparatively lower than the 2009 and 2012 scores, and scores in 2018 were not significantly different than scores in 2015 or 2003 (National Center for Education Statistics, 2016; National Center for Education Statistics, 2019). The PISA defines mathematics literacy as:

> An individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. (p. 7)

These results show that U.S. mathematics literacy scores have lied stagnant over several years of testing. From 2003 to 2018, 10 out of 36 education systems that participated in the PISA reported an increase in mathematics literacy scores, while 13 systems reported a decrease in the same scores (National Center for Education Statistics, 2019).

High school students need more support in both areas of mathematics and reading skills. Reading abilities influence children's growth in mathematics so, it is attractive to use reading interventions to tackle mathematical deficiencies (Jordan, Kaplan, & Hanich, 2002). Effective reading instruction includes phonemic awareness, phonics, fluency, vocabulary and comprehension (National Reading Panel, 2000). The focus of reading instruction in young students is on phonics, whereas older students benefit more from vocabulary and comprehension instruction (Roberts, Torgesen, Boardman, & Scammacca, 2008). The goal of comprehension is to understand what is read, construct memory representations of what is understood, and put the understanding to use (National Reading Panel, 2000). When comprehension is lacking, cognitive strategies can be employed to help students become competent and self-regulated readers. Students who are instructed with cognitive strategies make significant gains on measures of reading comprehension over students trained with conventional instruction procedures (National

Reading Panel, 2000). By providing students with methods to comprehend the components of a word problem, the application of cognitive reading strategies helps students build problem models and find accurate solutions.

Statement of the Problem and Problem Importance

It is important for students to be foundationally and mathematically literate in school and post-graduation, however many U.S. students do not develop the required mathematics skills for the college or the workplace. Many elementary age students enter school already behind in reading and mathematics skills (ACT, 2012). The achievement gap between these students and those that enter elementary school prepared widens over time because these students are at a disadvantage in learning new knowledge (ACT, 2012). Solving problems, especially those encountered in mathematics, is important as curricula become more focused on 21st century critical thinking. The National Council of Teachers of Mathematics (NCTM) (2000) describes process standards that should be incorporated into all mathematics classrooms: problem solving, reasoning and proof, communication, connections, and representations. NCTM (2014) calls for educators to implement actions to ensure that these processes are incorporated into teaching and learning including developing socially, emotionally, and academically safe environments, evaluating curricular materials for alignment with standards with coherent development across grade levels, incorporation of mathematical tools and technology in the classroom, providing feedback to students. The National Governors Association (NGA) published standards of mathematical practice, three of which say: make sense of problems and persevere in solving them, reason abstractly and quantitatively, and construct viable arguments and critique the reasoning of others (NGA, 2010). These process and practice standards and actions reflect the strong communication, judgment, problem-solving, decision-making, and critical thinking skills needed in today's post-secondary climate (Ozgen & Bindaka, 2011).

One of the mathematics skills needed for critical thinking and analysis is to be able to successfully read and solve word problems. Often, written word problems can be more difficult to solve than problems written in numeric form due to students needing to create a problem model from textual input before solving (Jonassen, 2003; Moran, Swanson, Gerber, & Fung, 2014; Pape & Smith, 2002). "A word problem requires a student to construct a problem model by identifying what information is missing, determining the number sentence that incorporate the given and the missing information, and deriving the calculation problem for finding the missing information" (Fuchs et al., 2008, p. 156). A problem solver decodes and analyzes the problem text for relationships among components to build a mental model which is then acted on by a procedure to solve the problem (Jonassen, 2003; Pape & Smith, 2002). In a study of first graders, all could solve an arithmetic problem presented as numbers, but only 29% could solve the same problem as a word problem. In addition, as students move up through the education system, they will be required to solve an increasing number of word problems (Cummins, 1988). Difficulty solving word problems in elementary school has a lasting effect on students and is a potential source of future mathematics anxiety. Four skills have been identified to solve word problems: a) ability to read the problem, b) ability to set it up so computational skills can be applied, c) performance of computations, and d) ability to combine reading, interpretation of the problem, and computation to solve the problem correctly (Ballew & Cunningham, 1982).

The American College Testing (ACT) program tracks students using the EXPLORE (8^{th} grade), PLAN (10^{th} grade), and ACT (11^{th} grade) tests. In 2010, the ACT published a report that summarized the performance of low-income twelfth grade students. Only 27% of these students met the ACT College Readiness Benchmark in reading, and 16% met the standard in mathematics (ACT, 2012). Tracking progress from eighth through twelfth grades, only 3% of students who scored more than one standard deviation below the eighth grade mathematics standard reached the College Readiness Benchmark by twelfth grade (ACT, 2012). These percentages imply that low performing students entering high school have a very difficult time catching up to the benchmarks by the time they graduate. Only 38% of eighth graders who did not meet the benchmark on EXPLORE will meet the benchmark by twelfth grade (ACT, 2012). Of course, the ACT can only track students who stay enrolled in school, thus this statistic does not include those who have dropped out before the twelfth grade.

There are too few interventions targeted at high school age students with the goal of increasing foundational literacy and mathematical proficiency by the time of graduation so that they may enter the workforce and college prepared to be competitive. Specifically, at the school of study, 17.5% of students scored proficient on the mathematics subsection of the ACT and 20.1% proficient on the reading subsection in 2016. The state offers a school-wide free administration of the ACT with a make-up day for absent students (412 out of 431 11^{th} grade students took the ACT at this school in 2016). Thus, students are graduating high school underprepared for the demands of college and the workplace.

Purpose of the Study

In an effort to increase proficiency in mathematics, by way of improving literacy skills, a cognitive and schema-based intervention developed by Marjorie Montague (2013), *Solve It!*, was prescribed to low performing mathematics students. Note that when the term low performing is used, it follows the terminology pattern of the school in this study and indicates scoring levels from students' state standardized mathematics assessments, specifically the Mathematics End-of-Grade middle school tests, that thereby resulted in a year-long Math I course. The term low performing does not denote the researcher's opinion or personal labeling of students involved in the study.

The purpose of this study was to assess the effects of *Solve It!*, an intervention which focuses on teaching students the processes and strategies needed to represent mathematics problems through a seven step process; read, paraphrase, visualize, hypothesize, estimate, compute, and check. *Solve It*! is meant to be used in inclusive classrooms of average and low achieving students as well as students with learning disabilities. Because *Solve It*! teaches students cognitive processes and strategies through explicit instruction and guided practice, students learn to better comprehend, represent, and plan to solve mathematic word problems (Montague, 2013). This not only increases accuracy of solutions, but also promotes perseverance in solving problems. Perseverance is defined as productive struggle through challenging mathematical ideas and relationships when students expend effort to make sense of something that is not immediately apparent (Brahier, Leinwand, & Huinker, 2014; Hiebert & Grouws, 2007).

As shown by ACT (2012), it is often too late to wait until high school to expect low reading and mathematical ability students to catch up to their peers in these areas. However, this does not mean that we should not implement some of the strategies used in elementary and middle schools into high school classrooms. Students need to be better prepared for college and/or the workforce, and an intervention applied during a first-year high school mathematics course could help students achieve this.

Study Design

Theoretical Framing

Two frameworks were used to develop the study. Research questions, hypotheses, and methods were formed based on the frameworks and discussion of the research questions was informed by them. The first is Kintsch and Greeno's (1985) processing model which explains the processes of translating textual arithmetic word problems into macrostructures that are then acted on by problem-solving strategies. Within this process, a student reads the text base, sorts propositions, makes sets, builds macrostructures through strategies, and uses problem-solving procedures. The second framework was Bandura's (1993) theory of self-efficacy in which people with high levels of efficacy will set goals for themselves and provide guides and supports to meet those goals, whereas people with low efficacy focus on failure and things that can go wrong. Kintsch and Greeno's (1985) processing model and Bandura's (1993) theory are explained in more detail in Chapter 2.

Research Questions

The questions that framed the study were structured to investigate and evaluate a literacy-focused mathematics word problem intervention called *Solve It!*. The questions were as follows:

1. What are the effects of the *Solve It!* intervention on mathematics word problem solution accuracy for year-long Math I students identified as low performing by a mathematics state achievement test?

2. What are the effects of the *Solve It!* intervention on mathematics word problem perseverance for year-long Math I students identified as low performing by a mathematics state achievement test?

2a. What are the differential effects of the *Solve It!* intervention on perseverance for one-, two-, and three-step mathematics word problems?

3. What are the effects of the Solve It! intervention on student self-efficacy around mathematics?

3a. To what extent does student self-efficacy correlate to changes in mathematics word problem solution accuracy?

3b. To what extent does student self-efficacy correlate to changes in mathematics word problem perseverance?

Effects were measured by statistically significant growth from pre- to post-tests using a General Linear Model SPSS analysis and correlations between factors to quantify significant relationships.

Two groups of student scores were analyzed; students who were in a class participating in the *Solve It!* Intervention, and students who were in a class not participating in the intervention. Participating students were matched to students who did not participate by gender identity group and racial identity group to compare differences in samples.

Hypotheses

The research hypotheses were stated in the null.

$H_0$1: Students who receive the *Solve It!* intervention will not have statistically significant gains in accuracy mathematics word problem solutions compared to students who did not when controlling for racial identity group and gender identity group.

$H_0$2: There will be no relationship (correlation) between perseverance and mathematics word problem solution accuracy using the *Solve It!* intervention.

$H_0$3: There will be no relationship (correlation) between self-efficacy and mathematics word problem solution accuracy using the Solve It! intervention.

The study controlled for significant differences in mathematics word problems scores attributed to racial identity group and/or gender identity group.

It was predicted that these hypotheses would be refuted.

Methods

The *Solve It!* program and its technique were used as an intervention in this study. The technique combined use of schema and cognitive strategy, and taught students the processes and strategies needed to represent mathematics problems via paraphrasing and visualization. The presumption was that as students become proficient with the routine, they build schema-based strategies for building connections and relationships between given propositions and cognitive strategies for monitoring self-learning. Pre- and post-tests of achievement, a checklist for perseverance, and pre- and post-assessment of self-efficacy were collected to assess the effects of the intervention. Quantitative analysis showed whether or not students who participated in *Solve It!* made statistically significant gains over non-participating peers in math word problem solution accuracy and the role of perseverance in solution accuracy. Further details of the study methods are described in Chapter 3.

Assumptions

An assumption of the study was that students enrolled at the high school in year-long Math I would stay enrolled for the duration of the semester to complete the intervention. Students at the school transfer frequently during the school year. A second assumption is that the students that remain enrolled would have to attend a majority of the classes to improve scores. It is important to note that other than the state truancy law, there was no enforced attendance policy at the school.

Limitations

One limitation of the study was that these students were not representative of the school population. Students chosen to participate in the intervention are identified as scoring a Level 1 on the 7^{th} and 8^{th} grade EOG mathematics test. The results are only generalizable to other low performing mathematics students. Another limitation on generalizability is the selection of one high school as the site for the study. The high school's population will not perfectly match the population of any high school wishing to adopt this intervention strategy. Additionally, due to the Family Rights and Education Act and the researcher's employment in the school district, the researcher could not record whether students had learning gifts or disabilities, emotional needs, or medical exceptions. This limited the data the researcher could collect and analyze in terms of differential effects of the intervention on students varying exceptionalities. Lastly, specific attendance data was not collected for each student in either the intervention or control classes. The researcher was able to determine if a student was present for at least half of the practice sessions but did not collect day-to-day attendance.

Chapter 1 Summary

This chapter outlined the background and argument for math interventions targeted at high school age students with the goal of increasing foundational literacy and mathematical proficiency by the time of graduation so that they may enter the workforce and college prepared to be competitive. If students are expected to graduate with critical thinking and problem-solving skills needed for success in a competitive job market, then educators need to incorporate instructional strategies in mathematics classrooms with the goal of honing these skills. This study explored the effects of a schema and cognitive strategy-based intervention, which focused on a technique for effectively solving mathematics word problems in a general education inclusive mathematics classroom.

The following chapters provide details about literature and study design. Chapter two presents a review of the current and relative literature and the theoretical and conceptual frameworks that forms the basis for this study. Chapter three presents the study design, methodology, and procedures for data collection and analysis. Chapter four provides a statistical analysis of the data collected during the study. Chapter five discusses the study's findings, limitations, and implications for future research on why schema- and cognitive-based word problem instruction methods should be used in the mathematics classroom. Before the next chapters, key terminology used throughout the dissertation are defined next.

Definition of Terms

Comorbid mathematics and reading learning disability – co-occurrence of a mathematics and reading learning disability.

Foundational literacy – proficiency in reading and writing text.

Mathematical literacy – "capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen" (Programme for International Assessment, 2006, p. 72).

Perseverance – productive struggle through challenging mathematical ideas and relationships when students expend effort to make sense of something that is not immediately apparent (Brahier, Leinwand, & Huinker, 2014; Hiebert & Grouws, 2007).

Self-efficacy – an individual's "judgments of how well one can execute courses of action required to deal with prospective situations" (Bandura, 1982, p. 122).

Solve It! – an instructional approach designed to help middle and secondary school students who have difficulty solving mathematical problems through teaching students the necessary cognitive and metacognitive processes and strategies that successful problem solvers use (Montague, 2013).

CHAPTER 2 – REVIEW OF THE LITERATURE

Introduction

This chapter is a discussion of the research surrounding the role of literacy and reading comprehension in doing general mathematics in the context of the Kintsch and Greeno's (1985) processing model and Bandura's (177) theory of self-efficacy. To frame the importance of literacy strategies in the mathematics classroom that help students read, interact with, and understand mathematical text, this chapter begins with a discussion of strategies common in the English Language Arts content area. Students who struggle with reading (non-proficient on End-of-Grade tests) often do not automatically use literacy strategies presented in English Language Arts in the context of reading mathematical text (Caprapro, Capraro, & Rupley, 2012). Thus, current literature surrounding the role of literacy and reading comprehension in doing mathematics and effective literacy strategies for mathematics instruction is presented. This body of literature explores strategies and interventions for teaching mathematics word problems to struggling readers using the theoretical framework in the K-12 setting.

The National Council of Teachers of Mathematics (NCTM) and Common Core State Standards-Mathematics (CCSSM) push for content area literacy in mathematics classrooms across states through process and practice standards (NGA, 2017; NCTM, 2017). Achievement of content area literacy, specifically in mathematics and English Language Arts, through the process and practice standards requires skills such as critical thinking and problem-solving skills, analyzing text, making inferences, drawing conclusions, and predicting probable consequences

(Porter et al., 2011). The change in both mathematics and English Language Arts standards highlight the need for critical thinking skills post-secondary school.

Generally, content area literacy is achieved when students are able to construct conceptual knowledge on a subject matter through reading strategies (Fordham, 2006 as cited in Smith & Angotti, 2012). In mathematics classrooms, content area literacy is honed through student interactions with mathematical texts and complex problem solving. These interactions include the use of reading strategies that are specific to comprehending mathematics texts as well as making in-depth mathematical connections using multiple models of solving, writing, and justifying answers to problems (Bernadowski, 2016; CCSSM Initiatives, 2017). In order for students to achieve literacy in mathematics, they must learn the skills necessary for reading and writing in this specific content area. Unfortunately, mathematics classrooms are where these practices are least likely to be implemented (Ippolito, Dobbs, & Charner-Laird, 2017). If students, with and without learning disabilities and difficulties, are expected to be successful critical thinkers and problem solvers in postsecondary education and the workforce, then educators need to effectively incorporate practices targeting these goals from kindergarten through high school. Achieving literacy across the content areas, and specifically in mathematics, prepares students for life beyond the high school classroom.

Mathematical text is different and more difficult to read than other content area text because of its use of precise symbols, longer and more complex sentence structure, dense concepts, and little redundancy to help with interpretation (Adams, Pegg, & Case, 2015). Students have to decode and comprehend signs, symbols, and graphics while reading mathematical text (Barton, Heidema, & Jordan, 2002). Once literacy in this content area is achieved, one sees that mathematical text presents, explains, describes, instructs, and guides students in doing mathematics as well as deepens understanding of concepts, supports independent learning of mathematics, and prepares students for advanced study (Adams & Lowery, 2007; Adams et al., 2015).

When teachers attempt to incorporate reading in mathematical classrooms it is typically done through using biographies, picture books, and word problems with real-life context (Adams, 2003). However, Adams (2003) warns that if the goal of reading and writing in the mathematics classroom is to increase mathematics learning, then these common practices do not suffice. Reading instruction requires sharpening metalinguistic awareness so students can reflect on and analyze mathematical texts. If mathematics is viewed as a language, then teachers can use reading strategies as they would in language arts courses to plan their instruction (Hamilton, 2017).

Ippolito et al. (2017) explored the findings of a group of mathematics teachers involved in a summer program who were looking for ways to successfully incorporate literacy strategies into their classrooms. After discussing the literacy goals of their students, they concluded there was a need to focus on academic discussion, the language of mathematics and multiple representations, and mathematical reading in the classroom. Academic discussion in mathematics classrooms is a collaborative practice in which students describe reasoning and thinking processes to others. This practice reflects CCSS Initiatives (2017), which states:

> They make conjectures and build a logical progression of statements to explore the truth of their conjectures…They justify their conclusions, communicate them to others, and respond to the arguments of others…Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. (Practice 3)

The NCTM (2017) process standards also include this practice, recommending students listen to the explanations of others in order to develop their own understandings and converse with other students to explore multiple perspectives, sharpen thinking, and make connections.

A focus on the language of mathematics and multiple representations supports the use of precise mathematical language and the impact of symbols and pictorial representations on mathematical concepts (Ippolito et al., 2017). “The language of math is written in symbols as well as words, and the symbols function somewhat like punctuation marks, clustering some numbers together, keeping others apart and defining relationships between numbers” (Hamilton, 2017, p. 47). Adams et al. (2015) and Barton et al. (2002) discuss what sets the reading of mathematical text apart from other content area text and describe the additional layer of comprehension skills that students must learn to be literate in mathematics. The CCSS Initiative (2017) states that students should use clear definitions in academic discussion, which attends to the use of precise mathematical language. Additionally, they should state the meaning of symbols they use. Mathematical texts read in the classroom that make use of precise language symbols can be used as model texts to help students know what their work should look like to convey meaning and understanding.

Students also need encouragement to read textbooks, dictionaries, encyclopedias, educational journals, science texts, biographies of mathematicians, and various other texts to understand real-world mathematics (Bossé & Faulconer, 2008; Ippolito et al., 2017). Reading a variety of texts in mathematics helps to break down the compartmentalization of traditional reading instruction and mathematics by allowing students to use the same cognitive processes learned to comprehend narrative and expository text in a new context (Capraro et al., 2012). The cognitive processes for comprehension include activating background knowledge to access and evaluate the reading, knowledge of goals and intentions for reading to sort relevant and irrelevant information, and knowledge of text structure to identify and organize information (Capraro et al., 2012). Cognitively, the processes used in reading comprehension are tantamount to those necessary to solve mathematics problems. Problem solvers must have background knowledge of the mathematical structures involved, know the goal of the problem to sort relevant and irrelevant information, then sort and organize information into macrostructures that represent the problem text (Capraro et al., 2012; Fuchs, Fuchs, Compton, Hamlett, & Wang, 2015; Jitendra, Dupuis, Star, & Rodriguez, 2016; Kintsch & Greeno, 1985; Wilson, 2013).

Organization

This literature review addresses the role of literacy practices in mathematics classrooms for the purpose of increasing and deepening mathematics learning and is comprised of four main components. First, this review examines Kintsch and Greeno's (1985) processing model for describing comprehension and processing of mathematics word problems into schemata and macrostructures, onto which problem solving strategies can then be applied. This schema theory serves as the overarching theoretical framework to which the rest of the literature was analyzed. Next is a review of the literature regarding the role of reading comprehension in doing mathematics. Following this review is an examination of the strategies for teaching a general mathematics curriculum to struggling readers, which includes general reading and writing development, comprehension with connection to reading and writing development, and comprehension of mathematic-specific text for learning and problem solving. Finally, strategies for teaching mathematics word problems to struggling readers are explored using the theoretical framework, followed by a review of reading and writing interventions used in elementary and middle schools to date.

A review of current practices and interventions highlights a dearth of information about successful implementation of reading and writing practices in general education inclusive high school mathematics classrooms for the goal of increasing mathematics learning.

Theoretical Framework

Processing Model

Kintsch and Greeno (1985) offer a model for the process of translating textual arithmetic word problems into macrostructures that are then acted on by problem solving strategies. During this process, they describe the reader converting a text base into an abstract-problem representation by using problem-relevant propositions and their own inferences. A proposition is a sentence that provides essential or nonessential information from the text base. Propositions and inferences are analyzed for relationship and these relations transform the text into a representation on which to use calculation strategies (Kintsch & Greeno, 1985).

The reader organizes the propositions into sets, which are the most basic structure of information in arithmetic word problems (Kintsch & Greeno, 1985). Each set has a slot for object, quantity, specification, and role that are filled by analyzing text propositions. During analysis, the reader finds propositional frames to help with sorting information into slots (Kintsch & Greeno, 1985). Kintsch and Greeno (1985) describe propositional frames as including proper names, quantity phrases (e.g. some, how many), possessive phrases (e.g. have, give, altogether), compare phrases (e.g. have-more-than, have-less-than), and indications of time (e.g. past, beginning, then, now). Figure 2.1 shows the sorting process.

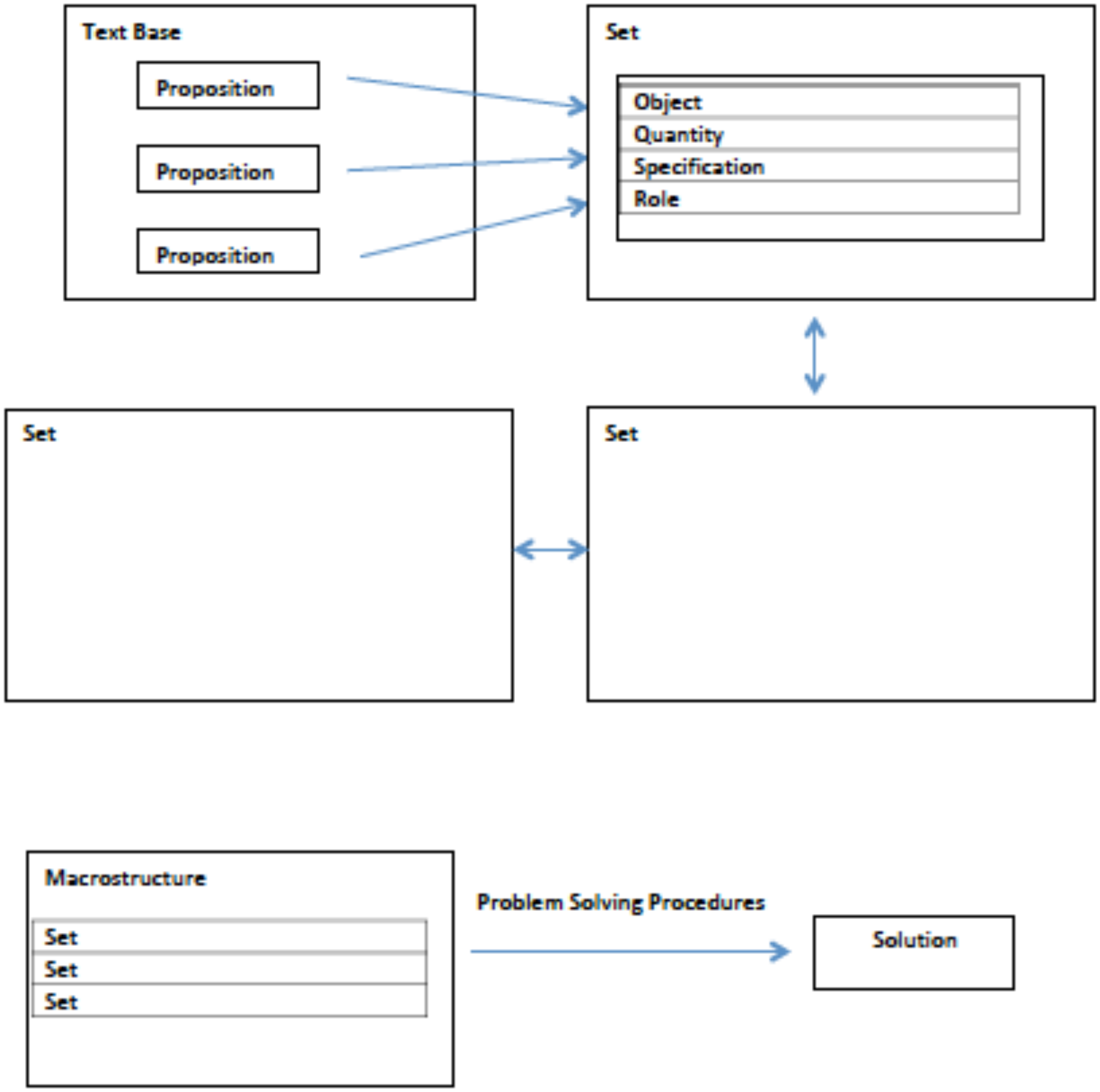

Figure 2.1. Translating textual word problems into macrostructures.

After sets are established, the reader begins to define relationships between sets in creating the macrostructure, or final representation, of the problem (Kintsch & Greeno, 1985). Kintsch and Greeno (1985) note that for arithmetic word problems there are six higher order schemata for defining relationships: transfer, part-whole, more-than, less-than, have-more-than, and have-less-than. The transfer schema begins with a start set of certain objects and a transfer set of objects of the same kind. The transfer set is given to the owner of the start set, creating the result set. The part-whole schema involves two subsets and a superset where the superset role is assigned to a set specified as belonging to the two individuals. More-than and less-than schemata have a largeset, smallset, and difference set. The have-more-than and have-less-than schemata specify the sets that should be assigned the roles of largeset and smallset (Kintsch & Greeno, 1985). Some of the propositional frame phrases above aid in assigning roles. For example, they specify "in the beginning" identifies a start set and "altogether" identifies a superset. Other assignments must come from inferences made by the reader during the comprehension process (Kintsch & Greeno, 1985).

Problem-solving procedures are used after the final representation of information is made. For arithmetic word problems, the procedures are basic counting methods: count-all, when the union of sets is counted; separate-from, when objects are removed and those that remain are counted; separate-to, when all the objects that were removed are counted; add-on, when the objects that were added are counted; and match-separate, when the members of the larger set that are not matched in the subset are counted (Kintsch & Greeno, 1985).

Reading the text base, sorting propositions, making sets, building macrostructures through strategies, and using problem-solving procedures requires a great deal of cognitive resources (Kintsch & Greeno, 1985). The most limiting cognitive resource used is the short-term buffer, which holds new propositions to be inserted into existing schema (Fuchs et al., 2015; Kintsch & Greeno, 1985). Once an existing schema for the new proposition is found and a revised schema is made, the short-term buffer discards the old schema and can accept new propositions. Poor comprehension will cause new propositions to build up in the short-term buffer without being sorted into sets and schema, causing the reader to never make it to the problem-solving stage (Fuchs et al., 2015; Kintsch & Greeno, 1985).

Self-Efficacy

Another aspect of persevering through the schematic and cognitive strategies of Kintsch and Greeno's (1985) processing model is the role of a student's self-efficacy in solving mathematics problems. Self-efficacy is defined as one's judgment of being successfully able to execute a behavior required to produce an outcome, rather than an estimate of a given behavior leading to a desired outcome (Bandura, 1977). An efficacy expectation determines how much effort one will expend in a productive struggle in the face of obstacles and aversive experiences. It is formed by observing others to see how new behavior patterns are performed, leading to response patterns and self-corrective adjustments based on performance feedback (Bandura, 1977).

Bandura (1977) described four sources of efficacy expectations – performance accomplishments, vicarious experiences, verbal persuasion, and emotional arousal. Performance accomplishments and vicarious experiences are the most pertinent sources for judging self-efficacy in the context of learning in the mathematics classroom. Performance accomplishments refer to a person's use of coping efforts in an experience perceived as threatening (Bandura, 1977). For many students of mathematics, being faced with solving word problems is a threatening experience. People with stronger self-efficacy will persevere, gaining corrective experiences that reinforce their sense of efficacy, eventually eliminating their defensive behavior (Bandura, 1977). Mathematics students who may initially be fearful of word problems sometimes give up before reading the whole problem as a defense mechanism. These struggling students need corrective experiences from teachers to help them increase their sense of efficacy and perseverance.

Students learn through observation of others, and in many classroom scenarios, the teacher provides modeling with guided practice. This modeling serves to show students how to successfully use certain skills and how to cope with setbacks. While teacher modeling is beneficial, student modeling can have a bigger impact on self-efficacy (Bandura, 1977). Bandura (1977, p. 197) states "phobics benefit more from seeing models overcome their difficulties by determined effort than from observing facile performances by adept models. If people of widely differing characteristics can succeed, then observers have a reasonable basis for increasing their own sense of self-efficacy." When a student sees their diverse group of classroom peers succeeding, their own sense of self-efficacy is likely to increase through vicarious experiences.

Bandura (1993) states that people with high levels of efficacy will set goals for themselves and provide guides and supports to meet those goals, whereas people with low efficacy focus on failure and things that can go wrong. Those with high efficacy work towards mastery of tasks, while those with low efficacy try to avoid tasks, thus it is integral to provide feedback about progress (Schunk, 2003). Once a person with high self-efficacy attains a goal that they have been working towards and sets a new, higher goal for themselves, then they have begun regulating their own learning (Bandura, 1993).

Relationship of Literacy and Mathematics

This section reviews current literature surrounding the role of literacy and reading comprehension in doing mathematics and effective literacy strategies for mathematics instruction. Strong literacy skills are important in mathematics education because children learn to read mathematics by reading words, numerals, and symbols to uncover messages (Adams, 2003). The sorting of these words, numerals, and symbols to form the overall structure of the problem is imperative to successfully doing mathematics (Kintsch & Greeno, 1985). Adams

(2003) suggests a focus on mathematical vocabulary because students need to decode word problems to gather information and answer questions. Polya (1948) developed a four-step process to solving a problem:

1. Understand the problem; see clearly what is required
2. See how the various items are connected to make a plan
3. Carry out the plan
4. Look back at the complete solution

Adams (2003) adapted Polya's (1948) four-step process for reading and solving a word problem.

1. Reads the problem in its entirety with no focus on key words or questions
2. Reads it again paying attention to vocabulary, context, setting, question, and extraneous information
3. Solves the problem
4. Looks back and checks for validity and accuracy

This has similarities to the repeated reading process of reading the text and then keying in on misread or misunderstood words before reading the passage again fluently.

Several studies have examined the role of reading performance in mathematics (Jordan, Hanich, & Kaplan, 2003; Hassinger-Das, Jordan, & Dyson, 2015; Rutherford-Becker & Vanderwood, 2009). Rutherford-Beck and Vanderwood (2009) sought to understand the extent to which reading performance related to mathematics performance by using oral reading fluency for passages and maze questions. Students read a passage aloud for one minute while the examiner scored the number of words read correctly, and then students completed a multiple-choice maze task silently. Students also took a mathematics test of basic mathematical concepts and applications on number concepts, measurement, names of numbers and vocabulary, grid reading, charts and graphs, decimals, fractions, word problems, and geometry. Rutherford-Becker and Vanderwood (2009) found that oral reading fluency and maze accounted for 17% of variance in applied mathematics. This result shows that students weak in both reading and mathematics would be good candidates for a reading intervention, as opposed to students who are weak in mathematics-only focusing on computational skills. These findings are similar to those found by Jordan et al. (2003) in their study of students with mathematics-only disabilities, reading-only disabilities, and comorbid mathematics and reading disabilities. Mathematics-only disabled students showed advantage over comorbid mathematics and reading disabled students in arithmetic combinations and story problems because they were required to comprehend the words of the problem and translate verbal information into mathematic representation. There were no significant differences in the other tested areas because calculation principles reflect a grasp of arithmetic operations, similar to Rutherford-Becker and Vanderwood (2009) finding that students weak in mathematics alone are only weak in computational skills (Jordan et al., 2003). It is likely that students only weak in mathematics, and not reading, are able to build an accurate macrostructure of the problem, but lack the computational skills needed to arrive at a correct answer (Kintsch & Greeno, 1985).

Hassinger-Das et al. (2015) studied whether providing help with number concepts in a storybook situation would improve learning outcomes for kindergarten students with low number knowledge. Children were randomly assigned to three groups; Story Number Competencies (SNC) intervention group, number sense intervention group, and business-as-usual control group. The SNC intervention included dialogic reading and direct instruction of vocabulary words repeated in different books to reinforce understanding. Students in the number sense group were specifically taught number sense without the use of storybooks. At the end of the study, test results showed that the SNC group knew a greater number of mathematics vocabulary words than peers, but did not show significant gains in mathematics achievement (Hassinger-Das et al., 2015). The SNC intervention included vocabulary instruction but did not use other reading strategies to increase fluency for developing readers. Although this work took place within a younger age group, it shows the importance of multiple exposures to and direct instruction of mathematical vocabulary in creating meaning across contexts, which helps students form conceptual relationships between words that will help them form the macrostructure of a mathematical word problem (Kintsch & Greeno, 1985; Smith & Angotti, 2012).

Through the presented research, it is recognized that reading instruction needs to be part of mathematics instruction (Phillips, Bardsley, Bach, & Gibb-Brown, 2009). One way of doing this is to use a reading intervention in the context of the mathematics classroom so students can experience the overlap of the skills needed to be functionally and mathematically literate. This method of instruction can support comorbid mathematics and reading disabled students in realizing success in both areas through improving reading comprehension.

Teaching Mathematics to Struggling Readers

In Foster's (2007) work with 4th grade students, a student shared the following about solving a mathematics word problems:

> The numbers fly around in my head…I just try to guess what would be a good answer…I block out all the words and just look at the numbers…I look for the clue words that tell me if I should add, subtract, multiply, or divide. But sometimes they aren't there and then I don't know what to do. (p. 196)

The reaction of this student, and others from Foster's class, towards mathematics word problems illustrate the sentiments of Bossé and Faulconer (2008) who found that students try to read and write as little as possible to finish homework and ignore the surrounding text when pictures or diagrams are involved. One reason that students ignore the words or context of a problem is that they are not trained in reading, and specifically, are not trained in reading mathematics text. It becomes the job of the mathematics teacher to teach reading strategies and how to apply those reading strategies to mathematical text (Foster, 2007). Capraro et al. (2012) assert that as students move into upper elementary school, they are expected to solve contextualized problems that require activation of prior knowledge, use text structure to locate and recall information, analyze and synthesize main ideas, learn new vocabulary through context, and use metacognitive strategies to monitor comprehension. Because of this shift from algorithmic solving to contextualized problems, it is important to teach broad reading strategies such as activation of prior content knowledge, mastery of vocabulary, and making sense of unfamiliar text styles (Barton et al., 2002). While some students use these strategies automatically and without prompting, struggling readers who are below proficiency levels on End-of-Grade reading tests need direct instruction in these areas. Improved reading development helps struggling students to cognitively represent complex problems en route to computing accurate solutions (Capraro et al., 2012). The following section describes how literacy practices common in English Language Arts classrooms can be used in the mathematics classroom to help students read, interact with, and understand mathematical text.

Activation of Prior Content Knowledge

Activating prior content knowledge aids students in making logical connections, drawing conclusions, and incorporating new ideas (Adams et al., 2015). Adams et al. (2015) and Barton et al. (2002) suggest using anticipation guides to help teachers determine what students know before reading and address common misconceptions and key points students will need as they read. An anticipation guide is a reading comprehension tool made up of statements related to concepts in the reading that students respond to before and after reading.

The goal of an anticipation guide for a mathematics text is to encourage students to read, think about, write, and discuss mathematical concepts meaningfully with peers and the teacher. There should be a variety of statements in the anticipation guide to provide scaffolding for students, including statements for which the answer is found in the text with different wording, statements whose answer requires interpretation of the text, statements that are true or false dependent on the case, and statements that require students to interpret examples and apply them to new situations (Adams et al., 2015).

After reading the text, responses to these types of statements should incorporate reading, writing, and communication supported by evidence from the text and discussion of decisions with peers (Barton et al., 2002). When students describe their reasoning and thinking processes to others, they engage in academic discussions which help them explore multiple viewpoints, clarify or improve their arguments, sharpen thinking, and make connections (CCSS Initiative, 2017; Ippolito et al., 2017; NCTM, 2017). Hearing multiple viewpoints can help a student incorporate new ideas into a current notion of a concept, thereby deepening understanding of that concept. Academic discussions also allow students to listen to others who may be more advanced in their arguments, providing a model for how clear thinking and reasoning should sound.

Mastery of Vocabulary

The most important reading strategy for any mathematics student is mastery of vocabulary. Matteston (2007) as cited in Capraro et al. (2012) identified a problem on a high-stakes mathematics test (e.g. End-of-Grade and End-of-Course tests) that 85% of sixth graders answered incorrectly. A single word was found to be responsible for 63% of the incorrect responses, and when changed to a more common word, only 23% of the students answered incorrectly. When reading mathematical text, it is highly important to develop meaningful, correct, and applicable definitions of mathematics terms for comprehension (Adams, 2003). Capraro et al. (2012) contrasted the impact of missing 10% of words in a narrative – where a reader would maintain adequate comprehension – to missing the same proportion of words in a mathematical text. In the mathematical text, meaning would be lost because the unknown words tend to be essential for solving the problem. Thus, it is extremely important that vocabulary instruction be a focus in teaching mathematics.

The approach to vocabulary instruction in the classroom is equally as important, as supported by research. Strategies include thoughtful lesson planning, providing opportunities for students to build definitions based on prior knowledge, and providing opportunities for students to use new vocabulary in academic discussion and writing prompts. In planning vocabulary instruction, teachers have to make decisions about which words to teach and how to teach them. Bay-Williams and Livers (2009) place academic mathematics vocabulary into three categories: words which have different use in everyday language as opposed to mathematics context, homonyms of words in everyday language, and words that may have a different meaning because of translation from another language.

Words that have a common meaning that students may already know, and also have a less common mathematical meaning, are called subtechnical words (e.g. pattern, rule, area) (Pierce & Fontaine, 2009). It is important for teachers to first explain the meaning of words in everyday language to help students build and refine definitions that draw on their prior knowledge. This type of interaction with the word encourages deep processing of the word's meaning. Students should then be provided with opportunities to encounter and practice using these words through classroom discussions and writing practices (Bay-Williams & Livers, 2009; Hamilton, 2017; Pierce & Fontaine, 2009). Technical words are those with a precise mathematical definition and should be taught explicitly to students (Pierce & Fontaine, 2009). Subtechnical and technical words are the most important to teach as they are generally the words in a text that carry meaning (Capraro et al., 2012; Pierce & Fontaine, 2009).

Smith and Angotti (2012) offer the 5 C's framework for teaching vocabulary in content area classes. This framework can help teachers decide which technical and subtechnical words need to be taught and the appropriate time to teach them during the unit. Using the 5 C's framework helps teachers identify the words students will need to understand the lesson as well as provides support for thoughtful planning of vocabulary instruction. The 5 C's are *Concepts, Content, Clarify, Cut,* and *Construct* and are described in Table 2.1.

Table 2.1. *5 C's Vocabulary Framework*

5 C's	Description
Concepts	Identify words essential to conceptual understanding of the lesson. Include new words and words that have multiple meanings.
Content	Identify non-mathematical terms that may be unfamiliar to students.
Clarify	Briefly review words not included in Concepts and Content that could cause confusion but are not crucial to the main idea of the lesson.
Cut	Remove or modify words not identified to reduce text complexity.
Construct	Plan lesson for six target words using definitions, contextual information, and timing of teaching each word.

Hamilton (2017) suggests that if the context will be new for the students, teachers should conduct vocabulary previewing activities at the beginning of the lesson and then allow students to practice using these new words in the new context. Previewing and practice may involve graphic organizers, pair-shares, and sentence starters (Hamilton, 2017). Graphic organizers, like vocabulary maps and webs, help students visualize how new ideas fit in with already familiar concepts (Barton et al., 2002). When students sort new words, they can make connections to parts of speech, recognize patterns, and discern examples from non-examples, which help to build formal definitions (Adams, 2003; Capraro et al., 2012). To aid in practicing new words, teachers should post vocabulary visibly throughout the room, encourage students to label answers, and write mathematical reflections describing their understanding of concepts (DiGisi & Fleming, 2005).

Using writing to learn mathematics is an opportunity for students to recall, clarify, and question what they already know and what they are still wondering about (Fisher & Frey, 2016). Writing in mathematics journals provides students an opportunity to practice expressing themselves using mathematical language and to make deeper connections with mathematical text (Hamilton, 2017). There are two types of writing that students can engage in: writing with and without revision. Writing without revision is a low risk way of writing because it is unedited and ungraded. This type of writing can be used to respond to prompts designed to activate prior knowledge or as a closure activity for students to reflect on a lesson (Fisher & Frey, 2016). Wilcox and Monroe (2011) suggest writing without revision in learning logs where students respond to a prompt for a few minutes before sharing out with the class. The sharing aspect of this practice is beneficial to all students because they are able to see and process multiple viewpoints and good examples of clear understanding (NCTM, 2017; Wilcox & Monroe, 2011). Outside of a mathematics journal or learning log, writing without revision can be incorporated into the note-making practice. Having students write reflections and perceptions into the notes they have taken helps them make connections between new concepts, prior knowledge, and personal experiences (Bossé & Faulconer, 2008; Wilcox & Monroe, 2011).

Writing with revision is more formal and engages students in the writing process. The writing process includes pre-writing activities, writing a draft, peer reviewing, revising, editing, writing the final draft, and publishing (Fisher & Frey, 2016). It is an iterative process that also invites students to share their work with others. Using writing with revision helps students explore a topic or concept by creating precise and descriptive writing. This is especially helpful when students are building precise definitions of vocabulary from vague ideas based on prior knowledge (CCSS Initiative, 2017; Pierce & Fontaine, 2009). Also, the repetition built in to this type of writing helps students uncover what is most important about a topic or concept (Wilcox & Monroe, 2011).

The depth of research focused on vocabulary acquisition highlights the importance of such a skill in learning mathematics. It is easy for teachers to omit words from text and curriculum materials to make reading easier for the students but doing so could take away a students' access to a deeper understanding of mathematics. New vocabulary in new contexts certainly increases the linguistic load on students, but by using careful planning, prior student knowledge, and opportunities to practice using academic mathematical language, students can build the precise definitions and context they need to be successful learners of mathematics content (Smith & Angotti, 2012).

Making Sense of Text Style

A mathematical text's unique style is another feature with which struggling readers grapple. In narrative and expository text, a main idea is often found in the first sentence of a paragraph, with the rest of the paragraph providing detail and explanation of the idea. Authors use cue words to show connections among ideas and relations among facts. Barton et al. (2002) note that in mathematical text, the first sentence of a paragraph is rarely the main idea, and many details need to be read before getting to the point of the problem. These details will be in long, complex sentences with little redundancy that the reader could use to help clarify ideas (Adams et al., 2015). Readers of mathematical text also must infer how sets of facts are related to one another in the absence of cue words.

Proper comprehension of mathematical text requires the reader to understand how the text is organized (Carter & Dean, 2006). By modeling thinking aloud, teachers can show students how they deal with mathematical text leading to comprehension (Barton et al., 2002; Bernadowski, 2016). Many students, especially struggling readers, read a text once, become discouraged by its difficulty, and give up. DiGisi and Fleming (2005) found that they had to teach their students how to read questions. Despite having adequate computational ability, they were unable to answer open-ended questions on a high stakes test because they found the text confusing. When the teacher reads texts multiple times, then students know they should do so also, thus modeling perseverance and best practices (DiGisi & Fleming, 2005; Hamilton, 2017). Bossé and Faulconer (2008) found that when teachers set the class tone by communicating clear purposes for reading and writing and providing proper scaffolds for assignments, students were more perceptive, participatory, and successful in their learning.

Strategies for Solving Word Problems

Different from the strategies described above for solving computational problems, word problems are a separate category of their own. This section provides research on schema-based and cognitive strategies specifically to aid in solving mathematics word problems. Solving mathematics word problems requires students to create a problem model from textual input before solving (Moran et al., 2014). “A word problem requires a student to construct a problem model by identifying what information is missing, determining the number sentence that incorporate the given and the missing information, and deriving the calculation problem for finding the missing information" (Fuchs et al., 2008, p. 156). The process reflects Kintsch and Greeno's (1985) processing model for translating propositional text base into a macrostructure that can be acted on by problem-solving strategies. Strong reading comprehension and perseverance are essential to this process, because the problem solver will need to read a problem presented in the context of a story or real-life scenario, decipher the problem situation, and represent the situation accurately before performing calculations (Adams, 2003; Fuchs et al., 2015). This build-up distinguishes word problem solving from simply computational problems. Students with adequate computation ability still struggle with word problems because they require the additional skill of understanding text-based statements to extract relevant information for the problem model (Capraro et al., 2012; Fuchs et al., 2015; Jitendra et al., 2016).

There are two ways that educators have traditionally taught students word problem solving skills: cognitive strategy instruction and schema-based instruction. Cognitive strategy instruction focuses on instruction in self-regulation, self-awareness, and metacognition while solving mathematics word problems (Capraro et al., 2012; Pape & Smith, 2002). Students take ownership of their learning by thinking about what they are doing and how they are doing it while reading and problem solving. This includes the approach of thinking out loud to achieve self-awareness of strategy use (Bernadowski, 2016). A prevalent cognitive strategy is the keyword method, in which students pick out keywords to determine the correct operation to use.

Schema-based instruction focuses on building conceptual understandings by teaching students to use schema-based strategies (Capraro et al., 2012). This type of instruction differs from the metacognitive approach because teachers "explicitly teach students to group problems into types with similar underlying mathematical structures and teach students problem-solution rules for each problem type" (Fuchs et al., 2008, p. 157). Grouping problems into problem and solution types requires students to break down word problems into components, or propositions and schema, to see structural similarities and differences (Wilson, 2013).

Effective word problem strategy instruction tends to use elements of both cognitive and schema-based approaches. Students organize information and analyze problem types and solutions while asking themselves questions for further clarification and self-regulation. One example of an instructional strategy that takes elements of both cognitive and schema-based approaches is The Math Frame, which "guides students to make sense of mathematics, reason abstractly, provide arguments and models, and attend to precision, structure, and repeated reasoning" (Wilson, 2013, p. 38). It also helps teachers assess student work by identifying the step with which a student is struggling. Table 2.2 shows the steps students take when using The Math Frame and potential difficulties for struggling readers.

Table 2.2. *The Math Frame*

Question	Purpose	Potential Difficulties for Struggling Readers
What information is given?	Pick out important facts and disregard extraneous information.	Difficult for students with limited vocabulary, poor attention to detail, poor organizational skills, weak comprehension and decoding.
What are you asked to solve?	Finding the problem statement, deciding what is known, what is unknown, and what information is needed to solve the problem.	Difficult for students who have trouble understanding that information to solve the problem is missing from the stated problem.
What strategy might help you solve this problem?	Visual depiction using drawings or tables to concretize problem and possible solutions.	
What steps are needed to find the solution?	Write a descriptive progression of steps to guide critical thinking about components of text base.	Difficult for students who struggle with writing thoughts into words using academic language.
What calculations are needed to find the solution?	Set up and compute problem.	Problem set up is difficult for students who miss the relationship between information in the text.
What is the solution to the problem? Does it make sense?	Compare the solution to the problem statement and check for reasonability.	Difficult for students with poor metacognitive awareness, contextual reasoning, or lack of background knowledge.

Cognitive strategy instruction prompts students to ask themselves self-regulating and metacognitive questions while using The Math Frame. While asking these questions, students are categorizing problem and solution types described by schema, finding a strategy that might help to solve the problem, and writing a description of the progression of steps needed. Teachers can help struggling readers and problem solvers by scaffolding steps of The Math Frame, including rewording and leaving out extraneous information, asking students to cross out extraneous information and circle important information, using manipulatives to concretize the problem, providing partial steps and calculations, and having students circle their answer and the question for comparison. The Math Frame extends beyond arithmetic word problems— free of extraneous information—described in Kintsch and Greeno's (1985) processing model and can be used for complex problems at every grade level (Wilson, 2013).

Survey, Question, Read, Question, Compute, and Question (SQRQCQ) is another instructional model, similar to The Math Frame, which uses a mixture of cognitive and schema-based strategy (Barton et al., 2002). This strategy is provided as a graphic organizer. First, the student reads the problem quickly for a general understanding, then asks herself what information the problem requires. This is followed by re-reading the problem to decipher relevant and irrelevant information. Subsequently, the student must ask herself what must be done to solve the problem. It is likely here that she would categorize the problem and solution by type before computing the answer. After computations, the student asks herself whether her solution process was correct and if the solution is reasonable (Barton et al., 2002). The self-questioning built into this model allows students to self-regulate by constantly checking for understanding while tackling a word problem. When understanding breaks down, the student can always go back to the previous step for reinforcement. Similar to The Math Frame, this type of instructional model can be used for more complex problems than basic arithmetic word problems. As students move through the mathematics curriculum, they are asked to consider increasingly more complex and real-world word problems. However, most research on struggling students has focused on arithmetic and arithmetic story problems (Fuchs & Fuchs, 2002; Jitendra et al., 2016). Complex story problems differ from arithmetic story problems in that they include nonessential details, but no irrelevant numbers, and require one to three step operations. Real-world problems add extended text with nonessential detail and irrelevant numbers for multiple, related questions in an introductory narrative remote from the questions (Fuchs & Fuchs, 2002). The addition of nonessential details and text complexity adds to cognitive demand because the reader has more propositions to sort into sets, as well as more information to hold in the short-term buffer (Kintsch & Greeno, 1985).

The following analyses of studies of elementary and secondary school mathematics instruction highlight the foundational roles that reading comprehension and perseverance has in successful completion of arithmetic and complex and real-world word problems.

Elementary School Studies

In this section, studies in the context of elementary school literacy-based mathematics instruction are presented. Many elementary age students enter school already behind in reading and mathematics skills, thus there are many interventions targeted at elementary school students (ACT, 2012). Literacy-based instruction in young students focuses on phonics and starts to incorporate vocabulary and comprehension instruction, which is more beneficial for older students (Roberts et al., 2008). The following discussion presents studies which highlight successful literacy-based mathematics interventions for elementary school students where the focus is on arithmetic word problems.

Children's literacy and basic reading skills. Hassinger-Das et al. (2015) investigated kindergartners with low number knowledge who were taught early number concepts and vocabulary through an intervention called Storybook Number Competencies (SNC). Proficiency in counting, number relations, and number operations in kindergarten is highly predictive of mathematics problem solving through third grade and middle school, so it is important to support students in optimizing proficiency at an early age. SNC uses dialogic reading – students explain in their own words, answer probing questions, and engage with illustrations – to help children learn new vocabulary by attaching personal meaning to words. This is an example of activating prior knowledge to make personal connections with vocabulary (Fisher & Frey, 2016). By using storybooks for instruction, children are exposed to mathematical vocabulary multiple times in non-mathematics related contexts (Hassinger-Das et al., 2016). Pierce and Fontaine (2009) and Smith and Angotti (2012) support the importance of multiple exposures to vocabulary across multiple contexts.

Kindergartners identified with early numeracy difficulties were randomly assigned to three groups: SNC intervention group, a number sense intervention group without storybooks, and a "business-as-usual" group. Children in the SNC group were exposed to seven different storybooks covering 34 total mathematical vocabulary words. The vocabulary words were learned through dialogic reading, explicit instruction, and guided play as review of the words and definitions.

On delayed posttest of the vocabulary words, the SNC group outperformed both the number sense and "business-as-usual" groups, demonstrating better understanding of "more than" and "less than," both being concepts that are essential to Kintsch and Greeno's (1985) processing model for solving arithmetic word problems. These children developed deeper understanding of number concepts and vocabulary—both general and contextual definitions—through interaction with storybook text.

Adams and Lowery (2007) examined more specific use of reading skills in children reading mathematical text and a children's trade book. The purpose of this study was to see how two 4th grade children, on different reading levels, exhibit elements of text comprehension and understanding of mathematics concepts when reading children's literature and their own mathematics textbook. Both students were asked to read a trade book first while the researchers asked them to pause in specific places to answer questions about the story. Then each student read a page aloud from their textbook including word problems and multiple-choice items. The student who was classified as reading above grade level started by reading the title page and the first page of the book, but read the text without prosody as one long sentence. She also replaced words she did not know with other words that did not necessarily have the same meaning. Despite this type of reading, she answered comprehension questions accurately and showed evidence of understanding the underlying mathematics concepts in the trade book, but did not use academic language to describe her understanding, which Ippolito et al. (2017) suggests shows contextual understanding of mathematics concepts. In reading the textbook selection about area and perimeter, with given formulas, she drew a picture, labeled the picture, and did some calculations reflecting that she understood the problem. However, she did not use the given formula precisely. Labeling her drawing helped convey her level of understanding of the concepts in the word problem (Adams & Lowery, 2007). According to DiGisi and Fleming (2005), labeling drawings is a way of showing understanding and comprehension of the problem base.

The student classified as an on grade level reader started the trade book by looking at the cover to predict what the book would be about before flipping directly to the first page of the story. While this student read, he attended to the illustrations. Similar to the above grade level student, he replaced some words, but also skipped other words he did not know. These actions did not seem to affect his comprehension of this particular story. However, when using the same strategies in the reading of the textbook, he ended up losing context of the problems and created his own version of the problems leading to incorrect calculations on multiple-choice items. Kintsch and Greeno (1985) would describe this as a breakdown in the building of sets and a representative macrostructure for these problems. For the area and perimeter problem he did not draw a picture, but did use correct calculations to arrive at a correct numeric answer without a label for units. While both students demonstrated some weaknesses, strength in reading comprehension did seem to lead to deeper conceptual understanding, especially when solving contextual mathematics problems.

Role of comprehension in word problem solving. Moran et al. (2014) assessed the impact of writing out propositions from word problems on solution accuracy in children with mathematics disabilities (MD). This study draws directly from Kintsch and Greeno's (1985) model by translating a problem text into a proposition-based representation. Additionally, writing out the propositions by restating and paraphrasing information could improve comprehension of the overall problem statement. To assess impact, Moran et al. (2014) separated a group of 3rd graders at risk for MD into three intervention groups: students in the 'restate condition' were instructed to only rewrite the question sentence in their own words; students in the 'relevant condition' rewrote all relevant, as opposed to irrelevant, propositions; students in the 'complete condition' rewrote all propositions with separation between relevant and irrelevant information. All problems were one-step word problems representing total, difference, and change problem types with the number of sentences in each word problem increasing across lessons. Scaffolding was provided to the intervention groups to help them learn this schema-based instructional strategy through repeated readings, explicit questioning, and corrective feedback to eventually solve problems independently.

The 'complete proposition' condition had the largest effect size of word problem solving performance requiring students to read problems independently. The 'complete' and 'relevant' proposition conditions had similar effect sizes on a measure of oral word problem solving. These two results suggest that access to a reader during testing may benefit students at risk for MD. The larger effect sizes of the 'complete' and 'relevant' proposition conditions compared to the 'restate' proposition condition shows that only restating the problem does not prompt students to examine information for relevance. Subsequently, considering a greater number of propositions influences text comprehension, leading to greater accuracy in answers (Moran et al., 2014).

Fuchs et al. (2015) sought to determine if word problem solving is a form of text comprehension, based on the fact that reading comprehension predicts development of both text comprehension and word problem solving separately. Fuchs et al. (2015) rely on Kintsch and Greeno's (1985) model that shows that word problem solving involves an interaction between language comprehension processes and problem-solving strategies that both depend on working memory and reasoning. Working memory span, nonlinguistic reasoning—being able to distinguish instances from non-instances of a class—and general language comprehension are all determinants of text comprehension. In order to determine if word problem solving is a form of text comprehension, the effects of these factors were measured on word problem solution accuracy in 2nd grade students. Additionally, Fuchs et al. (2015) considered whether the effects of these abilities are mediated by comprehension of word problem-specific language constructions.

To measure the effects of the cognitive and linguistic abilities on word problem solution accuracy, students were read 18 problems representing combine, compare, and change schemas with missing information in the first, second, and third positions (e.g. $x \pm y = z$), with and without irrelevant information. These problems were scored for correct mathematical computation and labeling of information, which reflects processing of the word statement and understanding of the problem's theme (Fuchs et al., 2015). DiGisi and Fleming (2005) support the use of labeling information in the problem and diagrams to show understanding. Text comprehension was measured by students pointing to pictures on the page that represented words, silently reading a sentence or passage, and identifying the missing words. These measures assessed the understanding of the propositional text structure and ability to build the situation model by using inferences based on prior background knowledge (Kintsch & Greeno, 1985).

In determining the effects of working memory span and nonlinguistic reasoning on word problem solution accuracy, findings showed that working memory's role in word problem solving and text comprehension is mediated by syntactic constructions that signal to the reader which object in a text is bigger (Fuchs et al., 2015). Being able to decipher these constructions is essential to building sets from propositions (Kintsch & Greeno, 1985). Working memory was not taxed by language that signals whether a word problem belongs to the compare or change schema. Due to mediation on working memory, Fuchs et al. (2015) suggest interventions designed to increase working memory span in the context of reading tasks that tax working memory. Nonlinguistic reasoning predicted both word problem solving and text comprehension ability suggesting that the ability to formulate rules about classes of objects plays a role in text comprehension beyond general language comprehension. General language comprehension was partially mediated by word problem-specific language in predicting word problem solving, suggesting that interventions focusing on word problem-specific language comprehension could be useful (Fuchs et al., 2015). DiGisi and Fleming (2005) addressed teaching procedural words and test vocabulary to help students develop fluency in mathematical text comprehension.

These results support the idea that word problem solving is a form of text comprehension. Due to this relationship, interventions focused on developing at-risk children's text processing may simultaneously benefit word problem solving and text comprehension. Furthermore, Fuchs et al. (2015) suggest exploring the effects of schema-based instruction on children's strategies for handling word problem-specific language in problems.

Improving comprehension for solving word problems. Development of word problem solving ability involves learning academic language and effective strategies for comprehending word problem passages, and then using this language and these strategies with peers and teachers to express understanding (Kong & Orosco, 2016). In order to improve comprehension, Kong and Orosco (2016) used Instruction Scaffolding (IS) with at-risk minority students. IS is defined as "an interactive process of learning that is constructed by teachers by gradually decreasing instructional support as students develop the skills necessary to become independent" (Kong & Orosco, 2016, p. 171). Kong and Orosco (2016) used IS principles to develop the Dynamic Strategic Math (DSM) intervention defined as the "tester modifying word-problem solving via a four-level vocabulary modification procedure to the students' level of word-problem-solving cognition, and then providing intervention with probes that assessed students' word-problem-solving ability" (Orosco, 2014, p. 45). Table 2.3 shows the four levels of vocabulary modification used in the study.

Table 2.3. *Levels of Vocabulary Modification*

Level	Description
1	Basic one-step word problems. Incorporate basic language used in everyday discourse.
2	Basic one-step word problems. Incorporate complex language by including irrelevant information.
3	Multi-step word problems. Incorporate basic language used in everyday discourse.
4	Multi-step word problems. Incorporate complex language by including irrelevant information.

Third graders were taught DSM and were considered adept at their level when they did not require any prompting in solving the word problem. The intervention took place in three phases. During Phase 1, instructors pre-taught concept and vocabulary using explicit instruction and modeling. Pre-teaching involved providing definitions and pictures linking student's prior knowledge, contextualization, and applied use of the vocabulary while solving problems. Phase 1 employed the principles of activating prior knowledge supported by Adams (2003) and using academic discussion to deepen understanding as shown by Ippolito et al. (2017).

In Phase 2, instructors taught seven problem-solving strategies: *Know, Find, Cross-Out, Draw a Picture, Math Words and Numbers, Set Up and Solve,* and *Check*. These strategies combine cognitive and schema-based instruction similar to The Math Frame (Wilson, 2013) and SQRQCQ (Barton et al., 2002). During Phase 3, students participated in cooperative learning where they checked each other's answers and improved understanding by asking questions.

The intervention was found to increase word problem solving comprehension by supporting necessary mathematical vocabulary and conceptual knowledge. All students started at a Level 1 despite having the basic computation skills to solve the problems correctly. Students who rose to a Level 3 or 4 increased reading comprehension to help them solve multi-step problems with complex language. Similar to the findings of Fuchs et al. (2015), these results support teaching reading comprehension strategies in mathematics content to teach word problem skills to students at-risk for MD.

Fuchs et al. (2008) also extended research in schema-based instruction by exploring the efficacy of a secondary preventative tutoring protocol on mathematics word problems using the RUN strategy. Students in this study were also in 3rd grade and were at-risk for comorbid mathematics and reading disabilities. The RUN strategy guides students to *Read* the problem, *Underline* the question, and *Name* the problem type. In order to name the problem type, the student must understand the underlying mathematical structure of the problem and recognize basic schema, which is the basis of Kintsch and Greeno's (1985) processing model. Following use of the RUN strategy, students would have to solve the problem type and transfer these skills to seemingly novel problems. Types of problems addressed in the tutoring protocol were one-step problems requiring addition and subtraction of single and double digit numerals, with and without irrelevant information, with and without relevant information embedded in charts, graphs, and pictures, and with missing information in all 3 positions.

Students were randomly assigned to receive tutoring. To start, students were given instruction for two weeks in foundational arithmetic skills, algebraic equation solving skills, and general mathematics problem solving strategies, which included checking answers for reasonability and labeling work with units. Next, tutors taught each problem type (total, difference, and change) independently using the RUN strategy and used concrete materials and role-playing to emphasize underlying mathematical structures. Algebra equation templates were posted in the room to help students recognize problems by underlying structure, flashcard sorting was used to help students identify problem type, and solution rules were taught for each problem type. Tutors also taught students how irrelevant information, charts, graphs, or pictures, and missing information in the first or second position of the equation presented in the problem narrative all can make a problem novel.

Students who received 15 hours of tutoring improved significantly on both a test of word problems with charts and graphs, and a test of story problems including total, difference, and change problem types, with missing information in all three positions. There was no statistically significant change in test scores on the Iowa Test of Basic Skills: Problem Solving and Data Interpretation despite similarity to one of the word problem tests. This may be explained by the multiple-choice format of the Iowa test suggesting students may need to be taught this during tutoring sessions.

This work by Fuchs et al. (2008) used the Kintsch and Greeno (1985) processing model for arithmetic word problems and extended it by including problems with and without irrelevant information and charts and graphs. This type of intervention included both cognitive strategy instruction and schema-based strategy instruction and was effective for students with both mathematical and reading difficulties.

Powell, Fuchs, Fuchs, Cirino, and Fletcher (2009) acknowledged that students with MD perform worse on word problems than peers without MD and examined how the features of word problems differentially affected problem difficulty as a function of MD status. This study also drew from Fuchs et al. (2008) using total, difference, and change problems with missing information in the first, second, and third positions with 3rd grade students. In addition, this study included students classified as no-MD, MD-only, and comorbid mathematical and reading disabilities (MDRD) to determine differences in performance as a function of MD status.

To measure ability, an examiner read similar word problems to the ones used in Fuchs et al. (2008) to students who followed along silently on their own tests. Students were able to ask the examiner to read a problem again before moving on to the next problem. MD-only students found total and difference problems to be comparably difficult and change problems to be easier than total problems. However, among MDRD students, change and difference problems were comparably difficult and total problems were easier than difference problems. One reason that total problems may be easiest for MDRD students is that these students tend to always add in word problems. Difference problems contain the words ‘more’ and ‘less,’ which are statements of relationship that should help students build sets and schema (Kintsch & Greeno, 1985). Change problems require students to distinguish between addition and subtraction, which is also built on the relationships between sets. These findings could explain the success of Fuchs et al.’s (2008) schema-based preventative tutoring with MDRD students. Additionally, there is support for Fuchs et al.’s (2015) claim that comprehension predicts word problem solving ability because MD-only students outperformed MDRD students on change problems, which require students to pay attention to and understand a narrative before distinguishing between addition and subtraction. There was no significant correlation between MD status and the position of missing information, suggesting that interventions should focus on comprehension of narratives and problem/solution type strategies rather than changing position of missing information.

Fuchs and Fuchs (2002) conducted a similar study, this time including arithmetic story problems, complex story problems, and real-world problem solving to describe functional performance of 4th grade students with MD-only and comorbid MDRD. Measurement of performance for arithmetic story problems was based on correct computation as well as correct numbers and operation. Complex story problems were scored for accurate mathematical work for the numbers used, correct problem solving strategies (e.g. finding relevant information), and correct operation. Real-world problem solving was scored for finding relevant information, correct work, correct operation, working the answer in distinct multiple parts, labeling at least half of the multiple parts, and labeling with units.

Total scores indicated that MD-only students answered correctly on 75% of arithmetic story problems as compared to 55% amongst MDRD students. For complex story problems, these percentages dropped to 14% and 8%, respectively. This was a large drop for both sets of students suggesting that the addition of more complex operations and narrative has a similar effect for both disability statuses. Scores for real-world problem solving were also quite low; 12% for MD-only and 5% for MDRD. Such low scores for both complex and real-world problems may suggest a floor effect for students at this age for these types of problems (Powell et al., 2009). These results point to a great need for implementation of interventions for students with MD and comorbid MDRD in complex and real-world problem solving tasks. Table 2.4 summarizes the studies presented in elementary school settings.

Table 2.4. *Summary of Studies: Elementary School*

Researcher/Author	Study	Key Findings	Date
Hassinger-Das et al.	Reading stories to learn math: Mathematics vocabulary instruction for children with early numeracy difficulties	• Storybook Number Competency group demonstrated better understanding of "more than" and "less than" • Developed deeper understanding of number concepts and vocabulary—both general and contextual definitions—through interaction with storybook text • No statistically significant gains in numeracy	2015
Adams and Lowery	An analysis of children's strategies for reading mathematics	• Above grade level reader replaced unknown words without changing meaning of story, used live experiences to help answer comprehension questions, drew pictures and used calculations reflecting understanding of problem • On grade level reader replaced and skipped unknown words which changed context of problem, did not draw a picture or label units, but did use correct calculations	2007
Moran et al.	The effects of paraphrasing interventions on problem-solving accuracy for children at risk for math disabilities	• Rewriting question and separating needed number sentences and irrelevant sentences had the most positive effect on scores • Distinguishing among a greater number of propositions of the problem during problem translation appears to lead to a greater degree of text comprehension	2014
Fuchs et al.	Is word-problem solving a form of text comprehension?	• Word problem solving is a form of text comprehension • Focus on word problem specific language and constructions rather than computation • Need to work on deliberately increasing working memory span in context of reading tasks	2015

Table 2.4. *Continued*

Researcher/Author	Study	Key Findings	Date
Kong and Orosco	Word-problem-solving strategy for minority students at risk for math difficulties	• Instruction scaffolding by pre-teaching concepts and vocabulary, explicitly teaching common problem-solving strategies, and cooperative learning helped students increase word problem difficulty and solution accuracy • Students has calculation skills needed to solve problems accurately, but required support in mathematical vocabulary and conceptual knowledge to solve correctly	2016
Fuchs et al.	Effects of preventative tutoring on the mathematical problem solving of third-grade students with math and reading difficulties	• Students given schema-broadening instruction RUN to a) understand underlying structure of problem type, b) recognize basic schema for problem type, c) solve problem, d) transfer to novel problems improved significantly on Story Problems and Peabody Word Problems	2008
Powell et al.	Do word-problem features differentially affect problem difficulty as a function of students' mathematics difficulty with and without reading difficulty?	• Problem type (i.e. total, difference, change) differentially affects performance of MD-only and MDRD students • Position of missing information did not significantly interact with MD status	2009
Fuchs and Fuchs	Mathematical problem-solving profiles of students with mathematics disabilities with and without comorbid reading disabilities	• When problems were read aloud and students had a copy of the text, MD-only students outscored MDRD students in arithmetic story problems potentially due to greater access to the text • Increased narrative and complexity affected MD-only and MDRD students similarly	2002

Secondary School Studies

In this section, studies in the context of secondary school literacy-based mathematics instruction are presented. The following studies presented highlight successful literacy-based mathematics interventions for secondary school students focusing on complex story problems and real-world problem solving requiring deeper analysis of mathematical text (Barton et al., 2002; Fuchs & Fuchs, 2002).

Features of difficult word problems. Parmar, Cawley, and Frazita (1996) hypothesized that students with mild disabilities would have trouble with arithmetic word problems of varying structure because of unfamiliarity with the language scheme involved in these problems. Barton et al. (2002) acknowledged that students need explicit instruction to learn how to understand mathematical text style, because it is unique from the narrative and expository texts more frequently studied. Students who participated in this study ranged from 3rd to 8th grade, thus including students at the middle school level. The varying structures of arithmetic word problems were comprised of all four arithmetic operations, direct and indirect problem statements, extraneous information, and requirement of one and two steps to reach a solution. However, the problems were written to require minimal reading, allowing Parmar et al. (1996) to focus on the effects of the structure rather than reading ability.

Results showed a considerable drop in performance from addition into the other arithmetic operations, even in middle school students. This differs from the findings of DiGisi and Fleming (2005) and Jitendra et al. (2016), who concluded that their students had adequate computation ability but struggled with word problems because the text was confusing. Fuchs et al. (2008) did not assume that their students had adequate computational ability and thus built it into their preventative tutoring protocol. Therefore, it may not be safe to assume that the computational abilities of older students are sound enough to ignore arithmetic and algebraic instruction in interventions. Teachers should consider this when designing and implementing a word problem intervention. Additionally, scores on problems with extraneous information were considerably lower for students with disabilities, and these students did not improve at the same rate as their peers without disabilities across grade levels. These results reflect those of Fuchs and Fuchs (2002) in that performance, regardless of student disability type, dropped considerably when nonessential details were included. Unfamiliarity with mathematical text structure including extraneous information taxes the short-term buffer that students use to help sort propositional text into sets to form the macrostructure of the word problem (Kintsch & Greeno, 1985).

While Parmar et al. (1996) sought to determine the cause of difficulty in arithmetic word problems in grades 3-8, Crisp (2015) studied features of exam questions on the General Certificate of Secondary Education exams that students take in high school in the UK. Specifically, she was interested in features that may make problems more difficult for students with disabilities. As these are high school level exams, the problems are more complex than the arithmetic word problems typical of elementary school curriculums. Crisp (2015) identified students likely to have reading difficulties, who were allowed to have a reader during exams, and explored the differential effects of the features of exam questions on those students.

Students who had access to a reader performed relatively well on questions in which the task was clearly defined in the final instruction, there was minimal reading and simple vocabulary, and diagrams were easy to interpret. Conversely, these students performed relatively poorly when asked to complete a sentence based on textual information provided in a table and on questions that required transforming a number expressed in words into figures. This may be the case because students are not able to form a correct macrostructure of the problem, thus they do not understand the overall concept that the text presents (Kintsch & Greeno, 1985). Crisp (2015) concluded that features that affect difficulty of exam questions may include amount of reading, difficulty of vocabulary, use of diagrams, rearranging information from a table, converting a number from words to figures, and algebraic equations involving subtraction. The number of sentences and propositional statements that are included in the text-base determines the amount of reading. Working memory would be taxed for students with reading difficulties when they attempt to fit these propositions into sets and schema to build the problem model (Fuchs et al., 2015; Kintsch & Greeno, 1985; Swanson & Beebe-Frankenberger, 2004). Knowledge of precise definitions of mathematical vocabulary can also greatly affect performance because these words typically carry essential information for solving the problem (Capraro et al., 2012). Bossé and Faulconer (2008) acknowledged that students often do not know how to relate textual information with information found in diagrams. These are all reasons that support Crisp's (2015) findings. The difficulties of translating numbers from words into figures and in algebraic equations using subtraction may be a point for further research with a population of older students.

Intervention strategies in the classroom. While many teachers acknowledge that reading comprehension is an integral part of solving word problems, some are reluctant to include this type of instruction in the classroom. One barrier is a lack of knowledge of how to effectively integrate reading instruction into mathematics instruction. Carter and Dean (2006) examined whether mathematics teachers at a summer mathematics clinic incorporated reading strategies for decoding, vocabulary, and comprehension into their lessons for students ranging from grades 5-11. Furthermore, they examined how these strategies, when used, helped students understand mathematical concepts. Instructors did not know they were being monitored for inclusion of reading strategies in their lessons.

The study found that when there was text to read, three of the instructors consistently read to the students, three instructors consistently prompted students to read aloud, and the other two instructors combined those techniques. Because the instructors read to the students most of the time, instruction in decoding was seldom observed. There were several instances of instruction focused on building vocabulary, suggesting that teachers understand the importance of vocabulary knowledge in understanding mathematics. This type of instruction was commonly manifested in the student building a definition from a vague notion into a precise definition through exploration and teacher questioning.

Studies by Jitendra et al. (2016), Montague, Enders, and Dietz (2011) and Bernadowski (2016) show how interventions in cognitive and schema-based strategy instruction helped students improve word problem solving ability. Bernadowski (2016) explored a cognitive reading strategy called ‘think-aloud’ with at-risk middle school students. This strategy was based on the teacher modeling what good readers do when they are reading by sharing their thought process out loud. Students were taught to verbalize their thoughts while reading and solving word problems and writing their thoughts, questions, and conclusions in mathematics journals. The mathematics teacher taught students the writing process—pre-writing, drafting, revising, and publishing—that is normally taught in language arts classes. According to Bossé and Faulconer (2008), teachers should model how to discuss read and written mathematics materials in the process of learning. Writing with revision in mathematics journals can help students create more precise and descriptive writing, in turn allowing students to acquire deeper understandings of mathematical concepts (Wilcox & Monroe, 2011). Both Bernadowski (2016) and Ippolito et al.

(2017) found that students who employed the think-aloud strategy and wrote reflectively in mathematics journals were more positive in their attitudes toward writing and had more productive discussions about mathematics with each other through describing reasoning and thinking processes. The teacher's modeling of thinking aloud and writing along with student observation of peers' successes using these strategies may have reinforced self-efficacy among students, pushing them to further engage with the mathematics and each other (Bandura, 1977).

Jitendra et al. (2016) used Standards Based Instruction (SBI) for proportional problem-solving with 7th grade MD-only and MDRD students. SBI combines metacognitive and schema-based strategies and incorporates four instructional practices: 1) model problem solving using DISC, 2) activate mathematical structure of problems, 3) use diagrams to represent information, and 4) develop procedural flexibility. The acronym DISC stands for *Discover* the problem type, *Identify* information to represent in a diagram, *Solve* the problem, and *Check* the solution. Discovering the problem type and representing information in a diagram reflects Kintsch and Greeno's (1985) processing model for solving arithmetic word problems. Understanding the relationship between propositions in the text base allows the student to make a model on which solution strategies may be applied (Kintsch & Greeno, 1985). This is also a metacognitive approach because teachers are instructed to ask students probing questions at each step. Another metacognitive feature of instruction is in development of procedural flexibility, as students are taught to look for methods of solving that are more efficient than others. In teaching an intervention group the SBI curriculum, Jitendra et al. (2016) sought to test the curriculum's effect on problem solving performance for students with MD relative to students with MDRD on immediate and retention posttests as well as a transfer assessment. Fuchs et al. (2008) also tested transfer skills in their exploration of the effectiveness of the RUN strategy intervention.

After completion of the SBI curriculum instructed by general education mathematics teachers, students took a posttest comprised of multiple-choice and short response questions graded with a rubric emphasizing correct reasoning. Kong and Orosco (2016) and Fuchs and Fuchs (2002) used similar scoring practices. The same test was also administered six weeks post intervention to test retention. In addition, students also took a test of transfer skills comprised of items not directly aligned with the content covered in the intervention.

Results showed that both MD and MDRD students who participated in the intervention made significant gains on the posttest and maintenance test. There were no statistically significant effects on the scores of the transfer test. Gains on the posttest and maintenance test may have been due to appropriate scaffolding, as highlighted by Kong and Orosco's (2016) IS intervention. Also, use of diagrams and checklists helped to reduce cognitive load on students by keeping the short-term buffer open to sort new propositions and reduce taxation on working memory (Fuchs et al., 2015; Kintsch & Greeno, 1985). Students may have also started realizing their own gains in achievement during the scaffolding process. This performance achievement combined with any vicarious experiences through seeing peers succeed may have enhanced self-efficacy for doing mathematics (Bandura, 1977).

Montague et al. (2011) were also interested in an intervention that could be given by general education mathematics teachers in an inclusive classroom. The intervention chosen in this study was *Solve It!*, a technique combining schema and cognitive strategy, focusing on teaching students the processes and strategies need to represent mathematics problems via paraphrasing and visualization. Moran et al. (2014) used the paraphrasing technique and found that the more propositions a student considered, the better understanding they had of the problem. *Solve It!* incorporates explicit instruction practices like cueing, modeling, rehearsal, and feedback. The feedback portion of the intervention allows students to track their progress, promoting metacognition in self-monitoring. *Solve It!* employs teacher modeling, group practice, individualized feedback, and progress monitoring which provide performance accomplishments and vicarious experiences. These two factors are known to bolster self-efficacy (Bandura, 1977). As students begin to see increased achievement on tasks, their self-efficacy for continued learning is enhanced (Schunk, 2003). Students with high efficacy work towards mastery of tasks while those with low efficacy try to avoid tasks, thus it is integral to provide feedback to students about their progress.

Krawec and Montague (2014) used the *Solve It!* intervention with 7^{th} and 8^{th} grade students with processing difficulties, limited cognitive strategies, low motivation, and low self-efficacy as part of a 3-year study. Three full days of explicit instruction were spent on introduction of the routine Read, Paraphrase, Visualize, Hypothesize, Estimate, Compute, Check (RPV-HECC) with Say, Ask, Check, establishing student mastery of the acronym, providing process modeling support, and supporting students as they worked both in groups and individually. The explicit instruction was followed by weekly practice sessions using the routine and monthly progress of the intervention group using Curriculum-Based Measures (CBMs).

Montague et al. (2011) found that students who received the intervention made significantly greater improvement in mathematics problem solving as compared to those in the control group. Differentially, students with learning disabilities consistently scored below low-achieving and average-achieving students; however, the intervention had a uniform impact across ability levels. Scores of students in the control group did not change appreciably over the school year, and interestingly, by the end of the year students with learning disabilities in the intervention group outperformed all students, regardless of ability, in the control group. Both students and teachers thought the feedback on student performance between CBM administrations was a helpful component of this intervention because it kept students motivated and allowed teachers to provide additional support where needed. Jitendra et al.’s (2016) and Montague et al.’s (2011) interventions, using cognitive and schema-based strategies, helped students with learning disabilities increase achievement in general education inclusive mathematics classes. Table 2.5 summarizes the studies presented in secondary school settings.

Table 2.5. *Summary of Studies: Secondary School*

Researcher/Author	Study	Key Findings	Date
Parmar et al.	Word problem-solving by students with and without mild disabilities	• Students with disabilities performed as a considerably lower rate in problems with extraneous information • Need analysis and interpretation instruction rather than cue word memorization for complex problems	1996
Crisp	Exploring the difficulty of mathematics examination questions for weaker readers	• Strong performance of weaker readers on questions with minimal reading, easy vocabulary, straightforward diagrams, and short, clear instructions • Features that may affect difficulty of items: amount of reading required, vocabulary, complexity of diagrams, rearranging information from a table, algebraic equations involving "subtraction from"	2015
Carter and Dean	Mathematics intervention for grades 5–11: Teaching mathematics, reading, or both?	• Instructors understand importance of vocabulary knowledge in understanding mathematics • Students need more opportunity to read text on their own for teachers to diagnose difficulties	2006
Bernadowski	"I can't evn get why she would make me rite in her class": Using think-alouds in middle school math for "at-risk" students	• Students became positive about ability and attitudes toward writing in mathematics after instruction and guided practice • Think-aloud process modeled by teacher helped students model behavior and reflect on their own interaction with mathematical text	2016

Table 2.5. *Continued*

Researcher/Author	Study	Key Findings	Date
Jitendra et al.	The effects of schema-based instruction on the proportional thinking of students with mathematics difficulties with and without reading difficulties	• Explicit and consistent procedures for solving word problems and use of schematic diagrams and checklists may have reduced cognitive load • Both MD and MDRD students improved in mathematical problem solving after taught DISC method and strategy for examining text	2016
Montague et al.	Effects of cognitive strategy instruction on math problem solving of middle school students with learning disabilities	• Students who participated in *Solve It!* made significantly greater growth in mathematics problem-solving • Intervention had uniform impact on average and low achieving students and students with LD • LD students in intervention grouped outperformed average and low achieving students in control group	2011

Summary of Chapter 2

In order to be successful in postsecondary education and the workplace, it is crucial for students to be literate in mathematics (CCSS Initiative, 2017; NCTM, 2017). Content area literacy in mathematics is defined as being able to read and learn from mathematical texts and solve complex problems. When students are literate in this area they can learn independently and are prepared for more advanced study (Adams & Lowery, 2007; Adams et al., 2015). Effective instruction aimed at increasing mathematics literacy for struggling readers involves activating prior content knowledge, vocabulary instruction, and understanding how mathematical text is written (Adams, 2003; Adams et al., 2015; Barton et al., 2002; Bay-Williams & Livers, 2009; Bossé & Faulconer, 2008; Capraro et al., 2012; Carter & Dean, 2006; DiGisi & Fleming, 2005; Hamilton, 2017; Smith & Angotti, 2012). In order to teach students how to read mathematical text and solve complex problems, mathematics teachers must use reading and comprehension strategies in their classrooms (Capraro et al., 2012). Strong reading comprehension and perseverance are essential to understanding the underlying mathematical structure of a problem in a text base and using the structure to carry out accurate mathematic computation to arrive at an answer.

Solving complex problems can differentially affect students with reading difficulties (Crisp, 2015; Fuchs & Fuchs, 2002; Jitendra et al., 2016; Powell et al., 2009). These students need explicit instruction in reading comprehension strategies that those students without disabilities might do automatically (Capraro et al., 2012). Teaching comprehension involves using cognitive and schema-based instruction strategies to help students break down text into propositions, find relationships between propositions and use them to understand the problem as a whole, use correct and efficient solving strategies, and check and label work (Capraro et al., 2012; Fuchs et al., 2008; Kintsch & Greeno, 1985). During each step of the way, students are engaged in taught cognitive strategies by asking themselves clarifying questions and thinking about their own thought processes.

The studies reviewed here provide strategies for teachers looking to implement interventions in their mathematics classrooms. However, there is a lack of studies investigating the use of reading interventions aimed at increasing mathematics learning and efficacy for high school students. Students at this level are required to solve complex and real-world problems that involve higher-order thinking skills that would greatly tax working memory. These problems are represented on high stakes tests such as the ACT and SAT. Research needs to be conducted on students with mathematics difficulties in high school to understand why these students struggle with solving word problems.

The current study assessed the effectiveness of the *Solve It!* instructional approach with year-long Math I students who historically struggled in mathematics using Montague et al.'s (2011) study as a template. This study differed from Montague et al.'s (2011) in that the participants were enrolled in year-long Math I, and the year-long time frame of Montague et al.'s (2011) study was reduced to eight weeks to fit into one semester. The year-long time frame of Montague et al.'s (2011) study was not feasible when considering the block scheduling of the school of study. This study also differed from Montague et al.'s (2011) in that perseverance and self-efficacy were measured and analyzed as opposed to only accuracy of solutions. When a student feels fear regarding word problems, they sometimes give up before reading the whole problem. The *Solve It!* approach gives these students a coping mechanism for dealing with mathematical word problems which helps in starting to increase problem solving ability (Montague et al., 2011). When students start to see performance accomplishment their self-efficacy toward solving mathematical word problems increases and in turn increases perseverance in solving (Bandura, 1977). Students also learn through observation of others, and in the classroom this looks like modeling with guided practice (Bandura, 1977). When a student sees their diverse group of classroom peers succeeding, their own sense of self-efficacy is likely to increase through vicarious experiences (Bandura, 1977). Including measures of perseverance and self-efficacy may provide insight into changes in scores to help inform key features of future interventions.

Chapter 3 describes the over-arching research approach for this study. The specific methodological approach and rationale for data collection is described, as well as the context of the study and data analysis plan.

CHAPTER 3 – RESEARCH DESIGN AND METHODOLOGY

Introduction

The purpose of this study was first to assess the effects of *Solve It!*, an instructional approach designed to help middle and secondary school students who have difficulty solving mathematical problems through teaching students the necessary cognitive and metacognitive processes and strategies that successful problem solvers use (Montague, 2013). The *Solve It!* approach incorporates explicit instruction practices like cueing, modeling, rehearsal, and feedback. This allows students to track their progress, promoting metacognition in self-monitoring. *Solve It!* combines schema-based strategy by helping students break text into propositions then finding relationships between propositions to understand the problem as a whole before solving, with cognitive strategies of asking clarifying questions and monitoring thought processes (Capraro et al., 2012; Fuchs et al., 2008; Kintsch & Greeno, 1985). The process for breaking text into propositions, finding relationships, and forming a macrostructure of the whole problem is described in detail in Chapter 2. The *Solve It!* approach has been found to be effective with students with and without learning disabilities in inclusive general education classrooms (Montague et al., 2011). Secondly, student perseverance and self-efficacy of solving mathematics word problems was measured to explore the extent that they play a role in solution accuracy.

This chapter provides a discussion of the over-arching research approach for this study. The specific methodological approach and rationale for data collection is described, as well as the context of the study and data analysis plan.

Research Design and Rationale

This study employed a nonequivalent control group quasi-experimental design. While a true experimental design is ideal for testing hypotheses, a quasi-experimental design, one that occurs in a natural social setting, is more feasible for research done in schools (Campbell & Stanley, 2015). Quasi-experimental designs include time series, equivalent time samples design, and nonequivalent control group design (Campbell & Stanley, 2015). An advantage to a quasi-experimental designed study in a school setting is that usual complexities of the real classroom are included (Hiebert & Grouws, 2007). Additionally, the pre-testing that takes place in this kind of study helps show similar entry knowledge and skills, uses similar content across groups except for different teaching methods, and classroom observations can be used to increase fidelity of treatment (Hiebert & Grouws, 2007). All of these designs expose a subject to an experimental variable and observations and/or measurements of the variable's effects.

A time series design relies on multiple observations before and after an exposure to a variable (Campbell & Stanley, 2015). This design was not chosen because it does not make use of pre- and post-test data or a control group. Additionally, multiple observations prior to the intervention would not enhance the study. An equivalent time samples design is similar to a time series design, however observations are taken both after giving an intervention and then taking the intervention away. This type of study assumes that the effects of the intervention are reversible (Campbell & Stanley, 2015). The reversible nature of an intervention does not apply to this study.

A nonequivalent control group design is similar to the pre-test post-test control group design with the exception of randomization of the subjects. The pre-test post-test control group design is used to help determine if a given intervention has an effect on subjects beyond what would normally have happened had they not been exposed to the intervention (Campbell & Stanley, 2015). According to Thyer (2012), in this design:

> Both groups are assessed at about the same point in time. The members of one group receive an intervention, whereas the members of the second group do not. Then, both are assessed at about the same point in time on a second occasion, after the first group receives intervention. If the treatment group changes and the no-treatment group does not, there is some modest logical justification to infer that it was the treatment, X, that produced these improvements. (p. 95)

As this study sought to explore the effects of an educational intervention in a classroom using pre-tests and post-tests, the appropriate research design was the nonequivalent control group design.

This study measured the effectiveness of the *Solve It!* instructional approach and, in turn, the usefulness of a schema- and cognitive-based literacy intervention for improvement in math word problem solving ability. The data collected for the study are quantitative in nature, using pre- and post-test scores and maintenance scores for measurement of solution accuracy, and teacher-monitored checklists for measurement of perseverance. The diagram in Figure 3.1 shows the overall design on the study. Analysis showed whether or not students who participated in *Solve It!* made statistically significant gains over non-participating peers in math word problem solution accuracy and the role of perseverance in solution accuracy.

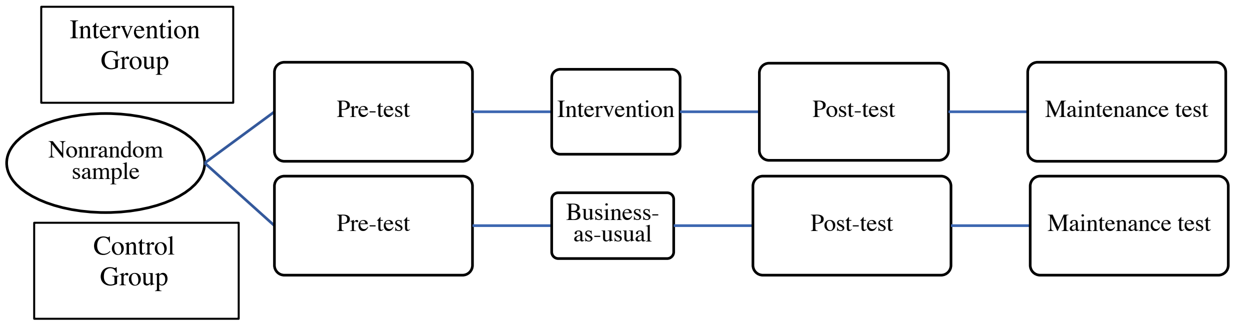

Figure 3.1. *Design of the study.*

Students who participated in *Solve It!* were part of a year-long Math I class and were placed in that class due to historical low performance on state achievement tests. Further description of the sample is provided later. These students were compared to others who were also in year-long Math I classes but were not instructed using *Solve It!*, making the study quasi-experimental.

Research Questions and Hypotheses

The questions framing this study were:

1. What are the effects of the *Solve It!* intervention on mathematics word problem solution accuracy for year-long Math I students identified as low performing by a mathematics state achievement test?
2. What are the effects of the *Solve It!* intervention on mathematics word problem perseverance for year-long Math I students identified as low performing by a mathematics state achievement tests?

2a. What are the differential effects of the *Solve It!* intervention on perseverance for one-, two-, and three-step mathematics word problems?

3. What are the effects of the *Solve It!* intervention on student self-efficacy around mathematics?

3a. To what extent does student self-efficacy correlate to changes in mathematics word problem solution accuracy?

3b. To what extent does student self-efficacy correlate to changes in mathematics word problem perseverance?

Two groups of students were studied to answer the research questions; a group of students who received the *Solve It!* intervention and a group of students who received typical classroom instruction. Before the start of the intervention, students in both groups took a pre-test, and immediately following the conclusion of the intervention, took a post-test, consisting of mathematics word problems using content from middle school mathematics. Students in both groups were given a maintenance test three months after the conclusion of the intervention. The pre- and post-tests and maintenance tests were assessed for solution accuracy as well as perseverance using a checklist for scoring. Prior to the start of the intervention and immediately following the intervention, students in both groups also took a self-efficacy assessment of perceived capabilities for correctly solving mathematics word problems.

The research hypotheses were stated in the null.

$H_0$1: Students who receive the *Solve It!* intervention will not have statistically significant gains in accuracy mathematics word problem solutions compared to students who did not when controlling for racial identity group and gender identity group.

$H_0$2: There will be no relationship (correlation) between perseverance and mathematics word problem solution accuracy using the *Solve It!* intervention.

$H_0$3: There will be no relationship (correlation) between self-efficacy and mathematics word problem solution accuracy using the *Solve It!* intervention.

It was predicted that these hypotheses would be refuted.

Sample and Population

The population of the study was students enrolled in year-long Math I at a high school in the Southeastern United States. Students who took year-long Math I were identified as low performers in mathematics based on 7th and 8th grade End-of-Grade assessment scores. These students were handpicked by the principal based on scores of Level 1 (of 5) on both 7th and 8th grade End-of-Grade Mathematics state assessments. The English Language Arts End-of-Grade assessment scores for individual students in the sample were not available to the researcher due to Family Educational Rights and Privacy Act (FERPA). During the first ten days of school, teachers of these classes gave in-class assessments to ensure that each student is properly placed in year-long Math I. The resulting year-long Math I classes were a mix of students with and without identified specific learning disabilities. Least restrictive environment policy for the district of study states that students with learning disabilities who have an Individualized Education Plan in place that does not include a resource class, participate with non-disabled peers at least 80% of the school day. Students whose individualized education plans specify the need for a resource class spend between 40-79% of the school day with non-disabled peers.

The sample size for the treatment group was 28 year-long Math I students in a class that received the *Solve It!* intervention. The sample size for the control group was another year-long Math I class of 26 students taught by the same teacher and did not use *Solve It!*. Both classes met in the morning before lunchtime. The treatment group was 60% male and 80% Hispanic. The control group was 61% male and 67% Hispanic. Determination for gender and racial identity group designations is addressed in the following School Context section.

Due to district policy, individual student data regarding learning disability, socio-economic status, and English proficiency could not be collected by the researcher. Aggregated school data for these areas is included in the next section. Both the treatment and control groups were taught by the same teacher to minimize variation due to teaching style. The teacher chosen for this study had 20 years of experience in teaching secondary and post-secondary mathematics, 11 of which were spent teaching the equivalent of Math I.

School Context

The population of the school of study was diverse with about 77% minority enrollment; the student body composition was 39.6% Black, 22.6% White, and 32.3% Hispanic. Student racial and gender identity group designations are determined by the school and provided to teachers by roster. The school identifies students as Hispanic or non-Hispanic, and this is not a teacher determination. Students with Limited English Proficiency made up 15.7% of the student population. Fifty-six percent of students were identified as economically disadvantaged through free and reduced lunch participation. With regard to academics, 10.4% of the school population had a learning disability and 22.1% were identified as Academically and Intellectually Gifted. Specific numbers of students with and without varying learning disabilities were not able to be recorded due to the employment of the researcher within the school district and the small sample size of the study. Table 3.1 shows the demographics of students at the school of study for the 2017-2018 school year.

Table 3.1. *Population Demographics (2017-2018)*

Characteristic	Percentage of the Population
Racial identity group	
Black	39.6
Hispanic	32.3
White	22.6
Multi-racial	2.8
Asian	2.5
Socioeconomic status	
Qualifies for free and reduced lunch	56
Does not qualify for free and reduced lunch	44
Academic	
Students with disabilities	10.4
Academically and Intellectually Gifted	22.1
Limited English proficiency	15.7
Gender identity group	
Male	54.6
Female	45.4

Testing data indicated that students at this large, urban high school were struggling in mathematics and reading. In the 2016-2017 school year, only 48.3% of students scored at grade level or higher on the Math I End-of-Course exam (38.9% met college and career readiness standard), as compared to 64.3% at grade level or higher statewide (54.1% met college and career readiness standard). English Language Arts at grade level or higher proficiency in the 2016-2017 school year, as measured by the English II End-of-Course, was 42.1% (34% met college and career readiness standard).

Overarching Study Social Context

The social context of the study site is important to describe as it may have had an effect on the diverse population of students at the school. As detailed in the previous sections, the school had a diverse population of students; specifically, the intervention and control groups were 80% and 67% Hispanic, as identified by the school. The researcher observed that many students were multilingual but was unable to collect more specific information such as student citizenship status, parent/guardian citizenship status, or date of entry to the United States. During the time of the study in the 2018-2019 school year, federal policy stated:

> aliens apprehended anywhere in the United States after unlawfully entering or re-entering the United States and who cannot establish to the satisfaction of an immigration officer that they have been physically present in the United States continuously since January 1, 2014 (Johnson, 2014, p. 4.).

Multiple students had been picked up by U.S. Immigration and Customs Enforcement officers in the years since this adoption of this policy, and these stories were prevalent in the daily news. This social context may have been a factor in student attendance during the time of the study.

Definition of Variables

The independent variable was the intervention *Solve It!* developed by Marjorie Montague (2013). Students were either in a class receiving the *Solve It!* intervention or in a class which did not. The *Solve It!* intervention was modified in length from that in Montague's (2013) handbook. Montague's (2013) *Solve It!* is designed for the duration of entire academic year of study. Her timeline includes professional development, explicit classroom instruction, weekly practice sessions, and maintenance testing with booster practice sessions through curriculum-based measures every six weeks for a total intervention time of approximately 36 weeks. This program was cut down in the area of maintenance testing to accommodate a district-wide policy of preserving the majority of the second 18 weeks of study to End-of-Course test preparation. The End-of-Course is a high stakes exam and scores are used to determine school report card grades. The first 18 weeks of study in the year-long Math I classroom is dedicated to reviewing middle school concepts and preparing students to learn Math I material. Thus, using a shortened version of *Solve It!* fit well into the district goals for Math I students. In Montague et al.'s (2011) study that sought to show validity of *Solve It!* for eighth-grade students in general education instructional classrooms, model-implied growth curves that controlled for student-level covariates suggested that students in the intervention group would start outperforming control group peers in December. Montague et al. (2011) started their study in October, at the same time of the present study, therefore, it is reasonable to believe that there would be some significant growth of the intervention group over the control group in this study using the shortened timeframe.

There were three dependent variables pertaining to this study. The first dependent variable was mathematics word problem achievement measured by accuracy. The second dependent variable was perseverance in solving mathematics word problems. The third dependent variable was the self-efficacy of students in solving mathematics word problems. The research hypotheses were stated in the null and thus predicted no significant gains in word problems from pre- to post-test and no changes in self-efficacy or perseverance.

Instrumentation

As stated, in order to collect the data around the study questions, the following instruments were used: pre- and post-test of achievement, a checklist for perseverance, and pre- and post-assessment of self-efficacy. The pre- and post-tests of achievement were from the *Solve It!* curriculum materials (Montague, 2008) and were validated by Montague et al. (2011). The pre- and post-test of achievement is provided in Appendix A. The checklist for perseverance was adapted from Warshauer's (2014) observations of productive struggle and is provided in Appendix B. The pre- and post-assessment of self-efficacy was created by Schunk (1981) and was validated by Schunk et al. (1987). The pre- and post-assessment of self-efficacy is provided in Appendix C.

The pre- and post-test of achievement was taken from the resources provided in the *Solve It!* curriculum materials and are also provided in Appendix A. The tests include a variety of different problem types to total 10 problems. The problem types are one, two, and three step word problems using the four arithmetic operations. The problems are a subset of “400 mathematical word problems extracted from middle school textbooks from nine mathematical basal series” (Montague, 1992, p. 233). Each set of 10 problems was calibrated using Item Response Theory to achieve equivalent difficulty level (Montague et al., 2011). Montague et al. (2011) also found the internal consistency of the measures ranged from .67 to .80.

In order to assess the word problem pre- and post-test for perseverance, the teacher used a checklist. The checklist, which is provided in Appendix B, is a list of characteristics of productive struggle, or perseverance, and is adapted from Warshauer (2014). These characteristics include getting started, carrying out a process, uncertainty in explaining and sense-making, and expressing misconception and errors. Each characteristic is listed and the teacher checked either ‘yes’ or ‘no’ based on what was shown for each problem on each student’s test.

The pre- and post- self-efficacy assessment is modeled from Schunk (1981). The purpose of assessment is to measure student self-efficacy regarding perceived ability to successfully solve mathematics word problems. The reason for measuring self-efficacy in this manner is because as students begin to see increased achievement on tasks, their self-efficacy for continued learning is enhanced (Schunk, 2003). As students begin to see improvement on mathematics word problems during weekly practice session, self-efficacy toward ability for successful solutions may increase, motivating the student to keep improving. The assessment is a scale with 10-unit intervals with verbal descriptors regarding the student’s perceived capabilities for correctly solving a problem; “not sure” (10), “maybe” (40), “pretty sure” (70), “really sure” (100). The test-retest reliability coefficient for this scale was 0.79 in Schunk et al.’s (1987) study. Prior to the test, there was a practice session for using the scale by judging the student's certainty of successfully jumping progressively longer distances. This helped the students learn the meaning of the scale's intervals (Schunk et al., 1987). After the practice, each student was shown a flashcard of a problem for several seconds and asked to make a judgment about their perceived capabilities. Each student did this for all 10 questions, circling the number on the scale that reflects their perceived capabilities for solving. Showing the problem for only a few seconds did not allow the student to try to work out a solution in their head, thus limited them to only judging problem difficulty (Schunk & Hanson, 1989).

Procedure and Data Collection

Krawec and Montague (2014) highlight the importance of providing professional development (PD) for teachers who will use the *Solve It!* technique in their classrooms. This PD took place over three sessions in the beginning of the semester. The purpose of the PD is to introduce teachers to instructional materials, approach and characteristics of the intervention, and provide examples of modeling with peer feedback (Krawec & Montague, 2014). These materials include cue cards to remind students of the steps (RPV-HECC), sample scripted lessons, word problems for teachers to draw from, and a fidelity checklist to be used during explicit instruction and weekly practice sessions. An example of the fidelity checklist is included in Appendix D. An example of problems to be used during a weekly practice session is in Appendix E.

After the PD sessions, students in both the treatment and control classes took a pre-test consisting of mathematics word problems featuring content from middle school mathematics. The purpose of this assessment was to get a baseline for ability in persevering through solving a word problem rather than assessing how well the student learned current computational methods. The pre-test was scored for solution accuracy. It was also assessed using a checklist for characteristics of perseverance; getting started, carrying out a process, uncertainty in explaining and sense-making, and expressing misconception and errors. At this time students were also given a self-efficacy assessment modeled from Schunk (1981) following a practice session for using the scale previously described. Figure 3.2 depicts a flowchart of the study.

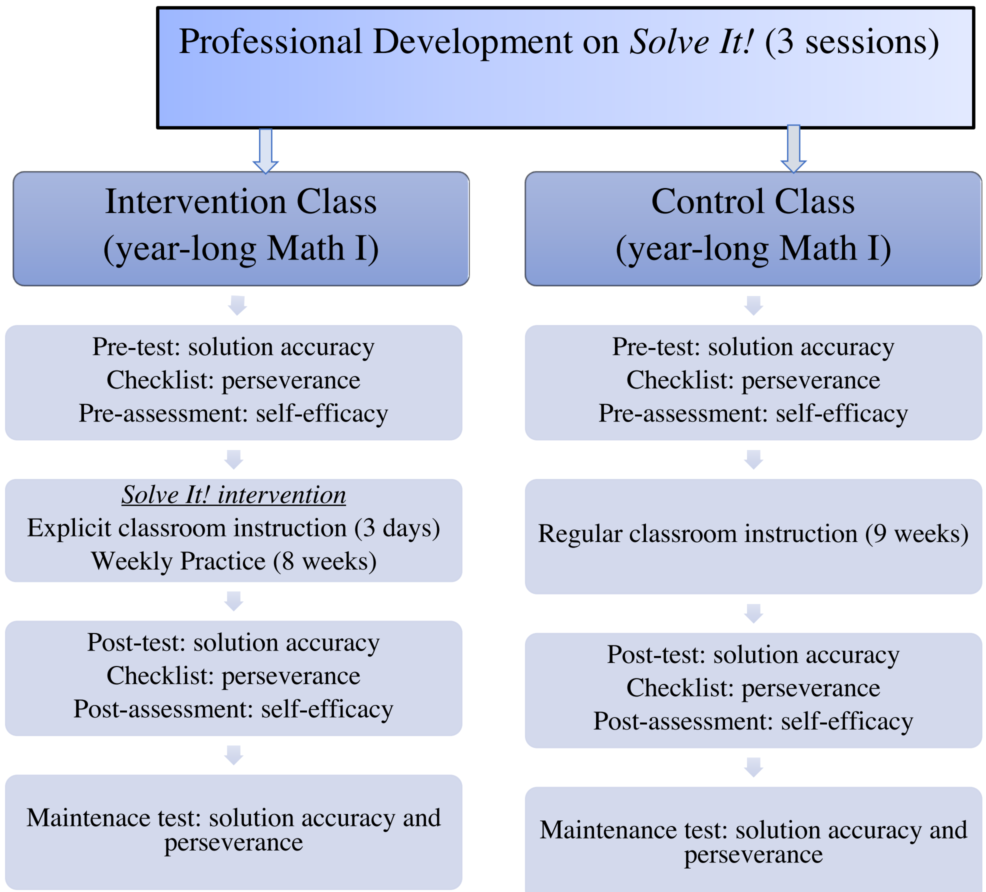

Figure 3.2. Flowchart of the study.

Following the pre-tests, the teacher spent three full days of explicit classroom instruction introducing the routine, establishing student mastery of the acronym RPV-HECC (Read, Paraphrase, Visualize, Hypothesize, Estimate, Compute, Check), providing process modeling, and supporting students as they used the routine to solve word problems (Krawec & Montague, 2014). The researcher was present during the explicit classroom instruction to ensure teacher fidelity. After the first three days there were weekly practice sessions where students practiced the routine for the continuation of the study. The results of the weekly practice sessions were used by the teacher to give individualized feedback to each student. The researcher evaluated treatment fidelity by using checklists when observing the weekly practice sessions. The checklists are included in Appendix D. If the teacher showed low fidelity, the researcher scheduled an immediate extra professional development session to help the teacher. Two of these sessions occurred; once after the first day of explicit instruction and again after the first weekly practice session.

At the end of eight weeks of practice sessions, students in both the treatment and control groups took a word problem post-test consisting of similar questions as the pre-test. Again, the purpose of the post-test was to assess perseverance of solving mathematics word problems rather than computational content taught during the class. Additionally, students took the same self-efficacy assessment after completing the intervention. Three months after the conclusion of the intervention, students were given a maintenance post-test that was scored for accuracy and perseverance. Table 3.2 depicts the timeline of the study. Table 3.3 depicts the data collection and how it informed each research question.

Table 3.2. *Timeline of Study*

Week	Dates	Activity
1	10/8-10/12	3 day professional development session Word problem/perseverance pre-test Self-efficacy pre-assessment
2	10/15-10/19	3 days explicit instruction (Read, Paraphrase, Visualization, Hypothesize, Estimate, Compute, Check 1st weekly practice session
3	10/22-10/26	Weekly practice session
4	10/29-11/2	Weekly practice session
5	11/5-11/9	Weekly practice session
6	11/12-11/16	Weekly practice session
7	11/26-11/30	Weekly practice session
8	12/3-12/7	Weekly practice session
9	12/10-12/14	Weekly practice session
10	12/17-12/21	Word problem/perseverance post-test Self-efficacy post-assessment
11	3/11-3/15	Maintenance test

Table 3.3. *Data Collection Crosswalk*

Research Questions	Data Sources		
	Student Pre/Post/Maintenance Word Problem Test Scores	*Student Pre/Post/Maintenance Word Problem Perseverance Checklist*	*Student Pre/Post Self-Efficacy Assessment*
Question 1: What are the effects of the *Solve It!* Intervention on mathematics word problem solution accuracy for year-long Math I students identified as low performing by a mathematics state achievement test?	X		
Question 2: What are the effects of the *Solve It!* intervention on mathematics word problem perseverance for year-long Math I students identified as low performing by a mathematics state achievement test?		X	
Question 2a: What are the differential effects of the *Solve It!* intervention on perseverance for one-, two-, and three-step mathematics word problems?		X	
Question 3: What are the effects of the *Solve It!* Intervention on student self-efficacy around mathematics?			X
Question 3a: To what extent does student self-efficacy correlate to changes in mathematics word problem accuracy?	X		X
Question 3b: To what extent does student self-efficacy correlate to changes in mathematics word problem perseverance?		X	X

Data Analysis

Data used were scores from the pre- and post-tests given before and after the intervention. First, an independent samples t-test determined whether there were significant differences between the treatment and control classes in terms of gender identity group and racial identity group. Gender identity group and racial identity group are determined by the school, and this information is provided to the teacher by roster. Gender and racial identity groups are not determined by the student. There were no significant differences between the two groups in terms of gender identity group and racial identity group. A general linear model in SPSS was used to analyze the instruments. The general linear model is inclusive of ANCOVA and includes the partial regression coefficient that predicts change in a dependent variable. Type III Sum of Squares was used to report mean square values as it is useful for both balanced and unbalanced data and because the data was input with no empty cells.   Gender identity group and racial identity group were used as covariates in each analysis as both the control and treatment groups were determined through an independent samples t-test to have similar variability with regards to these two variables.

Pre- and post-tests and maintenance tests were also scored by the teacher using the checklist for characteristics of perseverance. Additionally, students took a pre- and post- self-efficacy assessment of perceived capabilities for correctly solving mathematic word problems. The perseverance checklist and self-efficacy assessment were analyzed with pre- and post-test and maintenance test differences in solution accuracy scores using Spearman correlation. Spearman correlation is used rather than Pearson because 1) the data may not be normally distributed, and 2) the data from the checklist and self-efficacy assessments are ordinal. Table 3.4 provides a data analysis crosswalk to show how each form of data was analyzed.

Table 3.4. *Data Analysis Crosswalk*

Data Sources	Tools for Analysis	
	T-test/ANCOVA	Correlation (Spearman)
Student Pre/Post/Maintenance Word Problem Test Scores	X	X
Student Pre/Post/Maintenance Word Problem Perseverance Checklist	X	X
Student Pre/Post Self-Efficacy Assessment		X

Researcher Positionality

A researcher's positionality affects what they could see or not see in the design, implementation, analysis, and reflection in their research study (Foote & Gau Bartell, 2011). Finding one's positionality involves a reflection on philosophical, personal, and theoretical lenses through which the researcher sees, potential influence by the researcher on the research, and chosen or pre-determined positions held about the participants in the study (Foote & Gau Bartell, 2011).

This researcher conducted a study on the effectiveness of a word problem intervention in a mathematics classroom. Therefore, it is important to know that this researcher is a mathematics educator. She was taught mathematics with an emphasis on memorization of facts and formulas, and she considered this an effective way to learn mathematics through her undergraduate education, which was not in mathematics and not education. Her graduate studies, including an M.A.T. in Secondary Education, were focused on adolescent education and included courses with emphasis on reformative education practices and social justice in schools. These studies have influenced her beliefs on the purpose of education.

She has nine years of teaching experience in the K-12 educational setting, mostly in the high school setting, which is the same setting that the study occurred. She employs teaching methods that help students think critically about doing mathematics and how these critical thinking skills extend beyond the mathematics classroom. The researcher will use her findings and experience gained from completing this study to inform her own instructional practices. She plans to stay in the classroom as a mathematics teacher using cognitive and schema-based instructional practices, like those delineated in *Solve It!* to teach mathematics word problems.

The researcher is a doctoral student in a curriculum and instruction program and was interested in doing this study because of her first-hand experience with students who struggled to solve word problems because of their ability to read the problems. The population chosen for the study was the population with which the researcher has the most experience teaching. The researcher was familiar to the students from the school setting. The bias that this may have introduced is explained in chapter 5.

Summary of Chapter 3

This chapter provided a discussion of the over-arching research approach and rationale for this study. The specific methodological approach for data collection was described, as well as the context of the study and data analysis plan. This quantitative study explored the effects and usefulness of *Solve It!* for year-long Math I students as measured by changes in mathematics word problem achievement as well as perseverance in solving these types of problems. T-tests and correlation were used to analyze the effects of *Solve It!* program on year-long Math I students, low performing mathematics students' perseverance and accuracy in solving mathematics word problems. The population of year-long Math I participants was racially and economically diverse. Results from data analysis could inform future educational practices in high schools with similar populations.

CHAPTER 4 – RESULTS

Introduction

The purpose of this study was first to assess the effects of *Solve It!*, an instructional approach designed to help middle and secondary school students who have difficulty solving mathematical problems through teaching students the necessary cognitive and metacognitive processes and strategies that successful problem solvers use (Montague, 2013). Second, student perseverance and self-efficacy of mathematics word problems were measured to explore the extent to which perseverance and self-efficacy played a role in students' solution accuracy.

This study employed a nonequivalent control group quasi-experimental design. The data that collected for the study are quantitative in nature, using pre- and post-test scores and maintenance scores for measurement of solution accuracy, teacher-monitored checklists for measurement of perseverance, and a scale survey for measurement of self-efficacy.

The sample size for the treatment group was 28 year-long Math I students in a class that received the *Solve It!* intervention. The sample size for the control group was another year-long Math I class of 26 students taught by the same teacher that did not use *Solve It!*. The population of the school of study was diverse with about 77% minority enrollment; the student body composition was 39.6% Black, 22.6% White, and 32.3% Hispanic. Students with Limited English Proficiency composed 15.7% of the student population. Fifty-six percent of students were identified as economically disadvantaged through free and reduced lunch participation. With regard to academics, 10.4% of the school population had a learning disability and 22.1% were identified as Academically and Intellectually Gifted. Specific numbers of students with and without varying learning disabilities were not able to be recorded due to the employment of the researcher within the school district and the small sample size of the study.

Testing data indicated that students at this large, urban high school struggled in mathematics and reading. In the 2016-2017 school year, only 48.3% of students scored at grade level or higher on the Math I End-of-Course exam (38.9% met the college and career readiness standard), as compared to 64.3% at grade level or higher statewide (54.1% met college and career readiness standard). English Language Arts at grade level or higher proficiency in the 2016-2017 school year, as measured by the English II End-of-Course, was 42.1% (34% met college and career readiness standard).

The following research questions framed study:

RQ1: What are the effects of the *Solve It!* intervention on mathematics word problem solution accuracy for students identified as low performing by a mathematics state achievement test?

RQ2: What are the effects of the *Solve It!* intervention on mathematics word problem perseverance for students identified as low performing by a mathematics state achievement test?

RQ2a: What are the differential effects of the *Solve It!* intervention on perseverance for one-, two-, and three-step mathematics word problems?

RQ3: What are the effects of the *Solve It!* intervention on student self-efficacy around mathematics?

RQ3a: To what extent does student self-efficacy correlate to changes in mathematics word problem solution accuracy?

RQ3b: To what extent does student self-efficacy correlate to changes in mathematics word problem perseverance?

The hypotheses, stated in the null, of the study are as follows:

$H_0$1: Students who receive the *Solve It!* intervention will not have statistically significant gains in accuracy mathematics word problem solutions compared to students who did not when controlling for racial identity group and gender identity group.

$H_0$2: There will be no relationship (correlation) between perseverance and mathematics word problem solution accuracy using the *Solve It!* intervention.

$H_0$3: There will be no relationship (correlation) between self-efficacy and mathematics word problem solution accuracy using the *Solve It!* intervention.

The purpose of this chapter is to provide a statistical analysis of the findings of this study. First, there is a description of how each instrument was scored for data collection purposes and a timeline for when each of the measures was taken. Next, there is an independent samples t-test analysis to determine if there are statistically significant differences in baselines for accuracy scores between the control and treatment classes. Then, each research question and hypothesis are addressed and analyzed.

Scoring of Instruments

Table 4.1 reports the instruments in this study. The researcher solely scored the instruments against an earlier established key, so an interrater reliability protocol was not needed. Spanish versions of the instruments were provided when requested by students.

Table 4.1. *Instrumentation*

Measure	Instrument(s)
Accuracy	Pre-, post-, and maintenance tests
Perseverance	Pre- and post-tests
Self-efficacy	Pre- and post-tests

All variables on each instrument coded into dichotomous variables as shown in Table 4.2. All names were replaced with pseudonyms for each class.

Table 4.2. *Coding of Variables*

Variable	Coding
Gender identity group	Female=0; male=1
Racial identity group	Hispanic=0; non-Hispanic=1
Group	Control=0; treatment=1

All three tests for accuracy were scored out of 10 points as there were 10 questions on each test. Furthermore, each question was identified as a one-, two-, or three-step problem for further analysis. The breakdown of each test by number of different problem types is found in Table 4.3. A student was awarded one point for each correct final answer shown on the paper test.

Table 4.3. *Distribution of Problem Types*

	Number of problems		
Instrument	One-step	Two-step	Three-step
Pre-test	3	4	3
Post-test	1	6	3
Maintenance test	2	6	2

Self-efficacy was measured through a pre- and a post-test of 10 questions. The tests showed a scale of 10-100 in 10-point increments. Students were tested individually by being pulled into the hallway, shown a math word problem of similar variety to the accuracy tests for five seconds, and asked to make a judgment about their perceived capabilities by picking a number on the scale. Prior to the test, there was a whole class practice session for using the scale by judging the students' certainty of successfully jumping progressively longer distances. Ratings for each of the 10 problems were averaged for each student to have a single number to represent self-efficacy on a scale of 10-100.

Perseverance was measured using the pre- and post-test instruments for accuracy. The researcher collected these tests and awarded a point for each of the 10 problems for showing each of the 4 areas of productive struggle: getting started, carrying out a process, uncertainty in explaining and sense-making, and expressing misconception of errors. The total points in each area of productive struggle were summed and then averaged over the 10 questions for both the pre- and post-tests. For example, a student who scored nine points for 'getting started', eight points for 'carrying out a process', zero points for 'uncertainty and sense-making', and three points for 'expressing misconception' on the pre-test would receive an overall score of five for perseverance on the pre-test.

Timeline of Measures

The researcher provided three professional development sessions to the teacher of the control and treatment classes at the beginning of October, prior to any measurements being taken. The purpose of this professional development was to introduce the teacher to instructional materials, to introduce the approach and characteristics of the intervention, and provide examples of modeling with peer feedback (Krawec & Montague, 2014). The professional development occurred at the beginning of October because that is when most class rosters at the school had been finalized for the school year.

During this week, students in the control and treatment classes were given the pre-test for accuracy, which was also scored for perseverance, and the pre-test for self-efficacy. The researcher made every effort to administer the pre-test to students who were absent on the day of testing before the next week, when instruction began. In the next week, following the three days of professional development, the teacher explicitly taught the *Solve It!* framework of Read, Paraphrase, Visualize, Hypothesize, Estimate, Compute, and Check (RPV-HECC) to the treatment class while teacher as normal in the control class.

Eight weeks after the three days of explicit instruction followed by weekly practice sessions, all students were given the post-test for accuracy, which was also scored for perseverance, and the post-test for self-efficacy. If a student was absent on the day of testing, every effort was made by the researcher to post-test that student by the end of that week. Three months following the post-tests, students were given a maintenance test that was only scored for accuracy. The researcher allowed one week for students to take the maintenance test to accommodate for absences that may have occurred on the original date of testing.

Analysis of Baselines

An independent samples t-test was needed to determine whether there were significant differences in pre-test accuracy scores between the treatment and control classes based on gender identity group and racial identity group. First, an independent samples t-test was performed based on gender. Table 4.4 summarizes the descriptive statistics of the pre-test.

Table 4.4. *Means and Standard Deviations on Pre-Test for Accuracy by Gender Identity Group*

	Gender identity group	N	Mean	Std. deviation
Pre-Test	Female	16	3.8	2.2
	Male	23	4.6	3.2

Table 4.5 shows the results of the independent samples t-test based on gender.

Table 4.5. *Independent Samples Test for Pre-Test for Accuracy Based on Gender Identity Group*

		Levene's test for equality of variances		t-test for equality of means			
		F	Sig.	t	Sig. (2-tailed)	Mean difference	Std. error difference
Pre-test	Equal variances assumed	3.76	.06	-.92	.362	-.86	.93
	Equal variances not assumed			-.99	.330	-.86	.87

There was a not a significant difference in the means scores for the pre-test for accuracy for females (M=3.8, SD=2.2) and males (M=4.6, SD=3.2); t(37)=-.92, p=.06. Mean scores of pre-test accuracy were not statistically different between males and females in the control and treatment groups.

Next, an independent samples t-test was performed to compare pre-test accuracy scores of students in the control and treatment classes based on racial identity group. Table 4.6 shows the descriptive statistics for pre-test accuracy scores based on racial identity group.

Table 4.6. *Pre-Test for Accuracy by Racial Identity Group*

	Racial identity group	N	Mean	Std. deviation
Pre-Test	Hispanic	31	4.2	2.6
	Non-Hispanic	8	4.4	3.8

Table 4.7 shows the results of the independent samples t-test based on racial identity group.

Table 4.7. *Independent Samples Test for Pre-Test Accuracy Based on Racial Identity Group*

		Levene's test for equality of variances		t-test for equality of means			
		F	Sig.	t	Sig. (2-tailed)	Mean difference	Std. error difference
Pre-test	Equal variances assumed	1.86	.18	-.13	.897	-.15	1.15
	Equal variances not assumed			-.10	.919	-.15	1.43

There was a not a significant difference in the means scores for the pre-test for accuracy for Hispanics (M=4.2, SD=2.6) and non-Hispanics (M=4.4, SD=3.8); t(37)=-.13, p=.18. Mean scores of pre-test accuracy were not statistically different between Hispanics and non-Hispanics in the control and treatment groups.

There were not significant differences in pre-test accuracy scores between the treatment and control classes based on gender identity group or racial identity group.

Data Analysis

A general linear model in SPSS with covariates for gender identity group and racial identity group was used to analyze the data to answer the research questions. The general linear model is inclusive of ANCOVA and includes the partial regression coefficient that predicts change in a dependent variable. Type III Sum of Squares was used to report mean square values as it is useful for both balanced and unbalanced data and because the data were input with no empty cells. Gender identity group and racial identity group were used as covariates in each analysis as both the control and treatment groups were determined through an independent samples t-test to have similar variability with regards to these two variables.

RQ1: Effects on Solution Accuracy

First, the scores of the pre- and post-tests for accuracy between the control and treatment groups were compared. Table 4.8 shows that 18 students in the control class completed both a pre- and a post-test for comparison purposes. In the treatment class, 21 students completed both a pre- and a post-test for comparison purposes.

Table 4.8. *Means and Standard Deviations on Pre- and Post-test Accuracy Scores by Group*

Group	N	Mean	Std. deviation
Control	18		
Pre-test		3.9	2.4
Post-test		4.7	2.3
Treatment	21		
Pre-test		4.6	3.2
Post-test		5.5	2.7

The general linear model in SPSS was used to compare means separated by intervention to help explain gains controlling for gender identity group, racial identity group, and the intervention. Pre-test accuracy was used as a control for post-test accuracy and intervention, racial identity group, and gender identity group were used as covariates.

Table 4.9. *Significance and Partial Regression of Parameters for Post-Test for Accuracy Controlling for Pre-test Accuracy*

Parameter	Mean square	Significance	B (partial regression coefficient)
Pre-test	106.37	.000	.60
Group	.85	.639	.31
Gender Identity Group	7.54	.168	-.92
Racial Identity Group	.00	.989	.01

Note. R-squared=0.47. Computed using alpha=0.05.

Table 4.9 shows the *Solve It!* intervention did not have a significant impact (p=0.639) on change in accuracy scores from pre-test to post-test. The pre-test score was the only variable that had a significant impact on the post-test score (p<.001). Students who received the intervention scored 0.31 points higher (out of 10) than those that did not. In this model, 47% of post-test accuracy can be explained by pre-test accuracy, the intervention, gender identity group, and racial identity group.

Next, the scores of the post- and maintenance tests for accuracy between the control and treatment groups were compared. Table 4.10 shows that 15 students in the control class completed both a pre- and a post-test for comparison purposes. In the treatment class, 10 students completed both a post- and maintenance test for comparison purposes.

Table 4.10. *Means and Standard Deviations on Post- and Maintenance Test Accuracy Scores by Group*

Group	N	Mean	Standard deviation
Control	15		
Post-test		4.9	2.5
Maintenance		1.9	1.9
Treatment	10		
Post-test		6	2.8
Maintenance		2.4	2.8

The general linear model in SPSS was used to compare means separated by intervention to help explain gains controlling for gender identity group, racial identity group, intervention. Post-test accuracy was used as a control for maintenance test accuracy and the intervention, racial identity group, and gender identity group were used as covariates.

Table 4.11. *Significance and Partial Regression of Parameters for Maintenance Test Accuracy Controlling for Post-Test Accuracy*

Parameter	Mean square	Significance	B (partial regression coefficient)
Post-test	37.51	0.003	0.52
Group	1.77	0.480	0.57
Gender Identity Group	6.99	0.168	-0.17
Racial Identity Group	.	.	. .

Note. R-squared=0.43. Computed using alpha=0.05.

Table 4.11 shows the *Solve It!* intervention did not have a significant impact on change in accuracy scores from post-test to maintenance test (p=0.480). The post-test score was the only variable that significantly affected the maintenance test score (p=0.003). Students who received the intervention scored 0.57 points higher (out of 10) than those who did not. Significance of racial identity group was not measurable on this instrument because only Hispanic students completed both a post-test and a maintenance test in the treatment group. In this model, 43% of maintenance test accuracy can be explained by post-test accuracy, intervention, gender identity group, and racial identity group.

Summary of RQ1 and $H_0$1. The *Solve It!* intervention did not have significant effects on mathematics word problem solution accuracy for students identified as low performing by a mathematics state achievement test. Students who received the intervention did not have statistically significant gains in accuracy as compared to students who did not receive the intervention when controlling for racial identity group and gender identity group.

RQ2: Effects on Perseverance

First, the scores for the pre- and post-tests for perseverance between the control and treatment groups were compared. Table 4.12 shows that 18 students in the control class completed a pre- and a post-test for comparison purposes. Twenty-two students in the treatment class completed a pre- and a post-test for comparison purposes.

Table 4.12. *Means and Standard Deviations on Pre- and Post-Test Perseverance Scores by Group*

Group	N	mean	Std. deviation
Control	18		
Pre-test		6.6	2.1
Post-test		7.5	1.4
Treatment	22		
Pre-test		6.5	2.4
Post-test		7.7	1.3

The general linear model in SPSS was used to compare means separated by intervention to help explain gains controlling for gender identity group, racial identity group, intervention. Pre-test perseverance was used as a control for post-test perseverance and the intervention, racial identity group, and gender identity group were used as covariates.

Table 4.13. *Significance and Partial Regression of Parameters for Post-Test Perseverance Controlling for Pre-Test Perseverance*

Parameter	Mean square	Significance	B (partial regression coefficient)
Pre-test	37.52	0.000	0.46
Group	0.61	0.436	0.25
Gender Identity Group	0.85	0.358	-0.30
Racial Identity Group	0.21	0.650	-0.18

Note. R-squared=0.54. Computed using alpha=0.05.

Table 4.13 shows the *Solve It!* intervention did not have a significant impact (p=0.436) on change in perseverance scores from pre-test to post-test. The pre-test score was the only variable that had a significant impact on the post-test score (p=0.000). Students who received the intervention scored 0.25 points higher (out of 10) than those that did not. In the model, 54% of post-test perseverance scores can be explained by pre-test perseverance scores, the intervention, gender identity group, and racial identity group.

RQ2a: Differential Effects on Two-Step and Three-Step Perseverance

Next, questions from the pre- and post-tests were analyzed separately by problem type. Table 4.3 showed the distribution of each problem type on the pre- and post-tests. Because there was only a single one-step problem on the post-test, one-step problems were not analyzed for gains in perseverance. Because of the inconsistent quantity of each problem type from pre- to post-test, a proportion was used to identify student scores for comparison purposes. For example, a student who showed three areas of productive struggle on a two-step question would receive a 0.75 for that question. Then all scores for two-step questions were averaged. Table 4.14 shows the descriptive statistics for two-step problems by group.

Table 4.14. *Means and Standard Deviations on Pre- and Post-Test Two-Step Perseverance Scores by Group*

Group	N	Mean	Std. deviation
Control	18		
Pre-test		0.6	0.3
Post-test		0.7	0.1
Treatment	22		
Pre-test		0.6	0.3
Post-test		0.8	0.1

The general linear model in SPSS was used to compare means separated by intervention to help explain gains controlling for gender identity group, racial identity group, and the intervention. Pre-test two-step perseverance was used as a control for post-test two-step perseverance and the intervention, racial identity group, and gender identity group were used as covariates.

Table 4.15. *Significance and Partial Regression of Parameters for Post-Test Two-Step Perseverance Controlling for Pre-Test Two-Step Perseverance*

Parameter	Mean square	Significance	B (partial regression coefficient)
Pre-test	0.40	0.000	0.40
Group	0.00	0.786	0.01
Gender identity group	0.00	0.862	0.01
Racial identity group	0.01	0.326	-0.04

Note. R-squared=0.53. Computed using alpha=0.05.

Table 4.15 shows the *Solve It!* intervention did not have a significant impact (p=0.786) on change in two-step perseverance scores from pre-test to post-test. The pre-test score was the only variable that had a significant impact on the post-test score (p=0.000). Fifty-three percent of two-step post-test perseverance scores can be explained by pre-test accuracy, the intervention, gender identity group, and racial identity group.

Table 4.16 shows the descriptive statistics for three-step problems by group.

Table 4.16. *Means and Standard Deviations on Pre- and Post-Test Three-Step Perseverance Scores*

Group	N	Mean	Std. deviation
Control	18		
Pre-test		0.7	0.3
Post-test		0.8	0.2
Treatment	22		
Pre-test		0.7	0.2
Post-test		0.8	0.2

The general linear model in SPSS was used to compare means separated by intervention to help explain gains controlling for gender identity group, racial identity group, and the intervention. Pre-test three-step perseverance was used as a control for post-test three-step perseverance and the intervention, racial identity group, and gender identity group were used as covariates.

Table 4.17. *Significance and Partial Regression of Parameters for Post-Test Three-Step Perseverance Controlling for Pre-Test Three-Step Perseverance*

Parameter	Mean square	Significance	B (partial regression coefficient)
Pre-test	0.33	0.000	0.38
Group	0.00	0.726	0.02
Gender identity group	0.04	0.156	-0.07
Racial identity group	0.01	0.476	0.04

Note. R-squared=0.36. Computed using alpha=0.05.

Table 4.17 shows the *Solve It!* intervention did not have a significant impact (p=0.726) on change in three-step perseverance scores from pre-test to post-test. The pre-test score was the only variable to significantly effect on the post-test score (p=0.000). Thirty-six percent of three-step post-test perseverance scores can be explained by pre-test accuracy, the intervention, gender identity group, and racial identity group.

Summary of RQ2. The *Solve It!* intervention did not have significant effects on mathematics word problem perseverance for students identified as low performing by a mathematics state achievement test. Students who received the intervention did not have statistically significant gains in perseverance overall, or specifically on two- and three-step problem types, as compared to students who did not receive the intervention when controlling for racial identity group and gender identity group.

$H_0$2: Relationship between Accuracy and Perseverance

The general linear model in SPSS was used to look for a relationship between perseverance and mathematics word problem solution accuracy. Means from pre- and post-tests for accuracy and perseverance, separated by intervention, were compared to help explain gains when controlling for the intervention, racial identity group, and gender identity group. Table 4.18

shows the descriptive statistics for students who completed pre- and post-tests for accuracy which were also used to score perseverance.

Table 4.18. *Means and Standard Deviations on Pre- and Post-Test Accuracy and Perseverance Scores*

Group	N	Mean	Std. deviation
Control	18		
Pre-test accuracy		3.9	2.4
Post-test accuracy		4.7	2.3
Pre-test perseverance		6.6	2.1
Post-test perseverance		7.5	1.4
Treatment	22		
Pre-test accuracy		4.4	3.2
Post-test accuracy		5.4	2.6
Pre-test perseverance		6.5	2.4
Post-test perseverance		7.7	1.4

Table 4.19 shows the significance and partial regression coefficient of each variable on post-test accuracy controlling for pre-test accuracy, post-test perseverance, the intervention, gender identity group, and racial identity group.

Table 4.19. *Significance and Partial Regression of Parameters for Post-Test Accuracy Controlling for Post-Test Perseverance*

Parameter	Mean square	Significance	B (partial regression coefficient)
Pre-test accuracy	39.65	0.002	0.44
Post-test perseverance	14.91	0.042	0.56
Group	1.20	0.554	0.35
Gender identity group	2.41	0.403	-0.53
Racial identity group	0.47	0.710	-.267

Note. R-squared=0.53. Computed using alpha=0.05.

Table 4.19 shows there was a significant relationship between pre-test accuracy and post-test accuracy (p=0.002) and post-test perseverance and post-test accuracy (p=0.042) when control and treatment cases are included in the same model. This model was then separated to assess whether this relationship between perseverance and accuracy persisted within the control group and within the treatment group.

When only the control group was analyzed, Table 4.20 shows that a significant relationship between pre-test accuracy and post-test accuracy remained (p=0.004). There was not a significant relationship between post-test perseverance and post-test accuracy (p=0.703).

Table 4.20. *Significance and Partial Regression of Parameters for Post-Test Accuracy Controlling for Post-Test Perseverance for the Control Group Only*

Parameter	Mean square	Significance	B (partial regression coefficient)
Pre-test accuracy	30.35	0.004	0.87
Post-test perseverance	0.38	0.703	-0.17
Gender identity group	6.05	0.144	-1.39
Racial identity group	2.40	0.346	1.11

Note. R-squared=0.65. Computed using alpha=0.05.

When only the treatment group was analyzed, Table 4.21 shows that a significant relationship between pre-test accuracy and post-test accuracy remained (p=0.041). There was not a significant relationship between post-test perseverance and post-test accuracy (p=0.063).

Table 4.21. *Significance and Partial Regression of Parameters for Post-Test Accuracy Controlling for Post-Test Perseverance for the Treatment Group Only*

Parameter	Mean square	Significance	B (partial regression coefficient)
Pre-test accuracy	16.16	0.041	0.35
Post-test perseverance	0.38	0.063	0.74
Gender identity group	0.03	0.938	-0.08
Racial identity group	2.15	0.478	-0.75

Note. R-squared=0.52. Computed using alpha=0.05.

Summary of H_02. There was not a significantly significant relationship between perseverance and mathematics word problem solution accuracy for students identified as low performing by a mathematics state achievement test using the *Solve It!* intervention.

RQ3: Effects on Self-Efficacy

To assess effects of the *Solve It!* intervention on student self-efficacy around mathematics student were given a pre- and post-test. Table 4.22 shows the descriptive statistics for the control and treatment groups in the study.

Table 4.22. *Means and Standard Deviations on Pre- and Post-Test Self-Efficacy Scores*

Group	N	Mean	Std. deviation
Control			
Pre-test	16	74.3	18.1
Post-test	16	84.3	18.2
Treatment			
Pre-test	20	73.7	21.5
Post-test	20	87.6	15.7

The general linear model in SPSS was used to compare means separated by intervention to help explain gains controlling for gender identity group, racial identity group, and the intervention. Pre-test self-efficacy was used as a control for post-test self-efficacy and the intervention, racial identity group, and gender identity group were used as covariates.

Table 4.23. *Significance and Partial Regression of Parameters for Post-Test Self-Efficacy Controlling for Pre-Test Self-Efficacy*

Parameter	Mean square	Significance	B (partial regression coefficient)
Pre-test Self-efficacy	3722.79	0.000	0.59
Group	110.63	0.373	3.56
Gender identity group	69.84	0.478	3.23
Racial identity group	148.66	0.303	-4.62

Note. R-squared=0.57. Computed using alpha=0.05.

Table 4.23 shows the *Solve It!* intervention did not have a significant impact (p=0.373) on change in self-efficacy scores from pre-test to post-test. The pre-test score was the only variable to significantly affect the post-test score (p=0.000). Fifty-seven percent of the post-test self-efficacy scores can be explained by pre-test self-efficacy scores, the intervention, gender identity group, and racial identity group.

RQ3a and $H_0$3: Relationship between Self-Efficacy and Accuracy

The general linear model in SPSS was used to look for a relationship between self-efficacy and mathematics word problem solution accuracy. Means from pre- and post-tests for accuracy and self-efficacy, separated by intervention, were compared to help explain gains controlling for the intervention, racial identity group, and gender identity group. Table 4.24 shows the descriptive statistics for students who completed pre- and post-tests for accuracy and pre- and post-tests for self-efficacy.

Table 4.24. *Means and Standard Deviations on Pre- and Post-Test Accuracy and Self-Efficacy Scores*

Group	N	Mean	Std. deviation
Control	11		
Pre-test accuracy		3.3	1.7
Post-test accuracy		4.8	2.5
Pre-test self-efficacy		72.3	20.2
Post-test self-efficacy		82.5	21.2
Treatment	20		
Pre-test accuracy		4.6	3.4
Post-test accuracy		5.6	2.7
Pre-test self-efficacy		73.7	21.5
Post-test self-efficacy		85.8	15.7

Table 4.25 shows the significance and partial regression coefficient of each variable on post-test accuracy controlling for pre-test accuracy, post-test self-efficacy, intervention, gender identity group, and racial identity group.

Table 4.25. *Significance and Partial Regression of Parameters for Post-Test Accuracy Controlling for Post-Test Self-Efficacy*

Parameter	Mean square	Significance	B (partial regression coefficient)
Pre-test accuracy	66.19	0.001	0.58
Post-test self-efficacy	0.23	0.823	0.01
Group	0.23	0.822	0.19
Gender identity group	5.83	0.262	-0.99
Racial identity group	0.27	0.807	-0.23

Note. R-squared=0.46. Computed using alpha=0.05.

The pre-test accuracy scores had a significant relationship (p=0.001) with post-test accuracy scores. Post-test self-efficacy did not have a significant relationship (p=0.823) with post-test accuracy scores. The *Solve It!* intervention did not have a significant impact on post-test accuracy (p=0.822) when controlling for pre-test accuracy, post-test self-efficacy, gender identity group, and racial identity group.

Summary of $H_0$3. There was not a significantly significant relationship between self-efficacy and mathematics word problem solution accuracy for students identified as low performing by a mathematics state achievement test using the *Solve It!* intervention.

RQ3b: Relationship between Self-Efficacy and Perseverance

The general linear model in SPSS was used to look for a relationship between self-efficacy and mathematics word problem perseverance. Means from pre- and post-tests for perseverance and self-efficacy, separated by intervention, were compared to help explain gains controlling for intervention, racial identity group, and gender identity group. Table 4.26 shows the descriptive statistics for students who completed pre- and post-tests for perseverance and pre- and post-tests for self-efficacy.

Table 4.26. *Means and Standard Deviations on Pre- and Post-test Perseverance and Self-Efficacy Scores*

Group	N	Mean	Std. deviation
Control	11		
Pre-test perseverance		6.1	2.2
Post-test perseverance		7.1	1.4
Pre-test self-efficacy		72.3	20.1
Post-test self-efficacy		82.5	21.2
Treatment	20		
Pre-test perseverance		6.6	2.3
Post-test perseverance		7.9	1.3
Pre-test self-efficacy		73.7	21.5
Post-test self-efficacy		85.8	15.7

Table 4.27 shows the significance and partial regression coefficient of each variable on post-test perseverance controlling for pre-test perseverance, post-test self-efficacy, the intervention, gender identity group, and racial identity group.

Table 4.27. *Significance and Partial Regression of Parameters for Post-Test Perseverance Controlling for Post-Test Self-Efficacy*

Parameter	Mean square	Significance	B (partial regression coefficient)
Pre-test perseverance	17.75	0.000	0.40
Post-test self-efficacy	0.00	0.980	0.00
Group	3.18	0.093	0.69
Gender identity group	2.01	0.177	-0.62
Racial identity group	0.16	0.702	-0.18

Note. R-squared=0.55. Computed using alpha=0.05.

The pre-test perseverance scores had a significant relationship (p=0.00) with post-test perseverance scores. Post-test self-efficacy did not have a significant relationship (p=0.980) with post-test perseverance scores. The *Solve It!* intervention did not have a significant impact on post-test perseverance ($p=0.093$) when controlling for pre-test perseverance, post-test self-efficacy, gender identity group, and racial identity group.

Summary of RQ3. There was not a significantly significant relationship between self-efficacy and mathematics word problem solution accuracy for students identified as low performing by a mathematics state achievement test using the *Solve It!* intervention.

Chapter 4 Summary

Independent samples testing on the pre-test for solution accuracy did not show significant differences in scores between the treatment and control classes based on gender identity group and racial identity group. A general linear model in SPSS was used to analyze the data with covariates for gender identity group and racial identity group. The general linear model showed significance of factors such as the intervention, that is, pre-tests for accuracy, perseverance, and self-efficacy, and calculated partial regression coefficients that predict change in post-tests for accuracy, perseverance, and self-efficacy.

The results of the general linear model indicated that the intervention did not have a significant impact on change in accuracy scores from pre-test to post-test or post-test to maintenance test, change in perseverance scores, as a whole or by problem-type, from pre-test to post-test, or change in self-efficacy from pre-test to post-test. The results did not show a significant relationship between accuracy and self-efficacy or between self-efficacy and perseverance.

The results did show a significant relationship between post-test perseverance and post-test accuracy when both control and treatment cases are included in the same general linear model, however, this significance reached $p>0.05$ when control and treatment cases are analyzed in different general linear models.

Chapter 5 discusses the results, including their significance and implications. Next, challenges encountered during the study and limitations and potential modifications to the present study design are considered. Finally, implications and suggestions for future research are explored.

CHAPTER 5 – DISCUSSION

Introduction

The purpose of this chapter is to describe the significance and implications of the results presented in Chapter 4, the challenges encountered during the study, limitations of the current study design, and impetus for future research.

First, each research question is addressed and the results are interpreted through the lenses of Kintsch and Greeno's (1985) processing model of translating textual arithmetic word problems into macrostructures that are then acted on by problem-solving strategies, and Bandura's (1993) theory that people with high levels of efficacy will set goals for themselves and provide guides and supports to meet those goals, whereas people with low efficacy focus on failure and things that can go wrong. Following the interpretation of results are the challenges the researcher had in completing the data collection at the school. These challenges highlighted some limitations of the current study design which are each considered for change in future research. Lastly, the implications of the current study are weighed and ideas for future research are explored.

Significance and Implications of Results

Discussion of RQ1. RQ1 "What are the effects of the *Solve It!* intervention on mathematics word problem solution accuracy for students identified as low performing by a mathematics state achievement test?" was asked to determine whether this word problem solving intervention would increase accuracy on a test of word problems. The problem-solving routine central to *Solve It!*, RPV-HECC, aimed to help a student break down the text-base of the word problem into propositions with information about objects, quantities, specifications, and roles as modeled by Kintsch and Greeno (1983). The propositions are then compiled to make the macrostructure of the problem that can then be acted on by problem-solving procedures. Moran et al. (2014) asserted that mathematics word problems can be more difficult to solve because of the problem model that needs to be created from the textual input before solving, and that considering a greater number of propositions in the text influences comprehension, leading to greater accuracy in answers because of a better understanding of the problem. This research question assessed whether RPV-HECC played a significant role in aiding students in comprehending the text-base of a word problem and in turn increasing accuracy in solving mathematics word problems.

In this study, the results from RQ1 did not provide evidence that the *Solve It!* intervention significantly affected changes in accuracy scores from pre-test to post-test or from post-test to maintenance test in students who received the intervention when controlling for racial identity group and gender identity group. This result does not support the idea that better comprehension of the text-base of a word problem leads to increased accuracy of the problem solution. One possible reason that the results do not support the hypothesis is the small number of students who completed both a pre- and a post-test (39) and a post- and a maintenance test (25). There was a high level of absenteeism in the sample with only seven of 28 students in the intervention group attending at least 4 of the 8 intervention sessions. Small data sets decrease the power of a study, and this is a possible reason for statistically non-significant results in this study. The absentee rate is discussed further in the challenges section.

Discussion of RQ2. RQ2 "What are the effects of the *Solve It!* intervention on mathematics word problem perseverance for students identified as low performing by a mathematics state achievement tests?" was asked to determine whether this word problem solving intervention would increase perseverance on a test of word problems. Bandura (1977) asserts that students who started to see performance accomplishments in solving mathematical word problems would increase their perseverance in solving. The *Solve It!* instructional approach incorporates teacher modeling of reading word problems multiple times. When a student sees the teacher model this then they know that this is a practice they should also incorporate (DiGisi & Fleming, 2005; Hamilton, 2017).

In this study, the results from RQ2 did not provide evidence that the *Solve It!* intervention significantly affected changes in perseverance from pre-test to post-test in students who received the intervention when controlling for racial identity group and gender identity group. One reason for the nonsignificant impact may be the small sample size (40) of students who completed both a pre- and a post-test.

Discussion of RQ2a. RQ2a "What are the differential effects of the *Solve It!* intervention on perseverance for one-, two-, and three-step mathematics word problems?" was asked to determine if there were differing effects on perseverance based on the number of steps in the word problem. The data showed no significant change in perseverance for two-step or three-step word problems from pre-test to post-test for students in the intervention group when controlling for racial identity group and gender identity group. The post-test made up a single one-step question, so this measure was not analyzed for significance. The pre-test included four two-step questions and three three-step questions while the post-test included six two-step questions and three three-step questions. The small number of questions per type in tandem with the overall sample size may explain non-significant results for RQ2a.

Discussion of RQ3. RQ3 "What are the effects of the *Solve It!* intervention on student self-efficacy around mathematics?" was asked because it drew on Schunk's (2003) assertion that students with high efficacy work towards mastery of tasks while those with low efficacy try to avoid tasks. This efficacy is built from classroom strategy instruction and verbalization and performance feedback from the teacher (Schunk, 2001). This follows from Bandura's (1977) theory that when peers see each other succeed, their own sense of self-efficacy increases through vicarious experiences. The *Solve It!* intervention uses teacher modeling and guided practice where feedback is given to students about their work. Student observation of modeling and internalization of personal feedback could increase self-efficacy in solving mathematics word problems.

In this study, the results from RQ3 did not provide evidence that the *Solve It!* intervention significantly affected changes in self-efficacy from pre-test to post-test in students who received the intervention when controlling for racial identity group and gender identity group. One reason for the nonsignificant impact may be the small sample size (36) of students who completed both a pre- and a post-test. Another possibility is that students did not feel like the feedback they received from their teacher helped them feel successful. This breakdown in teacher-student rapport could greatly discount the prospective effectiveness of the intervention. Before pre-testing students using the self-efficacy instrument, the researcher held a practice session for using the instrument's scale by comparing it to judging the student's certainty of successfully jumping progressively longer distances. It is likely that not all students who were pre-tested were present for the practice session or did not thoroughly understand the purpose of it. This, and a possible want to impress the researcher, may have skewed their answers to the questions.

Discussion of RQ3a. RQ3a "To what extent does student self-efficacy correlate to changes in mathematics word problem solution accuracy?" was asked following the word of Bandura (1977) and Schunk (2003), who contend that increased self-efficacy enhances a student's want to learn more, and as student's strived to learn more, increased accuracy in problem-solving should follow. Moran et al.'s (2014) study employed an instructional model that provided scaffolding from the teacher to allow student's to eventually solve problems independently. The teacher provided feedback that allowed students to see their progress, monitor their own learning, and eventually solve mathematics word problems.

In this study, the results from RQ3a did not provide evidence that self-efficacy is correlated to changes in mathematics word problem solution accuracy for students identified as low performing by a mathematics state achievement test using the *Solve It!* intervention. A reason for this could be that there were no significant changes in self-efficacy or solution accuracy, thus there would be no significant correlation between changes in the two. Additionally, only 31 students completed pre- and post-tests for both self-efficacy and accuracy, so the sample size was small.

Discussion of RQ3b. RQ3b "To what extent does student self-efficacy correlate to changes in mathematics word problem perseverance?" was asked following Bandura (1977), who theorized that when students start to see performance accomplishments their self-efficacy toward solving mathematical word problems increases and in turn increases perseverance in solving.

In this study, the results from RQ3b did not provide evidence that self-efficacy is correlated to changes in mathematics word problem perseverance for students identified as low performing by a mathematics state achievement test using the *Solve It!* intervention. Similar to RQ3a, a reason for not finding a correlation could be that both self-efficacy and perseverance were not significantly changed in this study; thus, a correlation between their changes would not be shown.

Conclusion on Results

The results in this study failed to refute all three null hypotheses. Students who received the *Solve It!* intervention did not have statistically significant gains in solution accuracy compared to students who did not receive the intervention controlling for racial identity group and gender identity group. Additionally, there were no significant relationships found between perseverance and solution accuracy or between self-efficacy and solution accuracy using the *Solve It!* intervention.

Challenges

The most significant challenge encountered during the study was collecting data from students who had many absences. Only seven out of the 28 students in the intervention group attended at least half of the intervention practice sessions. Practice sessions occurred once per week, thus 75% of the intervention participants were absent at least once per week. Missing the sessions is a likely reason for the nonsignificant impact of the intervention on accuracy, perseverance, and self-efficacy, as well as non-significant relationships between accuracy and self-efficacy and perseverance and self-efficacy.

This absenteeism reduced the amount of data that could be collected from students. If a student was absent on the day of a pre- or a post-test, the researcher tried to find that student on several more occasions throughout the week to obtain that data; however, some students were absent for multiple consecutive days. A pre-test could not be given after the start of the intervention and a post-test could not be given too long after the end of the intervention or the results would lose meaning. These missing data points diminished the data set and reduced the power of the study, thereby making it more difficult to find significance in the analysis stage.

Response bias. The researcher was a teacher at the school of study. Specifically, the researcher taught mathematics in a classroom adjacent to the teacher's involved in the study. Because of this, the students in the control and intervention groups saw the researcher in the hallways every day before, during, and after the period of study. The researcher's position may have affected the way students responded on the self-efficacy instrument. Some students may have altered their answers to the self-efficacy questions in an effort to impress the researcher. For example, a student a student with low self-efficacy could have expressed high self-efficacy when the researcher asked about each question because the student was trying to impress the researcher.

Limitations and Potential Modifications to Present Research Design

In this section, limitations of the present research design are considered, as well as possible modifications for future iterations of this study.

Sample size. The current study design used two classes, a control and intervention, taught by one teacher. While this design choice limited inter-teacher variability it also limited the study to a small sample size. Modifying the design to include a second teacher would expand the data set and increase the power of the analysis. If a second teacher were added to the design, then a measure for inter-teacher reliability would need to also be created.

Instrumentation. The data set collected from the instruments for pre- and post-tests for accuracy, perseverance, and self-efficacy was small because of large numbers of absences from students. This made the analysis of the data difficult because there were so few complete data points, a pre-test and a post-test from the same student, to consider. The researcher was not able to determine the impact of absenteeism on the present study as both control and intervention classes reported high numbers of absences from school. One option for a future iteration of the study would be to use the instruments on a control group of students that regularly attends school to increase the size of the data set and thus power of the analysis.

The instruments measuring accuracy, perseverance, and self-efficacy were limiting. Each instrument comprised 10 items to score, and such a small number of items does not allow for much score variability, which also may have been a factor in the non-significance of the results. Increasing the number is a potential modification of this study; however, too many items may overwhelm students, whose participation and perseverance may wane.

Participant feedback. No surveys or interviews have been included in the present design of the quantitative study. However, if the *Solve It!* intervention were monitored again through a qualitative or mixed-methods study design, student interviews could provide feedback about not only students' buy-in of the program but also how students felt about their use of the strategies in the program, and how they felt their performance was perhaps affected by attendance. Robust interviews with students who attended all sessions could be a good basis for a case-study design in a modification to the present study.

Modifying the design to include teacher surveys and student interviews could provide valuable insight into the efficacy of *Solve It!*. A survey of participant teachers could explore their buy-in to the intervention program and their perspective on the uniqueness of it. Teacher buy-in is important as it affects the fidelity of classroom implementation. The cost of the program was $140 for all materials for one teacher. This cost should be weighed with teacher perspective in a math department or school's decision to implement *Solve It!*.

Future Research

The results showed a significant relationship between perseverance and accuracy when control and treatment cases were included in the same model. In exploration of that result, the researcher analyzed the data from the control and treatment groups in separate models. While the treatment-only analysis showed a relationship that approached significance (p=0.063), the significant relationship did not persist in either the control-only analysis or the treatment-only analysis, however in the treatment-only analysis it again approached significance. This may be because of the small sample sizes of each group which further lowered the power of the analysis and made significance of a statistic more difficult to obtain. Future research with an increased sample size should focus on this initial significant relationship between perseverance and accuracy as it relates to Bandura's (1977) theoretical framework. If an increased sample size results in significant p-values in analyses, then it can be concluded that correlations between variables are due to the intervention.

Additionally, practical significance should be explored by calculation of effect sizes. Cohen's *d* is a measure of effect size, is calculated as the difference between two observed sample means in standard deviation units, and is not affected by sample size. A medium or large effect size could impact a school's decision to implement the intervention. Repeated iterations of the intervention with medium to large effect sizes or larger sample sizes with significant correlations would suggest that *Solve It!* is an evidence-based practice.

A second area of future research could be a program evaluation of *Solve It!* The evaluation would not just measure student outcomes and changes in solution accuracy, perseverance, and self-efficacy, but also capture the perspective of the teachers implementing the intervention. This type of study could use a modification of the current study design to include teacher surveys to help gather feedback about teacher belief in the program. The level of teacher belief in its effectiveness may affect the fidelity with which the teacher carries out the program. Teachers who feel this program is useful will put more energy and focus into ensuring that they incorporate RPV-HECC effectively into instruction, whereas teachers who have little buy-in may avoid the intervention altogether.

An inclusion of student voice could highlight whether students have buy-in to the intervention as well. The intervention employed pre-made materials for pre- and post-tests and practice sessions from Montague (2013). Students who do not see relevance to their own lives in the pre-made word problems may not be as motivated to solve them. Students can also be asked about how they used RPV-HECC during class and testing sessions. Questions exploring frequency and timing of use could provide insight into how students use the intervention. Finally, students could be asked questions regarding the relationship between their attendance, use of RPV-HECC, and their thoughts about perceived effectiveness of *Solve It!*.

Summary of Chapter 5

This chapter provided a discussion of the results of each research question. While the results failed to refute all three null hypotheses of the study, the experience of data collection and analysis provided insight into study design modifications that could strengthen the impact of future iterations of this study. Challenges and limitations encountered during the present study included a high rate of student absenteeism, a small sample size, and low score variability of instruments. An idea for future research to explore the relationship between perseverance and accuracy was suggested. Additionally, modifications to study design were suggested to perform a program evaluation taking into consideration teacher and student voice.

Conclusion

The purpose of this study was to assess the effects of *Solve It!*, an intervention which focused on teaching students the processes and strategies needed to represent mathematics problems through a seven step process; read, paraphrase, visualize, hypothesize, estimate, compute, and check. Perseverance and self-efficacy of solving mathematics word problems was measured to explore the extent that they play a role in solution accuracy. Students who participated in *Solve It!* were part of a year-long Math I class and were placed in that class due to historical low performance on state achievement tests. Results indicated that the *Solve It!* intervention did not have a significant effect on the change in accuracy scores from pre-test to post-test or post-test to maintenance test, the change in perseverance scores, as a whole or by problem-type, from pre-test to post-test, or change in self-efficacy from pre-test to post-test. The results showed a significant relationship between perseverance and solution accuracy, although the significance disappeared when the control and intervention groups were analyzed separately. Future research should be directed to further explore this initial significant relationship. The results do not show a significant relationship between accuracy and self-efficacy or between self-efficacy and perseverance. While not significant, results show that the intervention positively affected all measures of this study. This provides an impetus for future research to explore whether the effects on the measures become significant with a larger sample. Teacher and student voice can be collected in future research to capture perspectives on *Solve It!* and factor into a program evaluation of the intervention. Findings from this future research can add to the literature of the effectiveness of schema- and cognitive-based literacy interventions for improvement in math word problem-solving ability.

APPENDIX A: PRE- AND POST-TEST FOR SOLUTION ACCURACY AND PERSEVERANCE

These resources are reproduced from the *Solve It!* curriculum materials (Montague, 2008).

Pretest #1

Name ______________________________

1) Two schools plan a trip to the science museum. There are 1,044 people. Each bus holds 58 people. How many buses are needed?

2) Two cantaloupes and three honeydew melons cost a total of $1.75. The cantaloupes cost $.50 each. How much did each honeydew melon cost?

(x÷=−+) (x÷=−+) **Appendix** A-1 (x÷=−+) (x÷=−+)

Pretest #1 (continued)

Name ____

3) In one week, the local newspaper printed 762,954 copies. From Monday to Saturday, a total of 255,960 morning papers were printed and 396, 475 evening papers were printed. How many copies of the Sunday paper were printed?

4) After working 24 weeks, Julio saved $50.90. With this money, he bought a radio that cost $49.50 and four batteries that cost $.35 each. How much money did he have left?

Pretest #1 (continued)

Name ______________________________

5) **Marcus rents out his fishing boat for $50 per day. If he rented it out 20 days last summer, how much money did he earn?**

6) **There are four trays of jelly-filled donuts on the shelf. Each tray holds 72 donuts. A customer buys 45 donuts for a party. How many donuts are left?**

Pretest #1 (continued)

Name ____

7) **Tanya earns $6.25 an hour. D'Andra earns $7.90 an hour. How much more than Tanya does D'Andra earn in an eight-hour work day?**

8) **It is 1,458 miles from Chicago to Denver. It is 1,530 miles from Denver to San Francisco. What is the total distance from Chicago to San Francisco, going through Denver?**

Pretest #1 (continued)

Name ____

9) **We need stamps to send 22 packages. Each package needs a $.60 stamp and a $.32 stamp. How much money for stamps do we need in total?**

10) **Chris bought six books at $1.95 each. She used a 20 dollar bill to pay for the books. What change did she receive?**

Posttest #1

Name ______________________________

1) Two schools plan a trip to the science museum. There are 1,044 people. Each bus holds 58 people. How many buses are needed?

2) Two cantaloupes and three honeydew melons cost a total of $1.75. The cantaloupes cost $.50 each. How much did each honeydew melon cost?

(x÷=−+) (x÷=−+) **Appendix** A-2 (x÷=−+) (x÷=−+)

Posttest #1 (continued)

Name ____

3) In one week, the local newspaper printed 762,954 copies. From Monday to Saturday, a total of 255,960 morning papers were printed and 396, 475 evening papers were printed. How many copies of the Sunday paper were printed?

4) After working 24 weeks, Julio saved $50.90. With this money, he bought a radio that cost $49.50 and four batteries that cost $.35 each. How much money did he have left?

Posttest #1 (continued)

Name ____

5) Marcus rents out his fishing boat for $50 per day. If he rented it out 20 days last summer, how much money did he earn?

6) There are four trays of jelly-filled donuts on the shelf. Each tray holds 72 donuts. A customer buys 45 donuts for a party. How many donuts are left?

Posttest #1 (continued)

Name ____

7) Tanya earns $6.25 an hour. D'Andra earns $7.90 an hour. How much more than Tanya does D'Andra earn in an eight-hour work day?

8) It is 1,458 miles from Chicago to Denver. It is 1,530 miles from Denver to San Francisco. What is the total distance from Chicago to San Francisco, going through Denver?

Posttest #1 (continued)

Name ____

9) **We need stamps to send 22 packages. Each package needs a \$.60 stamp and a \$.32 stamp. How much money for stamps do we need in total?**

10) **Chris bought six books at \$1.95 each. She used a 20 dollar bill to pay for the books. What change did she receive?**

APPENDIX B: PERSEVERANCE CHECKLIST

Use one checklist per problem per student on the pre-test, weekly practice problems (e.g. WP#1, WP#2, etc.), and post-test.
Check 'yes' if you observe evidence of the technique in the student's work. Check 'no' if you do not observe evidence of the technique in the student's work.
Student Name:_________________________ **Assessment:_________________**

Problem #1			
Kind of Struggle	**Example**	**Yes**	**No**
Get started	Paper not left blank		
Carry out a process	Plan for achieving goal present Algebraic procedure or geometric formula carried out		
Uncertainty in explaining and sense-making	Description of a reasonable answer in the context of the problem Units used in answer		
Express misconception and errors	Understanding of relationship among numbers and units in problem		

Problem #2			
Kind of Struggle	**Example**	**Yes**	**No**
Get started	Paper not left blank		
Carry out a process	Plan for achieving goal present Algebraic procedure or geometric formula carried out		
Uncertainty in explaining and sense-making	Description of a reasonable answer in the context of the problem Units used in answer		
Express misconception and errors	Understanding of relationship among numbers and units in problem		

Problem #3			
Kind of Struggle	**Example**	**Yes**	**No**
Get started	Paper not left blank		
Carry out a process	Plan for achieving goal present Algebraic procedure or geometric formula carried out		
Uncertainty in explaining and sense-making	Description of a reasonable answer in the context of the problem Units used in answer		
Express misconception and errors	Understanding of relationship among numbers and units in problem		

Problem #4			
Kind of Struggle	**Example**	**Yes**	**No**
Get started	Paper not left blank		
Carry out a process	Plan for achieving goal present Algebraic procedure or geometric formula carried out		
Uncertainty in explaining and sense-making	Description of a reasonable answer in the context of the problem Units used in answer		
Express misconception and errors	Understanding of relationship among numbers and units in problem		

Problem #5			
Kind of Struggle	**Example**	**Yes**	**No**
Get started	Paper not left blank		
Carry out a process	Plan for achieving goal present Algebraic procedure or geometric formula carried out		
Uncertainty in explaining and sense-making	Description of a reasonable answer in the context of the problem Units used in answer		
Express misconception and errors	Understanding of relationship among numbers and units in problem		

Problem #6			
Kind of Struggle	**Example**	**Yes**	**No**
Get started	Paper not left blank		
Carry out a process	Plan for achieving goal present Algebraic procedure or geometric formula carried out		
Uncertainty in explaining and sense-making	Description of a reasonable answer in the context of the problem Units used in answer		
Express misconception and errors	Understanding of relationship among numbers and units in problem		

Problem #7			
Kind of Struggle	**Example**	**Yes**	**No**
Get started	Paper not left blank		
Carry out a process	Plan for achieving goal present Algebraic procedure or geometric formula carried out		
Uncertainty in explaining and sense-making	Description of a reasonable answer in the context of the problem Units used in answer		

Express misconception and errors	Understanding of relationship among numbers and units in problem		

Problem #8			
Kind of Struggle	**Example**	**Yes**	**No**
Get started	Paper not left blank		
Carry out a process	Plan for achieving goal present Algebraic procedure or geometric formula carried out		
Uncertainty in explaining and sense-making	Description of a reasonable answer in the context of the problem Units used in answer		
Express misconception and errors	Understanding of relationship among numbers and units in problem		

Problem #9			
Kind of Struggle	**Example**	**Yes**	**No**
Get started	Paper not left blank		
Carry out a process	Plan for achieving goal present Algebraic procedure or geometric formula carried out		
Uncertainty in explaining and sense-making	Description of a reasonable answer in the context of the problem Units used in answer		
Express misconception and errors	Understanding of relationship among numbers and units in problem		

Problem #10			
Kind of Struggle	**Example**	**Yes**	**No**
Get started	Paper not left blank		
Carry out a process	Plan for achieving goal present Algebraic procedure or geometric formula carried out		
Uncertainty in explaining and sense-making	Description of a reasonable answer in the context of the problem Units used in answer		
Express misconception and errors	Understanding of relationship among numbers and units in problem		

For researcher use:

	Totals	
Kind of Struggle	**Yes**	**No**
Get started		
Carry out a process		
Uncertainty in explaining and sense-making		
Express misconception and errors		

APPENDIX C: SELF-EFFICACY ASSESSMENT

<u>Circle</u>: Pre-test Post-test

Student Name:____________________

Answer the following statement for each word problem using the scale: *I have the ability to solve this problem*.

Problem #	**Not sure**			**Maybe**			**Pretty sure**			**Really sure**
1	10	20	30	40	50	60	70	80	90	100
2	10	20	30	40	50	60	70	80	90	100
3	10	20	30	40	50	60	70	80	90	100
4	10	20	30	40	50	60	70	80	90	100
5	10	20	30	40	50	60	70	80	90	100
6	10	20	30	40	50	60	70	80	90	100
7	10	20	30	40	50	60	70	80	90	100
8	10	20	30	40	50	60	70	80	90	100
9	10	20	30	40	50	60	70	80	90	100
10	10	20	30	40	50	60	70	80	90	100

APPENDIX D: FIDELITY CHECKLIST

These resources are reproduced from the *Solve It!* curriculum materials (Montague, 2008).

Solve It! Observation Protocols

Lesson 3– Strategy Master Verbalization

Code the occurrence of each instructional component using the following keys:
Yes = The behavior is observed.
No = The behavior is not observed.

Preparation

Did the teacher:	Coding	Notes
Prepare transparency of problem?	❒ Yes ❒ No	
Display Master Class Charts?	❒ Yes ❒ No	

Implementation

Did the teacher:	Coding	Notes
Check all students for verbalization of strategy?	❒ Yes ❒ No	
Prompt students about RPV-HECC?	❒ Yes ❒ No	
Record whether students met 100% correct criterion?	❒ Yes ❒ No	
Model problem solving by **Reading** the problem?	❒ Yes ❒ No	

Facilitator Tool 6-5 (continued)

Did the teacher:	Coding	Notes
Model problem solving by **Paraphrasing** the problem?	❒ Yes ❒ No	
Model problem solving by **Visualizing** (emphasizing relationships among problem parts)?	❒ Yes ❒ No	
Model problem solving by **Hypothesizing**?	❒ Yes ❒ No	
Model problem solving by **Estimating**?	❒ Yes ❒ No	
Model problem solving by **Computing**?	❒ Yes ❒ No	
Model problem solving by **Checking**?	❒ Yes ❒ No	
Use the group problem solving routine?	❒ Yes ❒ No	
Guide students in interaction session?	❒ Yes ❒ No	
Provide strategy rehearsal practice by covering the Master Class Charts?	❒ Yes ❒ No	
Check for mastery of verbalization and record student performance?	❒ Yes ❒ No	
Additional notes:		

APPENDIX E: WEEKLY PRACTICE PROBLEMS

These resources are reproduced from the *Solve It!* curriculum materials (Montague, 2008).

Solve It! Curriculum-Based Measures (CBMs)

CBM-1

Student Name		
Date	Class Period	Grade

Directions: **Read** the problem. **Think** about solving the problem. Then **solve** the problem.

1) Mr. Sanchez has 40 students in the first period, 30 in his second period, 35 in his third period, 27 in his fourth period, and 32 in his fifth period. What is the total number of students?

2) Mr. Garton bought four rolls of film in the museum gift shop. Each roll cost $4.10. He paid with a $20 bill. How much change did he receive?

CBM-1 (continued)

Student Name

3) There are six trays of jelly-filled donuts on the shelf. Each tray holds 52 donuts. Ari comes in to pick up 46 of them for an after-school party. How many donuts are left?

4) Mary earns $7.25 an hour. Jennifer earns $8.75 an hour. How much more than Mary does Jennifer earn in an eight-hour work day?

CBM-1 (continued)

Student Name

5) When Ralph works overtime he gets paid $8.50 per hour. The regular hourly rate is $4.20. How much does he make for a regular week of 40 hours plus five hours of overtime?

6) The camp leader bought 36 gallons of milk for summer camp. Summer camp is three weeks long. The third week she bought 13 more gallons. How many gallons did she buy?

Student Name

7) Warren bought a camera tripod for $39.95 and an electronic flash for $59.95. He also spent $2.89 for batteries. The total cost, including sales tax, was $109.47. How much was the sales tax?

8) A broadcasting company has three maintenance workers for every 50 office workers. There are 12 maintenance workers. How many office workers are there?

Student Name

9) Beth received $54.00 for nine hours of work. Erin received $36.00 for the same number of hours. How much more than Erin did Beth make per hour?

10) Three pens and four notebooks cost $5.50. If each pen costs $.50, how much does each notebook cost?

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