Theoretical and Experimental Investigation of 30K Single Stage GM-Type Pulse Tube Cryocooler

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ABSTRACT

Theoretical modeling of thermodynamic performance plays an important role in the design and development of pulse tube cryocoolers. In the present work, a thermodynamic model of a GM-type double-inlet pulse tube cryocooler has been developed. It includes calculation of ideal refrigeration power, various losses, and hence net refrigeration power. An experimental setup was designed and fabricated to carry out experimental investigations on the GM-type double-inlet pulse tube cryocooler. Experiments were conducted, and the results obtained were used for comparison with the isothermal model developed in the present work. The effect of orifice valve opening, double-inlet valve opening, and frequency on the performance of cryocooler was studied in terms of net refrigeration power and no-load temperature.

NOMENCLATURE

OPTC	Orifice Pulse Tube Cryocooler	m_{I0}	Constant
DIPTC	Double Inlet Pulse Tube Cryocooler	R	Gas constant
A_0	Cross sectional area of orifice, m^2	T	Temperature, K
A_{DI}	Cross sectional area of double inlet, m^2	t	Time ,s
CII_0	Constant	T_C	Temperature at the cold end , K
m	Mass flow rate kg/s	To	Room temperature, K
m_0	Mass flow rate at the orifice, kg/s	K	Ratio of specific heats
m_{0DI}	Mass flow rate at the double inlet valve, kg/s	V	Specific volume, m^3/kg
P	Pressure, MPa	V	Volume, m^3
P_H	High pressure, MPa	V_P	Volume of pulse tube, m^3
P_L	Low pressure, MPa	P_R	Average pressure, MPa
τ	Time period	Φ	Frequency, Hz
μ	Flow coefficient	$t_{1,2,3,4}$	Cycle time

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INTRODUCTION

Applications of pulse tube cryocoolers are increasing day by day. This includes cooling of infrared sensors, low noise amplifiers, MRI, SQUIDS, small capacity nitrogen liquefiers, etc. Theoretical modeling of the pulse tube cryocooler plays an important role in its design and development. The model should be simple and clear in revealing the essentials of internal physical processes and reliable in predicting the performance of pulse tube cryocoolers.

The first remarkable work in this direction was reported by Storch et al. [1] and Radebaugh et al. [2]. The model developed by them describes the internal processes of a Stirling-type orifice pulse tube (OPT) cryocooler using enthalpy flow analysis and investigated the performance for various operating parameters. Wang et al. [3] proposed a numerical model using conservation of energy, momentum, and continuity for performance prediction of an OPT cryocooler. Ravex et al. [4] developed an algorithm based on the wall heat pumping effect and enthalpy flow to calculate the dominant cooling processes in both orifice and double-inlet pulse tube cryocoolers. Zhu et al. [5] proposed an isothermal model for a pulse tube cryocooler. Liang et al. [6] has discussed the thermodynamic non-symmetry effect. They derived a mathematical formulation considering orifice and double-inlet valves as impedance devices. Zhu et al. [7] has developed an improved numerical model for simulation of the oscillating fluid flow of orifice and double-inlet pulse tube cryocoolers based on conservation of mass, momentum, and energy. The governing equations include the pressure gradient, inertia, viscous, and convection terms. Atrey et al. [8] applied this model to carry out cyclic simulations of Stirling-type orifice pulse tube cryocoolers including the losses.

In a double-inlet GM-type pulse tube cryocooler, a bypass tube connects a pressure-wave generator, i.e. the GM distribution valve, to the hot end of the pulse tube (see Fig. 1). The bypass reduces the mass flow rate in the regenerator, which reduces the irreversibility of the system, thereby improving the performance. Several comparative experiments have verified that the addition of a double-inlet valve can lower the no-load temperature and increase the refrigeration power of a pulse tube cryocooler. Desai et al. [9] developed a theoretical model for a GM-type orifice pulse tube refrigerator based on a second-order isothermal model developed by Atrey et al [8]. Further Investigation on orifice pulse tube refrigerators was carried out by Desai et al. [10].

In the present work, the earlier theoretical models for orifice pulse tube cryocoolers are extended to a double-inlet pulse tube cryocooler followed by experimental investigations. The effects of various operating parameters like, frequency, orifice opening, double-inlet opening, etc. on the performance of system are studied.

MODEL DESCRIPTION

The isothermal model for a Stirling-type orifice pulse tube cryocooler has been developed by Zhu et al. [5] and extended by Atrey et al. [8] and Desai et al. [9]. It is further modified in the present work for a GM-type double-inlet pulse tube cryocooler as presented here.

A schematic diagram of a double-inlet pulse tube cryocooler is shown in Figure 1. The pulse tube can be divided into three parts: the portion of gas in the cold region of the pulse tube that flows from the regenerator into the pulse tube and expands to give out work is defined as Gas III. The portion of the gas that flows from the orifice into the pulse tube from the reservoir or vice versa is defined as Gas I. And, the gas in the central portion of the pulse tube, which never flows out of the

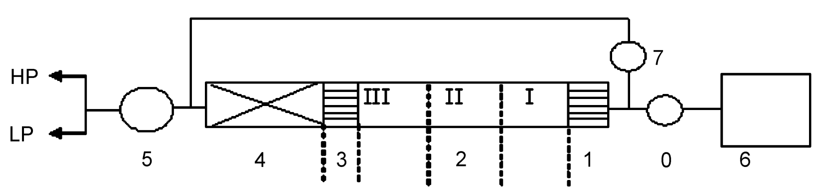

Figure 1. Schematic diagram of double-inlet pulse tube cryocooler: 0 - Orifice, 1 - Hotend heat exchanger, 2 - Pulse tube, 3 - Coldend heat exchanger, 4 - Regenerator, 5 - Distribution valve, 6 - Reservoir, 7 - Double Inlet valve, HP - High pressure, LP - Low pressure.

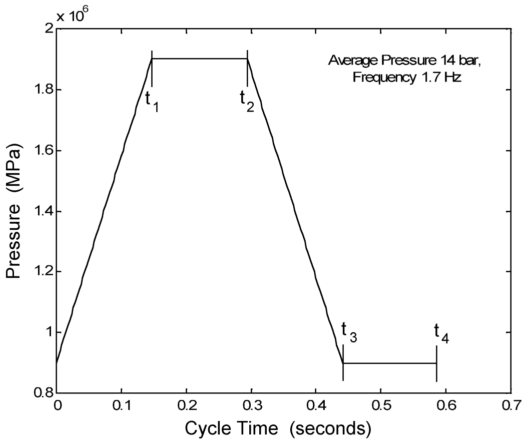

Figure 2. A typical trapezoidal waveform.

pulse tube and acts similar to a displacer of split Stirling cryocooler, is defined as Gas II. The other assumptions made for the analysis include:

1. Working gas is ideal gas
2. The variation of pressure is trapezoidal as shown in Fig. 2.
3. The pressure in the system remains uniform at any instant in time
4. Gas I & III are isothermal and gas II is adiabatic.

The governing equations are defined below, starting with the pressure variation, which is given by Eq (1) using the time nomenclature indicated in Fig. 2:

$$P(t)=\begin{cases} P_L+((P_H-P_L)/(t_1))t & (0\le t\le t_1) \\ P_H & (t_1\le t\le t_2) \\ P_H+((P_L-P_H)/(t_3-t_2))\,(t-t_2) & (t_2\le t\le t_3) \\ P_L & (t_3\le t\le t_4) \end{cases} \tag{1}$$

Mass flow rate at the orifice is given by Eq (2) where P_6 is the reservoir pressure:

$$\dot{m}_0=\mu\, A_0\sqrt{2\frac{k\ \ P}{(k-1)v_1}\left[\left[\left(\frac{P_6}{P}\right)^{2/k}-\left(\frac{P_6}{P}\right)^{(k+1)/k}\right]\right]} \qquad for\ \ P>P_6$$

$$\dot{m}_0=-\mu\, A_0\sqrt{2\frac{k\ \ P_6}{(k-1)v_6}\left[\left(\frac{P}{P_6}\right)^{2/k}-\left(\frac{P}{P_6}\right)^{(k+1)/k}\right]} \qquad for\ P<P_6 \tag{2}$$

Mass flow rate at the double-inlet valve is given by Eq (3):

$$\dot{m}_{0DI}=\mu\, A_{DI}\sqrt{2\frac{k\ \ P}{(k-1)v_1}\left[\left[\left(\frac{P_L}{P}\right)^{2/k}-\left(\frac{P_L}{P}\right)^{(k+1)/k}\right]\right]} \qquad for\ P>P_L$$

$$\dot{m}_{0DI}=-\mu\, A_{DI}\sqrt{2\frac{k\ \ P_6}{(k-1)v_6}\left[\left(\frac{P}{P_H}\right)^{2/k}-\left(\frac{P}{P_H}\right)^{(k+1)/k}\right]} \qquad forP<P_H \tag{3}$$

Pressure in the orifice reservoir (P_6) is given by Eq (4):

$$\frac{dP_6}{dt} = \frac{RT_0\,\dot{m}_0}{V_0} \tag{4}$$

Mass flow rate at section 1 is calculated by Eq (5):

$$\dot{m}_1 = \dot{m}_0 + \dot{m}_{0DI} + \frac{V_1}{RT_0}\frac{dP}{d\tau} \tag{5}$$

Mass of Gas I is given by Eq (6):

$$m_1 = m_{10} + \int_0^t (-\dot{m}_1)\,dt \tag{6}$$

where, m_{I0} can be obtained by ensuring $(m_I)_{min} = 0$.

The volume of Gas I is given by Eq (7):

$$V_1 = \frac{m_I RT_0}{P} \tag{7}$$

Volume of Gas II is calculated by Eq (8):

$$V_{II} = C_{II0}\,P^{-(1/k)} \tag{8}$$

Volume of Gas III is calculated by Eq (9):

$$V_{III} = V_P - V_{II} - V_I \tag{9}$$

where, C_{II0} can be obtained from $(V_{III})\min = 0$.

Eq (10) gives Gross Refrigeration Power:

$$Q = \frac{1}{\tau}\oint P\,dV_{III} \tag{10}$$

The effect of various losses is significant on the net refrigeration power obtained from the pulse tube cryocooler. The method of calculating such losses in the case of Stirling cryocoolers is well explained by Atrey et al. [11].

Heat losses due to regenerator ineffectiveness, temperature swing, pressure drop and solid conduction loss are calculated as per Atrey et al. [11]. The net refrigeration power is calculated by subtracting all the losses from the gross refrigeration power as given in Eq (11).

$$Q_{NET} = Q_{GROSS} - \sum Q_{LOSSES} \tag{11}$$

Many researchers have used a sinusoidal pressure wave form for such investigations, which is essentially a feature of a Stirling-type cryocooler. Figure 2, which was plotted for 14 bar average pressure, gives the nature of the wave form used in the present work. The model is solved using MATLAB.

DESIGN OF GM PULSE TUBE CRYOCOOLER

The model has been used to design a 100 W capacity GM-type double-inlet pulse tube cryocooler having a U-tube configuration, operating at 80 K temperature, 14 bar average pressure, and 1.75 Hz frequency. The design and operating parameters are listed in Table 1, while the different losses calculated using the theoretical model are shown in Table 2.

Table 1. Design and operating parameters.

Regenerator Matrix Material = SS 250 mesh
Pulse Tube = Dia. 25.4 mm, 250 mm long
Regenerator = Dia. 38 mm, 175 mm long
Reservoir Volume = 1000 cm^3
Average Pressure = 14 bar
Pressure ratio = 1.9
Frequency = 1.7 Hz

Table 2. Losses predicted by model.

A. Ideal refrigeration power	135 W
B. Regenerator ineffectiveness loss	1.35 W
C. Temperature swing loss	6.21 W
D. Regenerator pressure loss	25.87 W
E. Solid conduction loss	1.7 W
F. Net refrigeration power A-(B+C+D+E)	100 W

The refrigeration capacity of a cryocooler decreases because of losses such as the loss due to regenerator ineffectiveness, temperature swing, regenerator pressure drop, solid heat conduction etc. If the effects of all individual losses identified separately are combined, as they are in the real situation, then the net refrigerating power is substantially reduced from that pertaining to the ideal case. The loss analysis of a cryocooler is very important to estimate its efficiency and performance.

The amount of gas that passes through the regenerator is quite large and consequently the load on the regenerator is also large. Hence, the regenerator is a critical component that influences the performance of a pulse tube refrigerator, and the regenerator efficiency is critical to its overall thermodynamic performance. The regenerator performance varies with different mesh structures used and also with the properties of the mesh matrix material. Regenerator ineffectiveness causes the incoming gas to be cooled to a slightly higher temperature than with an ideal regenerator, thereby causing loss of refrigeration power.

Temperature swing loss accounts for the temperature changes in the matrix of the regenerator during the cycle. It is the heat taken up by the matrix due to its finite heat capacity. The drop in temperature of the regenerator matrix is calculated along the length of the regenerator.

Due to pressure drop in the regenerator and other parts, the actual pressure in the pulse tube will be lower than the supply pressure. During the expansion process, the gas can't be expanded in the pulse tube to the lowest possible pressure for the same reason. This results in a reduction of P-V area (of gas III), and hence a reduction of the refrigeration power. The pressure drop depends on flow acceleration and core friction. This loss is estimated based on the viscosity of the gas, gas density, mass velocity, Reynolds' number, and friction factor. This is a major loss among all the other types of losses.

Solid conduction loss is due to heat conduction from high temperature regions to low temperature regions. It occurs down the length of the pulse tube, down the regenerator tube liner, and down the length of the regenerator matrix. These losses also reduce the net refrigeration power and can be calculated from basic heat transfer equations.

EXPERIMENTAL INVESTIGATION

A double-inlet GM-type pulse tube cryocooler was developed using the above mentioned design procedure. The unit was fabricated and instrumented to carry out experimental investigations. A Leybold 6 kW helium compressor was used for the experiments, together with a rotary valve developed to deliver the pressure pulse. The dimensions of the experimental setup are listed in Table 1.

Results and Discussion

The computed movements of the interfaces between the three gas regions (I, II and III) in the double-inlet pulse tube over one cycle are shown in Figure 3. This gives some idea about the gases

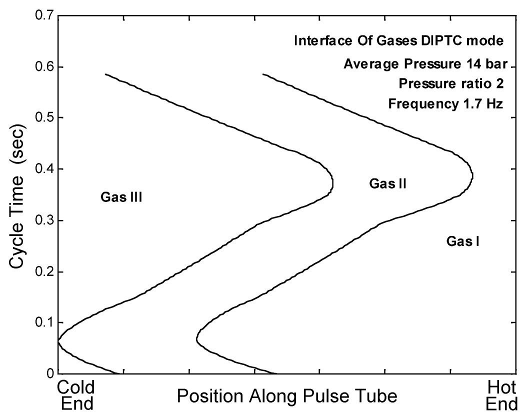

Figure 3. Gas interface motion over one cycle.

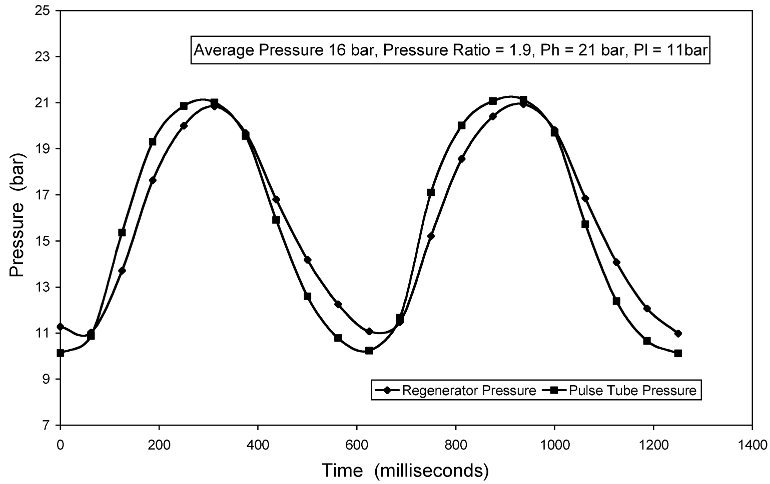

Figure 4. Recorded pressure waveform.

that are undergoing compression and expansion over a cycle. In the double-inlet mode, a bypass is introduced between the distribution valve and the hot end of the pulse tube through a double-inlet valve. The introduction of a double-inlet valve reduces the gas III entering the pulse tube at the cold end. This reduces the flow of gas through the regenerator, which reduces irreversibility in the system. However the refrigerating effect per unit mass of gas increases, thereby contributing to improved performance for a pulse tube cryocooler with a double-inlet bypass.

The pressure waveforms recorded at the warm end of the regenerator and pulse tube are shown in Figure 4. It can be seen from the figure that the pressure at the hot end of the pulse tube is lower than that at the hot end of regenerator due to a drop in pressure. It was found experimentally that the steady-state pressure ratio remains around 1.9 to 2 for different average pressures and frequencies.

The no-load temperatures measured at the cold end of the pulse tube are shown in Figure 5 for different operating pressures. The coldend temperature decreases linearly with time and then becomes steady. The lowest no-load temperature of 28 K was recorded for an average pressure of 12 bar with an operating frequency of 1.7 Hz. As the average pressure increases, the compression heat and friction heat will also increase, resulting in lower performance at 14 bar and 16 bar average pressure. The coldend no-load temperature reached 30 K in 30 minutes.

The net refrigeration power measured at the cold end is shown in Figure 6 for various average pressures. Refrigeration power increases with increasing coldend temperature. A net refrigeration

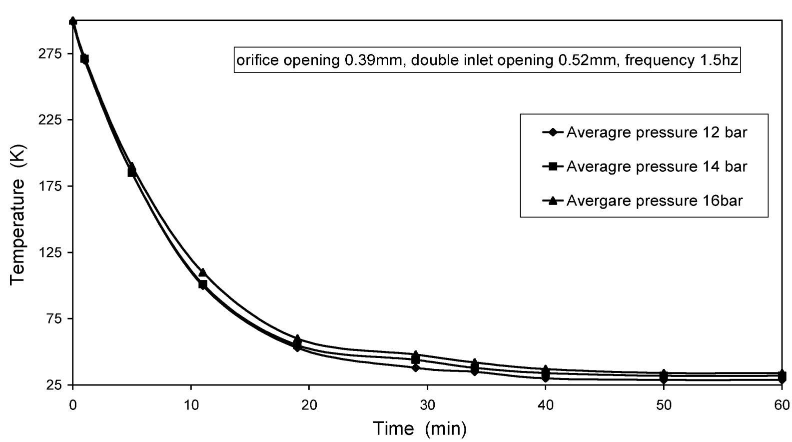

Figure 5. Cooldown curve.

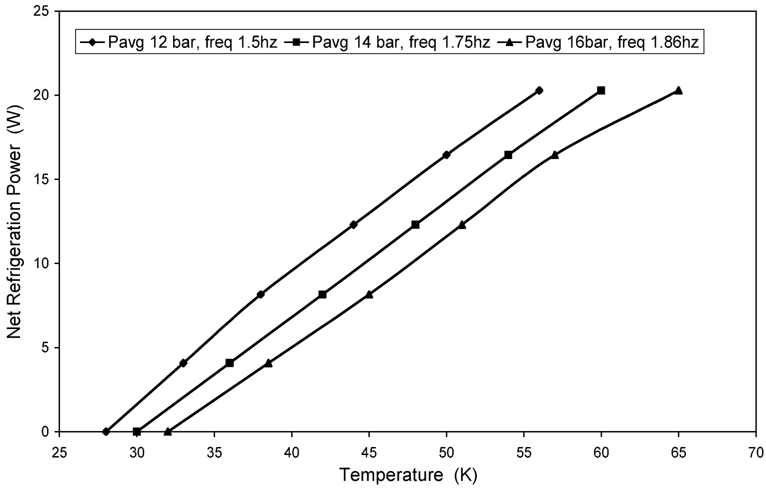

Figure 6. Net refrigeration power at various average pressure.

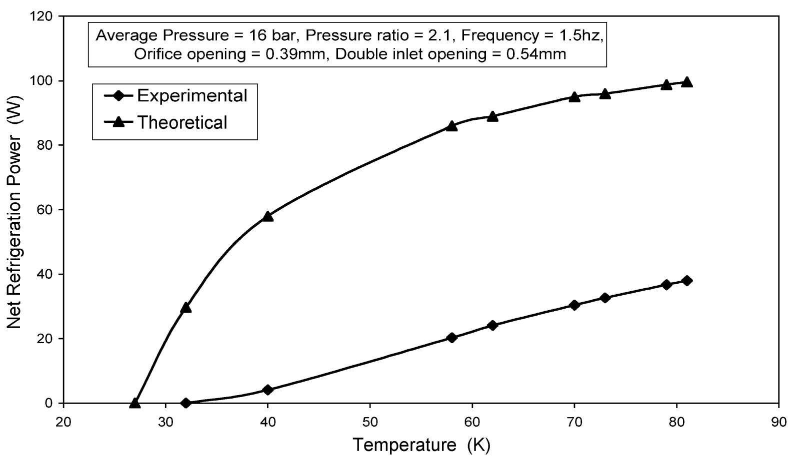

Figure 7. Theoretical and experimental results for net refrigeration power.

power of 20 W was measured at 56 K with an operating frequency of 1.5 Hz.

A comparison of experimental and theoretical results for net refrigeration power versus cold end temperature is shown in Figure 7. Each curve used the operating frequency that maximized its performance. A net refrigeration power of 37 W was measured at 80 K with 16 bar average pressure and an operating frequency of 1.5 Hz. The theoretical model, which used the actual recorded pressure waveform as an input, predicts 99 W of net refrigeration power at 80 K for the same operating conditions. The difference between the theoretical and experimental results is due to the fact that the theoretical model does not calculate other losses viz. shuttle heat loss, pumping loss, DC flow heat loss, and unaccounted losses.

Generally, GM-type pulse tube cryocoolers are operated at low frequencies i.e. in the range of 1 Hz to 5 Hz. For a particular orifice and double-inlet opening there exist an optimum operating frequency for which the no-load temperature is minimum. The effect of frequency on no-load temperature is depicted in Figure 8. The no-load temperature decreases with frequency, passes through a minima, and then increases with frequency. With an increase in frequency, the losses also increase. This is due to the fact that, with an increase in frequency, initially the losses reduce marginally, and thereafter they increase. This is because, at higher frequency, the cycle time reduces,

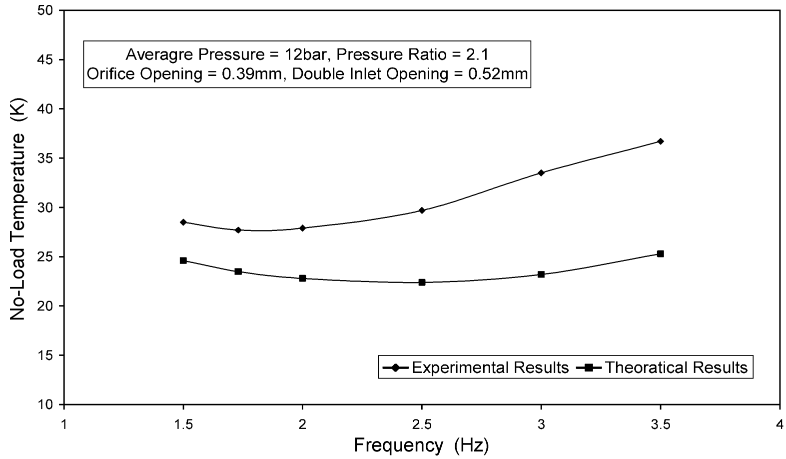

Figure 8. Effect of frequency on no load temperature.

which in turn reduces the time available for the gas to flow in and out of the pulse tube. This results in higher losses. Hence, there exists an optimum frequency at which the no-load temperature is minimum. At 12 bar average pressure, the theoretical model predicts an optimum frequency of 2.7 Hz, in contrast to the experimentally recorded value of 1.7 Hz.

CONCLUSION

The proposed isothermal model is compared with experimental results. The model is useful in understanding the performance of double-inlet pulse tube cryocoolers through the behavior of the gas. A lowest no-load temperature of 28 K was recorded using an average pressure of 12 bar and an operating frequency of 1.7 Hz. A net refrigeration power of 37 W was measured at 80 K at 16 bar average pressure and 1.7 Hz frequency.

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