# Size vs. Number: Assigning Number Words to Discrete and Continuous Quantities 

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## Mathematical Cognition and Learning Society 2021 Conference

## Symposium Title

## Learning to count: New insights on the acquisition of symbolic numerical knowledge

## Symposium Abstract

Humans have developed symbolic systems to represent and efficiently manipulate numerical information. Children gradually learn the rules and principles characterising the number system and how number words and numerals represent exact numerical quantities. The acquisition of symbolic number knowledge is a long and error-prone process that occupies children for several years, starting from the age of two until the first years of primary school, and it represents a crucial stepping stone for future mathematical achievement. In the symposium, we provide new insights on the acquisition of the symbolic numerical knowledge throughout development, including in how children initially learn number words, how they understand that they refer to number (as opposed to other dimensions of quantity), and how this achievement eventually leads to benefits in how number words relate to symbolic mathematics.
The first talk (Slusser) explores the developmental trajectory of children's understanding that number words refer to discrete numerical quantities, rather than continuous dimensions (e.g., "small", "a lot"). Children fully grasp the discrete nature of number words when they are 3-knowers, at an intermediate stage toward the mastering of the cardinality principle.
The second talk (Krajcsi) revalidates the widely used Give-a-Number task, in which children create numerical sets according to a required number (e.g., "give me four apples"). The current literature describes a sudden change in performance and strategy implementation when children extend their cardinal knowledge beyond four (i.e., cardinal-principle knowers). Contrary to this prevailing view, the proposed re-evaluation highlights the presence of a smooth, rather than sudden, change in performance as children slowly acquire the numerical meaning of number words beyond four, challenging traditional accounts of number words being learned through a sudden insight into general counting principles.
The third talk (Sella) describes how the mastering of different numerical concepts relate to the understanding of the exact numerical magnitude represented by number words and Arabic numbers. The mastering of the predecessor knowledge (i.e., removing one item from a set leads to the preceding number word in the counting list) and the knowledge of the spatial order of numbers relate to the performance in number comparison tasks beyond the acquisition of the cardinality principle and the later-greater principle.

The fourth talk (Odic) shows how the eventual mastery of number words leads to other benefits in children's numerical skills, most notably in children's ability to associate symbolic number words with their intuitive sense of number (the Approximate Number System; ANS), with non-numeric dimensions (e.g., estimating the length of a line), and as a source of error detection in symbolic mathematics.

The combination of the four talks provides an overview of the development of symbolic numerical knowledge from children who have just begun learning number words to those who have largely mastered it while giving new insights challenging current theoretical views.

## Organiser

## Francesco Sella

Centre for Mathematical Cognition
Loughborough University, UK
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## Talk 1

Speaker: Emily Slusser
Position: Associate Professor and Department Chair
Institution: San Jose State University, US.
Authors: Slusser, Emily ${ }^{1}$; Cravalho, Patrick ${ }^{1}$
${ }^{1}$ San Jose State University

## Size vs. Number: Assigning Number Words to Discrete and Continuous Quantities

Brief description of the talk: This talk explores when and how children come to understand that number words refer specifically to discrete numerosities (e.g., 1, 2, and 7), rather than various dimensions of continuous quantity (e.g., "small" in terms of surface area or "a lot" in terms of volume). Findings from a study of 2- to 4 -year-old children indicate that number words are assigned exclusively to discrete numerosities at an intermediate stage of number word acquisition - after they have learned at least three number words ('three-knowers') - with evidence suggesting that executive functioning skills play an important role in this developmental trajectory.

## Talk 2

Speaker: Attila Krajcsi
Position: Professor
Institution: ELTE Eötvös Loránd University, Hungary
Authors: Krajcsi, Attila ${ }^{1}$
${ }^{1}$ ELTE Eötvös Loránd University, Hungary

## Revalidating the Give a Number task

Brief description of the talk: The Give a Number task is an essential tool to measure preschoolers' symbolic number knowledge. In a series of studies, reevaluating the original proposals by Wynn (1990, 1992), we found that contrary to the original findings there is no sudden increase in the task performance after understanding number 4. Additionally, strategy change between grabbing and counting can be observed, because only large numbers have to be counted; furthermore, the strategy does not change radically after understanding number 4 . Based on these results, it might be reasonable to reevaluate the use of the task and former related findings of the literature.

## Talk 3

Speaker: Francesco Sella
Position: Lecturer
Institution: Loughborough University, UK
Authors: Sella, Francesco ${ }^{1}$; Lucangeli, Daniela ${ }^{2}$; Cohen Kadosh, Roi $^{3}$; Zorzi, Marco ${ }^{4}$
${ }^{1}$ Centre for Mathematical Cognition, Loughborough University, UK
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## Learning the number sequence: preschool children make sense of numbers

I will describe the acquisition of the cardinality principle, the mastering of successor and predecessor knowledge, the mapping between number words and external numerosities (ANS-to-word mapping), the spatial mapping of numbers and how all these abilities relate to the performance in number words and Arabic numbers comparison tasks. Our results revealed that, beyond the acquisition of the cardinality principle, the mastering of the predecessor knowledge and spatial order of digits relate to the understanding of the exact numerical magnitude represented by number words and Arabic digits.

## Talk 4

Speaker: Darko Odic
Position: Assistant Professor
Institution: University of British Columbia, Canada
Authors: Odic, Darko ${ }^{1}$; Dramkin, Denitza ${ }^{1}$; Wong, Harris ${ }^{1}$
${ }^{1}$ University of British Columbia, Canada

## Developing the interface between number word and the intuitive number sense: challenges and benefits

Young children have access to two distinct representations of number: an intuitive, but imprecise, perceptual number sense (ANS), and the slowly developing symbolic number words. In this talk, I show that the development of this interface is initially slow and protracted but, once children master the ability to translate between number words and the ANS, this gives them two key advantages for other numerical skills: (1) children can immediately form a novel number word interface with nonnumeric dimensions, such as estimating area; (2) children can use their intuitive number sense as a source of error detection in formal mathematics.

Together, the talks argue that: a ) Children's understanding of the discrete nature of number words only emerges at an intermediate stage toward mastering the cardinality principle; b) widely used tasks and their underlying models should be carefully reconsidered to obtain a more detailed understanding of the developmental stages children go through when learning number words; c) Learning the directional property of the counting list and the spatial order of numbers can scaffold the understanding of symbolic numerical magnitude; d) The ability to translate between number words and the ANS gives children crucial advantages for other numerical skills.

## Size vs. Number

 Assigning Number Words to Discrete and Continuous Quantities

Emily Slusser and Patrick Cravalho San José State University MCLS Conference August 7, 2020

## Overview

- Background
- Early Representations of Number
- Number Knower-Levels
- Quantity vs. Numerosity
- Methods \& Results
- Give-N (Knower-Level)
- Elephant Crocodile (Executive Functioning)
- Transform Sets (Quantity v. Number)
- Conclusions and Future Directions


## Early Representations of Number



Small-Exact
e.g. Feigenson \& Carey, 2003


Large-Approximate

e.g. Halberda et al. 2013

## Large, Exact Representations



## Knower Levels



Sarnecka, Goldman, \& Slusser, 2014

## Sarnecka \& Gelman (2004)



## Take Aways...

The number word does not change when shaken or rotated, but does change when an item is added or removed.

Slusser \& Sarnecka (2011)
"This Picture has four turtles.Find another picture with four turtles.


Sample picture


Correct response picture (matches sample picture on number)


Incorrect response picture (matches sample picture on summed contour length)

## Slusser \& Sarnecka (2011)



## Take Aways...

Match when prompted with color or mood ("find another picture with green turtles").

Do not match on numerosity when prompted with a number word until CP level.

## Size vs. Number

- When and how do children come to understand that number words refer to discrete numerosity?
- Do children think the number word changes as the individual items grow or shrink in size?
- Influence of executive functioning (domain-general) and language and counting (domain-specific) skills?

$$
\text { 2- to 4-year-olds }\left(n=45 ; 25 \text { males; } M_{\text {age }}=3 ; 10\right)
$$

## Give-N (Wynn 1990)

## can you put two fish in the pond?



Determine how many number words a child knows (Knower-Level) and whether they understand the Cardinality Principle.

## Elephant-Crocodile (Leoonard et oll, 2014)

When the elephant presses a button, press the same button that he presses.


When the crocodile presses a button, press the other button, the button that he didn't press.

Measure children's executive functioning skills (impulse control and cognitive flexibility).

## Transform Sets (Sarnecka \& Gelman ‘04)



Assess children's understanding of number words do number words refer to (discrete) numerosity?

## Transform Sets x Knower Level



Three/Four and CP Knowers understand that number labels change if, only if, an item is added or subtracted from a set.

## Number Trials x Knower Level



This understanding is first applied to known numbers ("two" and "three" for Three-Knowers).

## Quantity Trials x Knower Level



One Knowers may change number word labels in response to changes in continuous quantity (grow/shrink).

## Number Trials x Executive Function



EF correlates with performance on number trials. EF does not predict performance on quantity trials.

## Conclusions

Children draw on AMS representations as they construct number concepts.
AMS supports representation of discrete and continuous quantities.
$\therefore$ Children must figure out whether number words label numerosity or continuous quantity (often conflated).

- Children appear to link number words to numerosity as they learn/induce the Cardinality Principle.
- Executive functioning influences children's developing understanding of number, but...?



## Thank You!

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