An Improved Complex Signal Based Calibration Method for Beam-Steering Phased Array

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An Improved Complex Signal Based Calibration Method for Beam-Steering Phased Array

Fengchun Zhang, Huaqiang Gao, Zhengpeng Wang and Wei Fan

***Abstract*—Phased array calibration is essential to ensure accurate array radiation performance. This paper proposes an improved calibration method based on complex signals in an anechoic chamber for phased arrays that can only operate in its default beam-steering mode with limited beam-steering angular ranges. The basic principle is to convert the array factors associated with the multi-probe locations into the beam-steering factors of the array, which can virtually widen the beam-steering angular range. The broadened angular range can lower the condition number of the phase setting matrix and therefore improve the calibration accuracy. Both theoretical derivations and measurement validation are provided to demonstrate the effectiveness and enhancement of the proposed method.**

***Index Terms*—Over-the-air testing, millimeter-wave phased array, array calibration, beam-steering angular range.**

I. Introduction

Phased arrays have found increasing use in various key applications, e.g. 5G, radar, and satellite communications, to provide high-speed and reliable wireless data links. To ensure accurate and optimal radiation performance for practical phased arrays, array calibration is often required to compensate for the imhomogenities among radio frequency (RF) chains associated with phased array elements.

Phased array calibration has been a long standing research topic and a variety of measurement techniques have been proposed in the literature [1]–[11]. The calibration methods can be grouped into two categories: on-off method and all-on methods. The on-off method is typically adopted in the industry, where the RF responses of array elements are measured one by one via activating the target element and deactivating other elements. In the industry, on-off is often done in the near-field with the help of a scanner [12]. However, on-off measurements might not truthfully reflect the states of the array elements in their default all-on mode, especially for mmWave phased arrays [6]. In all-on calibration measurements, all elements radiate simultaneously while we can tune the phase excitation for individual array elements. Based on the measured data, calibration methods have been developed for both complex signal data [4], [5] and amplitude-only signal data [1], [3], [8], [10], [11].

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However, dedicated phase tuning operations per element might not be supported by commercial mmWave radios, making the above mentioned all-on methods inapplicable. It would be desirable if array calibration can be done in the default beam-steering mode, i.e. via simultaneously tuning phases of all elements according to beam-steering vectors, which is generally supported by device under test (DUT).

An array calibration method is proposed in [13] for phased arrays that can only operate in beam-steering mode. However, the algorithm is restricted to uniform linear array (ULA)-type phased array, whose beam-steering matrix has the unique Vandermonde structure. The algorithm would deteriorate and fail for the DUTs with a limited beam-steering range and it can not be directly applied for phased arrays with arbitrary array configurations. To address these problems, a multi-probe strategy is proposed in this paper to virtually increase beam angular interval steered by the DUT array, thus improving the calibration accuracy. The proposed strategy necessitates repetitive measurements for multiple DUT orientations or multiple probe antenna directions, which can be easily realized and automated in practical systems. The proposed method also works for phased arrays with arbitrary array configurations.

In this paper, we firstly review the reference calibration method presented in [13] and highlight the limitation of the existing solution. After that, a strategy to tackle the problem with ill-conditioned beam-steering matrix is discussed. We then experimentally validate the proposed algorithm in a compact antenna testing range (CATR) measurement setup. Our proposed solution is essentially an extension and improvement of the reference method presented in [13].

II. Method

A. Signal Model

The system diagram for calibrating a N-element DUT array is illustrated in Fig. 1, where $Q = 2M + 1$ probes are symmetrically distributed around the boresight direction and numbered from $-M$ to M with θ_m denoting the angular location of the m-th probe, with $\theta_{-m} = -\theta_m$ and $\theta_0 = 0^o$. The complex signal received by the m-th probe is given by:

$$\mathbf{s}(\theta_m) = \mathbf{B} \cdot \mathbf{A}(\theta_m) \cdot \mathbf{c}, \tag{1}$$

where the vectors and matrices are explained below:

- The vector $\mathbf{s}(\theta_m) \in \mathbb{C}^{P\times 1}$ is the complex signal vector received by the m-th probe antenna when P phase shifter settings are implemented for the DUT antennas.
- The matrix $\mathbf{B} \in \mathbb{C}^{P\times N}$ is the phase setting matrix with its entry b_{pn} denoting the p-th phase shifter setting

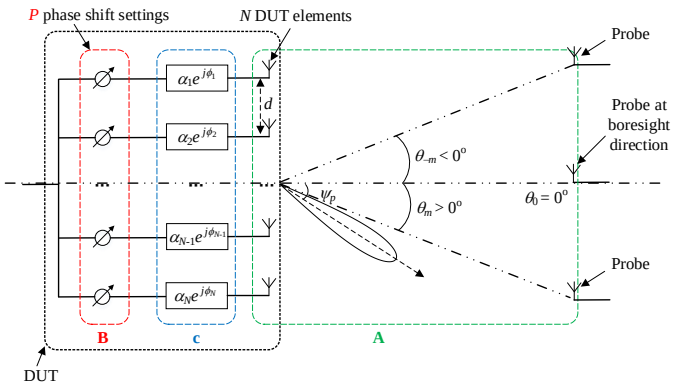

Figure 1. System diagram of a multi-probe setup for phased array calibration.

Table I
CONDITION NUMBER OF $\mathbf{B}$ FOR VARIOUS Ψ VALUES.

Ψ[deg]	10	20	30	40	50 - 90
Condition number	241	19.6	4.4	1.7	1

implemented for the n-th DUT antenna. To steer a beam in direction ψ_p with a ULA, b_{pn} should be set as:

$$b_{pn} = e^{-jk(n-1)d\sin\psi_p}, \tag{2}$$

where k and d represent the wave number and the element spacing, respectively. The beam directions ψ_p varies within $[-\Psi, \Psi]$ with $\Psi \leq 90^o$. Note that the proposed principle can be applied for arbitrary array configuration, though ULA is investigated in this work for simplicity.

- The matrix $\mathbf{A}(\theta_m) \in \mathbb{C}^{N\times N}$ is a diagonal matrix with the diagonal elements representing the coupling coefficients between the DUT antenna ports and the m-th probe antenna port. Assuming free space propagation scenario under the far-field condition and a constant complex gain value among elements over a limited angular region for both the DUT elements and probe antennas, the coupling coefficient between the n-th DUT antenna port and the m-th probe antenna port can be simplified as:

$$a_n(\theta_m) = G\underline{G}\frac{e^{jk(n-1)d\sin\theta_m}}{2kr}, \tag{3}$$

where G and $\underline{G}$ denote the complex radiation patterns of the DUT antennas and the probe antennas, respectively. The above assumption is valid since the DUT arrays are typically composed of identical elements with broad main beams in design. The parameter r is the distance between the center of the DUT array and the probe array.

- The vector $\mathbf{c} \in \mathbb{C}^{N\times 1}$ denotes the calibration vector with the entry c_n denoting the complex initial excitations of the n-th DUT antennas. The goal of the calibration is to detect the complex initial excitations among the DUT antennas, i.e. the vector $\mathbf{c}$, and further to compensate for the excitation discrepancies among the DUT elements.

B. Problem Statement

In [13], an array calibration method is proposed for ULAs with a single probe located in the boresight direction of the DUT, i.e. with $m = 0$ and $\theta_0 = 0^o$. According to (1) and (3), the calibration vector $\mathbf{c}$ can be solved as:

$$\mathbf{c} = \frac{2kr}{G\underline{G}} \cdot \mathbf{B}^+ \cdot \mathbf{s}(0^o), \tag{4}$$

where $()^+$ denotes the pseudo-inverse operator.

However, several factors would affect the calibration accuracy:

- The approximation errors in radiation patterns of DUT antennas and probe antennas in matrix $\mathbf{A}$. The radiation patterns are not identical in a practical array, due to the fabrication error and mutual coupling effect among the elements. Therefore, the pattern approximation in (3) for matrix A will introduce errors in the estimated c_n.
- Implementation errors in the phase setting matrix $\mathbf{B}$. Although the target matrix $\mathbf{B}$ used in (4) is ideally determined, the actual matrix $\mathbf{B}$ in (1) implemented with phase shifters will suffer from the quantization error and stepping error (tuning error) in both amplitude and phase terms. Thus, the matrix $\mathbf{B}$ implemented with phase shifters will introduce errors in the estimated c_n.
- The measurement noise in the complex signal vector $\mathbf{s}(\theta_m)$. In a practical measurement, the measurement noise is unavoidable, which will introduce estimation errors to the calibration coefficients as well.

As concluded in [13], the calibration accuracy, i.e. the estimation accuracy of the vector $\mathbf{c}$, relies on the condition number of the phase setting matrix $\mathbf{B}$. The phase setting matrix $\mathbf{B}$ in beam-steering mode defined in (2) is mainly ruled by the steering angle interval Ψ of the DUT array, as illustrated in Table. I for a 4-element ULA with a half-wavelength spacing. It can be observed that the condition number of matrix $\mathbf{B}$ decreases as Ψ increases from 10° to 50° and converges to the minimum value 1 for $\Psi \geq 50^o$.

However, in many deployment scenarios, phased arrays can only steer beams within a small angular range, e.g. up to 15° for LTE base station antennas, leading to a poor calibration accuracy with the method in [13] due to the large condition number of matrix $\mathbf{B}$. Therefore, a more generic calibration method for beam-steering phased arrays is highly demanded.

C. Proposed solution

In this work, we propose an improved calibration method for beam-steering phased arrays. The basic idea is to virtually increase the DUT steering range, therefore reducing the condition number, by employing multiple probes or measuring the DUT in multiple orientation angles (which are equivalent in principle). The product of matrix $\mathbf{B}$ and $\mathbf{A}(\theta_m)$ can be denoted by matrix $\mathbf{H}(\theta_m)$, where its entry $h_{pn}(\theta_m)$ can be calculated by following (2) and (3):

$$\begin{aligned} h_{pn}(\theta_m) &= b_{pn} \cdot a_n(\theta_m) \\ &= \frac{G\underline{G}}{2kr} e^{-jk(n-1)d\cdot(\sin\psi_p - \sin\theta_m)} \\ &= \frac{G\underline{G}}{2kr} e^{-jk(n-1)d\cdot\sin[\phi_p(\theta_m)]}, \end{aligned} \tag{5}$$

where we define

$$\sin[\phi_p(\theta_m)] = \sin\psi_p - \sin\theta_m. \tag{6}$$

A new signal vector $\tilde{\mathbf{s}} \in \mathbb{C}^{QP\times 1}$ can be constructed by stacking the signal vectors received by the probes as:

$$\tilde{\mathbf{s}} = [\mathbf{s}^T(\theta_M), ..., \mathbf{s}^T(\theta_0), ..., \mathbf{s}^T(\theta_{-M})]^T, \tag{7}$$

where $()^T$ denotes the transpose operator. Accordingly, a new matrix $\tilde{\mathbf{H}} \in \mathbb{C}^{QP\times N}$ can be written as:

$$\tilde{\mathbf{H}} = [\mathbf{H}^T(\theta_M), ..., \mathbf{H}^T(\theta_0), ..., \mathbf{H}^T(\theta_{-M})]^T. \tag{8}$$

Based on (1), (7) and (8), the signal model with a multi-probe setup can be expressed by:

$$\tilde{\mathbf{s}} = \tilde{\mathbf{H}} \cdot \mathbf{c}, \tag{9}$$

where the entry of the matrix $\tilde{\mathbf{H}}$ can be given as:

$$\begin{aligned} \tilde{h}_{in} &= \frac{G\underline{G}}{2kr} \cdot e^{-jk(n-1)d\cdot \sin\tilde{\psi}_i} \\ &= \frac{G\underline{G}}{2kr} \cdot \tilde{b}_{in}, \end{aligned} \tag{10}$$

where $\tilde{b}_{in}$ is expressed by:

$$\tilde{b}_{in} = e^{-jk(n-1)d\cdot \sin\tilde{\psi}_i}. \tag{11}$$

Comparing the above equation with (2), we can see that the phase of $\tilde{b}_{in}$ is equal to the phase shift set for the n-th DUT antenna to steer a beam in direction $\tilde{\psi}_i$. Based on (9), (10) and (11), the calibration vector $\mathbf{c}$ can be solved as:

$$\mathbf{c} = \frac{2kr}{G\underline{G}} \cdot \tilde{\mathbf{B}}^+ \cdot \tilde{\mathbf{s}}. \tag{12}$$

Again, the estimation accuracy of vector $\mathbf{c}$ is determined by the condition number of the matrix $\tilde{\mathbf{B}}$.

When ψ_p and θ_m are small, the i-th virtual beam direction $\tilde{\psi}_i$ in (11) can be approximated according to (6), (8) and (10):

$$\tilde{\psi}_i = \phi_p(\theta_m) \approx \psi_p - \theta_m, \tag{13}$$

where the index $i = (M-m)P + p$ via setting $m = M, ..., -M$ and $p = 1, ..., P$, respectively.

For a DUT array with the steering angles $\psi_p \in [-\Psi, \Psi]$, the virtual steering angle range is extended to $[-\theta_M - \Psi, \theta_M + \Psi]$ with $\theta_M > 0^o$ denoting the angular location of the edged probe. The range extension is accomplished via converting the DUT array factors in the directions of the probes into the beam-steering factors. The widened beam-steering angle range can decrease the condition number of matrix $\mathbf{B}$ and thereby improve the calibration accuracy. Hence, the proposed multi-probe setup is capable of calibrating phased arrays even with a limited steering range.

As explained, the matrix $\tilde{\mathbf{B}}$ should be designed to have a minimum condition number in principle to avoid magifying the errors, which requires a beam-steering range $\geq 50^o$. To achieve a larger virtual beam-steering range with a multi-probe setup, the edged probes should be located further away from the boresight direction, i.e. 0°, which however, will introduce larger approximation errors in the radiation patterns in (3). Therefore, the selection of the probe locations is a trade-off between the virtual beam-steering range (determining the condition number of matrix $\mathbf{B}$) and the approximation errors in the radiation patterns of the DUT and the probe antennas.

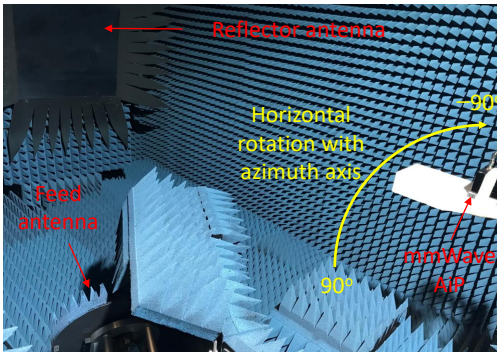

Figure 2. A photo of the measurement setup seen inside the CATR chamber (VNA and control computer are outside the chamber and not shown).

III. Measurement Validation

A. Measurement Setup and Procedure

The validation measurements were performed in a standard CATR setup, where a 4×4 mmWave phased array antenna-in-package (AiP) platform detailed in [6] and [13] was employed as the DUT. As shown in Fig. 2, the measurement setup consisted of the phased array AiP, the CATR setup, VNA, a DC power supply for the AiP and a control computer to automate measurements and store the measured data.

The mmWave AiP was placed in the quiet zone of the CATR where the polarization of feed antenna and mmWave AiP was aligned. The azimuth axis of 0° represents the boresight direction of the AiP pointing to the CATR reflector center. To validate the efficiency of the proposed calibration method in practice, the following two measurements were conducted.

- Calibration measurement using the well-known rotating element electric field vector (REV) method. Same as [13], the REV method was adopted to obtain the ground truth coefficients of the AiP elements. The REV calibration procedure was detailed in [14].
- Beam-steering calibration measurement for the AiP with multiple orientations. The AiP was horizontally rotated around the azimuth axis from 90° to -90° with a step of -10°, which is equivalent to multiple probes located in cooresponding directions as shown in Fig. 1. For each orientation of the AiP (i.e. each probe location), the AiP performed horizontal beam-steering operations from -90° to 90° with a total of 65 beam-steering angles. The complex S-parameters were recorded in the VNA for each beam-steering operation in each orientation.

As explained in [13], the calibration coefficients of the 1 × 4 ULA elements can be determined based on horizontal beam-steering measurements, with each ULA element being a subarray composed of 4 × 1 antenna elements.

B. Measurement Results

To measure the estimation accuracy of a set of complex calibration coefficients, the root-mean-square deviation (RMSD) is taken as a figure of merit in this letter, which is defined as

$$\text{RMSD} = \sqrt{\frac{\sum_{n=1}^{N} |c_n - \hat{c}_n|^2}{N}}, \tag{14}$$

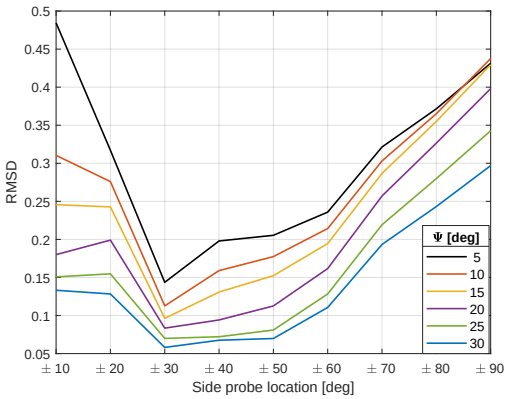

Figure 3. RMSD with the proposed method in a 3-probe setup via locating side probes in different directions for various Ψ.

where c_n represents the coefficients obtained with REV method and $\hat{c}_n$ denotes the coefficients obtained with beam-steering mode methods, for various settings, e.g. the beam-steering range and the configuration of the probe array.

To investigate the impact of the side probe locations on the calibration accuracy, an analysis is done for a 3-probe setup based on the data measured with the side probes in various directions for DUTs with different beam-steering ranges Ψ, as plotted in Fig. 3. It shows that the minimum RMSD is achieved when the side probes are located in $\pm 30^o$ for DUTs with different beam-steering ranges. For the side probes located from $\pm 10^o$ to $\pm 30^o$, the RMSDs decrease due to the reduced condition number of matrix $\mathbf{B}$ and small approximation errors in the radiation patterns for matrix $\mathbf{A}$. However, when the side probes located further away from each other, i.e. from $\pm 30^o$ to $\pm 90^o$, the condition number of matrix $\mathbf{B}$ is small, and the RMSDs increase mainly due to increased approximation errors in the radiation patterns of the DUT antennas and the probe antennas. The results demonstrate that locating side probes in $\pm 30^o$ is a good trade-off between the condition number of matrix $\mathbf{B}$ and the approximation accuracy of matrix $\mathbf{A}$. The figure further illustrates that a better calibration accuracy can be achieved for the DUT with a larger beam-steering range Ψ, which is expected due to the larger beam-steering range and more measurement data are used in the calibration.

Furthermore, the minimal RMSD achieved with an optimal 3-probe setup is compared to that with the method in [13] for different Ψ, as illustrated in Table II. It can be observed that:

- The RMSDs achieved with the proposed method in the 3-probe setup are all obviously lower than those with the method in [13] for various Ψ, which further demonstrates the performance improvement with the proposed method.
- With the proposed method, the RMSD decreases as Ψ increases due to the reduced condition number of matrix $\mathbf{B}$ and more measurement data used in the calculation while constant approximation errors remain in radiation patterns of the DUT antennas and the probe antennas .
- For small beam-steering ranges, i.e. $\Psi \leq 20^o$, the RMSDs with the proposed method are slightly higher than those with the method in [13] with $\Psi = 90^o$ due to the slightly higher condition number of matrix $\mathbf{B}$ for the virtual beam-steering range within $[35^o, 50^o]$. While for $\Psi > 20^o$, the RMSDs with the proposed method are almost the same to those with the method in [13] with $\Psi = 90^o$ since the condition number of matrix $\mathbf{B}$ converges to 1 when the virtual beam-steering range larger than 50°, as illustrated in Table I.

Table II
RMSD COMPARISON BETWEEN THE 1-PROBE AND 3-PROBE SETUP.

Ψ[deg]	RMSD for 1-probe setup	RMSD for 3-probe setup
5	1.276	0.144
10	0.574	0.113
15	0.489	0.096
20	0.155	0.083
25	0.216	0.070
30	0.138	0.058
35	0.129	0.044
40	0.143	0.037
45	0.109	0.029
50	0.086	0.026
55	0.076	0.029
60	0.075	0.024
65	0.073	0.038
70	0.073	0.042
75	0.073	0.042
80	0.072	0.042
85	0.072	0.042
90	0.072	0.042

The results demonstrate that the proposed method can efficiently enhance the calibration accuracy for the DUTs with limited beam-steering ranges. The achieved calibration performance is comparable to that of the DUTs with a full steering range, i.e. $[-90^o, 90^o]$. Thus, a 3-probe setup, which can achieve acceptable calibration accuracy with a cost-effective setup, is employed for the 1×4 DUT array in the letter.

IV. CONCLUSION

An array calibration method is proposed in [13] for phased arrays operating only in beam-steering mode. However, the calibration performance of the method deteriorates as the beam-steering range shrinks, making it inapplicable to calibrate the phased arrays with limited beam-steering ranges. Therefore, an improved calibration method is proposed in this work. The proposed method employing multiple probes can virtually extend the angle range steered by the DUT array via converting the array factors associated with the probe locations into the beam-steering factors. For a multi-probe setup, the proposed method can virtually enlarge the beam-steering range to $\pm(\theta_M + \Psi)$ with θ_M denoting the angular location of the edged probe and $\pm\Psi$ being the steering range of the DUT array. The widened beam-steering range will weaken the error amplification effect introduced by the phase setting matrix and thereby improve the calibration accuracy. The performance of the proposed method is validated in a mmWave AiP. The array calibration results demonstrate the effectiveness and an obvious enhancement of the proposed method for the calibration of phased arrays with restricted steering angle ranges.

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