Proceedings of the 40th IEEE
Conference on Decision and Control
Orlando, Florida USA, December 2001

A Control Lyapunov Function Approach to Multi-Agent Coordination[1]

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Abstract

In this paper, the multi-agent coordination problem is studied. This problem is addressed for a class of robots for which control Lyapunov functions can be found. The main result is a suite of theorems about formation maintenance, task completion time, and formation velocity. It is also shown how to moderate the requirement that, for each individual robot, there exists a control Lyapunov function. An example is provided that illustrates the soundness of the method.

1 Introduction

In this paper we investigate the problem of how to coordinate a collection of robots in such a way that they maintain a given formation relative to each other. The main assumption about the dynamics of the individual robots that we initially make in this paper is that they can be globally, asymptotically stabilized. Based on this assumption, an abstract and theoretically sound coordination strategy can be developed based on the theory of controlled Lyapunov functions.

Multi-agent formation control problems have been extensively studied in the literature and our main contribution is that we use control Lyapunov functions to define the formation. By doing this we convert the formation control problem, typically a constrained motion control problem of multiple systems, into a stabilization problem for one single system. By this approach we neither cast the problem without real dynamics , nor with an explicit nonlinear robot model [7]. Instead we believe that by requiring the existence of control Lyapunov functions, we can capture the essential some aspects of the platform dynamics, while not having to spend our main effort on nonlinear robot control. We can thus focus on the coordination problem at a higher level.

In addition to the control Lyapunov function approach we use the idea of virtual vehicles introduced in [3]. Concepts of similar flavor are the "action reference" suggested by Kang et. al. [2] and the "dynamic coordination variable" proposed by Beard et. al.[4]. Furthermore, in the terminology of Beards in [5], our approach would fall into the category of "virtual structures".

The motivation for studying this type of multi-agent coordination problem mainly stems from the observation that there is robustness and strength in numbers. If more than one agent is asked to carry out a given task, e.g. search a disaster area, the likelihood of success increases as more agents are included in the mission. In other situations, cost and energy efficiency indicates that using many small robots might be more beneficial than using one big robot.

The outline of this paper is as follows: In Section 2 we define what we mean by a formation, and show how this can be done implicitly through the null-set of a formation function. In Section 3 we then prove a suite of theorems about bounded formation errors, task completion times, and group velocities. This is done for the case where each individual robot can be globally, asymptotically stabilized. In the following section, we relax this requirement, but show how the theory holds for this case as well. We conclude, in Section 5, with an example, illustrating the usefulness of our proposed method.

2 Formation Functions

Our primary object of study is a collection of m robots, whose dynamics can be described by the following set

[1]The work by the first and third authors was sponsored in part by the Swedish Foundation for Strategic Research through its Centre for Autonomous Systems at KTH, in part by TFR. The work by the second author was supported in part by the US Army Research Office, Grant number DAAG 5597-1-0114, and in part by the Sweden-America Foundation 2000 Research Grant.

of controlled differential equations

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i,\ i = 1,\ldots,m, \quad (1)$$

where $f_i, g_i \in C^\infty$, $x_i \in \mathbb{R}^n$, and $u_i \in \mathbb{R}^{p_i}$.

Now, a desired formation in $\mathbb{R}^{nm}$ is simply a set $\{x_{10}, \ldots x_{m0}\} \in \mathbb{R}^{nm}$, and we define this set implicitly through the null-set of a so called formation function.

Definition 2.1 (Formation Function) *Given a formation $\{x_{10}(s), \ldots, x_{m0}(s)\} \in \mathbb{R}^{nm}$ parametrized by a scalar s. We say that a positive definite, continuously differentiable map $F : \mathbb{R} \times \mathbb{R}^{nm} \to \mathbb{R}$ is a formation function to the set $\{x_{10}(s), \ldots, x_{m0}(s)\}$ if $\{s, x_1, \ldots, x_m\} \in F^{-1}(0)$ gives a unique $\{x_1, \ldots, x_m\}$ for each choice of $s \in \mathbb{R}$.*

In this paper, we connect the formation function with the concept of control Lyapunov functions. In this way we can simplify the controller design for each agent and focus on the high-level coordination issues. We assume:

Assumption 2.1 *Each of the m subsystems can be globally, asymptotically stabilized.*

Since there is no *a priori* reason why the stabilizing controller has to be smooth, we can take advantage of the following result, found for example in [1, 8].

Theorem 2.1 (Arstein-Sontag Theorem) *Given $f, g \in C^\infty$ and an $x_0 \in \mathbb{R}^n$. Then there exists a feedback law $u = \alpha(x)$ (smooth everywhere except at x_0 where it is continuous), which globally, asymptotically stabilizes $\dot{x} = f(x) + g(x)u$ to x_0 if and only if there exists a control Lyapunov function, i.e. a smooth and positive definite function $V(x, x_0)$ (zero only at x_0) such that the following holds:*

$$\frac{\partial V}{\partial x} g(x) = 0 \Rightarrow \frac{\partial V}{\partial x} f(x) < 0,\ \forall x \neq x_0. \quad (2)$$

We refer to [8] for the proof. Theorem 2.1 moreover ensures us that we can choose a control such that

$$\frac{\partial V}{\partial x}\dot{x} < 0,\ \forall x \neq x_0.$$

Assumption 2.2 *Given a curve $x_0(s)$ of equilibrium points in the state space the control Lyapunov function of the system for each point can be written as a smooth function $V(x, x_0(s))$.*

Remark 2.1 *This is a reasonable assumption since most velocity controlled, planar, mobile robotic systems have a translational invariance in position coordinates and thus the $V(x, x_0(s))$ above just contains a coordinate translation in $x_0(s)$.*

With this additional assumption it is possible to formulate the formation function to a given set in terms of the convex cone spanned by the control Lyapunov functions from the previous paragraphs.

Lemma 2.1

$$F(s, x_1(t), \ldots, x_m(t)) = \sum_{i=1}^{m} \beta_i V_i(x_i(t), x_{i0}(s)) \quad (3)$$

is a formation function to the formation $\{x_{10}(s), \ldots x_{m0}(s)\}$ as long as $\beta_i > 0, i = 1, \ldots, m$.

Proof: The proof follows directly from the fact that the individual V_i:s only vanishes at $x_i(t) = x_{i0}(s)$. ■

Remark 2.2 *The choice of coefficients in Equation (3) reflects how large deviations from zero are allowed for each V_i. By specifying an upper bound, F_U, on $F(s, x)$, each particular choice of β_i implies an upper bond on each V_i, proportional to F_U/β_i.*

3 Coordinated Control

By establishing these somewhat straightforward observations about the formation functions derived from the individual control Lyapunov functions, we can now shift our attention to actually controlling the evolution of the formation. The one parameter that we can control is the s-parameter, i.e. the parameterization of the time evolution of the desired positions. We do this by specifying the trajectory that we want the so called *virtual leader* (or *leaders*), $x_0(s(t))$, to follow. This nonphysical leader is a reference point in the state space with respect to which we can define the rest of the formation. We denote the trajectory executed by the virtual leader by $x_0(s(t)) = p(s(t))$. Intuitively one might want to set $s(t) = t$. But, due to robustness considerations we incorporate error feedback into the time evolution of s and let $\dot{s}$ be given by

$$\dot{s} = \min\left\{ \frac{v_0}{\delta + \|\frac{\partial p(s)}{\partial s}\|}, \frac{-\left(\frac{\partial F}{\partial x}\right)^T \dot{x}}{\delta + |\frac{\partial F}{\partial s}|} \left(\frac{\sigma(F_U)}{\sigma(F(s,x))}\right) \right\}. \quad (4)$$

Here $\delta > 0$ is a small positive constant that prevents $\dot{s}$ from becoming singular and F_U is a user defined upper bound on the formation function $F(s, x)$. The idea is to say that the formation is being respected as long as $F(s, x) \leq F_U$. Furthermore, v_0 is the nominal velocity that we want the formation to move with, and as we will see later, it holds that $\|\dot{x}_0(s(t))\| \approx v_0$ when F is small. Finally, σ is a function of class $\mathcal{K}$, i.e. $\sigma : [0, k) \to \mathbb{R}_+$ is a continuous function that satisfies $\sigma(0) = 0$, $\sigma(x) > 0$, $\forall x > 0$, and σ is nondecreasing everywhere.

Before we can proceed to actually showing the main results in this paper, the following assumption about σ in Equation (4) must be made:

Assumption 3.1 *Given the class $\mathcal{K}$ function σ in Equation (4). We assume that there exists a control $u(s,x)$, and a constant $L > 0$, such that*

$$\frac{-\left(\frac{\partial F}{\partial x}\right)^T \dot{x}}{\sigma(F(s,x))} \geq L \;\forall\; s \in \mathbb{R}, x \in \mathbb{R}^{nm} \mid 0 < F(s,x) \leq F_U.$$

Furthermore, this expression goes to infinity as F approaches zero.

Remark 3.1 *Assumption 3.1 essentially means that $-\left(\frac{\partial F}{\partial x}\right)^T \dot{x}$, can be bounded from below by a nondecreasing function $\sigma(F(s,x))$ when $F(s,x) \neq 0$. The limit property will furthermore be used when proving Theorem 3.3 about the nominal group velocity.*

We move on to showing that Assumption 3.1 actually is satisfied by a group of systems as long as it is satisfied by each of the individual subsystems. As a special case we also show that Assumption 3.1 is satisfied when the individual systems are globally asymptotically stabilizable.

Lemma 3.1 *If Assumption 3.1 holds for the individual control Lyapunov functions V_i then it holds for the convex cone*

$$F = \sum_{i=1}^{m} \beta_i V_i.$$

Proof: By the hypothesis it holds that

$$\frac{-\left(\frac{\partial V_i}{\partial x}\right)^T \dot{x}}{\sigma_i(V_i(s,x))} \geq L_i \;\forall\; s \in \mathbb{R}, x \in \mathbb{R}^{nm} \mid F(s,x) \leq F_U.$$

We thus have that

$$\begin{aligned} -\left(\frac{\partial F}{\partial x}\right)^T \dot{x} &= \sum_{i=1}^{m} \beta_i \left(-\frac{\partial V_i}{\partial x}\right)^T \dot{x} \\ &\geq \sum_{i=1}^{m} \beta_i L_i \sigma_i(V_i(s,x)) \qquad (5) \\ &\geq \min_i(\beta_i L_i) \min_i \sigma_i\left(\frac{F}{\beta_{max} m}\right) \\ &= L\sigma(F(s,x)), \end{aligned}$$

where

$$L = \min_i(\beta_i L_i)$$

and

$$\sigma(F(s,x)) = \min_i \sigma_i\left(\frac{F}{\beta_{max} m}\right).$$

The second inequality in (6) follows since there exists a j such that $V_j \geq \frac{F}{\beta_{max} m}$, and thus

$$\sigma_j(V_j(s,x)) \geq \min_i \sigma_i\left(\frac{F}{\beta_{max} m}\right).$$

The limit property also follows from this last inequality. ■

Lemma 3.2 *Assumption 3.1 holds for globally, exponentially stabilizable systems with $\sigma(z) = z^2$.*

Proof: If the m subsystems are globally, exponentially stabilizable the control Lyapunov function $V_i(x_i(t), x_{i0}(s))$ satisfies

$$c_{i1}\|x_i(t) - x_{i0}(s)\|^2 \leq V_i(x_i(t), x_{i0}(s)) \leq c_{i2}\|x_i(t) - x_{i0}(s)\|^2$$

$$\left(\frac{\partial V_i}{\partial x_i}\right)^T \dot{x}_i(t) \leq -c_{i3}\|x_i(t) - x_{i0}(s)\|^2,$$

along trajectories for some choice of positive scalars $c_{ij}, i = 1, \ldots, m, j = 1,2,3$. This directly gives that

$$-\left(\frac{\partial V_i}{\partial x_i}\right)^T \dot{x}_i(t) \geq c_i V_i(x_i(t), x_{i0}(s)), \qquad (6)$$

for some $c_i > 0$. Thus a choice of $\sigma(z) = z$ would fulfill the lower bound property but to achieve the limiting behavior we choose $\sigma(z) = z^2$. ■

3.1 Theoretical Properties

In the following paragraphs we will investigate what theoretical properties the evolution of the multi-agent formation exhibits when letting $\dot{s}$ be given by Equation (4). We will show that if $F(s(t_0), x(t_0)) \leq F_U$ then F remains bounded by F_U along trajectories for all times greater than t_0. We will also show that if $s \in [s_{start}, s_{final}]$ and if $\dot{s}$ is governed by Equation (4) then s reaches s_{final} in finite time. We will conclude our theoretical investigations by showing that $\|\dot{x}_0(s(t))\| \approx v_0$ if the formation function is small enough.

Theorem 3.1 *If $F(s(t_0), x(t_0)) \leq F_U$, then*

$$F(s,x) \leq F_U, \quad \forall t \geq t_0,$$

i.e. the formation function will never exceed F_U.

Proof. We directly have

$$\begin{aligned} \dot{F} &= \left(\frac{\partial F}{\partial x}\right)^T \dot{x} + \frac{\partial F}{\partial s}\dot{s}(s,x) \\ &\leq \left(\frac{\partial F}{\partial x}\right)^T \dot{x} + \frac{\partial F}{\partial s} \frac{-\left(\frac{\partial F}{\partial x}\right)^T \dot{x}}{\delta + \left|\frac{\partial F}{\partial s}\right|} \left(\frac{\sigma(F_U)}{\sigma(F(s,x))}\right). \end{aligned}$$

Now, assume that $F(s(t_0), x(t_0)) \geq F_U$, which gives

$$\dot{F}(s(t_0), x(t_0)) \leq 0$$

since

$$\frac{\sigma(F_U)}{\sigma(F(s(t_0), x(t_0)))} \leq 1,$$

$$\frac{\frac{\partial F(s(t_0),x(t_0))}{\partial s}}{(\delta + |\frac{\partial F(s(t_0),x(t_0))}{\partial s}|)} \leq 1,$$

and

$$(\frac{\partial F(s(t_0), x(t_0))}{\partial x})^T \dot{x}(t_0) \leq 0.$$

This directly gives that if $F(s(t_0), x(t_0)) \geq F_U$ then $\dot{F}(s,x) \leq 0$ along trajectories. Thus $F(s(t), x(t)) \leq F_U$ for all $t \geq t_0$. ∎

Theorem 3.2 *If $s \in [s_{start}, s_{final}]$ (user given), then by using the controller in Equation (4) we can find an upper bound $M < \infty$ such that*

$$T_{final} \leq M.$$

In other words, there is an upper bound on the completion time.

Proof. Let the completion time be defined in such a way that

$$\int_{T_{start}}^{T_{final}} \dot{s} = s_{final} - s_{start}.$$

If there exists a constant $v_s > 0$ such that $\dot{s} \geq v_s$ then we obviously have

$$T_{final} \leq \frac{s_{final} - s_{start}}{v_s} + T_{start}.$$

The proof thus consists of finding such a lower bound on $\dot{s}$. Let

$$K_1 = \max_{s \in [s_{start}, s_{final}]} \left\{ \delta + \|\frac{\partial p(s)}{\partial s}\| \right\}$$

and

$$K_2 = \max_{s \in [s_s, s_f], x \in \mathbb{R}^{nm} \mid F(s,x) \leq F_U} \left\{ \delta + |\frac{\partial F(s,x)}{\partial s}| \right\}.$$

Now

$$\begin{aligned} \dot{s}(s,x) &= \min\left\{ \frac{v_0}{\delta + \|\frac{\partial p(s)}{\partial s}\|}, \frac{-(\frac{\partial F}{\partial x})^T \dot{x}}{\delta + |\frac{\partial F}{\partial s}|} \left(\frac{\sigma(F_U)}{\sigma(F(s,x))} \right) \right\} \\ &\geq \min\left\{ \frac{v_0}{K_1}, \frac{L\sigma(F_U)}{K_2} \right\} \\ &= v_s > 0, \end{aligned}$$

since $\frac{F_U}{F(s,x)} \geq 1$. Thus

$$T_{final} \leq \frac{s_{final} - s_{start}}{v_s} + T_{start} = M,$$

which concludes the proof. ∎

Theorem 3.3 *If the formation error is small, $F(s,x) \ll F_U$ and $\delta \ll \|\frac{\partial p(s)}{\partial s}\|$ then*

$$\|\frac{dx_0}{dt}\| \approx v_0,$$

i.e. the formation velocity is v_0

Proof. We will start by showing that the right hand term in the brackets of equation (4) grows to infinity as F approaches zero.

$$\begin{aligned} & \frac{-(\frac{\partial F}{\partial x})^T \dot{x}}{\delta + |\frac{\partial F}{\partial s}|} \left(\frac{\sigma(F_U)}{\sigma(F(s,x))} \right) \\ & \geq \frac{\sigma(F_U)}{K_2} \frac{-(\frac{\partial F}{\partial x})^T \dot{x}}{\sigma(F(s,x))} \\ & \to \infty, \text{ as } F(s,x) \to 0 \end{aligned}$$

by Assumption 3.1. Thus the left hand term governs $\dot{s}$ when F is small. In that case we have

$$\begin{aligned} \|\frac{dx_0}{dt}\| &= \|\frac{\partial p(s)}{\partial s}\| \dot{s} \\ &= \|\frac{\partial p(s)}{\partial s}\| \frac{v_0}{\delta + \|\frac{\partial p(s)}{\partial s}\|} \\ &= v_0 - \frac{\delta v_0}{\delta + \|\frac{\partial p(s)}{\partial s}\|} \\ &\approx v_0 \end{aligned}$$

∎

4 Generalization

The assumption from the previous section that each individual robot is globally, asymptotically stabilizable might seem a bit restrictive. The many nonholonomic platforms encountered in the robotics literature do not exhibit this property. Despite this fact, we believe that our proposed approach sheds some light on the general multi-agent coordination problem. One observation to be made is that Assumption 3.1 is all we need in order to have Theorems 3.1, 3.2, and 3.3 go through. We state this as a corollary:

Corollary 4.1 (Generalization) *Given any continuously differentiable, positive semidefinite function $F : \mathbb{R} \times \mathbb{R}^{nm} \to \mathbb{R}$ (a semidefinite formation function). If there are controls $u_i \in \mathbb{R}^{p_i}$, $i = 1, \ldots, m$ such that Lemma 3.1 holds then Theorem 3.1, 3.2, and 3.3 also hold.*

The proof of this is just a repetition of the proofs of the previous theorems, and is thus omitted.

It should be noted that by allowing semidefiniteness in F we allow formations with some degrees of freedom, e.g. polygons with fixed arc lengths but free orientations.

Theorem 4.1 *If for each $s \in \mathbb{R}, x \in \mathbb{R}^{nm}$ there exists a control $u \in \mathbb{R}^{p_1} \times \ldots \times \mathbb{R}^{p_m}$ such that Assumption 3.1 holds, then the assumption also holds for the control*

$$u = argmin_{u \in U} \left(\frac{\partial F}{\partial x}\right)^T \dot{x}(u).$$

where U is the set of admissible controls.

Proof. Since Assumption 3.1 states that

$$\frac{-\left(\frac{\partial F}{\partial x}\right)^T \dot{x}}{\sigma(F(s,x))} \geq L \,\forall\, s \in \mathbb{R}, x \in \mathbb{R} \mid 0 < F(s,x) \leq F_U \quad (7)$$

and

$$-\min_{u \in U} \left(\frac{\partial F}{\partial x}\right)^T \dot{x}(u) \geq -\left(\frac{\partial F}{\partial x}\right)^T \dot{x}$$

the proof follows directly. ∎

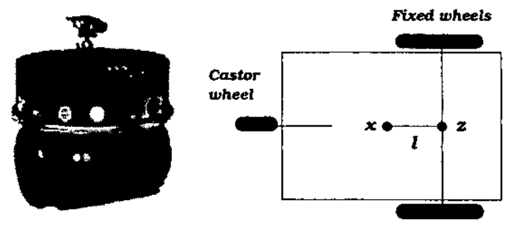
5 Example: Two Robots Carrying a Beam

Inspired by the picture on page 25 in the latest issue of the *IEEE Robotics & Automation Magazine* [9] we consider the coordination problem of two robots carrying a beam on a Mars construction site. This motion is constrained by the fact that the distance between the two robots should be "close" to the length of the beam. We moreover want the center of gravity of the beam to follow a given path $x_0(s)$. It is thus possible to use a formation function like $F(x,s) = ||x_A - (x_0(s) + (0, 0.5)||^2 + ||x_B - (x_0(s) - (0, 0.5)||^2$. This choice is positive definite and fulfills the conditions in Definition 2.1. To make matters more interesting we take advantage of the generalization described earlier and let F be a semidefinite formation function with a rotational degree of freedom:

$$F(x,s) = (||x_A - x_B||^2 - R^2)^2 + ||\frac{x_A + x_B}{2} - x_0(s)||^2.$$

Setting the length of the beam to $R = 1m$ and $F_U = 0.1$ we can guarantee that the distance $||x_A - x_B|| \in (0.82, 1.15) = (\sqrt{1-\sqrt{0.1}}, \sqrt{1+\sqrt{0.1}})$ which we assume can be handled by the way in which the beam is mounted on the robots.

We model the robots using the standard unicycle model (see for example [6, 7]). Such a model is applicable to e.g. the Nomadic Scout (as seen in Figure 1(a)) and most syncro drive robots as well as all-terrain caterpillar vehicles.



(a) (b)

Figure 1: Nomadic Technologies' Scout (a) and the unicycle model (b).

The equations of motion are

$$\begin{aligned} \dot{z}_1 &= v\cos\theta \\ \dot{z}_2 &= v\sin\theta \\ \dot{\theta} &= \omega, \end{aligned}$$

with the output $x_1 = z_1 + l\cos(\theta)$, $x_2 = z_2 + l\sin(\theta)$ being the position of the center of gravity. ((x_1, x_2) must lie between the wheel axis and the castor wheel for stability reasons, as shown in Figure 1(b).) We assume that the beam bearing device is mounted at (x_1, x_2). This gives the output dynamics

$$\begin{aligned} \dot{x}_1 &= v\cos\theta - \omega l \sin(\theta) \\ \dot{x}_2 &= v\sin\theta + \omega l \cos(\theta). \end{aligned}$$

(A similar argument is carried out for the more complex Hilare-type robot model in [5].) The limited velocities available on the Scout are approximately given by $|v| \leq 0.5m/s$ and $|\omega| \leq 2.5s^{-1}$.

We choose the control of robot $\alpha \in \{A, B\}$ as

$$\begin{pmatrix} v_\alpha \\ \omega_\alpha \end{pmatrix} = \begin{pmatrix} \cos(\theta_\alpha) & \sin(\theta_\alpha) \\ -\sin(\theta_\alpha)/l & \cos(\theta_\alpha)/l \end{pmatrix} \begin{pmatrix} -\frac{\partial F}{\partial x_{\alpha 1}} \\ -\frac{\partial F}{\partial x_{\alpha 2}} \end{pmatrix},$$

with the velocity saturations implied. With this choice of control it is straight forward to check that

$$\sigma(z) = z^2$$

together with the proposed $F(x,s)$ satisfies Assumption 3.1. Furthermore, $V = ||x - x_0(s)||^2$ is a control Lyapunov function for this system.

The desired motion of the beam is just that its center of mass is translated eastwards. The expression for the virtual leader is thus $x_0(s) = (s, 0.5)$.

If we would execute this motion as it stands the two robots would just move eastwards (to the right) while maintaining a constant relative distance. To make the example more interesting we let one of the robots be exposed to a partial battery failure such that its maximal speed is lowered from $V_{Bmax} = 0.5$ to $V_{Bmax} = 0.1$ while $V_{Amax} = 0.5$ all the time. We set (as already mentioned) the distance between the robots to $R = 1$ and let the formation error bound be $F_U = 0.1$. Finally, we let $\delta = 0.01$ in Equation (4).

This example will show the workings of both Theorem 3.1 and Theorem 3.3 as well as demonstrating how the unspecified degrees of freedom (in this case rotation) can be taken advantage of.

Running the simulation we see from the top view in Figure 2 that the reduced speed of robot B causes the formation to rotate as robot A tries to keep the center of gravity of the beam moving at a high speed (v_0). Then the rotation levels out and robot A must slow down to the velocity of robot B. Thereby the whole formation velocity approaches that of robot B, as can be seen in Figure 4. Meanwhile the attempts to drive the group at the (now unavailable) velocity v_0 has forced the formation function up to the bound of F_U (Figure 3). The reason that F does not approach zero before the battery failure is not inherent in the method, it is due to the fact that our chosen controller is proportional to $\frac{\partial F}{\partial x}$.

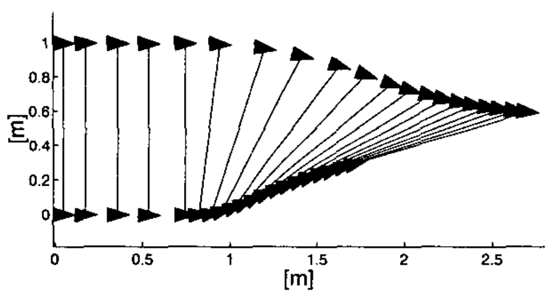


Figure 2: Top view of the two robot positions. Robot A starts at $(0,1)$ and robot B at $(0,0)$. Notice how the reduced speed of robot B (after the battery failure) is compensated for by a rotating motion.

6 Conclusions

In this paper we propose a stable coordination strategy for a team of formation constrained autonomous agents. If the individual robots can be globally, asymptotically stabilized, a formation function can be formed from the convex cone spanned by the individual control Lyapunov functions. This formation function furthermore decreases along trajectories, which shows that the robots maintain the formation as time evolves.

The assumption that each individual robot can be globally, asymptotically stabilized can, however, be relaxed.

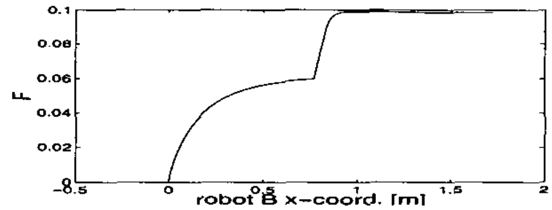


Figure 3: The formation function: First F is driven towards an equilibrium around 0.06 but after the battery failure the value approaches (but never exceeds) the upper bound $F_U = 0.1$ as stated in Theorem 3.1.

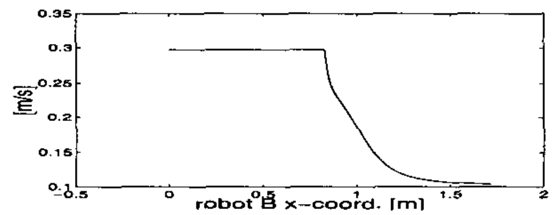


Figure 4: The formation velocity: Notice that at first $||dx_0/dt|| \approx v_0 = 0.3$ as stated in Theorem 3.3 but then, as F grows, $V \searrow V_{Bmax} = 0.1$.

In fact, as long as it is possible to construct a formation function and individual controls such that the formation function decreases, theorems about bounded formation errors, task completion times, and nominal formation velocities can be proved.

In this paper, we also present an example that illustrates the soundness of our method.

References

[1] Z. Arstein. Stabilization with Relaxed Controls. *Nonlinear Anal.*, 7(11):1163-1173, 1983.

[2] W. Kang, N. Xi and A. Sparks. Formation Control of Autonomous Agents in 3D Workspace. *IEEE International Conference on Robotics and Automation*, San Francisco, CA, 2000.

[3] M. Egerstedt, X. Hu, and A. Stotsky. Control of a Car-Like Robot Using a Virtual Vehicle Approach. *37th IEEE Conference on Decision and Control*, Tampa, Florida, Dec. 1998, a revised version is to appear in IEEE Trans. Aut. Control.

[4] Randal W. Beard, Jonathan Lawton, Fred Y. Hadaegh. A Coordination Architecture for Spacecraft Formation Control. *IEEE Transactions on Control Systems Technology, (to appear).*

[5] J.R.T. Lawton, B.J. Young and Randal W. Beard. A Decentralized Approach to Elementary Formation Maneuvers. *IEEE Transactions on Robotics and Automation, (to appear).*

[6] C. Canudas de Wit, B. Siciliano, and G. Bastin. *Theory of Robot Control.* Springer Verlag, 1996.

[7] J. Desai, J. Ostrowski, and V. Kumar. Control of Formations for Multiple Robots. Proceedings of the *IEEE International Conference on Robotics and Automation, Leuven*, Belgium, May 1998.

[8] A. Isidori. *Nonlinear Control Systems*, pp. 449. Third Edition, Springer-Verlag 1996.

[9] *IEEE Robotics & Automation Magazine*, pp. 25. Vol. 7, No. 4 December 2000.