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**WORKING PAPER**

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**“Can personalized pricing be a winning strategy  
in oligopolistic markets with heterogeneous  
demand customers? Yes, it can.”**

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# Can personalized pricing be a winning strategy in oligopolistic markets with heterogeneous demand customers? Yes, it can.\*

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July 2021

## Abstract

This paper aims to understand under what market conditions, can competing symmetric firms employ personalized pricing as a winning strategy. A key departure of our paper from the literature is that we introduce customer heterogeneity in demand. If firms' data discloses only vertical information (demand heterogeneity), firms can only employ group pricing. This is always a winning strategy. When data discloses horizontal information (consumer preferences) and vertical information, perfect personalized pricing (PPP) becomes feasible. If data only discloses horizontal information, firms can only employ imperfect personalized pricing (IPP). By comparing uniform pricing (UP) with personalized pricing, we show that if the share of high demand customers in the market is greater than the share of low demand consumers, firms are always better off with no discrimination. More importantly, we show that if heterogeneity in purchase quantity is sufficiently high, then personalized pricing can be a winning strategy for all symmetric practice firms. If heterogeneity in consumer value is high and the share of high demand consumers is sufficiently low, in comparison to UP, both firms are better off under IPP. For an intermediate share of high demand consumers, firms can get higher profits under PPP than under UP and IPP.

*JEL Code: D43, D80, L13, L40*

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\*This research has been financed by Portuguese public funds through FCT (Fundação para a Ciência e a Tecnologia) in the framework of project PTDC/EGE-ECO/28540/2017. Any errors are my own responsibility.

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# 1 Introduction

*“Personalized Pricing is the practice where businesses may use information that is observed, volunteered, inferred, or collected about individuals’ conduct or characteristics, to set different prices to different consumers (whether on an individual or group basis), based on what the business thinks they are willing to pay.”* (OFT, 2013[3])

The growth of the digital economy and the availability of big data allows businesses to look into consumer preferences, volume purchasing habits, loyalties and other quantifiable differences among customers. As data analytics and pricing algorithms become a common business practice in digital markets, companies are increasingly able to use such tools to estimate more accurately consumers’ willingness-to-pay and engage in personalized pricing, a form of price discrimination that involves charging different prices to consumers with different valuations.

A fundamental, but not a new question, which is of interest to practitioners, economic and marketing scholars alike is the following: *Can personalized pricing be a winning strategy in oligopolistic markets?*

Although in monopolistic markets, we can offer a quick ‘yes’ to this question, the complexity of oligopolistic markets suggests that in this case the answer to the question is not that easy. As stated by Zhang (2009) on a revision on targeted/personalized pricing, the simple answer to a question like this is ‘it depends’. This is, of course, the easy part of the answer. The difficult part is to figure out what it depends on.

The aim of this paper is to complement the extant literature on personalized pricing, by assessing the profit effects of personalized pricing in markets where businesses face consumers, which are heterogeneous in terms of tastes and purchase quantity. Many researchers such as Thisse and Vives (1988), Shaffer and Zhang (1995), Bester and Petrakis (1996) and Matsumura and Matsushima (2015) have already looked at this question, focusing on personalized pricing in horizontal product differentiated markets with equally valuable consumers. All of these theoretical studies have come to the same conclusion: Personalized pricing based on data about consumer preferences, leads symmetric practicing firms to become worse off. As pointed out by Corts (1998, p. 321), “Competitive price discrimination may intensify competition by giving firms more weapons with which to wage their war.” When competing symmetric firms all have access to the required data to employ personalized pricing, they can target each other’s customers with great accuracy and efficiency, and consequently, each individual customer is a market to be contested. For that reason, the intensity of price competition reduces all prices to the detriment of firms.

Back in 2017, The Economist published a story titled, “The world’s most valuable resource is no longer oil, but data” (Parkins, D., 2017). No one ignores that data is an important input of firms’ targeted pricing and/or advertising strategies. However, why would companies be so

eager to collect data for pricing if by doing so they become worse off? Of course, because, under some market conditions, data-based pricing is employed by firms as a winning strategy. The extension of previous studies to asymmetric firms with regard to cost of targeting, more loyal customers, vertical differentiation, cost reducing activities, have shown that personalized pricing can be a winning strategy, at least for one of the firms (e.g. Shaffer and Zhang, 2002; Ghose and Huang, 2009; Matsumura and Matsushima, 2015).

While the economic and marketing literature has generated important theoretical insights regarding personalized pricing in oligopolistic markets, there still are a number of unanswered questions. To our knowledge little is known about the profit implications of quoting personalized prices in *symmetric* markets where not all customers are equally valuable for firms, i.e., when some customers purchase more units than others. In light of this, the aim of this paper is to investigate under what conditions “yes” can arise as the answer to the previous question in contexts where symmetric firms face a proportion of (*H*)igh and (*L*)ow-type demand customers with horizontal preferences for their products.

Modeling customer heterogeneity in terms of preferences and purchase quantity allows firms to gain a new dimension of information, in comparison to previous studies (e.g. Thisse and Vives, 1988; Matsumura and Matsushima, 2015). In other words, in addition to data revealing “*what brand they prefer*” (horizontal information), businesses’ data can also reveal information pertaining to “*how much they buy*” (vertical information).

Fudenberg and Villas-Boas (2006) note that more information in symmetric markets where firms price discriminate according to consumer preferences data, will lead to more intense competition between firms at the expense of profits. An additional contribution of this paper is to investigate under what conditions are practice firms better off with less or more information about consumers. In other words, are firms better off with access to data disclosing vertical or horizontal information or both?

With this goal in mind, we build a game theoretical model with two firms selling a horizontal differentiated product to consumers, which are heterogeneous with regard to two dimensions: (i) brand preferences (horizontal dimension) and (ii) purchase quantity (vertical dimension). Consumer preferences for each product is represented by  $x \in [0, 1]$ . Firm A is located at 0 and B at 1. A key departure of our paper from the personalized pricing literature is that we adapt the Hotelling model to introduce customer heterogeneity in demand: a *H*(igh)-type consumer purchases  $q > 1$  units of the good and a *L*(ow)-type segment that purchases only one unit,  $q = 1$ .

We compare the no information/uniform pricing benchmark, with three different data-driven price discrimination schemes. Most often personalized pricing is associated to first-degree or “perfect” price discrimination. We take this view in this paper too. In the extreme case where firms’ data discloses perfect information about the two dimensions of consumer heterogeneity—demand types and individual preferences—firms are able to employ a Perfect Personalized Pricing (henceforth, PPP) scheme. Obviously, we do not exclude (perhaps) more

realistic pricing practices where firms' data does not reveal full information about the two dimensions. In light of this, we consider the case where firms' data reveals information about consumer preferences but not about demand heterogeneity. Because each consumer demand type is not revealed, firms can only employ an Imperfect Personalized Pricing (henceforth, IPP) strategy. Likewise, when data available is more limited, it is also possible that data-based pricing discriminates groups instead of individuals, thus resulting in group pricing (henceforth, GP). We also allow for this possibility, when each firm's user data discloses information about consumer demand heterogeneity but not about individual preferences.

Our analysis highlights that whether price discrimination based on consumer data is good or bad for profits, depends on the type of information disclosed by firms' data and the market conditions. We show that symmetric competing firms might all become better off with information about consumers. If firms' data is limited and discloses only vertical information (demand heterogeneity), then, in comparison to no discrimination, group pricing is always a winning strategy. The benefit of this price discrimination scheme is higher in markets characterized by high demand heterogeneity and a low (high) share of H(L)-type consumers. Furthermore, firms are always strictly better off under group pricing than under imperfect or perfect personalized pricing.

More importantly, we add to the literature a model showing that personalized pricing (perfect or imperfect) might be a winning strategy for all symmetric practice firms. When heterogeneity in purchase quantity is low and/or the share of H-type consumers is higher than the share of L-type consumers, practice firms are always worse off with perfect/imperfect personalized pricing than with uniform pricing. This result is consistent with existing theoretical models.

In contrast, when heterogeneity in purchase quantity is sufficiently high and the proportion of H-type consumers is sufficiently low, in comparison to no discrimination, all firms can be better off under personalized pricing. The intuition behind this result is as follows. Under uniform pricing, when heterogeneity in purchase quantity increases, firms charge lower prices and profits fall. A lower price due to higher  $q$  leads to a small reduction in profits from the H-type segment, because the increase in demand compensates the reduction in price. In contrast, the reduction in profits in the L-type segment is stronger because the lower price is not compensated by more demand. Thus, the negative effect of an increase in  $q$  on overall profits is greater the lower (higher) is the proportion of H(L)-type consumers in the market. Under these market conditions, access to data for personalized pricing might act to soften price competition to the benefit of firms. When  $q$  is high and the proportion of H (L)-type consumers is sufficiently small (high), practice firms are better off under imperfect than under perfect personalized pricing, i.e., when data only reveals horizontal information, i.e. information about tastes. However, profits can also be higher under perfect personalized pricing (horizontal and vertical information) than under imperfect personalized pricing. This happens when  $q$  is high and the share of H-type consumers is intermediate (but lower than the share of L-type consumers). Summing up, in oligopolistic

markets characterized by a high enough heterogeneity in consumer value and a higher share of L-type consumers, PPP can be the winning strategy if the share of H-type consumers is intermediate; IPP is the winning strategy when the share of H-type (L-type) consumers is sufficiently small (high).

The rest of the paper is organized as follows. Section 2 describes the related literature. The model is presented in Section 3. The benchmark case of uniform pricing is discussed in Section 4. Section 5 presents the equilibrium analysis for the three different price discrimination schemes, namely group pricing, imperfect and perfect personalized pricing. Section 6 discusses the price and profit effects of different pricing schemes. Final remarks appear in Section 7. All the proofs are relegated to the Appendix.

## 2 Relevant Literature

This paper is related to the broad literature on competitive price discrimination based customer recognition (i.e., based on consumer data). Some studies have focused the analysis in static frameworks, while others in dynamic settings where firms recognize consumers after the first-period purchase behavior is revealed. This later form of price discrimination has been termed Behavior-Based Price Discrimination (e.g. Chen, 1997; Villas-Boas, 1999; Fudenberg and Tirole, 2000; Esteves, 2010, to name few). In all of these approaches, profits fall down with price discrimination. Under BBPD, one explicitly models how past purchase behavior provides firms with information about preferences, which are then used to determine discriminatory prices in the future. Unlike BBPD settings, models of personalized/targeted prices are in general static, and firms discriminate among consumers based on perfect or noisy information about their underlying preferences.

Our work aligns closely to the static theoretical literature on personalized pricing (Thisse and Vives, 1988; Shaffer and Zhang, 1995; Bester and Petrakis, 1996; Matsumura and Matsushima, 2015). So far, the literature has mostly focused on preference-based pricing, which means that, according to the terminology of Corts (1998), the market exhibits best-response asymmetry. Like Thisse and Vives (1988) we assume that there are two firms located at the extremes of the segment  $[0, 1]$ . Consumers are uniformly distributed in the line segment and firms can observe the location (or brand preference) of each individual consumer and price accordingly. With symmetric firms and no consumer heterogeneity in demand, offering personalized prices, while being optimal for each practicing firm, makes all firms worse off. Thus, in either static or dynamic contexts, price discrimination based on consumer preferences data is generally bad for profits when all practice firms are symmetric and learn the same about consumers.

Other studies tell us that this conclusion is not inevitable. In static settings, the rationale for the positive effect of competitive price discrimination on profits may lie on firms' asymmetry (e.g.

Shaffer and Zhang, 2002; Ghose and Huang, 2009; and Matsumura and Matsushima, 2015),<sup>1,2</sup> multi-dimensional product differentiation (e.g. Esteves, 2009b)<sup>3</sup> or imperfect targetability (Chen et al, 2001).<sup>4</sup> In dynamic settings, profits may increase with BBPD due to imperfect correlated preferences across time (Chen and Percy, 2010, and Shin and Sudhir, 2010), consumers' demand heterogeneity (Shin and Sudhir, 2010), heterogeneity in price sensitivity (Colombo, 2018), non-uniform distribution of consumer preferences (Esteves et al, 2020), imperfect informed consumers (Chen and Zhang 2009, Esteves, 2009a, and Esteves and Resende, 2016, 2019).

The aim of this article is to extend the analysis of personalized pricing to a situation where there are two sources of consumer heterogeneity: (i) horizontal preferences for firms and (ii) purchase quantity. Shin and Sudhir (2010) use a similar approach for consumer heterogeneity in demand in a dynamic BBPD model. Our model is different from theirs in several respects. First, they explicitly model how past purchase behavior provides firms with information about preferences and demand heterogeneity, which are then used to determine discriminatory prices in the future. We do not model how firms obtain such information. In their BBPD model, first-period choices allow firms to segment consumers into a strong (own customers) and a weak (rival's customers) segment customers. Additionally, in the second-period, information about the vertical dimension is asymmetric, because each firm recognizes H and L type consumers only in its own customers. Therefore, customer heterogeneity in purchase quantity confers an endogenous information advantage of companies about their current customers.<sup>5</sup> As we allow firms to rely on the same piece of information about existing/potential customers' preferences and/or demand types, we exclude information asymmetry from our analysis. Finally, while in their model, firms offer a price to its own and rival's customers, here depending on the ex-ante available data, firms can offer group or personalized prices. By so doing, our analysis sheds light about the impact of consumer value heterogeneity and the share of Low and High demand

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<sup>1</sup>Shaffer and Zhang (2002) consider both horizontal and vertical differentiation, with a positive cost of targeting customers. They show that the firm with more loyal customers can earn higher profits in equilibrium when both firms engage in one-to-one promotions.

<sup>2</sup>Matsumura and Matsushima (2015) allow firms to engage in cost-reducing activities after determining their pricing policies, personalized or uniform pricing. They show that when the ex ante cost difference between the two firms is large, employing personalized pricing harms the less-efficient firm even when employing this pricing is costless. This result does not hold when the firms do not engage in cost-reducing activities.

<sup>3</sup>In Esteves (2009b) consumer horizontal preferences are two-dimensional. When firms quote prices based on partial information, they might become better off under personalized pricing than under uniform pricing.

<sup>4</sup>Chen et al. (2001) consider the case where consumer information is noisy; hence, targeting is imperfect. At low levels of accuracy, the positive effect of price discrimination on profit is stronger, whereas at high levels, the negative effect of competition on profit is stronger. Overall, profits are greatest at moderate levels of accuracy.

<sup>5</sup>Another important paper in the context of BBPD is Colombo (2018). Consumers are heterogeneous both in tastes and in price sensitivity. Each firm is able to distinguish between the consumers that have bought from it and those that have bought from the rival. Information about price sensitivity is asymmetric, because each firm can only learn the price sensitivity of its own consumers. The author shows that using this additional information may yield higher profits than uniform pricing provided that consumers are heterogeneous enough with respect to price sensitivity.

consumers, on the profitability of each information/pricing settings, namely (i) group pricing (data discloses information only about demand types), (ii) imperfect personalized pricing (data discloses information only about preferences) and (iii) perfect personalized pricing (data discloses information about demand types and preferences).<sup>6</sup>

### 3 The model

Consider a market with two firms,  $A$  and  $B$ , producing at zero marginal cost<sup>7</sup> a differentiated product. The product produced by firm  $A$  and  $B$  is located at 0 and 1, respectively. There is a continuum of consumers with mass normalized to 1. Consumers are heterogeneous with regard to two dimensions: (i) brand preferences (horizontal dimension) and (ii) demand or purchase quantities (vertical dimension). Consumer preferences for each product is represented by  $x$  uniformly distributed along the Hotelling line,  $x \sim U[0, 1]$ . A consumer located at  $x$  incurs a desutility cost equal to  $x$  if she buys product  $A$ , while this cost is  $(1 - x)$  if she buys product  $B$ . A key departure of our model from the literature on personalized pricing focusing on brand preferences (e.g. Thisse and Vives, 1988; and Matsumura and Matsushima, 2015) is that we adapt the Hotelling model to introduce customer heterogeneity in demand. Specifically, we assume there are two types of consumers in the market,  $j \in \{L, H\}$ : a  $H$ (igh)-type segment that purchases  $q > 1$  units of the good and a  $L$ (ow)-type segment that purchases only one unit, i.e.,  $q = 1$ . Shin and Sudhir (2010) use a similar assumption to model consumer value heterogeneity but in a behavior-based price discrimination model. The proportion of  $L$  and  $H$  type consumers in the market is, respectively,  $\alpha$  and  $1 - \alpha$ , with  $0 < \alpha < 1$ . A consumer of type  $j$  purchasing from  $A$  at price  $p_A$  obtains a surplus of

$$Q^j(v - p_A) - x, \tag{1}$$

while if purchasing from  $B$  at price  $p_B$  she obtains a surplus equal to

$$Q^j(v - p_B) - (1 - x), \tag{2}$$

where  $Q^H = q$  and  $Q^L = 1$ . We also assume that the gross utility from consuming the differentiated product  $v$  is large enough so that the market is covered.

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<sup>6</sup>Colombo (2016) allows BBPD to be perfect and imperfect. By “perfect” BBPD he means that, once consumers have done their initial choice by choosing one firm, the firms are then able to recognize from which firm each consumer has bought. Under imperfect BBPD the information firms receive about consumers’ past purchases is incomplete, that is, it does not cover all consumers. Imperfect price discrimination in Chen et al. (2001) and Esteves (2014) means that all consumers are classified with some noise as loyal to one firm or to the other. In Liu and Serfes (2004) and Colombo (2011) all consumers are classified (correctly) into a number of subsegments within the Hotelling line and the imperfectness stems from the dimension of each subsegment.

<sup>7</sup>The assumption of zero marginal costs can be relaxed without altering the basic nature of the results derived throughout the model.



Firms set their prices simultaneously and non-cooperatively. Based on what firms can learn from their available data, we consider three symmetric data-based pricing strategies. In the extreme case where firms' data discloses perfect information about the two dimensions of consumer heterogeneity—demand types and individual preferences—firms are able to employ a Perfect Personalized Pricing (PPP) scheme. In this case, each firm charges each consumer a personalized price, namely  $p_i^j(x)$ , with  $j = L, H$  and  $i = A, B$ . We also take into account perhaps more realistic practices, in which firms' data does not reveal full information about the two dimensions of consumer heterogeneity. In light of this, we consider the case where firms' data only reveals information about one of the two dimensions. When data discloses information about individual tastes but not about demand types, each firm can only employ an Imperfect Personalized Pricing (IPP) strategy. In this setting, each firm chooses an individual preference-based price, i.e.,  $p_i(x)$ ,  $i = A, B$ . Finally, when information is even more limited in the sense that the available data only reveals information about consumer types, firms can only price discriminate on a group rather than on an individual basis, thus employing a group pricing (GP) strategy. We look at this possibility when each firm's data discloses information about consumer demand heterogeneity but not about individual preferences. Firms segment consumers in two groups and price accordingly. In other words, firm  $i$  chooses price  $p_i^j$  with  $j = L, H$  and  $i = A, B$ .

## 4 Benchmark: Uniform pricing

To isolate the effect of group and personalized pricing on profits, let us first establish the benchmark of uniform pricing where each firm can only charge one price to all consumers. Let  $p_A^u$  and  $p_B^u$  denote the uniform prices set by firm A and B, respectively. A  $j$ -type consumer is indifferent between buying from the two firms if she is located at  $\hat{x}^j$ , such that

$$Q^j(v - p_A) - \hat{x}^j = Q^j(v - p_B) - (1 - \hat{x}^j).$$

This yields

$$\hat{x}^L = \frac{1}{2} + \frac{p_B - p_A}{2}. \quad (3)$$

$$\hat{x}^H = \frac{1}{2} + \frac{q(p_B - p_A)}{2}. \quad (4)$$

Each firm profit is

$$\pi_A^u(p_A^u, p_B^u) = p_A^u \left[ \alpha \left( \frac{1}{2} + \frac{p_B^u - p_A^u}{2} \right) + (1 - \alpha)q \left( \frac{1}{2} + \frac{q(p_B^u - p_A^u)}{2} \right) \right], \quad (5)$$

$$\pi_B^u(p_A^u, p_B^u) = p_B^u \left[ \alpha \left( \frac{1}{2} + \frac{p_A^u - p_B^u}{2} \right) + (1 - \alpha)q \left( \frac{1}{2} + \frac{q(p_A^u - p_B^u)}{2} \right) \right]. \quad (6)$$

**Proposition 1:** *Under uniform pricing in the symmetric NE:*

(i) each firm charges  $p^u$  equal to

$$p^u = \frac{\alpha + q(1 - \alpha)}{\alpha + q^2(1 - \alpha)}, \text{ and,} \quad (7)$$

(ii) each firm profit is

$$\pi^u = \frac{1}{2} \frac{(\alpha + q(1 - \alpha))^2}{\alpha + q^2(1 - \alpha)}. \quad (8)$$

**Proof.** See the Appendix.

When firms have no information about consumers' types and preferences and/or when price discrimination is not permitted they are forced to quote the same price to all consumers. As expected, when all consumers demand  $q = 1$  we get the standard Hotelling results, the uniform price is  $p^u = 1$  and profits per firm are  $\pi^u = \frac{1}{2}$ .

From the equilibrium uniform pricing, we conclude that when H-type consumers demand more units ( $q$  increases) firms charge a lower uniform price (i.e.,  $\frac{\partial p^u}{\partial q} < 0$ ).<sup>8</sup> Therefore, when heterogeneity in purchase quantity increases, firms price more aggressively, reducing the equilibrium uniform price. In terms of profits, the reduction in price is compensated by selling more units to H-type consumers. Thus, a reduction in  $p^u$ , due to higher  $q$ , has a smaller effect on overall uniform profit if the share of H-type consumers is high ( $\alpha$  low). Regarding the effect of  $\alpha$  on the uniform price, we conclude that  $\frac{\partial p^u}{\partial \alpha} > 0$ .<sup>9</sup> The reason is that an increase in the proportion of L-type consumers in the market softens price competition, thus for the same  $q$  firms have fewer incentives to reduce  $p^u$  when the proportion of H-type consumers falls.

Under uniform pricing, when heterogeneity in purchase quantity increases, firms charge lower prices and profits fall ( $\frac{\partial \pi^u}{\partial q} < 0$ ). A lower price due to higher  $q$  leads to a small reduction in profits from the H-type segment, because the increase in demand compensates the reduction in price. In contrast, the reduction in profits in the L-type segment is stronger because the lower price is not compensated by more demand. Therefore, the negative effect of an increase in  $q$  on overall profits is greater the lower (higher) is the proportion of H(L)-type consumers in the market. Specifically, regarding the effect of  $\alpha$  on profits, we find that  $\frac{\partial \pi^u}{\partial \alpha} < 0$  when  $\alpha < \tilde{\alpha}$ , while  $\frac{\partial \pi^u}{\partial \alpha} > 0$  when  $\alpha > \tilde{\alpha}$ , with  $\tilde{\alpha} = \frac{q}{q+1}$ . As  $q > 1$  then  $\tilde{\alpha} > \frac{1}{2}$ . As the proportion of L-type consumers increases, firms have lower incentives to reduce the uniform price. In fact, when  $\alpha \rightarrow 1$ ,  $p^u \rightarrow 1$  and  $\pi^u \rightarrow \frac{1}{2}$ .

## 5 Equilibrium analysis of different price discrimination schemes

While for a long time perfect price discrimination was considered a highly theoretical concept, firms are nowadays more capable to use consumer data and information technologies in an

<sup>8</sup>Specifically,  $\frac{\partial p^u}{\partial q} = \frac{\alpha(\alpha-1)(q-1)(\alpha+q(1-\alpha))}{(\alpha-q^2\alpha+q^2)^2} < 0$  because  $\alpha < 1$  and  $q > 1$ .

<sup>9</sup>Note that  $\frac{\partial p^u}{\partial \alpha} = \frac{q(q-1)}{(\alpha-q^2\alpha+q^2)^2} > 0$  for  $q > 1$ .

attempt to improve their knowledge of customers. To perfectly price discriminate, a firm must be able to use an algorithm that base on firm's user data can identify each consumer's exact valuation. So one important impediment to perfect personalized pricing is insufficient data.

As aforementioned, our aim is to offer a complete picture of firms' price decisions and profit implications under *insufficient* data—group pricing or imperfect personalized pricing— and under *sufficient* data to produce perfect estimates of consumers' valuations—perfect personalized pricing.

In what follows we consider three information/pricing settings. In the first two cases, we assume that firms' data is insufficient, in other words data discloses information only about one of the two dimensions of consumer heterogeneity. When data only reveals consumer demand types, firms discriminate by groups and not by individuals (GP). Then, we consider the case where firms' data reveals information about consumer preferences but not about demand heterogeneity. In this case, each firms directly observes the location of each consumer in the preference line, i.e.  $x$ , and can offer a preference-based individual price to each consumer, i.e.  $p_i(x)$ . As demand types are unknown, we call this price strategy, as Imperfect Personalized Pricing (IPP). Finally, we consider the extreme (perhaps less realistic) case where firms' data is rich enough to disclose perfect information about the two dimensions of consumer heterogeneity, i.e., consumers' preferences and demand types, meaning that algorithms identify perfectly each customer's reservation value. In this situation, Perfect Personalized Pricing (PPP) becomes possible.

## 5.1 Group pricing

Under group pricing firms segment consumers in two groups ( $H$  and  $L$ ), so each firm sets a price to a  $H$  and a  $L$  type consumer, respectively denoted  $p_i^H$  and  $p_i^L$ ,  $i = A, B$ . Using equations (3) and (4) it is straightforward to obtain each firm's profit. Consider the case of firm A, its profits per segment are:

$$\pi_A^L = \alpha p_A^L \left( \frac{1}{2} + \frac{p_B^L - p_A^L}{2} \right) \text{ and } \pi_A^H = (1 - \alpha) q p_A^H \left( \frac{1}{2} + \frac{q(p_B^H - p_A^H)}{2} \right) \quad (9)$$

Similar expressions hold for firm B. Taking the first order conditions and solving for prices we can establish the following proposition.

**Proposition 2 (Group pricing):** *When firms' data only reveals consumer demand types, firms compete with group pricing and in the NE:*

(i) *each firm quotes, respectively, the following prices to Low and High type consumers:*

$$p^L = 1 \text{ and } p^H = \frac{1}{q}. \quad (10)$$

(ii) *Each firm's profits are*

$$\pi^{GP} = \frac{1}{2}. \quad (11)$$

**Proof.** See the Appendix.

In this situation, with price discrimination there is *best-response symmetry*. Following Corts (1998), best-response symmetry simply requires that both firms rank the same group of consumers as the strong market. Under group pricing, both firms are unanimous with regard to the price targeted to low and high type consumers. As in the traditional literature, in models exhibiting best-response symmetry, price discrimination leads to an increase in price to one group and to a decrease in price to the other group. In line of this, as established in Corollary 1, the comparison between no discriminatory and discriminatory prices is clear: with group pricing firms price high to the Low demand consumers and price low to the High demand consumers, exactly as one would expect. Thus, with group pricing H-type consumers are better off, while L-type consumers are worse off.

**Corollary 1:** *In comparison to uniform pricing, when firms' data only allow for group pricing:*

- (i) *Low demand consumers pay higher prices and High demand consumers pay lower prices.*
- (ii) *Profits increase at the expense of consumers.*

**Proof.** See the Appendix.

## 5.2 Imperfect personalized pricing

Now we assume that firms' data discloses perfect information about each consumer location  $x$  but discloses no information about consumers' demand types. In line of this, each firm tailors offers based on each consumer's location  $x$ , i.e.  $p_i(x)$ ,  $i = A, B$ . Now following Corts (1998), with price discrimination the model exhibits *best-response asymmetry* because one firm's "strong" market is the other's "weak" market. Consumers with preferences in the interval  $x \leq \frac{1}{2}$  belong to firm A' strong market and to firm B's weak market. The reverse happens to consumers located at  $x > \frac{1}{2}$ . Thus, with price discrimination, each firm wants to increase the price to its nearby consumers (strong market) and to reduce the price to far away consumers (weak market).

With no loss of generality consider the case where  $x \leq \frac{1}{2}$ . Given  $p_A(x)$  and  $p_B(x)$ , a consumer of type  $L$  is indifferent between A and B as long as

$$p_A(x) = p_B(x) + (1 - 2x) \text{ or } p_A(x) = p_B(x) + \gamma(x)$$

with  $\gamma(x) = (1 - 2x)$ . A consumer of type  $H$  is indifferent between A and B as long as

$$p_A(x) = p_B(x) + \frac{1 - 2x}{q} \text{ or } p_A(x) = p_B(x) + \frac{\gamma(x)}{q}.$$

Note that  $\gamma(x) > \frac{\gamma(x)}{q}$ .

We first consider the case where the market is composed by a high enough share of High demand consumers.

**High proportion of H-type consumers in the market (low  $\alpha$ ):** The next proposition shows that when the proportion of High demand consumers in the market is high ( $\alpha$  is low), firm A tailors  $p_A(x) = \frac{\gamma(x)}{q}$  to consumers in its strong market, while firm B quotes  $p_B(x) = 0$  to consumers in its weak market. Given the symmetry of the model, similar arguments apply to firms' price decisions to consumers located at  $x > \frac{1}{2}$ .

**Proposition 3. (Imperfect Personalized Pricing— $\alpha$  low):** *Suppose  $\alpha \leq \hat{\alpha}$  with  $\hat{\alpha} = \frac{q}{2q-1}$ . If firms can only personalize prices based on consumer preferences, there is a NE in pure strategies in which:*

(i) *Each firm quotes a consumer located at  $x$  the following prices:*

$$\begin{aligned} p_A^{IPP}(x) &= \begin{cases} \frac{1-2x}{q} & \text{for } x \leq \frac{1}{2} \\ 0 & \text{for } x > \frac{1}{2} \end{cases}, \\ p_B^{IPP}(x) &= \begin{cases} \frac{2x-1}{q} & \text{for } x \geq \frac{1}{2} \\ 0 & \text{for } x < \frac{1}{2} \end{cases}. \end{aligned}$$

(ii) *Each firm profit is*

$$\pi^{IPP} = \frac{\alpha + (1 - \alpha)q}{4q}. \quad (12)$$

**Proof.** See the Appendix.

Because each firm cannot distinguish a L/H-type consumer in its strong market, it prices low, i.e., according to the highest price that just prevents H-type consumers from being tempted by the rival offer (in the case of  $x \leq \frac{1}{2}$ ,  $p_A(x) = \frac{1-2x}{q}$ ). This happens as long as the gain from selling to both types of customers through a low price is greater than the gain from pricing high to L-type consumers and selling for sure to these group of customers. This depends of course, on the share of each consumer type, specifically when the share of High (Low) value consumers in the market is sufficiently high (low), i.e, when  $\alpha \leq \hat{\alpha} = \frac{q}{2q-1}$ . Note that  $\frac{\partial \hat{\alpha}}{\partial q} < 0$  and  $\lim_{q \rightarrow +\infty} \hat{\alpha} = \frac{1}{2}$ .

When the proportion of L and H types in the market is the same ( $\alpha = \frac{1}{2}$ ), firms always behave as in Proposition 3. Before proceeding note that consistent with Thisse and Vives (1988), when  $x \leq \frac{1}{2}$  and consumers are equally valuable ( $q = 1$ ) we get  $p_A(x) = 1 - 2x$  and  $p_B(x) = 0$ .

Next we look at the case in which the proportion of High demand consumers in the market is low or, equivalently, the proportion of Low demand consumers is high.

**Low proportion of H-type consumers (high  $\alpha$ ):** Following a reasoning similar to Narasimhan (1988), when  $\alpha > \hat{\alpha} = \frac{q}{2q-1}$  we can show that a pure strategy in prices fails to exist. There is, however, an asymmetric mixed strategy equilibrium in prices, the existence of which is proved by construction. In this equilibrium, each firm uses a price strategy that prevents its opponent from predicting its price setting behavior, which in turn makes undercutting less likely.

With no loss of generality consider next each firm price decisions for consumers located at  $x \leq \frac{1}{2}$  (firm A's strong market and firm B's weak market). (Similar reasoning holds for consumers located at  $x \geq \frac{1}{2}$ , firm B's strong market and firm A's weak market). The minimum price firm A is willing to charge to a consumer located at  $x$  in an attempt to serve a L/H type consumer in its strong market should satisfy the condition  $p_{A \min}(x) [\alpha + (1 - \alpha)q] = \gamma(x)\alpha$ . This yields:

$$p_{A \min}(x) = \frac{\gamma(x)\alpha}{\alpha + (1 - \alpha)q}. \quad (13)$$

Firm B takes into account firm A's behavior and so the minimum price it is willing to quote to the same consumer in an attempt to sell to  $H$ -type consumers is  $p_{B \min}(x) = p_{A \min}(x) - \frac{\gamma(x)}{q}$ . Suppose that firm  $i$  selects a price randomly from the c.d.f  $F_i(p_j(x))$ . For a consumer located at  $x < \frac{1}{2}$  in the MSNE we must observe:

$$p_A(x) [\alpha + (1 - \alpha)q] \left[ 1 - F_B \left( p_A(x) - \frac{\gamma(x)}{q} \right) \right] + \alpha p_A(x) F_B \left( p_A(x) - \frac{\gamma(x)}{q} \right) = \alpha \gamma(x), \quad (14)$$

$$p_B(x)(1 - \alpha)q \left[ 1 - F_A \left( p_B(x) + \frac{\gamma(x)}{q} \right) \right] = (1 - \alpha)q p_{B \min}(x). \quad (15)$$

After some computations presented in the Appendix we can establish the following proposition.

**Proposition 4 (Imperfect Personalized Pricing— $\alpha$  high):** *When the proportion of Low-type consumers in the market is sufficiently high, i.e. when  $\alpha > \hat{\alpha}$  with  $\hat{\alpha} = \frac{q}{2q-1}$  in the MSNE for a consumer located at  $x < \frac{1}{2}$ :*

(i) *Firm A chooses the price  $p_A(x)$  randomly from the distribution function:*

$$F_A [p(x)] = 1 - \frac{\alpha(2q - 1) - q}{[p(x)q - \gamma(x)] [q(1 - \alpha) + \alpha]} \gamma(x) \quad (16)$$

with a mass point  $m = \frac{\alpha(2q-1)-q}{(q-1)(q(1-\alpha)+\alpha)}$ . With

$$p_{A \min}(x) = \frac{\alpha \gamma(x)}{\alpha + q(1 - \alpha)}, \quad (17)$$

$$p_{A \max}(x) = \gamma(x). \quad (18)$$

(ii) *Firm B chooses the price  $p_B(x)$  randomly from the distribution function:*

$$F_B [p(x)] = \frac{\alpha + (1 - \alpha)q}{q(1 - \alpha)} - \frac{\alpha \gamma(x)}{[qp(x) + \gamma(x)](1 - \alpha)} \quad (19)$$

with

$$p_{B \min}(x) = \frac{\alpha \gamma(x)}{\alpha + q(1 - \alpha)} - \frac{1}{q} \gamma(x),$$

$$p_{B \max}(x) = \gamma(x) - \frac{\gamma(x)}{q} = p_{A \max}(x) - \frac{\gamma(x)}{q}.$$

**Proof.** See the Appendix.

Proposition 4 highlights that in firm A's strong market, prices fall with  $x$ , i.e., consumers with stronger preferences for firm A will be charged higher prices. In other words, firm A will set a high price to nearby consumers (lower  $x$  and higher  $\gamma(x)$ ) in order to exploit those consumers' unwillingness to travel so far to the other firm. Like in Thisse and Vives (1988) consumers in the middle, with  $\gamma(x) = 0$ , will always receive the better deals, in fact they will pay the marginal cost price, i.e.  $p(x = \frac{1}{2}) = 0$ . The same happens here.

Interestingly, Proposition 4 highlights that consumers with strong preferences for each firm will be charged the highest price by *both* firms. Note that the highest price in the support of firm A's equilibrium price targeted to a consumer located at  $x$ , only depends on the consumer preference parameter  $x$ . The stronger is the consumer preference for firm A (lower  $x \Rightarrow$  higher  $\gamma(x)$ ) the higher will be the expected price quoted by firm A to that consumer. In contrast to what happens in Thisse and Vives (1988) and Proposition 3, in this case firm B charges a higher expected price to a consumer located near the rival than to a consumer located near the centre. Note that for  $x \leq \frac{1}{2}$ , the minimum and maximum prices of each firm support are higher the lower is  $x$ .

From the expression of the mass point, we observe that  $\frac{\partial m}{\partial q} = q \frac{1-\alpha}{(q-1)^2} \frac{q-2\alpha(q-1)}{(q+\alpha-q\alpha)^2} < 0$ ,<sup>10</sup> suggesting that the lower is the heterogeneity in purchase quantity (lower  $q$ ), the higher will be the probability of firm A charging the highest price  $\gamma(x)$  to a consumer located at  $x$ . On the other hand, taking into account the support of firm B's equilibrium price distribution, we conclude that  $p_{B \max}(x)$  always increases with  $q$ . Note that the higher is  $q$  the lower is  $\frac{\gamma(x)}{q}$  and so the higher is  $p_{B \max}(x) = \gamma(x) - \frac{\gamma(x)}{q}$ . Regarding the effect of changes in  $q$  on  $p_{B \min}(x)$ , we conclude that when  $q < 4$ ,  $\frac{\partial p_{B \min}(x)}{\partial q} > 0$ , while the reverse happens when  $q > 4$ .<sup>11</sup> Thus, for high enough heterogeneity in consumer demand, an increase in  $q$  also increases  $p_{B \min}(x)$ .

Additionally, since  $\alpha > \hat{\alpha}$ , further increases in  $\alpha$  only affect the minimum price of the support of each firm's equilibrium price distribution. Specifically, an increase (decrease) in the proportion of L (H) type consumers increases the minimum price firm A is willing to charge in equilibrium and also firm's B minimum price. From the expression of the mass point we can see that  $\frac{\partial m}{\partial \alpha} = \frac{q^2}{(q-1)(q+\alpha-q\alpha)^2} > 0$ . Therefore, when  $\alpha > \hat{\alpha}$ , an increase in the proportion of L type consumers, increases the probability with which firm A charges the highest price to the consumer located at  $x$ .

From Proposition 4, it follows that for a consumer with preference  $x$  located at  $x \leq \frac{1}{2}$ , firm A's expected profit is:

$$\pi_A(x) = \alpha \gamma(x).$$

<sup>10</sup>This inequality holds as long as  $\alpha > \frac{q}{2(q-1)}$ , which is always true given that  $\alpha > \hat{\alpha}$ .

<sup>11</sup>Specifically, when  $q > 4$  and  $\frac{q}{2q-1} < -\frac{q+\frac{1}{2}\sqrt{q^3(q-4)}-\frac{3}{2}q^2}{-2q+2q^2+1} < \alpha < \frac{-2q+\sqrt{q^3(q-4)+3q^2}}{-4q+4q^2+2}$ .

while firm B's expected profit is:

$$\pi_B(x) = \frac{(1 - \alpha) [\alpha(2q - 1) - q]}{\alpha + q(1 - \alpha)} \gamma(x),$$

Therefore, with regard to consumers located at  $x \leq \frac{1}{2}$ , firm A's expected profit from its strong market is

$$\pi_A^s = \int_0^{\frac{1}{2}} \pi_A(x) dx = \frac{\alpha}{4}, \quad (20)$$

while firm B's expected profit from its weak market is:

$$\pi_B^w = \int_0^{\frac{1}{2}} \pi_B(x) dx = \frac{1}{4} \frac{(1 - \alpha) (\alpha(2q - 1) - q)}{\alpha + q(1 - \alpha)}. \quad (21)$$

Under personalized prices with equally valuable consumers (and also here under Proposition 3), in spite of lower prices offered to consumers in a firm's weak market, no consumer switches to the more distant firm. Thus, each firm's profit from its weak market is zero. Here, in contrast, with some probability H-type consumers with a preference say for firm A decide to purchase from firm B. Thus, when  $\alpha > \hat{\alpha}$ , firm B's profit from its weak market is no longer null.

A similar reasoning applies to both firms' price offers and respective profits to consumers located at  $x > \frac{1}{2}$ . In line of this, due to symmetry, we can establish that when the proportion of Low (High) type consumers in the market is sufficiently high (low), i.e., when  $\alpha > \frac{q}{2q-1}$ , each firm's overall profit under imperfect personalized pricing is

$$\pi_i^{IPP} = \frac{\alpha}{4} + \frac{1}{4} \frac{(1 - \alpha) (\alpha(2q - 1) - q)}{\alpha + q(1 - \alpha)}. \quad (22)$$

### 5.3 Perfect personalized pricing

Now we assume that both firms' user data discloses full information about consumers' preferences and demand types, which implies that perfect personalized pricing becomes a possibility to both firms. Hence, for a consumer located at  $x$  each firm quotes  $p_i^j(x)$ , with  $j = L, H$ . Consider, for instance, a consumer located at  $x \leq \frac{1}{2}$ . Given  $p_A^L(x)$  and  $p_B^L(x)$ , a consumer of type  $L$  located at  $x$  is indifferent between buying from A or B as long as  $p_A^L(x) + x = p_B^L(x) + (1 - x)$ . The best price firm B can offer to a consumer with a preference for A is  $p_B^L(x) = 0$ . Therefore, in order not to lose this consumer, firm A needs to quote  $p_A^L(x) = \gamma(x)$ . Doing the same for a consumer located at  $x > \frac{1}{2}$ , we get that  $p_B^L(x) = -\gamma(x)$ . We do the same for a consumer of type  $H$ . Then, we can establish the following proposition.

**Proposition 5:** *When both firms have perfect information about each consumer demand type and tastes, they are able to employ perfect personalized pricing and in the NE:*

(i) *each firm quotes:*

$$p_A^L(x) = \begin{cases} 1 - 2x & \text{for } x \leq \frac{1}{2} \\ 0 & \text{for } x > \frac{1}{2} \end{cases} \quad \text{and} \quad p_B^L(x) = \begin{cases} 2x - 1 & \text{for } x \geq \frac{1}{2} \\ 0 & \text{for } x < \frac{1}{2} \end{cases} \quad (23)$$



$$p_A^H(x) = \begin{cases} \frac{1-2x}{q} & \text{for } x \leq \frac{1}{2} \\ 0 & \text{for } x > \frac{1}{2} \end{cases} \quad \text{and} \quad p_B^H(x) = \begin{cases} \frac{2x-1}{q} & \text{for } x \geq \frac{1}{2} \\ 0 & \text{for } x < \frac{1}{2} \end{cases} \quad (24)$$

(ii) each firm profit equals

$$\pi^{PPP} = \frac{1}{4}. \quad (25)$$

## 6 Price and profit effects

This section discusses the price and profit effects of data-based pricing in the three information/pricing schemes presented above.

### 6.1 Price effects

Most of the existing literature on competitive price discrimination suggests that when the market exhibits best-response asymmetry, the optimal choice for each firm is to offer a lower price to consumers in its weak market (with a preference for the rival) than to consumers in its strong market (e.g. Thisse and Vives, 1988; Fudenberg and Tirole, 2000). Furthermore, in comparison to uniform pricing, due to the *all-out competition* result, all prices fall under price discrimination. In what follows we compare firms' price behavior under price discrimination and uniform pricing.

Corollary 1 (U versus GP) highlights that when firms' data discloses vertical information (H/L types), both firms agree in their price decisions to each segment: they want to raise the price targeted to L-type consumers and reduce the price targeted to H-type consumers. In line of this, in comparison to uniform pricing, Low demand consumers pay higher prices and High demand consumers pay lower prices (i.e.,  $p^H < p^U < p^L$ ). Consequently, under group pricing, H-type consumers are made better off, while L-type consumers are made worse off.

Consider now the extreme case where firms have all the required information to engage in perfect price discrimination. The relation among prices targeted to H-type consumers is

$$p^H(x) < p^u. \quad (26)$$

Furthermore, from the comparison among uniform, group and perfect personalized pricing, it follows that with the exception of consumers with  $x = 0$  and  $x = 1$ , who pay the same price under GP and PPP the relation between prices targeted to H-type consumers is as follows:

$$p^H(x) \leq p^H < p^u. \quad (27)$$

Concerning the price charged to L-type consumers under PPP and GP it follows that  $p^L(x) \leq p^L$ . Again, with exception of consumers with  $x = 0$  and  $x = 1$ , who pay the same price under GP and PPP, all the others pay lower prices under PPP than under GP. If we compare the price charged to Low demand consumers under uniform and perfect personalized pricing we conclude

that for consumers located at  $x \leq \frac{1}{2}$  :

$$p^u < p^L(x) \text{ when } 0 < x \leq \hat{x} \quad (28)$$

$$p^u > p^L(x) \text{ when } \hat{x} < x < \frac{1}{2}. \quad (29)$$

with  $\hat{x} = \frac{1}{2} \frac{q(1-\alpha)(q-1)}{\alpha+q^2(1-\alpha)} < \frac{1}{2}$ .

In sum, although H-type consumers are always better off under PPP than under UP,<sup>12</sup> the same is not always true at least for some of L-type consumers who can be harmed by the ability of firms to recognize them and price accordingly, either under GP or PPP. More specifically, L-type consumers with preferences  $x \in [0, \hat{x}]$  are better off with UP than with PPP, while consumers with preferences  $x \in ]\hat{x}, \frac{1}{2}[$  are better off with PPP than with UP. Like in Thisse and Vives (1988), when  $q = 1$ , all consumers are better off under PPP than under UP (indeed, consumers with  $x = 0$  pay the same price under PPP and UP).

As  $\frac{\partial \hat{x}}{\partial \alpha} < 0$ , the higher is the proportion of H-type consumers in the market (lower  $\alpha$ ), the lower will be  $p^u$  and so the higher is the threshold  $\hat{x}$ . When this happens a higher share of Low type consumers are better off if firms cannot use their data for price personalization. The same happens when consumer value heterogeneity i.e.,  $q$  increases ( $\frac{\partial \hat{x}}{\partial q} > 0$ ).

In what follows we compare prices under UP, GP and IPP. Consider first the case in which the proportion of H-type consumers is sufficiently high. In other words, suppose  $\alpha < \hat{\alpha} = \frac{q}{2q-1}$  holds and firms behave as in Proposition 3. A consumer located at  $x$ , with  $x \leq \frac{1}{2}$  is charged  $p_A^{IPP}(x) = \frac{1-2x}{q}$  and  $p_B(x) = 0$ . In this case, H-type consumer with preference  $x$  is offered the same price under IPP and PPP, then she is better off under personalized pricing than under uniform pricing. Put differently:

$$p^{IPP}(x) \leq p^H < p^u,$$

with  $p^{IPP}(x) = \frac{\gamma(x)}{q}$ . Because in this situation L-type consumers are also charged  $p^{IPP} = \frac{\gamma(x)}{q}$ , they are also better off under IPP. Thus, L-type consumers can buy at lower prices as  $q$  increases. Looking for instance at consumers located at  $x \leq \frac{1}{2}$ , we can show that:

$$\begin{aligned} p^{IPP}(x) &< p^L(x) < p^u \text{ if } \hat{x} < x < \frac{1}{2}, \\ p^{IPP}(x) &< p^u < p^L(x) \text{ if } 0 < x \leq \hat{x}. \end{aligned}$$

Therefore, in comparison to UP and IPP, L-type consumers pay always lower prices under IPP than under UP if the share of H-type consumers in the market is sufficiently high. Finally, we look at the case where the market is composed by high (low) enough share of L-type (H-type) consumers. In other words, suppose that  $\alpha > \hat{\alpha}$  with  $\hat{\alpha} = \frac{q}{2q-1}$  holds. We have seen that in this

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<sup>12</sup>Consider for instance a consumer located at  $x \leq \frac{1}{2}$ . Specifically,  $p^u > p^H(x)$  as long as  $x > -\frac{\alpha(q-1)}{\alpha+q^2(1-\alpha)}$ , which is always true.

situation, firms set their prices randomly. Due to symmetry, consider firms' pricing decisions to consumers with preferences located at  $x < \frac{1}{2}$ . (Note that consumers located at  $x = \frac{1}{2}$  are charged 0 by both firms in equilibrium.)

The support of firm A's equilibrium price is  $p_A(x) \in [p_{A \min}(x), p_{A \max}(x)]$  with

$$p_{A \min}(x) = \gamma(x) \left( \frac{\alpha}{\alpha + q(1 - \alpha)} \right), \quad (30)$$

and

$$p_{A \max}(x) = \gamma(x).$$

The support of firm B's equilibrium price is  $p_B(x) \in [p_{B \min}(x), p_{B \max}(x)]$  with

$$p_{B \min}(x) = \gamma(x) \left( \frac{\alpha(2q - 1) - q}{q(q(1 - \alpha) + \alpha)} \right) > 0,$$

and

$$p_{B \max}(x) = \gamma(x) \left( \frac{q - 1}{q} \right). \quad (31)$$

Because  $\alpha > \hat{\alpha}$  then  $p_{B \min}(x) > 0$  for consumers located at  $x < \frac{1}{2}$  ( $\gamma(x) > 0$ ). Interestingly, as consumer preferences for brand A are stronger (lower  $x$ ),  $\gamma(x)$  also increases and the minimum and maximum prices in the support of both firms' equilibrium price distributions increase.

The uniform price is  $p^u = \frac{\alpha + q(1 - \alpha)}{\alpha + q^2(1 - \alpha)}$ . Note that  $\frac{\partial p^u}{\partial \alpha} > 0$  when  $\alpha > \tilde{\alpha}$ , with  $\tilde{\alpha} = \frac{q}{q+1}$ ; otherwise if  $\alpha < \tilde{\alpha}$  then  $\frac{\partial p^u}{\partial \alpha} < 0$ . When  $q > 2$ , we conclude that  $\hat{\alpha} < \tilde{\alpha}$ , therefore an increase in  $\alpha$  reduces the uniform price if  $\hat{\alpha} < \alpha < \tilde{\alpha}$ . When  $\alpha > \tilde{\alpha}$ , then  $p^u$  can increase with  $\alpha$ .

Consider both firms' price behavior for consumers located at  $x \leq \frac{1}{2}$ . Firm A uses a "Hi-Lo" pricing strategy. L-type consumers will always buy from A. With probability equal to  $m$  they pay  $\gamma(x)$ , otherwise they can pay a lower price. Nevertheless, depending on  $x$ ,  $q$  and  $\alpha$  the discriminatory price can be above or below its uniform counterpart. When demand heterogeneity and the proportion of Low type consumers are high, the uniform price falls drastically. This suggests that for small  $x$  (high  $\gamma(x)$ ) the discriminating price can be above the uniform one.

Even though it is not possible to establish a general stochastic ordering between  $F_A[p(x)]$  and  $F_B[p(x)]$ , we plot the cumulative distribution functions for  $x < \frac{1}{2}$ , assuming that  $\alpha > \frac{q}{2q-1}$ . The figures are plotted for  $x = \{0, 0.25\}$ ,  $q = \{6, 15\}$  and  $\alpha = \{0.6, 0.8\}$ .

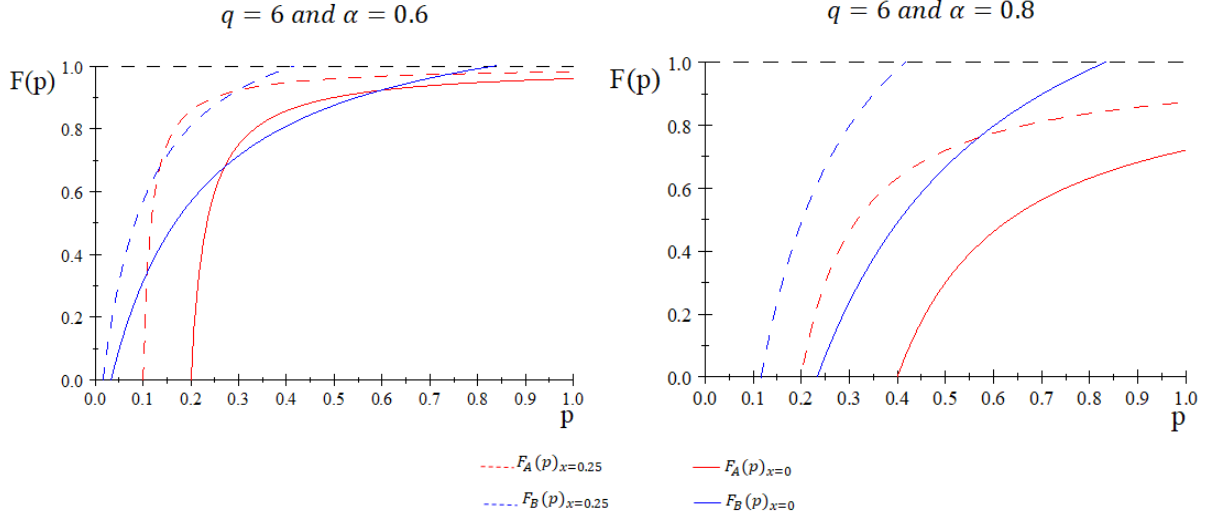


Fig. 1: Cumulative distributions when  $q = 6$

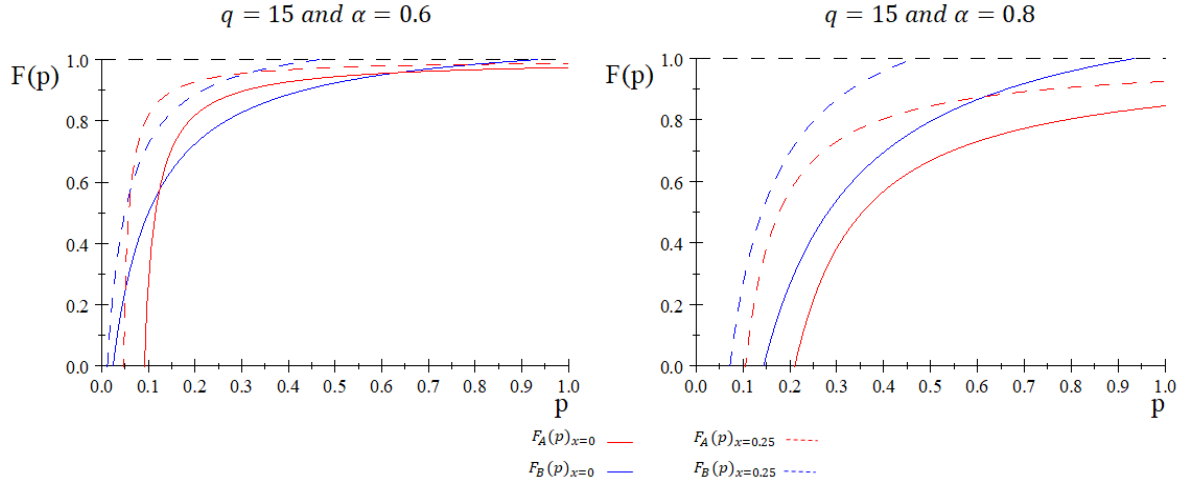


Fig.2: Cumulative distributions when  $q = 15$

Figures 1 and 2 shed some light about the effects of  $\alpha$  and  $q$  on firms' price decisions under IPP, when  $\alpha > \hat{\alpha}$ . When the share of L-type consumers in the market is sufficiently high, for instance  $\alpha = 0.8$ ,  $F_A[p(x)] < F_B[p(x)]$ , that is  $F_A$  first-order stochastically dominates  $F_B$ . In this case,  $p_A(x)$  is stochastically larger than  $p_B(x)$ , because it assumes large values with higher probability. Thus, on average firm A charges higher prices than its competitor to consumers located at  $x < \frac{1}{2}$ . Additionally, considering the mass point, the greater is the size of the L-type segment in the market, the higher is the probability of firm A charging the highest price to L-type consumers. The same happens when  $q$  falls. When  $\alpha > \frac{q}{2(q-1)}$ , from the expression of

the mass point we observe that  $\frac{\partial m}{\partial q} < 0$ , suggesting that the lower is the demand heterogeneity in the market (lower  $q$ ), the higher is the probability of firm A charging price  $\gamma(x)$  to a consumer located at  $x$ . The reverse happens when  $\frac{q}{2q-1} < \alpha < \frac{q}{2(q-1)}$ .

When the proportion of low demand consumers is not sufficiently high, for instance  $\alpha = 0.6$ , there is no stochastic order between  $F_A$  and  $F_B$ , the mass point is smaller, and firms compete more aggressively for each consumer. Average prices fall.

Regarding the effect of consumer preferences on firms' price decisions, it follows that  $F_i [p(\underline{x})] < F_i [p(\bar{x})]$  with  $0 \leq \underline{x} < \bar{x} < \frac{1}{2}$ ,  $i = A, B$ . As expected, in its strong market, firm A charges on average higher prices to consumers with small  $x$  ( $x$  close to zero) than to consumers with  $x$  close to  $\frac{1}{2}$ . As explained before, in its weak market, firm B charges on average higher prices to consumers with small  $x$  ( $x$  close to zero) than to consumers with  $x$  close to  $\frac{1}{2}$ .

### 6.1.1 Profit effects

We can now try to provide an answer to our initial question: *Can personalized pricing be a winning strategy for practice firms?*

As stated by Zhang (2009), here the answer to this question is also "it depends". Specifically, answering "Yes" or "No" depends on (i) the share of Low and High demand customers in the market ( $\alpha$ ) and (ii) the level of heterogeneity in purchase quantities ( $q$ ).

We first compare profits with different price discrimination schemes. The next proposition summarizes our main findings.

**Proposition 6 (Profits with different price discrimination schemes):**

(i) *Profits with group pricing are always above their counterparts with imperfect and perfect personalized pricing.*

(ii) *When  $\alpha < \hat{\alpha}$  with  $\hat{\alpha} = \frac{q}{2q-1}$ , profits are higher with perfect personalized pricing than with imperfect personalized pricing.*

(iii) *When  $\alpha > \hat{\alpha}$ , profits are higher with perfect personalized than with imperfect personalized pricing as long as  $\frac{q}{2q-1} < \alpha < \frac{2q}{3q-2}$  (with  $q > 2$ ). In contrast, if  $\alpha > \frac{2q}{3q-2}$  then profits are greater with imperfect personalized pricing than with perfect personalized pricing.*

**Proof.** See the Appendix.

Next we compare profits under the different price discrimination strategies with profits under uniform pricing. As mentioned before, moving from UP to GP (the less demanding price discrimination scheme in terms of information) is always a winning strategy for practice firms. Additionally, it is important to stress that when the proportion of Low and High volume customers in the market is equal (i.e.,  $\alpha = \frac{1}{2}$ ), in comparison to uniform pricing, any price discrimination strategy based on data about consumer preferences is never a winning strategy, because firms' profits fall when moving from UP to IPP or PPP.

In contrast, when there are sufficiently more L than H type consumers in the market, and demand heterogeneity is sufficiently high, then the access to data about consumers' preferences for personalized pricing schemes can, indeed, be a winning strategy for practice firms in competitive markets. The next proposition summarizes our main findings.

**Proposition 7 (Comparison of profits under UP and PP schemes):**

- (i) When  $\alpha < \hat{\alpha}$  with  $\hat{\alpha} = \frac{q}{2q-1}$ , IPP always reduces profits in comparison to UP.
- (ii) When  $\alpha > \hat{\alpha}$ , then

$$\pi^u - \pi_i^{IPP} = \frac{\chi(\alpha, q)}{(\alpha + q^2(1 - \alpha))(\alpha + q(1 - \alpha))}$$

with

$$\chi(\alpha, q) = -\alpha^3 q(5q - 3)(q - 1) + \alpha^2(2q - 15q^2 + 13q^3 + 1) - q\alpha(-7q + 11q^2 - 1) + 3q^3.$$

Then when  $q$  and  $\alpha$  are sufficiently high  $\chi(\alpha, q) < 0$ , suggesting that IPP boosts profits in comparison to UP.

- (iii) If heterogeneity in purchase quantities is sufficiently low ( $q < 2\sqrt{2} + 3$ ), profits with UP are above profits with PPP.

- (iv) If heterogeneity in purchase quantities is sufficiently high, i.e.  $q > 2\sqrt{2} + 3$ , in comparison to UP, PPP boosts profits as long as  $\alpha_1 < \alpha < \alpha_2$ , with  $\alpha_1 = \frac{1}{4(q-1)}(3q - 1 - \sqrt{-6q + q^2 + 1})$  and  $\alpha_2 = \frac{1}{4(q-1)}(3q - 1 + \sqrt{-6q + q^2 + 1})$ . The reverse happens when  $\hat{\alpha} < \alpha < \alpha_1$ . As  $q \rightarrow +\infty$ ,  $\alpha_1 \rightarrow 0.5$  and  $\alpha_2 \rightarrow 1$ .

**Proof.** See the Appendix.

In order to shed light about our main profit results, Figures 3, 4 and 5 plot three pictures based on different levels of demand heterogeneity, namely  $q = \{2, 6, 15\}$ .

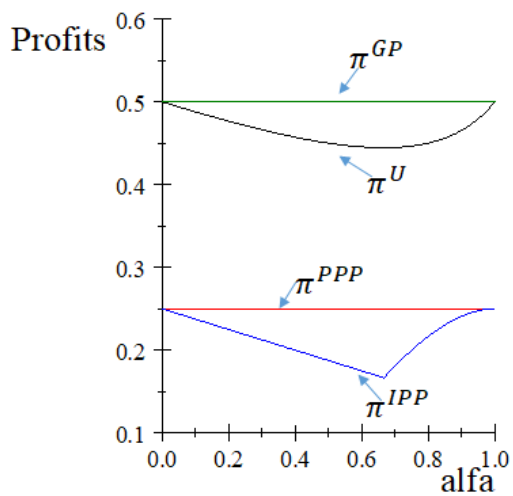


Fig.3: Profits when  $q = 2$

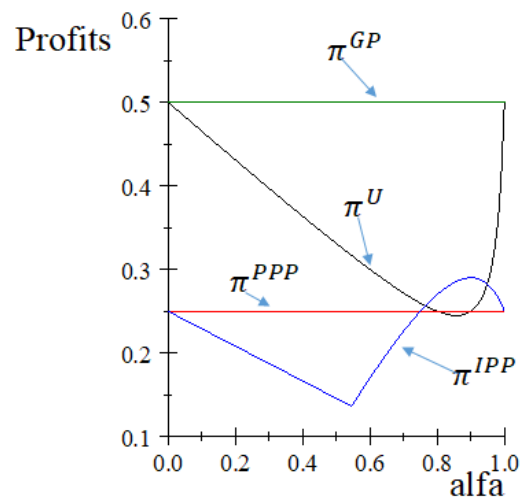


Fig.4: Profits when  $q = 6$

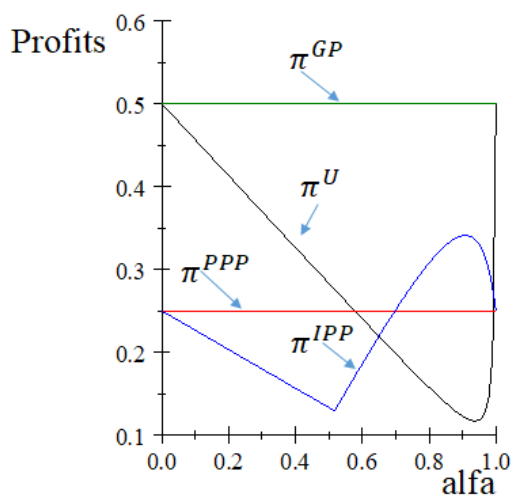


Fig.5: Profits when  $q = 15$

The figures presented above confirm that if firms have only information about consumer demand types, and price discrimination based on this information (GP) is permitted, profits are always above their counterparts with UP and IPP/PPP.

Practice firms are also better off under UP than under any form of personalized pricing when (i) the share of H-type consumers is higher than the share of L-type consumers, *and/or* (ii) when the heterogeneity in purchase quantity is sufficiently low. Thus our analysis complements the existing theoretical models. As aforementioned, when there is no demand heterogeneity in the market, i.e.,  $q = 1$ , and all firms have the required data to engage in personalized pricing,

the intensity of price competition for each consumer increases, and profits fall (Thisse and Vives (1988)). The same happens in this model heterogeneity of demand is low and when the proportion of H-type (L-type) consumers in the market is sufficiently high (low).

Our analysis offers new insights when the proportion of H-type consumers in the market falls and they demand increasingly more units. In this situation, in comparison to UP, firms might be better off under IPP or PPP. The pictures show that IPP is the worst strategy for industry profits when the proportion of H-type consumers in the market is sufficiently high. In this case, under IPP firms quote lower personalized prices to L and H type consumers with preference  $x$ . When PPP is permitted, in comparison to IPP, firms can recognize L and H types. Hence, although they charge the same price to H-type consumers, they can charge higher prices to L-type consumers. In contrast, when the proportion of H-type consumers further reduces such that  $\alpha > \hat{\alpha}$ , and demand heterogeneity is high enough, due to insufficient information (i.e, no information about demand types), IPP can act to soften price competition in the market and boost industry profits, either in comparison to UP or PPP.

Summing up, in the context of this model GP is the most profitable price discrimination strategy. Apart from it, when we compare UP, IPP and PPP, we conclude that a relatively small share of H type consumers, purchasing a sufficient high number of units, might allow firms to compete with personalized prices in a profitable way.

## 7 Final remarks

This paper complements the extant literature looking at the profitability of personalized pricing in oligoplistic markets. Like most of the literature, consumers are heterogeneous with respect to their preferences for firms. However, we incorporate another simple but important feature of customer heterogeneity in several markets, by assuming that not all customers are equally valuable to firms. In other words, we assume that some consumers purchase more than others. As stated by Shin and Sudhir (2010), widespread empirical support in various categories confirms the 80/20 rule, i.e., the idea that a small share of customers contributes to most of the purchases and profit in a category.

Our model offers a reasonable abstraction of many real-world markets in which consumer tastes and consumer heterogeneity in demand are important features (e.g. airlines, grocery stores, hotels, department stores, retail) and which businesses use data for price discrimination purposes.

We show that (i) the heterogeneity in consumer value, (ii) the share of L/H demand consumers and (iii) the type of data available for pricing, play an important role on the profit effects of price discrimination. When firms' data fully reveals consumer demand types (vertical information), but gives no information about consumer tastes (horizontal information), businesses can only employ group pricing. This is always a winning strategy. Indeed, in comparison to UP,



IPP and PPP, in this model GP is the more profitable price discrimination strategy.

When firms' data fully reveals consumer tastes but discloses no information about demand types, firms can quote prices on an individual basis and, under certain market conditions, imperfect personalized pricing may yield higher or lower profits than uniform pricing. More specifically, in markets where the heterogeneity in purchase quantity is sufficiently high and the proportion of Low (High) demand consumers is sufficiently high (low), in comparison to UP, profits are higher under imperfect personalized pricing. This suggests that in comparison to no discrimination, the existence of a small share of H-type consumers allows businesses to get greater profits when they employ imperfect personalized pricing. The reverse happens, however, when the proportion of High demand customers in the market is greater than the proportion of Low demand consumers.

Additionally, our analysis also suggests that if firms have access to perfect information about demand types and tastes, then, perfect personalized pricing can also be a winning strategy in comparison to UP and IPP. For instance, when  $q = 15$ , profits are greater under PPP than under UP as long as  $0.58 < \alpha < 0.99$ . And they are greater under PPP than under IPP as long as  $0.52 < \alpha < 0.70$ . This simple example suggests that when  $q = 15$ , in comparison to UP and IPP, PPP is better for profits as long as  $0.58 < \alpha < 0.70$ . If  $q = 15$  and the share of H-type consumers is 20%, then firms are better off under PPP than under UP, however IPP is more profitable than PPP. If the share of H-demand consumers is rather 35% of the market, then profits are higher under PPP than under UP or IPP.

Notwithstanding the model addressed in this paper is far from covering all complex aspects of real markets, it provides a theoretical strategic rationale for the increasingly use of consumer data for personalized pricing strategies only possible in the context of digital markets. In light of this, we show that when consumer heterogeneity is sufficiently high, the existence of a small share of H-type consumers can help businesses to employ personalized pricing as a winning strategy, even if they are symmetric. Therefore, our model offers critical information about the value of using consumer data for personalized pricing in oligopolistic markets relatively well represented by the features of this model. As the theoretical model provides empirically testable hypotheses, we hope it can be used for further empirical research.

## Appendix

This Appendix collects the proofs that were omitted from the text.

**Proof of Proposition 1:** Under uniform pricing each firm profit is

$$\begin{aligned}\pi_A^u(p_A, p_B) &= p_A \left[ \alpha \left( \frac{1}{2} + \frac{p_B - p_A}{2} \right) + (1 - \alpha)q \left( \frac{1}{2} + \frac{q(p_B - p_A)}{2} \right) \right] \\ \pi_B^u(p_A, p_B) &= p_B \left[ \alpha \left( \frac{1}{2} + \frac{p_A - p_B}{2} \right) + (1 - \alpha)q \left( \frac{1}{2} + \frac{q(p_A - p_B)}{2} \right) \right]\end{aligned}$$

Consider the case of firm A. From the FOC we obtain:

$$p_A = \frac{(q + \alpha - q\alpha + \alpha p_B + q^2 p_B - q^2 \alpha p_B)}{2\alpha - 2q^2\alpha + 2q^2}$$

Imposing symmetry it is straightforward to obtain:

$$p^u = \frac{\alpha + q(1 - \alpha)}{\alpha + q^2(1 - \alpha)}$$

Thus

$$\pi^u = \frac{1}{2} \frac{(\alpha + q(1 - \alpha))^2}{\alpha + q^2(1 - \alpha)}. \blacksquare$$

**Proof Proposition 2:** Under group pricing considering firm each firm sets a price to a  $H$  and a  $L$  type consumer, respectively denoted  $p_i^H$  and  $p_i^L$ ,  $i = A, B$ . Using equations (3) and (4) it is straightforward to obtain each firm profit. For firm A's profits are:

$$\pi_A^L = p_A^L \left[ \alpha \left( \frac{1}{2} + \frac{p_B^L - p_A^L}{2} \right) \right] \text{ and } \pi_A^H = p_A^H (1 - \alpha) q \left( \frac{1}{2} + \frac{q(p_B^H - p_A^H)}{2} \right)$$

From the FOCs we get that  $p_A^L = \frac{1}{2}(p_B^L + 1)$  and  $p_A^H = \frac{1}{2q}(qp_B^H + 1)$ . Imposing symmetry yields  $p_A^L = 1$  and  $p_A^H = \frac{1}{q}$ .  $\blacksquare$

**Proof of Corollary 1:** It is straightforward to see that  $p^u - p^L = q(\alpha - 1) \frac{q-1}{\alpha+q^2(1-\alpha)} < 0$  and  $p^u - p^H = \alpha \frac{q-1}{q(\alpha+q^2(1-\alpha))} > 0$ . Thus all consumers pay lower prices under uniform than under group pricing. In line of this personalized pricing based on consumer heterogeneity boosts both firms profits at the expense of consumer surplus ( $\pi^{GP} - \pi^u = \alpha(1 - \alpha) \frac{(q-1)^2}{\alpha+q^2(1-\alpha)} > 0$ ).  $\blacksquare$

**Proof of Proposition 3:** With no loss of generality consider firms' price decisions to consumers locate at  $x \leq \frac{1}{2}$ . If  $p_A(x) = \frac{\gamma(x)}{q}$  retailer A serves for sure all consumer types at  $x$  and its profit from consumers located at  $x$  is

$$\pi_A(p_A(x), p_B(x)) = \frac{\gamma(x) [\alpha + (1 - \alpha) q]}{q}$$

If firm A deviates to  $p_A^d(x) = \gamma(x)$  its guarantee profit is  $\pi_A^d(\gamma(x)) = \alpha\gamma(x)$ . Thus, firm A has an incentive to deviate to price  $\gamma(x)$  as long as  $\gamma(x)\alpha > \frac{\gamma(x)}{q} [\alpha + (1 - \alpha) q]$ , which implies  $\alpha > \frac{q}{2q-1}$ . As long as  $\alpha \leq \frac{q}{2q-1}$  there is a pure strategy equilibrium in prices with  $p_A(x) = \frac{\gamma(x)}{q}$  and  $p_B(x) = 0$  for all consumers with  $x \leq \frac{1}{2}$ . In this case overall profits are

$$\pi_A = (\alpha + (1 - \alpha) q) \int_0^{\frac{1}{2}} \frac{1 - 2x}{q} dx = \frac{\alpha + (1 - \alpha) q}{4q}. \blacksquare$$

**Proof of Proposition 4:** When  $\alpha > \frac{q}{2q-1}$  following a reasoning similar to Narasimhan (1988) we can show that a PSNE in prices fails to exist. There is however a MSNE the proof

of which is done by construction. With no loss of generality consider firms' price behavior to consumers located at  $x \leq \frac{1}{2}$ . Note that the minimum price firm A is willing to charge in an attempt to serve all consumers in its strong market should satisfy the condition  $p_{A \min}(\alpha + (1 - \alpha)q) = \gamma(x)\alpha$ . This yields:

$$p_{A \min} = \frac{\gamma(x)\alpha}{\alpha + (1 - \alpha)q}.$$

Firm B takes into account firm A's price behavior and so by pricing slightly below  $p_{A \min} - \frac{\gamma(x)}{q}$  it can sell to the H-type consumers. Thus  $p_{B \min} = p_{A \min} - \frac{\gamma(x)}{q}$ . For a consumer located at  $x \leq \frac{1}{2}$  in the MSNE we must observe:

$$p_A(x)(\alpha + (1 - \alpha)q) \left[ 1 - F_B \left( p_A(x) - \frac{\gamma(x)}{q} \right) \right] + \alpha p_A(x) F_B \left( p_A(x) - \frac{\gamma(x)}{q} \right) = \gamma(x)\alpha, \quad (32)$$

$$p_B(x)(1 - \alpha)q \left[ 1 - F_A \left( p_B(x) + \frac{\gamma(x)}{q} \right) \right] = p_{B \min}(x)(1 - \alpha)q. \quad (33)$$

Solving for  $F_B$  we obtain  $F_B \left( p_A(x) - \frac{\gamma(x)}{q} \right) = \frac{(\alpha + (1 - \alpha)q)}{q(1 - \alpha)} - \frac{\gamma(x)\alpha}{p_A(x)q(1 - \alpha)}$ , which yields

$$F_B(p_A(x)) = \frac{(\alpha + (1 - \alpha)q)}{q(1 - \alpha)} - \frac{\alpha\gamma(x)}{\left( p_A(x) + \frac{\gamma(x)}{q} \right) q(1 - \alpha)}.$$

From  $F_B(p_{\min}(x)) = 0$  and  $F_B(p_{\max}(x)) = 1$  we get

$$\begin{aligned} p_{B \min}(x) &= \frac{\alpha\gamma(x)}{\alpha + q(1 - \alpha)} - \frac{1}{q}\gamma(x) \\ p_{B \max}(x) &= \frac{1}{q}(q - 1)\gamma(x). \end{aligned}$$

Solving next for  $F_A$  we get  $F_A \left( p_B(x) + \frac{\gamma(x)}{q} \right) = 1 - \frac{p_{B \min}}{p_B(x)}$ . This yields

$$F_A(p(x)) = 1 - \gamma(x) \left[ \frac{q + \alpha - 2q\alpha}{(-\gamma(x) + p(x)q)(-q - \alpha + q\alpha)} \right]$$

From  $F_A(p_{A \min}(x)) = 0$  we get

$$p_{A \min} = \frac{\alpha\gamma}{\alpha + q(1 - \alpha)}.$$

There is a mass point at  $\gamma(x)$  equal to

$$m = 1 - F_A(\gamma(x)) = \frac{q^2(1 - \alpha)}{(q - 1)(q(1 - \alpha) + \alpha)}$$

Note that  $m < 1$  as long as  $\alpha > \frac{q}{2q - 1}$  which is true under our initial assumption. Therefore:

$$F_A(p(x)) = 1 - \frac{\alpha(2q - 1) - q}{(p(x)q - \gamma(x))(q(1 - \alpha) + \alpha)}\gamma(x)$$

with a mass point  $m = \frac{q^2(1-\alpha)}{(q-1)(q(1-\alpha)+\alpha)}$ .

Firm B expected profits for a consumer with preference  $x$  located at  $x \leq \frac{1}{2}$  (B's weak market) is:

$$\begin{aligned}\pi_B^w(x) &= \left( \frac{\alpha}{\alpha + q(1-\alpha)} - \frac{1}{q} \right) (1-\alpha) q \gamma(x) \\ &= \gamma(x) \frac{(1-\alpha)(\alpha(2q-1)-q)}{\alpha + q(1-\alpha)},\end{aligned}$$

and its overall profit is

$$\pi_B^w = \int_0^{\frac{1}{2}} \pi_B^w(x) dx = \frac{1}{4} \frac{(1-\alpha)(\alpha(2q-1)-q)}{\alpha + q(1-\alpha)}.$$

Firm A expected profits for a consumer with preference  $x$  located at  $x \leq \frac{1}{2}$  (A's strong market) is:

$$\pi_A^s(x) = \gamma(x)\alpha$$

thus its overall profits from this segment is

$$\pi_A^s(x) = \int_0^{\frac{1}{2}} \pi_A^s(x) dx = \frac{\alpha}{4}.$$

Summing up when  $\alpha > \frac{q}{2q-1}$ , due to symmetry, each firm profit with IPP

$$\pi^{IPP} = \frac{1}{4} \frac{(1-\alpha)(\alpha(2q-1)-q)}{\alpha + q(1-\alpha)} + \frac{\alpha}{4}. \blacksquare$$

### Proof of Proposition 6:

Consider the expressions for profits under GP, IPP and PPP, respectively equal to:

$$\begin{aligned}\pi^{GP} &= \frac{1}{2} \\ \pi^{IPP} &= \frac{\alpha + (1-\alpha)q}{4q} \text{ if } \alpha < \hat{\alpha} \\ \pi^{IPP} &= \frac{\alpha}{4} + \frac{1}{4} \frac{(1-\alpha)(\alpha(2q-1)-q)}{\alpha + q(1-\alpha)} \text{ if } \alpha > \hat{\alpha} \\ \pi^{PPP} &= \frac{1}{4}\end{aligned}$$

The proof of part (i) is straightforward, as profits with IPP and PPP are always below  $\frac{1}{2}$ . To prove part (ii) note that when  $\alpha < \hat{\alpha}$ :  $\pi^{IPP} - \pi^{PPP} = -\frac{1}{4q}\alpha(q-1) < 0$  which is always true for  $q > 1$ . Look next at part (iii). When  $\alpha > \hat{\alpha}$  with  $\hat{\alpha} = \frac{q}{2q-1}$ :

$$\pi^{IPP} - \pi^{PPP} = \frac{1}{4} (1-\alpha) \frac{3q\alpha - 2q - 2\alpha}{\alpha + q(1-\alpha)}$$

Thus  $\pi^{IPP} - \pi^{PPP} > 0$  if  $3q\alpha - 2q - 2\alpha > 0$ , which implies  $\alpha > \frac{2q}{3q-2}$ . As  $\alpha > \frac{q}{2q-1}$  and  $\frac{2q}{3q-2} > \frac{q}{2q-1}$  then  $\pi^{IPP} - \pi^{PPP} > 0$  as long as  $\alpha > \frac{2q}{3q-2}$ . The reverse happens, i.e.,  $\pi^{IPP} - \pi^{PPP} < 0$  when  $\frac{q}{2q-1} < \alpha < \frac{2q}{3q-2}$ . ■

**Proof of Proposition 7:**

With no discrimination profits equal

$$\pi^u = \frac{1}{2} \frac{(\alpha + q(1 - \alpha))^2}{\alpha + q^2(1 - \alpha)}. \quad (34)$$

When  $\alpha < \hat{\alpha}$ ,  $\pi^{IPP} = \frac{\alpha + (1 - \alpha)q}{4q}$  thus

$$\pi^u - \pi^{IPP} = \frac{(\alpha + (1 - \alpha)q)}{2} \left( \frac{1}{2} \frac{\alpha(2q - 1) + q^2(1 - \alpha)}{q(\alpha + q^2(1 - \alpha))} \right) > 0.$$

To prove parte (ii) when  $\alpha > \hat{\alpha}$ :

$$\pi^u - \pi_i^{IPP} = \frac{-\alpha^3 q(5q - 3)(q - 1) + \alpha^2(2q - 15q^2 + 13q^3 + 1) - \alpha q(-7q + 11q^2 - 1) + 3q^3}{(\alpha + q^2(1 - \alpha))(\alpha + q(1 - \alpha))}$$

with

$$\chi(\alpha, q) = [-\alpha^3 q(5q - 3)(q - 1) + \alpha^2(2q - 15q^2 + 13q^3 + 1) - \alpha q(-7q + 11q^2 - 1) + 3q^3]$$

$$\pi^u - \pi_i^{IPP} > 0 \text{ when } \chi(\alpha, q) > 0$$

When for instance  $q = 2$  then we need to impose that  $\alpha > \frac{2}{3}$ . In this range for  $\alpha$ ,  $\chi(\alpha, 2) > 0$ . When  $q = 6$  and  $\alpha > \frac{6}{11}$ , it follows that  $\chi(\alpha, 6) > 0$  as long as  $\frac{6}{11} < \alpha < 0.7648$ , otherwise if  $\alpha > 0.7648$  the reverse happens. When for instance  $q = 15$  and  $\alpha > \frac{15}{29}$ , we have that  $\chi(\alpha, 15) > 0$  as long as  $\frac{15}{29} < \alpha < 0.65$  and  $\chi(\alpha, 15) < 0$  when  $0.65 < \alpha < 0.995$ .

Next we prove part (iii). It follows that  $\pi^u - \pi^{PPP} = \frac{1}{2} \frac{2\alpha^2(q-1)^2 - \alpha(3q-1)(q-1) + q^2}{\alpha + q^2(1-\alpha)}$ . Look at the sign of the numerator. It has no roots when  $q < 2\sqrt{2} + 3$ , suggesting that in this case  $\pi^u - \pi^{PPP} > 0$ . In contrast when  $q > 2\sqrt{2} + 3$  and  $\alpha > \hat{\alpha}$ , there are two roots:  $2\alpha^2(q-1)^2 - \alpha(3q-1)(q-1) + q^2 < 0$  as long as  $\alpha_1 < \alpha < \alpha_2$ . Because  $\alpha_1 > \hat{\alpha}$  always holds for  $q > 2\sqrt{2} + 3$  then  $\pi^u - \pi^{PPP} < 0$  when  $\alpha_1 < \alpha < \alpha_2$ . The reverse happens when  $\hat{\alpha} < \alpha < \alpha_1$ . With:

$$\alpha_1 = \frac{1}{4q-4} \left( 3q - \sqrt{-6q + q^2 + 1} - 1 \right) \text{ which tends to } 0.5 \text{ when } q \rightarrow \infty$$

$$\alpha_2 = \frac{1}{4q-4} \left( 3q + \sqrt{-6q + q^2 + 1} - 1 \right) \text{ which tends to } 1 \text{ when } q \rightarrow \infty. \blacksquare$$

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