Nonlocal Basis Pursuit and the Cost of Antisymmetric Fluxes

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We consider a nonlocal version of the classical optimal design problem of distributing a limited amount of conductive material in a given design domain [1, 2]. In the local case, such problems belong to the class of optimal control problems in the coefficients of PDEs.
Of particular interest for these problems is the study of the limiting case, which appears when the amount of available material is driven to zero. Such a limiting process is of both theoretical and practical interest and continues to be a subject of active study [3, 4]. For example in the context of linear elasticity the resulting limiting problem corresponds to the celebrated case of Michell trusses [3, 4]. The limiting problem can be formulated in terms of fluxes only and is convex. Unfortunately, it admits solutions in the space of Radon measures, and as such is not easy to solve numerically.
With this in mind we focus on identifying the vanishing material limit for the corresponding nonlocal optimal design problem. The resulting limiting nonlocal optimization problem is convex, can be expressed in terms of nonlocal antisymmetric two-point fluxes, and admits solutions in Lebesgue spaces with mixed exponents. When the nonlocal interaction horizon is driven to zero, the "vanishing material limit" nonlocal problems provide a one-sided estimate for the corresponding local measure-valued optimal design problem.
The surprising fact is that in order to transform the one-sided estimate into a true limiting process it is sufficient to disregard the antisymmetry requirement on the two-point fluxes. This result relies on duality and requires generalizing some of the nonlocal estimates established in [5] to the case of mixed Lebesgue exponents.

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