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The Impact of Math Teachers' Circles on Teacher Dispositions toward Inquiry-based Learning: A Comparison between a Three-day and a One-day Summer Workshop

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High-quality professional development for K-12 teachers is a critical need for both teachers and their students. For teachers to provide more engaging and powerful learning opportunities for their students, researchers suggest that we provide similar opportunities for teachers. That is, professional development should model high-impact instructional strategies. Math Teachers' Circles provide one such model for this type of training. In this paper, we discuss the impact on participants of a one-day and participants of a three-day Math Teachers' Circle workshop. In particular, we compare how teacher dispositions regarding the teaching of mathematics and inquiry-based learning changed between the workshops.

Keywords: Inquiry-based Learning, K-12 Professional Development, Math Teachers' Circles

The purpose of this paper is to provide an analysis on research conducted on a Math Teachers' Circle (MTC) as a form of professional development. Specifically, we studied the impact of participation in workshops run through

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the Southwest Chicago Math Teachers' Circle on the teachers' dispositions toward the teaching of mathematics and toward inquiry-based instruction. We expect that having experienced and observed an inquiry-based instructional model, having gained access to resources that guide inquiry-based instruction, and having access to collaborators with whom they can share ideas and struggles, teacher-participants would increase their confidence in implementing inquiry-based instruction. Additionally, we compare and contrast the impact of a one-day summer MTC workshop and a three-day summer MTC workshop. Depending on the goals of the prospective professional development organizers, this information could assist in determining whether to hold a longer workshop, which requires more materials, funds, and effort.

1 Introduction: Math Teachers' Circles as Professional Development

Providing high quality and effective professional development and training for K-12 teachers is an increasingly urgent need for both teachers and their students [5, 9, 15, 17, 19, 20]. According to Borko et al. [5, p. 555], "A majority of teachers in the United States are engaging in [professional development], and there is a push for more [professional development] opportunities across the country." They assert that the focus of research in this area has changed from creating theories about high quality professional development to designing programs based on these theories, with limited research being conducted on these new programs. Additionally, in the context of their study, Lee found that the demand for professional development for certain grade groupings was greater than what they were able to supply [9]. Addressing this need is especially important for STEM teachers since schools struggle to find qualified teachers [15]. In order to meet the call for teachers to provide more engaging and powerful learning opportunities for their students, we must, in turn, provide the same powerful learning opportunities for teachers [5]. According to Borko et al., high quality professional development should focus on student learning and engage teachers in active learning [5]. Furthermore, it should model high impact instructional strategies in a way that allows teachers to experience being the learners in the classroom. This allows them to reflect and make connections between their learning and their classroom pedagogy [5, 9, 11]. McNeil and Knight emphasized the importance of having teachers discuss student responses to different instructional techniques and ideas since teachers encounter this in the classroom [11]. It is also useful for professional development to provide teachers with a sustainable ongoing professional community which gives teachers opportunities to engage in cycles of experi-

mentation and reflection [5]. Indeed, Lee and Scher state that professional development is more effective if it forms a coherent part of a wider set of connected and ongoing opportunities for teacher learning and development rather than disjointed "one and done" workshops [9, 15]. Zarske et al. and Lee shared the usefulness of bringing professors and K-12 teachers together through pedagogical and content-based workshops [9, 20]. Zarske et al. further found that providing teachers with opportunities to work with other teachers on real world problems through these workshops deepened their knowledge of science and mathematical content [20]. They also recommended a two-day workshop (15 hours) over a four-day workshop. Scher recommended professional development that is very content focused (for example, focusing on just math or science rather than on both math AND science) so that it has a larger impact on student learning [15]. Scher also found that ongoing mathematical professional development had a stronger positive effect on student performance than interventions which only took place over one academic year [15]. She concluded that further research and analysis of professional development schemes and programs is needed in order to make generalizations regarding the impact of different professional development programs in order to determine which ones work best [15].

1.1 Inquiry-based Learning

Inquiry-based instruction is an approach meant to engage students in "an authentic scientific discovery process" [14, p. 48]. This requires that students pose questions, investigate, and take ownership of their learning with little guidance from instructors to construct their knowledge [2]. In other words, the role of the instructor is to create a learning environment where students fully engage as active learners. This involves the careful creation and structuring of open-ended questions, allowing students to explore independently or in small groups, and listening to the students to determine what has or has not been understood, all before the content is presented rather than afterwards [6]. Collaboration is one of the key aspects of inquiry-based learning, which often comes in the form of group work, though it can take other forms such as student presentations of mathematical solutions accompanied by peer review and revision [7]. In any case, inquiry-based learning emphasizes student justification of their observations/conclusions, both orally and in writing. It is thought that learning in this manner creates knowledge that can be retained since the learner has learned through their actual experience [2]. In fact, compared to different forms of instruction, Alfieri et al. suggests that inquiry-based learning

that includes feedback, scaffolding, and elicited explanations results in better learning outcomes [3].

1.2 Math Teachers' Circles

In 2006, as part of a National Science Foundation grant, the American Institute of Mathematics (AIM) developed Math Teachers' Circles (MTCs), a K-12 teacher professional development model loosely based on the Eastern European student math circles [16]. MTCs bring teachers and mathematicians together to meet regularly to work on deep mathematics problems. Through this collaboration these mathematicians and teachers are able to form enriching relationships and provide support for each other. The Math Teachers' Circle Network (https://www.mathteacherscircle.org/) helps support and organize over 100 Math Teachers' Circles around the country.

The Southwest Chicago Math Teachers' Circle was formed to provide opportunities for middle and high school mathematics teachers in the southwest Chicago suburbs to grapple with deep and interesting mathematical problems which, in many cases, relate to the topics that they teach in their classroom. This ongoing professional development opportunity consists of summer workshops and regular monthly meetings which rotate between four colleges in the area: Lewis University, Saint Xavier University, Trinity Christian College, and University of St. Francis. These sessions are participant-centered and are designed to model an inquiry-based learning approach to teaching and learning. With a renewed emphasis on developing deep understanding in the classroom, as well as a push to include more active learning, teachers are often left to themselves to struggle with these new expectations of teaching. By attending the Math Teachers' Circle, teachers are joined by fellow peers as well as college faculty in attempting to resolve some of these struggles. Our current participants come from a variety of areas and backgrounds, within public schools and within private and Catholic schools. Participants come from a variety of the suburbs outside of Chicago and teach from second grade through college. These teachers all bring a new perspective that can connect each topic to a new mathematical concept and audience, making each activity that much richer. The goals for the Southwest Chicago Math Teachers' Circle follow the goals of the MTC Network. These goals are to:

- create a community of learners that encourages communication and collaboration between college professors and 5-12 grade math teachers;
- provide math instructors a supportive network to discuss mathematical and pedagogical issues that arise in the classroom;

- help teachers develop their students' problem-solving skills;
- give teachers the confidence, support and resources to initiate more student-centered, inquiry-based pedagogies in their classrooms; and
- rekindle the enjoyment and passion for solving interesting and challenging math problems in a supportive group environment.

The Southwest Chicago Math Teachers' Circle hopes to achieve these goals by providing opportunities for participants to work on creative and challenging mathematics problems that require them to further develop and refine their own problem solving skills. During these sessions, participants also have chances to observe (as a participant) possible models for student-centered inquiry-based pedagogy. The circle also provides access to materials, lesson plans, information on connections to the Common Core State Standards, and other resources available through the Southwest Math Teachers' Circle website. Finally, this circle gives participants a network of teachers who can collaborate together to determine how to address pedagogical issues.

1.3 Current Research on Math Teachers' Circles

Some research has already been conducted on Math Teachers' Circles [1]. Most of this research provides evidence about how the program may affect the participating teachers. Marle et al. found that teachers felt more pedagogically prepared and have increased self-efficacy as math teachers after participating in Math Teachers' Circles [10]. Furthermore, Marle et al. found that the teachers used more inquiry-based teaching practices in their classrooms [10]. In addition to pedagogical benefits, White et al. found that the teachers' mathematical knowledge for their subjects increased after participating in a Math Teachers' Circle workshop [17]. Additionally, on national surveys with hundreds of respondents, teachers reported increased enthusiasm for mathematics; more interactive, student-centered problem solving in their classrooms; and an increased belief that all their students are capable of doing mathematics. Many teachers say they have started seeing themselves as mathematicians [16]. Hill et al. found that students had better achievement outcomes as teachers' mathematical knowledge increases [8]. Math Teachers' Circles also seem to increase the likelihood of teachers using inquiry-based and active learning methods in their classrooms [10].

2 Methods

Data for this study was collected over two Math Teachers' Circles summer immersion workshops. The first workshop was a three-day event. The second workshop was a one-day event that was conducted one year following the three-day workshop. Both workshops were held in early August of their respective years when teachers would be preparing to return to their classrooms (but before they had actually returned). Participants consisted of primarily in-service mathematics teachers from grades 5-12 from throughout the Chicago suburbs, as well as college faculty; however, college faculty were not included in this study. To encourage collaboration on the sessions during the workshops, tables were arranged into groups of four and participants were asked to move tables (if necessary) in order to ensure group sizes of three to four.

2.1 Three-Day Workshop

The three-day Math Teachers' Circle immersion workshop hosted by the Southwest Chicago Math Teachers' Circle was held at Trinity Christian College in Palos Heights, Illinois. The workshop consisted of a total of eight mathematics sessions over three days. The workshop ran 1:00 pm - 8:00 pm on the first day, 8:30 am - 7:15 pm on the second day, and 8:30 am - 2:00 pm on the third day. It consisted of time for meals, snacks, and reflection each day. Of the eight mathematics sessions run, there were two sessions the first day, four sessions the second day, and two sessions the third day. Each was 1.5 to 2 hours in length, and questions were posed afterwards to encourage teachers to reflect on what they learned and how they could use similar activities in their own classrooms. Each session was led by two individuals (though facilitators varied from session to session). Most of the facilitators were from the leadership team of the Southwest Chicago Math Teachers' Circle and the remainder were from the Twin Cities Math Teachers' Circle leadership team. Additionally, the session facilitators were primarily college or university mathematics faculty, with the exception of one teacher-leader who taught 5th grade mathematics.

The pre-survey was administered after the first session on the first day and also included questions to obtain background information. The post-survey was administered after the last session on the third day of the workshop. See Tables 3 and 4 for survey questions.

2.2 One-Day Workshop

The one-day Math Teachers' Circle immersion workshop was held at the University of St. Francis in Joliet, Illinois. It ran from 9:00 am to 7:00 pm. This

workshop consisted of four sessions with meals, snacks, and reflection time included. Each of the four sessions was led by two individuals (facilitators varied from session to session) and was 1.5 hours in length (with the exception of one session which ran 1 hour and 20 minutes). Teachers were asked reflection questions after each session. All of the session facilitators were from the leadership team of the Southwest Chicago Math Teachers' Circle (as opposed to the three-day workshop which included facilitators from the Twin Cities MTC). Once again, the session facilitators were primarily college or university mathematics faculty, with the exception of two teacher-leaders who each taught 5th grade mathematics. For the one-day workshop, teacher-participants were given the pre-survey before the first session (but after breakfast) and the post-survey after the last session (but before dinner).

2.3 Participants

Study participants consisted of attendees of the Southwest Chicago Math Teachers' Circles workshops. All attending in-service or pre-service teachers were invited to take part in the study. The three-day workshop had a total of 30 participants. The count varied over each of the three days, with 30 teachers attending the first day, 25 on the second day, and 22 on the third day. Of those 30 participants, 13 were teachers at grade levels 5-12 who elected to participate in this study, one of which was a pre-service teacher. These teachers attended all sessions over the three days. The one-day workshop had 25 participants; 13 of those (all in-service teachers) elected to participate in this study.

Table 1 provides information regarding participants' grade level, years of teaching, and highest degree earned. These features are emphasized as we expect those categories could impact teachers' willingness to implement a new teaching style. Additionally, it is important to note that the backgrounds of the participants do vary between the groups in some key ways. In particular, the one-day workshop had a larger percentage of teachers with 10 or more years of experience (46.2% versus 15.4%) and with master's degrees (53.8% versus 30.8%). In future studies (with more data), it would be worthwhile to determine whether those features do, in fact, influence the results.

In addition to the teaching backgrounds of participants, the survey included questions on their school environment. Table 2 provides results from these questions. We note that the one-day participants were more likely to indicate that they worked in an underserved community (with 46.2% indicating so versus 23.1% for the three-day workshop) and that poverty was a serious problem at their current school (with 30.8% at the one-day workshop versus

Table 1

Demographics of study participants. The percentages are taken out of a total of 13 participants each.

Number of years taught:	**Three-Day**	**One-Day**
Less than 1 year	15.4%	0%
1 to 4.9 years	30.8%	30.8%
5 to 9.9 years	7.7%	23.1%
10+ years	15.4%	46.2%
Not answered	30.8%	0%
Grade levels taught:	**Three-Day**	**One-Day**
Elementary School Only (Grades 2-5)	15.4%	7.7%
Elementary and Middle School	7.7%	0%
Middle School Only (Grades 6-8)	46.2%	69.2%
Middle and High School	7.7%	0.0%
High School Only (Grades 9-12)	15.4%	23.1%
Not Answered	7.7%	0%
Highest degree:	**Three-Day**	**One-Day**
Some College	7.7%	0%
Bachelor's	61.5%	15.4%
Master's	30.8%	53.8%
Not answered	0%	30.8%

7.7% at the three-day workshop). Again, with limited data, we cannot at this time determine whether these factors influence the impact of the workshop; however, we include them here for completeness.

2.4 Survey

The original survey included three parts, each representing a six-point Likert-style questionnaire. These three parts were meant to evaluate teachers' dispositions toward inquiry-based instruction, toward mathematics itself, and toward the teaching and learning of mathematics. In addition, it included demographic questions and feedback questions (about the participants' perceptions of the workshop). Many of the demographic and feedback questions were open-ended; some are included in this paper (see Table 2). In order to reduce the length of the survey (from its first use during the three-day workshop to its next implementation in the one-day workshop), many of the demographic and feedback questions were reduced and the second (out of three) Likert-style portion of the questionnaire (on the disposition of teachers toward

Table 2

School environment of teacher-participants. The percentages are taken out of a total of 13 participants each.

Do you teach in an underserved community?	**Three-Day**	**One-Day**
Yes	23.1%	46.2%
No	69.2%	53.8%
No answer	7.7%	0%
To what extent is poverty a problem at your current school?	**Three-Day**	**One-Day**
Serious Problem	7.7%	30.8%
Moderate Problem	30.8%	15.4%
Minor Problem	15.4%	30.8%
Not a Problem	30.8%	23.1%
No answer	15.4%	0%
To what extent is students' lack of English-proficiency a problem at your current school?	**Three-Day**	**One-Day**
Serious Problem	7.7%	0%
Moderate Problem	23.1%	30.8%
Minor Problem	23.1%	15.4%
Not a Problem	30.8%	53.8%
No answer	15.4%	0%

mathematics itself) was removed. Additionally, during the one-day workshop, the survey was separated into a pre-survey and post-survey to eliminate unnecessary repetition (on demographic information) and so that participants were not being asked for feedback prior to full participation in the program. Note that the two remaining Likert-style portions were identical between the pre- and post-surveys. Both sections of the original survey were validated by three researchers (two of whom were from institutions external to the authors' home institutions). The credentials of these reviewers include a doctorate in mathematics education, a doctorate in mathematics (with specialization in mathematics education), and a doctorate in education. They found that the survey does, in fact, evaluate its desired objectives.

The first portion of the survey was designed to assess characteristics identified in the literature as being fundamental to inquiry-based instruction. For example, the characteristics of problem solving, exploration, and discover-

ing knowledge [14, 18, 13] were assessed through items such as "provide exploratory problems or projects," "engage students in inquiry-oriented activities," "engage students in problem-based activities," and "provide hands-on experiences before introducing new concepts." Furthermore, some items were reverse-worded; for example, "provide students with step-by-step procedures to solve problems," "present new content through lectures," and "immediately assist struggling students." Note that the last item was reverse-scored, as a goal in the inquiry process is for students to experience discovery as opposed to being told how to do the problem and so we view immediate assistance as disruptive to this discovery process. The use of open-ended questions [18, 4] was assessed through the item "create open-ended problems." Application of collaborative work [18, 7] was assessed with items "encourage social interaction as a means of learning," "encourage students to discuss their reasoning with each other," and "create opportunities for students to communicate mathematics verbally." See Table 3 for other items included in the survey.

For this first portion, the items were presented as a six-point Likert-style questionnaire that addressed three facets of the teachers' disposition toward inquiry-based pedagogy: how important they felt it was for effective mathematics instruction (with items being scored as 1 for not important to 6 for very important), how confident they felt in applying it in the classroom (1 being not confident to 6 being very confident), and how frequently they implement it (1 being not frequent to 6 being very frequent). Since the workshop was held over the summer, teachers did not have the opportunity to immediately change their instructional implementation. During the three-day workshop, several teachers did indicate a change in their implementation level. Therefore, given that teachers were in the midst of preparing for the upcoming school year, we interpreted this as a change in the intent to implement those strategies; however, we also acknowledge that some instructors may have changed their responses due to a different understanding of what that item may have meant. These will be discussed in further detail in the Results section. This ambiguity was alleviated in the one-day workshop by the change of the phrase "how frequently you implement it" to "how frequently you plan to implement it." Note that this adds another element of caution in comparing the two groups with regard to this particular feature.

During the three-day workshop, there were 28 items assessed on "importance," "confidence," and "implementation" of techniques related to inquiry-based instruction. For the one-day workshop, we reduced this part of the survey from 28 items to 17 items. For these 17 items, internal consistencies among each of the three categories were analyzed using Cronbach's alpha, and the internal reliability over all items was good (Cronbach's alpha for "impor-

tance" = 0.800; Cronbach's alpha for "confidence" = 0.976; Cronbach's alpha for "implementation" = 0.834). Note that in determining the Cronbach's alpha for confidence, we excluded the three items "provide students with step-by-step procedures to solve problems," "immediately assist struggling students," and "present new content through lectures" since one's confidence (or lack thereof) in implementing these techniques is unrelated to one's confidence in implementing inquiry-based instruction. Additionally, those items were reverse-scored in computing Cronbach's alpha for importance and implementation. Finally, the confidence towards those three items was excluded from analysis (and, so, discussion in this paper). Thus, this part of the survey contains a total of 48 items that were analyzed.

For the second portion of the survey, we aimed to assess the disposition of the teacher towards the teaching of mathematics. This includes items such as "I feel competent at facilitating effective group work," "I encourage students to pose their own questions in class," "I encourage students to share their methods with each other," "It is important to learn more than one way to solve math problems," and "I believe all students can understand mathematics through an investigative approach," among others. Furthermore, many items were reverse-worded; for example, "mathematics is an activity which is done alone, not with others," "the average math student can, at best, memorize math procedures and apply formulas," "I avoid open-ended problems in class," "mathematics is primarily memorization and computation," "lecturing is the best way to teach mathematics," etc. See Table 4 for other items included in that portion of the survey. The instrument structure was modeled after the design in the research by Dr. Patel [12], which was used to evaluate the mathematical disposition of students. For this portion of the survey, participants were asked to indicate the number that best described their agreement with each statement with 1 being strongly disagree, 2 disagree, 3 somewhat disagree, 4 somewhat agree, 5 agree, and 6 strongly agree.

As mentioned above, for the one-day workshop, we reduced some of the Likert-style questions from two distinct parts of the original survey (on the disposition of teachers toward mathematics and toward the teaching and learning of mathematics) to form one part focusing on the disposition toward the teaching and learning of mathematics. This reduced the number of total items for these portions from 57 to 31. Since 25 of these 31 items were contained on both surveys (using the exact same language), their outcomes can be compared across the two workshops. Once again, internal consistencies among these 25 items were analyzed using Cronbach's alpha, and the internal reliability over all items was acceptable (Cronbach's alpha = 0.696). To emphasize, this por-

Table 3

Survey questions for determining participant disposition toward inquiry-based instruction.

Item no.	Item wording
1	Provide exploratory problems or projects.
2	Connect mathematics to real-world applications.
3	Immediately assist struggling students.
4	Engage students in inquiry-oriented activities.
5	Provide students with step-by-step procedures to solve problems.
6	Create opportunities to develop students' conceptual understanding of mathematics.
7	Encourage students to discuss their reasoning with each other.
8	Present new content through lectures.
9	Encourage social interaction as a means of learning.
10	Engage students in problem-based activities.
11	Encourage students to persevere in solving problems.
12	Allow students to work at their own pace.
13	Create opportunities for students to communicate mathematics verbally.
14	Incorporate multiple representations (e.g., computational, graphical, geometric, algebraic, etc.) when introducing a concept.
15	Provide hands-on experiences before introducing new concepts.
16	Provide applications that require mathematics to solve real-world problems.
17	Create open-ended problems.

tion of the survey contained 25 items to analyze; thus, combined with part one, this makes a total of 73 items in the survey that were examined.

Table 4

Survey questions for determining participants' disposition toward the teaching and learning of mathematics.

No.	Item wording
1	A math teacher should be able to solve any mathematics problem or puzzle.
2	I feel inventive when solving an unfamiliar math problem.
3	Mathematics is primarily memorization and computation.
4	Mathematics is an activity which is done alone, not with others.
5	There are no new concepts to discover in mathematics.
6	A student who cannot solve a mathematics problem is not good at math.
7	I like to give students math problems they have not seen before.
8	Lecturing is the best way to teach mathematics.
9	The math I teach in school is useful in everyday life.
10	In my experience, group activities are not productive.
11	I prefer to teach math problems that do not take long to solve.
12	I often feel unprepared to teach the topics I'm expected to teach.
13	I like teaching math problems using new approaches.
14	As adults, my students will not have the opportunity to use the math I teach.
15	It is important to learn more than one way to solve math problems.
16	I do not enjoy teaching mathematics.
17	If I don't fully understand a mathematics concept, I avoid using it in the classroom.
18	I avoid open-ended problems in class.
19	I believe all students can understand mathematics through an investigative approach.
20	I feel competent at facilitating effective group work.
21	The typical student cannot conceptualize mathematics.
22	I avoid using instructional techniques if I am unfamiliar with it.
23	The average math student can, at best, memorize math procedures and apply formulas.
24	I find pleasure in teaching mathematics.
25	I encourage students to pose their own questions in class.

3 Results

In this section, we present the significant results we observed in the individual workshops and also in comparing the two workshops (three-day versus one-day). Both the workshops observed statistically significant changes on multiple items (16 for the three-day and 13 for the one-day workshop). In addition, changes within individuals were observed. In this section, we will share some of the most notable observations from our survey outcomes, starting with results from the individual workshops and then a comparison of the two workshops.

In order to determine whether there was a significant change in individual workshops from pre-workshop to post-workshop, we ran a Wilcoxon signed rank test for each of the 73 relevant items in the survey. The p-values reported were generated via SPSS using exact significance. Throughout, we report a statistic as significant if it satisfies $p < 0.1$. We have organized our significant results into three classifications.

- **Type A:** Larger, more significant changes from the pre- and post-survey which move in a direction which aligned with the goals for the workshop;
- **Type B:** Definite change but not necessarily practically important;
- **Type C:** Changes in a direction which did not align with the goals of the workshop.

To clarify the distinction between types A and B, we classify an item as type A if at least 50% of participants observed positive changes, including at least three with changes of two levels or more, or if there were large changes in individual participants (changing by three or more levels). In contrast, for those classified as type B, most of the change in individual responses was by one level only, with at most two participants changing by two levels, and no extreme changes (of three or more levels) among any individual (with one exception in the one-day workshop, which is discussed below).

3.1 Three-Day Workshop

There were 16 total items out of 73 (21.92%) that resulted in statistically significant changes. Of these, 15 items changed in a direction aligned with the goals of the Math Teachers' Circle workshop. In particular, changes were observed in 5 out of the 14 items (35.71%) assessing confidence in inquiry-based instruction, 7 out of the 17 items (41.18%) assessing implementation in inquiry-based instruction, and 4 out of the 25 items (16%) assessing disposition of teachers toward the teaching and learning of mathematics. The items,

their p-values, and the pre-workshop and post-workshop medians can be seen in Table 5. Note that none of the items assessing importance of inquiry-based instruction produced significant changes. This could be a result of the participants already having a perspective aligned with inquiry-based instruction. In fact, for all but 3 of the 17 importance items, the pre-workshop medians were already solidly in agreement with an inquiry-based instructional pedagogy.

We can further restrict the analysis of the survey to only those items whose pre-workshop scores were not already solidly aligned with the goals of the workshop. In particular, we examined only those items whose pre-workshop median was 4 or below (for those items which were forward-scored), or 3 or above (for those items which were reverse-scored). That leaves three items on the importance of inquiry-based learning (IBL), 10 items on the confidence in applying IBL, 12 items on implementation of IBL, and eight items on the disposition of teachers toward the teaching and learning of mathematics. Of these, zero (0%), four (40%), six (50%), and two (25%), respectively, changed in the desired direction. Additionally, one of the items which indicated potential room for improvement pre-workshop ("if I don't fully understand a mathematics concept, I avoid using it in the classroom") went in the undesired direction (we consider this item to be of type C). Furthermore, despite solid pre-workshop scores, the following three items saw an even greater alignment with inquiry-based teaching post-workshop: implementation of "encourag[ing] students to discuss their reasoning with each other," confidence in "incorporating multiple representations," and "I feel inventive when solving an unfamiliar math problem." Below we will further categorize and discuss the significant items.

3.1.1 Type A

We consider 4 out of the 16 significant results to be sizable enough to represent possible practical changes in participants.

1. For the intent to implement "creat[ing] opportunities for students to communicate mathematics verbally," movement was exclusively toward the agree side of the scale post-workshop, with 7 out of 12 respondents selecting agree or strongly agree post-workshop. In particular, 8 out of 12 respondents increased their agreement with this statement (with three increasing by two levels). This is evidenced by the fact that all but four data points in the matched pairs plot in Figure 1 lie above $y = x$, the line that represents no change, and the remaining lie precisely on that line. Additionally, we observed a change from three participants disagreeing with the statement pre-workshop to just one participant disagreeing

post-workshop. See Figure 2 for a groupwise comparison between pre-workshop and post-workshop results.

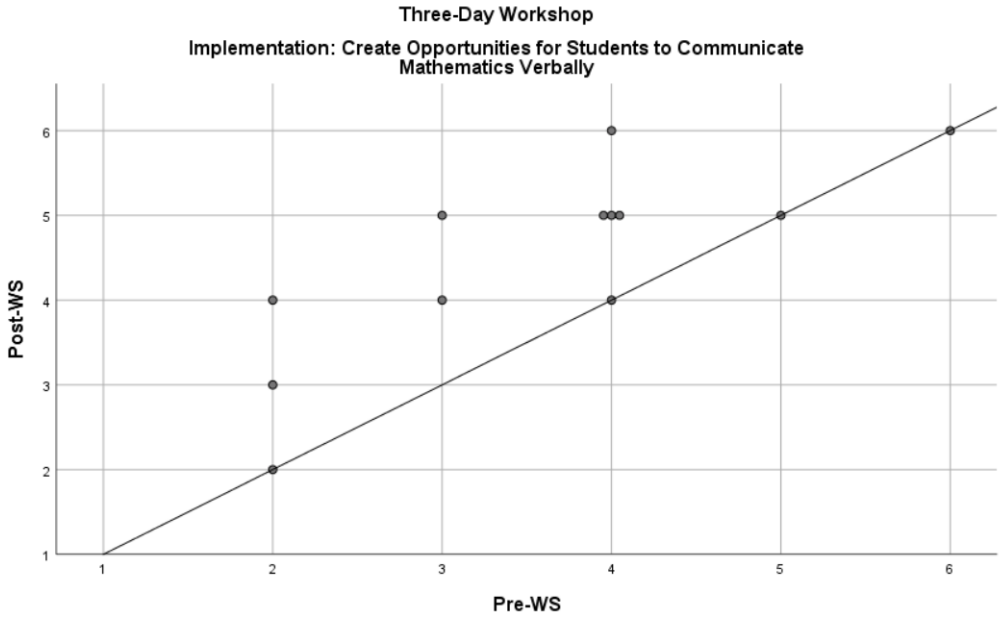

Figure 1. Three-day workshop matched pairs plot: Implementation of creating opportunities for students to communicate mathematics verbally.

2. For the intent to implement "incorporat[ing] multiple representations when introducing a concept," half of the participants responded at the very frequent level post-workshop (compared to just 25% pre-workshop), with 66.67% increasing in frequency level. Five of the 12 participants increased one frequency level, three increased two frequency levels, the remaining participants did not change. The results, which are shown in Figure 3, clearly demonstrate the shift from just five participants selecting a frequency level of five or six pre-workshop to nine participants selecting those levels post-workshop.

3. There were significant increases in the responses to the item "I feel inventive when solving an unfamiliar math problem" from pre-workshop to post-workshop. Specifically, two participants moved from disagree to strongly agree (an increase of four levels). Additionally, one participant increased from sometimes disagree to agree (which is two levels). As seen in the matched pairs plot in Figure 4, six of the seven participants who did not change were already at agree or strongly agree, and all but one of those who could have improved (i.e., who rated this item 4 or below pre-workshop) did. Additionally, the plot shows an improvement among six (46.15%) participants (two each increased by one, two, and

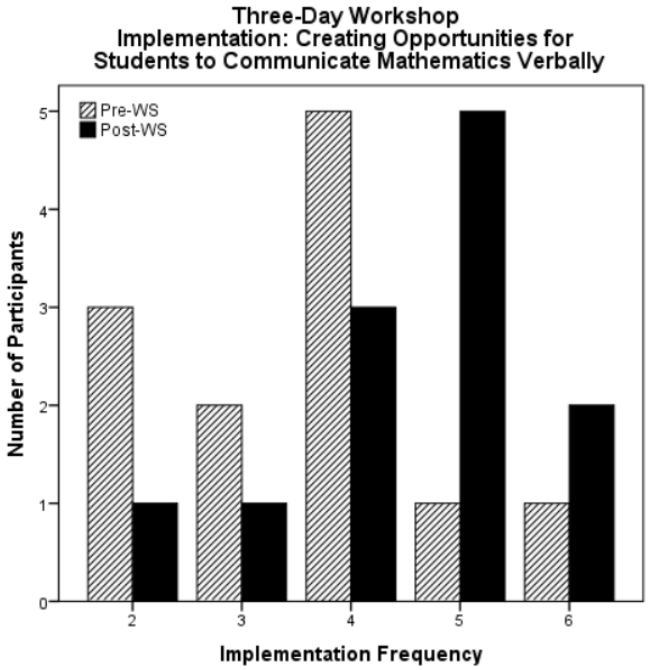

Figure 2. Three-day workshop: Implementation of creating opportunities for students to communicate mathematics verbally.

four levels in agreement). Figure 5 provides a groupwise comparison between pre-workshop and post-workshop results, which indicates a shift to a strongly left-skewed distribution post-workshop.

4. While the item "I avoid using instructional techniques if I am unfamiliar with it" had a p-value of 0.063 (which might be considered moderate significance), we include it in the type A category because of the fact that 5 out of 13 respondents decreased their agreement by two levels for this item (with only one increasing their agreement, and that by just one level). Additionally, all five of these respondents switched from the agree (4-6) to the disagree (1-3) side of the scale. This suggests a practical change in participant perspective regarding this item.

3.1.2 Type B

There were 11 items (out of the 16) that resulted in significant p-values with changes in a direction congruent with the goals of the workshop but for which we question the practical implication of these items due to the small size of changes observed. These items can be found in Table 6. Note that, in this

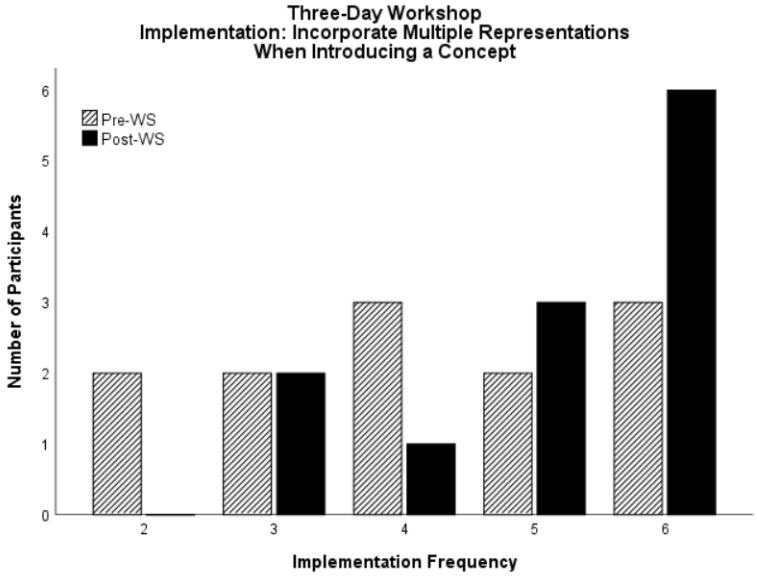

Figure 3. Three-day workshop: Implementation of incorporating multiple representations when introducing a concept.

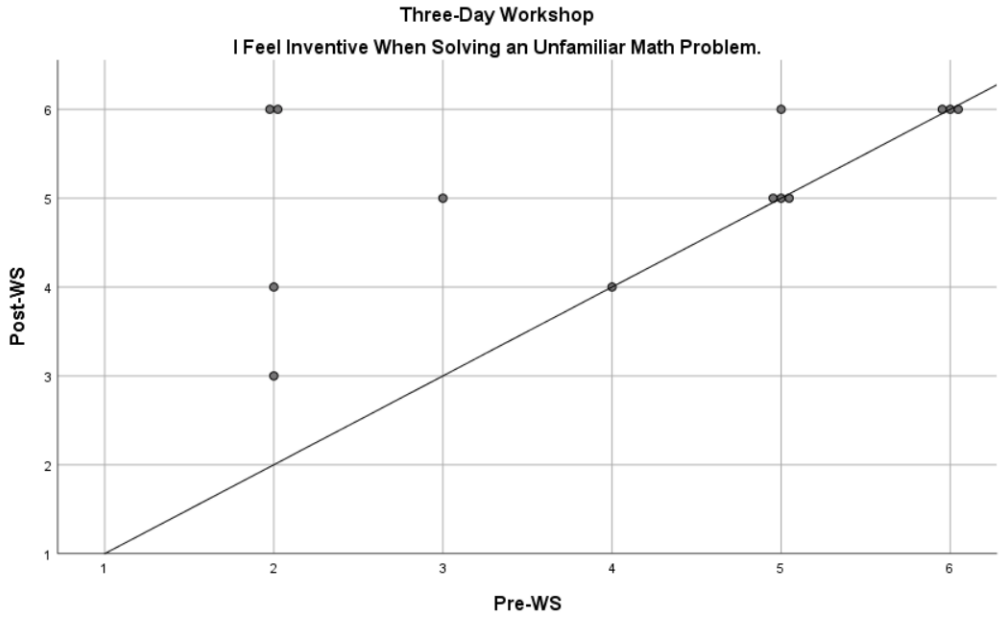

Figure 4. Three-day workshop matched pairs plot: I feel inventive when solving an unfamiliar math problem.

table, we only included the number of participants for those who changed responses; all remaining participants did not indicate a change for these items.

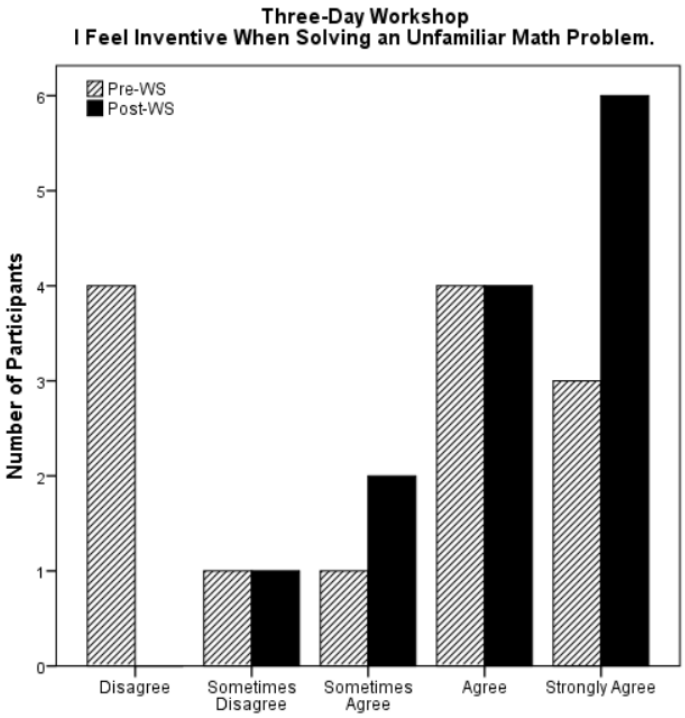

Figure 5. Three-day workshop: I feel inventive when solving an unfamiliar math problem.

As noted in the table, most changes observed were by one level (with two or fewer by two levels, with one exception). Also, observe that for most items, all participant changes were in the same direction (a direction in alignment with the goals of the workshop), with four exceptions, where a single individual moved one level in the opposite direction. So, again, while this indicates an observable change among the group, the size of the change (mostly by one level) suggests the change may not be large enough to have useful implications.

As noted above, for all but one item listed as type B, two or fewer participants changed their responses by two (or more) levels. The item "[confidence in] allow[ing] students to work at their own pace" is the exception. Five out of 12 respondents increased their agreement (all others were unchanged). Of these, three increased two levels and two increased one level. Even though three increased by two levels, we consider this type B instead of type A since the proportion of respondents who changed was less than 50%. We note that two of the participants changed from the disagree side to the agree side of the scale (from three to five). This may suggest strong impact within individuals,

even if the overall impact is minor. See Figure 6 for a matched pairs plot that shows all changes among the individual participants at the workshop.

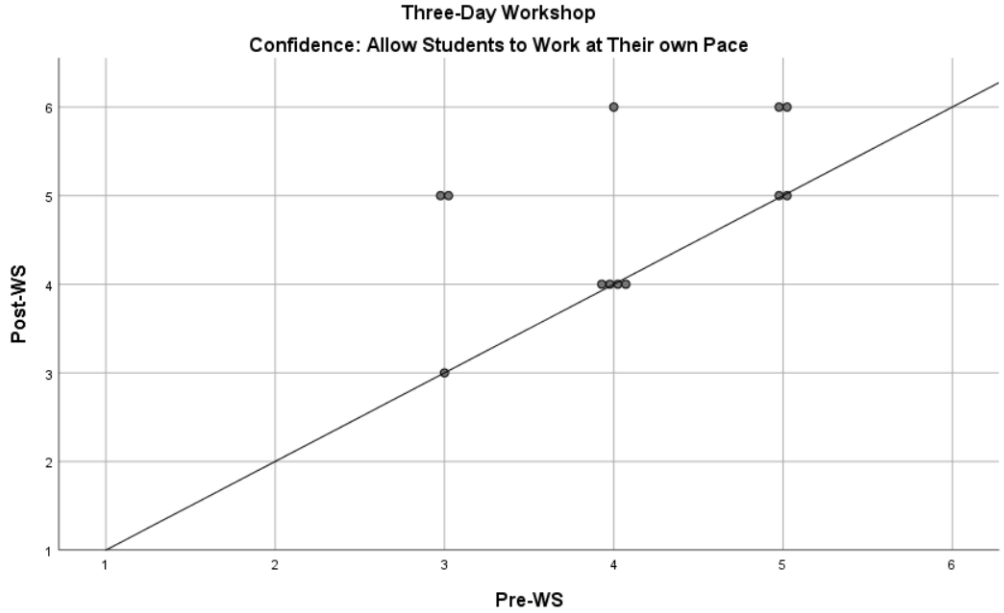

Figure 6. Three-day workshop matched pairs plot: Confidence in allowing students to work at their own pace.

3.1.3 Type C

There was only one item that we categorized as type C, which indicated a statistically significant change but in the direction opposite the goals of the workshop. For the item "if I don't fully understand a mathematics concept, I avoid using it in the classroom" all six participants who changed increased their agreement by one level. This was unexpected. However, since the change was by one level only, this result is not necessarily an indication of a measurable change in the participants' perspective with regard to this item. The matched pairs plot can be seen in Figure 7.

Table 5

Significant changes for the three-day workshop.

Item	**P**	**M Pre**	**M Post**	**Type**
Create opportunities to develop students' conceptual understanding of mathematics (confidence).	0.031	4	5	B
Create opportunities to develop students' conceptual understanding of mathematics (implementation).	0.016	3.5	4.5	B
Encourage students to discuss their reasoning with each other (implementation).	0.031	4.5	5	B
Allow students to work at their own pace (confidence).	0.063	4	5	B
Allow students to work at their own pace (implementation).	0.008	4	4	B
Create opportunities for students to communicate mathematics verbally (confidence).	0.016	4	4.5	B
Create opportunities for students to communicate mathematics verbally (implementation).	0.008	4	5	A
Incorporate multiple representations (e.g., computational, graphical, geometric, algebraic, etc.) when introducing a concept (confidence).	0.063	4.5	5	B
Incorporate multiple representations (e.g., computational, graphical, geometric, algebraic, etc.) when introducing a concept (implementation).	0.008	4	5.5	A
Provide hands-on experiences before introducing new concepts (implementation).	0.07	4	4	B
Create open-ended problems (confidence).	0.094	3	3.5	B
Create open-ended problems (implementation).	0.063	3	3	B
I feel inventive when solving an unfamiliar math problem.	0.031	5	5	A
If I don't fully understand a mathematics concept, I avoid using it in the classroom.	0.031	3	3	C
I avoid using instructional techniques if I am unfamiliar with it.	0.063	4	3	A
The average math student can, at best, memorize math procedures and apply formulas.	0.094	3	2	B

Table 6

Three-day workshop: Changes among participants for type B significant items. Note that an increase in the level corresponds to an increase in agreement with the statement.

Item	**Two Down**	**One Down**	**One Up**	**Two Up**
Create opportunities to develop students' conceptual understanding of mathematics (confidence).	0	0	4	2
Create opportunities to develop students' conceptual understanding of mathematics (implementation).	0	0	5	2
Encourage students to discuss their reasoning with each other (implementation).	0	0	6	0
Allow students to work at their own pace (confidence).	0	0	2	3
Allow students to work at their own pace (implementation).	0	0	7	1
Create opportunities for students to communicate mathematics verbally (confidence).	0	0	7	0
Incorporate multiple representations (e.g., computational, graphical, geometric, algebraic, etc.) when introducing a concept (confidence).	0	1	6	1
Provide hands-on experiences before introducing new concepts (implementation).	0	1	7	0
Create open-ended problems (confidence).	0	1	4	2
Create open-ended problems (implementation).	0	0	4	1
The average math student can, at best, memorize math procedures and apply formulas.	2	4	1	0

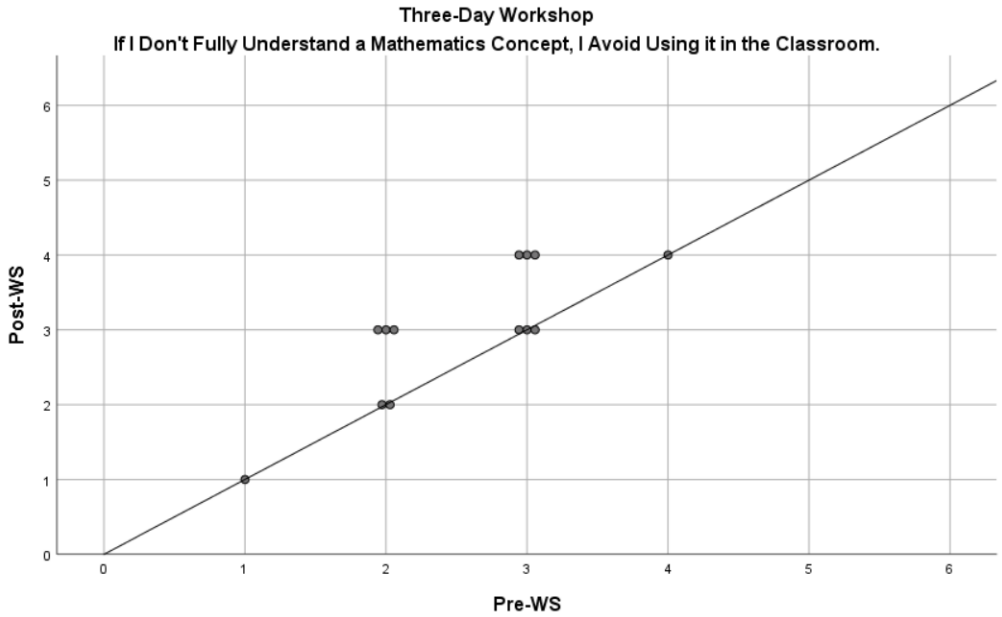

Figure 7. Three-day workshop matched pairs plot: If I don't fully understand a mathematics concept, I avoid using it in the classroom.

3.2 One-Day Workshop

There were 13 total items out of 73 (17.81%) that resulted in statistically significant changes. Of these, 12 items changed in a direction aligned with the goals of the Math Teachers' Circle workshop. In particular, changes were observed in 3 out of the 14 items (21.43%) assessing confidence in inquiry-based instruction, 7 out of the 17 items (41.18%) assessing implementation in inquiry-based instruction, and 3 out of the 25 items (12%) assessing disposition of teachers toward the teaching and learning of mathematics. The items, their p-values, and the pre-workshop and post-workshop medians can be seen in Table 7. Note that, just as with the three-day workshop, none of the items assessing importance of inquiry-based instruction produced significant changes. Once again, this could be a result of the participants already having a perspective aligned with inquiry-based instruction since for all but 3 of the 17 importance items, the pre-workshop medians were already solidly in agreement with an inquiry-based instructional pedagogy.

As we did with the three-day workshop, in order to eliminate those items which had little room for improvement pre-workshop, we restrict the analysis of the survey to only those items whose pre-workshop scores were not already solidly aligned with the goals of the workshop. As a reminder, we mean those

items whose pre-workshop median was four or below (for those items which were forward-scored), or three or above (for those items which were reverse-scored). That leaves three items on the importance of inquiry-based learning (IBL), six items on the confidence in applying IBL, 11 items on implementation of IBL, and 11 items on the disposition of teachers toward the teaching and learning of mathematics. Of these, zero (0%), two (33.33%), seven (64.64%), and two (18.18%), respectively, changed in the desired direction. Additionally, one of the items assessing the disposition of teaching and learning ("lecturing is the best way to teach mathematics") went in the undesired direction (we consider this item to be of type C), and one item assessing implementation of IBL ("encourag[ing] students to persevere in solving problems") had an increase in agreement post-workshop despite already having high agreement pre-workshop. Below we will further categorize and discuss the significant items.

3.2.1 Type A

We classified four survey question responses as type A results. For these items, either several (three or more) participants observed a change of two or more levels or particular individuals observed a change of three or more levels (and at least half of the respondents overall also reported a change).

1. For the intent to implement "encourag[ing] students to persevere in solving problems," six of the 13 respondents increased their intended frequency (with three increasing by two levels, from four to six, and one increasing by three levels, from three to six) and only one decreased their intended frequency (by one level). These results can be seen in the matched pairs plot in Figure 8. It is clear that the workshop had a strong impact, particularly on those four individuals who increased two or more levels to the "very frequent" end of the scale. Note that the pre-workshop median was already high (five out of six), but the post-workshop median among participants was even higher (a six, meaning "very frequent").

2. As seen in Figure 9, four out of the 12 respondents addressing confidence in "provid[ing] hands-on experiences before introducing new concepts" indicated an increase in their response by two or more levels (with two indicating an increase in two confidence levels, from three to five and four to six, and two indicating an increase in three confidence levels, from three to six). Notice that three of these individuals went from the lower portion of the confidence scale (with a rating of three or lower) to securely

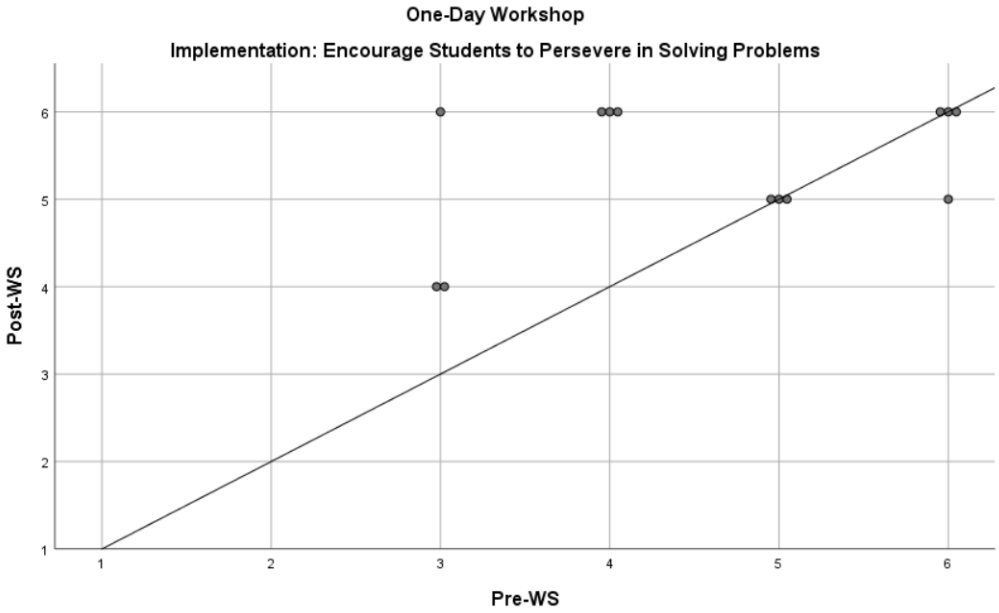

Figure 8. One-day workshop matched pairs plot: Implementation in encouraging students to persevere in solving problems.

confident (with a rating of five or six). The other changes observed were by one level only (with two increasing one level and two decreasing one level of confidence). Since the change in the four individuals mentioned was substantial, this seems to suggest that the observed significance in this item may indicate a sizable impact.

3. In addition to the confidence in "provid[ing] hands-on experiences before introducing new concepts," participants also changed regarding their intended implementation frequency for this item. In particular, two individuals changed by one level (one in either direction of the scale), three went up two frequency levels, and one went up three frequency levels. Among the individuals changing by two or more frequency levels, two changed from three to five, one changed from two to four, and the other changed from three to six. Note that the pre-workshop scores for these participants suggest they infrequently used hands-on experiences before introducing new concepts, but the post-workshop scores indicate a strong intention to apply this approach in their classrooms. Again, these differences are large enough to suggest a realizable change.

4. The last item we list as type A is "a math teacher should be able to solve any mathematics problem or puzzle." When implementing open-

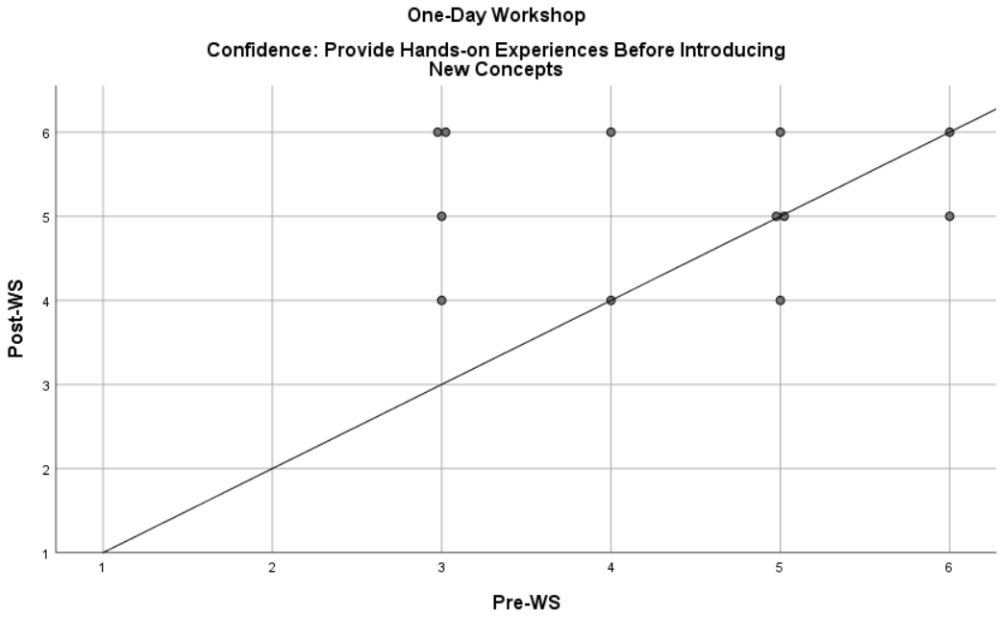

Figure 9. One-day workshop matched pairs plot: Confidence in providing hands-on experiences before introducing new concepts.

ended problems or projects, teachers are sometimes faced with questions to which they do not know the answers. Students may see a problem from a different perspective than what was intended or may simply try to determine what would happen if the problem were changed slightly. In order to have the confidence to run such open-ended problems, it may be helpful for a teacher to be willing to acknowledge and accept some level of uncertainty. Thus we consider it a positive result that one teacher disagreed with this statement by four full levels (from "strongly agree" (6) to "disagree" (2)). In addition, two participants changed from "somewhat agree" (4) to "disagree" (2) and one changed from "somewhat disagree" (3) to "strongly disagree" (1). The sizable change in these four respondents is the reason we consider this item type A. In addition to these, two other participants increased agreement by one level and two decreased agreement by one level. The change in individual participants can be seen in the matched pairs plot in Figure 10. Finally, we also note that the number of participants who securely disagreed with this item (either "strongly disagreed" or "disagreed") changed from three (23.08%) pre-workshop to six (46.15%) post-workshop.

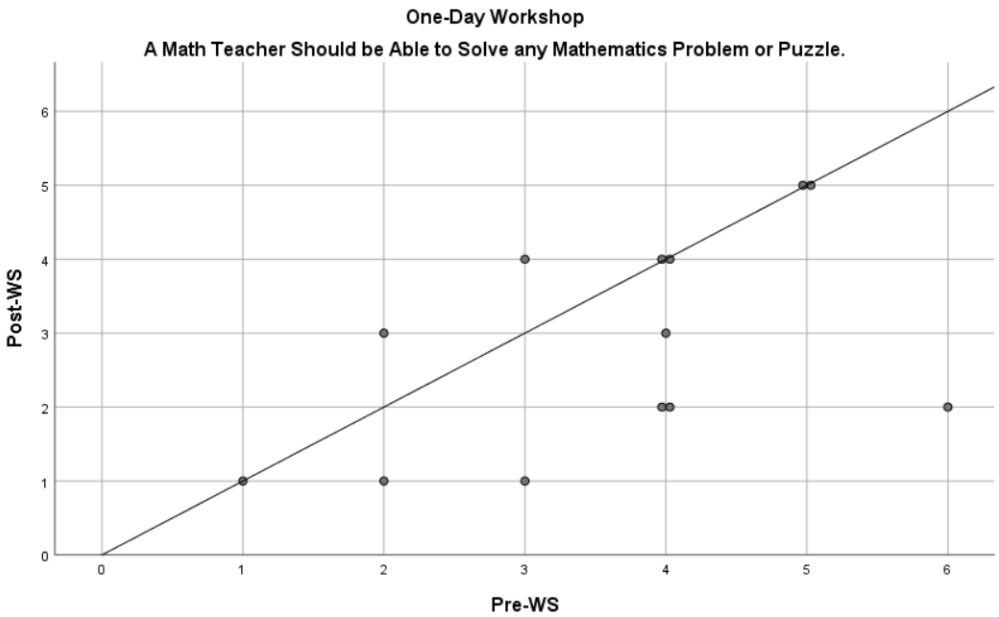

Figure 10. One-day workshop matched pairs plot: A math teacher should be able to solve any mathematics problem or puzzle.

3.2.2 Type B

There were eight items (out of the 13) that we consider to be of type B. These items can be found in Table 8. Note that, in this table, we only included the number of participants for those who changed responses; all remaining participants (out of 12 or 13 respondents, depending on the question) did not indicate a change for these items. As the table demonstrates, most changes were by one level (with two or fewer additional changes by at most two levels, with one exception). Also, observe that for most items, most of the participant changes within that item were in a direction in alignment with the goals of the workshop. So, again, while this indicates a definite change among the group, the size of that change (mostly by one level) suggests it may not be large enough to have useful implications. We'll discuss some of these items below.

First, we highlight the item "the math I teach in school is useful in everyday life." While there was one individual who increased agreement to this item by three levels, we consider this type B because fewer than half of the respondents (46.15%) indicated a change in this item and, of those, all other changes were by one level only. This is the only item we list as type B that observed a change of more than two levels among any one participant.

One of the goals of this workshop was to help give teachers the confidence to implement the use of open-ended questions in the classroom. In the pre-survey, over half of the participants reported that they do not often implement "creat[ing] open-ended problems" (with seven out of 13 rating their frequency level as three and the others rating four or five). In the post-survey all participants reported a frequency level of four or above (with six indicating a five or six frequency level). Despite this overall movement in the more frequent direction, the classroom implications of this change are questionable considering that eight of the participants only moved one level and only one participant moved two levels in the desired direction.

We saw a similar trend with the participants reporting how confident they were in "creat[ing] open-ended problems." In fact, the distribution of pre-workshop responses was exactly the same as the distribution for the implementation of "creat[ing] open-ended problems," with seven participants rating their confidence a three, two rating their confidence a four, and four rating their confidence a five. By the end of the workshop, all participants reported feeling some level of confidence with "creat[ing] open-ended problems." In fact, half reported feeling confident or very confident (with six out of 12 indicating a confidence level of five or six) after the workshop. Once again, though, the changes within the individuals are relatively small, with six increasing one level of agreement and two increasing two levels of agreement. We include this discussion to demonstrate that there is strong evidence that some shift in perspective is occurring but to caution from assuming this change in perspective is large enough to translate into classroom practice. The results of this item can be found in Figure 11.

3.2.3 Type C

The most surprising result was the response to how much participants agreed with the statement "lecturing is the best way to teach mathematics." As seen in Figure 12, one of the four participants who had strongly disagreed with this statement pre-workshop moved completely to the agree side of the scale post-workshop, and five others increased their agreement by one level. This was not the case with the three-day workshop (which saw no change in this item overall, with two participants increasing one level and two decreasing one level and no other change). We wonder if this is due to the fact that they did not have enough time to participate in productive struggle with successes. Possible reasons could be the condensed slightly shorter sessions in the one-day workshop. It would be interesting to see if having fewer slightly longer sessions in a one-day workshop would have resulted in the same trend.

Table 7

Significant changes for the one-day workshop.

Item	**P**	**M Pre**	**M Post**	**Type**
Engage students in inquiry-oriented activities (implementation).	0.072	4	5	B
Encourage social interaction as a means of learning (confidence).	0.063	4	5	B
Encourage social interaction as a means of learning (implementation).	0.063	4	5	B
Engage students in problem-based activities (implementation).	0.031	4	5	B
Encourage students to persevere in solving problems (implementation).	0.063	5	6	A
Provide hands-on experiences before introducing new concepts (confidence).	0.086	4	5	A
Provide hands-on experiences before introducing new concepts (implementation).	0.094	4	5	A
Provide applications that require mathematics to solve real-world problems (implementation).	0.094	4	5	B
Create open-ended problems (confidence).	0.008	3	4.5	B
Create open-ended problems (implementation).	0.004	3	4	B
A math teacher should be able to solve any mathematics problem or puzzle.	0.086	4	3	A
Lecturing is the best way to teach mathematics.	0.031	2	3	C
The math I teach in school is useful in everyday life.	0.031	4	5	B

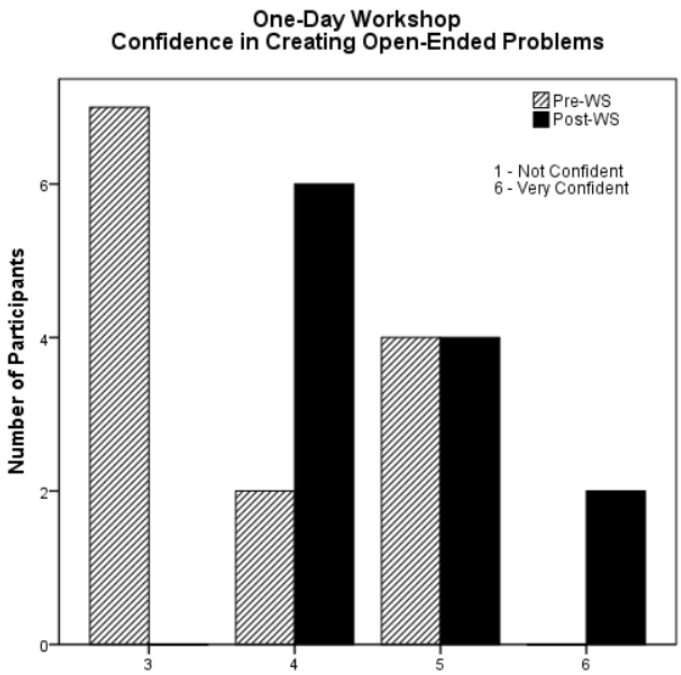

Figure 11. One-day workshop: Confidence in creating open-ended problems.

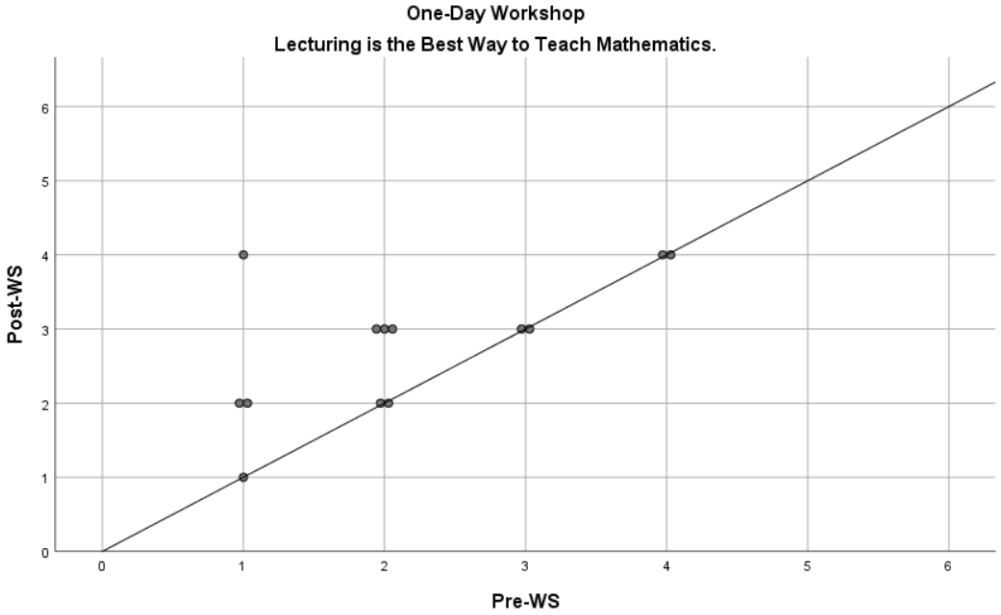

Figure 12. One-day workshop matched pairs plot: Lecturing is the best way to teach mathematics.

Table 8

One-day workshop: Changes among participants for type B significant items. Note that an increase in the level corresponds to an increase in agreement with the statement.

Item	**One Down**	**One Up**	**Two Up**	**Three Up**
Engage students in inquiry-oriented activities (implementation).	2	6	2	0
Encourage social interaction as a means of learning (confidence).	1	6	1	0
Encourage social interaction as a means of learning (implementation).	1	6	1	0
Engage students in problem-based activities (implementation).	0	5	1	0
Provide applications that require mathematics to solve real-world problems (implementation).	1	4	2	0
Create open-ended problems (confidence).	0	6	2	0
Create open-ended problems (implementation).	0	8	1	0
The math I teach in school is useful in everyday life.	0	5	0	1

3.3 Differences between three-day and one-day workshops

We compare the two workshops in two ways. First, we discuss the results of running a Mann-Whitney test for each of the 73 relevant items. Then, we discuss how the two workshops compare when examining overall changes in individuals as opposed to changes item-by-item.

3.3.1 Mann-Whitney test outcomes

In order to determine whether there was a difference between the two groups (representing participants of the two different workshop formats) in the *change* in responses from pre-workshop to post-workshop, we ran a Mann-Whitney test for each of the 73 relevant items. The p-values reported were generated via SPSS using exact significance. Throughout, we report a statistic as significant if it satisfies $p < 0.1$. With this, there were four items (8.33%) that had

statistically significant differences in the change in responses between the two workshop types. While statistical significance was observed, some of these may not translate into a practically important difference. We discuss each of these four items below.

1. Between the one-day workshop and the three-day workshop, there was a significant difference in the intent to implement "creat[ing] opportunities to develop students' conceptual understanding of mathematics" ($p = 0.052$). The three-day workshop saw an increase in median from 3.5 to 4.5 (with 58.3% reporting increased intent post-workshop and a mean rank of 15.96), while the one-day workshop had a pre- and post-workshop median of 5 (with 30.8% reporting increased intent post-workshop and a mean rank of 10.27). Since the medians for the one-day workshop were already high, it is not surprising that their change was not as significant as the three-day workshop results. The difference in responses can be seen in Figure 13. It is important to note that all changes for the three-day workshop were toward the more frequent side of the scale, but half of the one-day workshop participants who changed score did so in the less frequent direction.

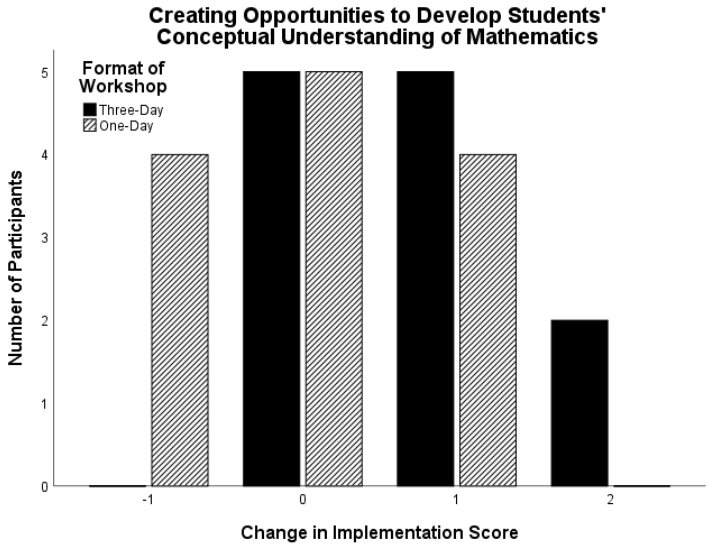

Figure 13. Three-day vs. one-day: Implementation in creating opportunities to develop students' conceptual understanding of mathematics.

2. Between the one-day workshop and the three-day workshop, there was a significant difference in the intent to implement "allow[ing] students to work at their own pace" ($p = 0.089$). In this case, the three-day workshop saw a larger percentage of participants increase their agreement

to this statement post-workshop (66.67% of the 12 respondents, with a mean rank of 14.96) versus the one-day workshop (with 25% of the 12 respondents increasing agreement and a mean rank of 10.04). However, only one individual from each workshop changed two levels (all other changes were by one level only), and those both increased two levels. Since the change in level was by only one level for most participants, the distinction between these two groups may not be practically important.

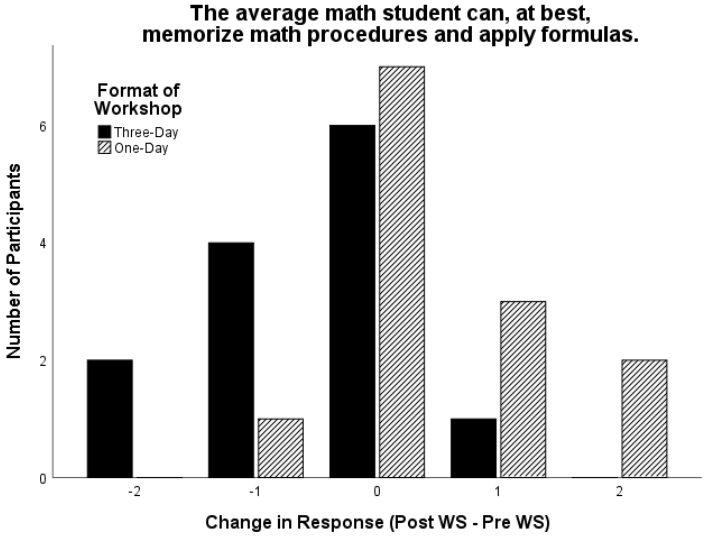

Figure 14. Three-day vs. one-day: The average math student can, at best, memorize math procedures and apply formulas.

3. While the item "if I don't fully understand a mathematics concept, I avoid using it in the classroom" was statistically significant ($p = 0.016$), looking at the individual workshops, the practical importance of this result is questionable. The three-day workshop had a larger mean rank (at 17.08) than the one-day workshop (at 9.92), meaning the changes for the three-day workshop tended to be larger (more positive). In other words, the three-day workshop saw an overall greater increase in agreement to this statement than the one-day workshop. In fact, examining Figure 15, we see that the one-day workshop had four participants decrease (and only one increase) their agreement, while the three-day workshop had about half (46.15%) increase in their agreement. The reason we say these might not translate to practically important differences is that both workshops had a pre- and post-workshop median of three, and in both cases, no participants outright agreed with this statement (five or six) after the workshop. Additionally, for both workshops, the majority

of participants did not change their position, and those that did, changed by one level only (with one exception).

Finally, we should emphasize that using open-ended problems can result in questions/concepts/discussions that instructors have not anticipated. Additionally, there are mathematics problems that are unresolved (and sharing this with students allows them to have a more evolving vision of mathematics) and so disagreement with this statement could demonstrate an instructor's willingness to pursue such open-ended (and open) problems with students. Thus, we view the outcome from the one-day workshop in this item to be more aligned with this instructional approach than the outcome from the three-day workshop.

4. The outcomes for the statement "the average math student can, at best, memorize math procedures and apply formulas" demonstrate one of the most significant differences between the workshops ($p = 0.016$). Although both workshops had a pre-workshop median of three, the post-workshop medians changed in opposite directions (a median of two, with mean rank of 9.96, for the three-day and a median of four, with mean rank of 17.04, for the one-day workshop). As seen in Figure 14, for the three-day workshop, only one teacher increased in agreement with the statement whereas 46.15% decreased in agreement (with two teachers disagreeing by two more levels than they did pre-workshop). For the one-day workshop, only one teacher decreased in agreement, whereas five teachers increased in agreement. Again, it is notable that between the two workshops, teachers' dispositions toward the statement changed in opposite directions. This may be due to the difference in number of days the workshop spanned, the amount of time allotted per session, or individual differences among participants. We note here that the disposition of those attending the three-day workshop changed in a direction that was more in line with conceptually-based instruction than did the disposition of those attending the one-day workshop.

3.3.2 Overall changes in individuals between the two workshops

In addition to changes in specific items, we also examined changes in individuals overall with regard to their perspective in the importance of, their confidence in, and their implementation frequency towards using inquiry-based learning in their classroom. To do this, for each individual, we determined the percentage of items for which a participant's response had changed toward

Table 9

Three-day versus one-day: statistically significant results from Mann-Whitney tests

Three-Day Versus One-Day: Statistically Significant Results					
Item	**P-value**	**M Pre 3-day**	**M Post 3-day**	**M Pre 1-day**	**M Post 1-day**
Implementation in creating opportunities to develop students' conceptual understanding of mathematics.	0.052	3.5	4.5	5	5
Implementation in allowing students to work at their own pace.	0.089	4	4	4	5
If I don't fully understand a math concept, I avoid using it in the classroom.	0.016	3	3	3	3
The average math student can, at best, memorize math procedures and apply formulas.	0.016	3	2	3	4

the direction more aligned with inquiry-based teaching. We also computed the percentage of items for which a participant's response had changed in the direction opposite what we would expect for inquiry-based teaching. We then compared these two percentages: assigning a 1 if the percentage toward inquiry-based teaching was higher than that representing a change in the opposite direction, assigning a 0 if these two percentages were the same, and assigning a -1 otherwise. These values provided a simple summary to represent an overall change within the individuals in one direction or the other (more aligned or less aligned with inquiry-based teaching). Finally, we computed the percentage of participants with a summary score of 1 versus a summary score of -1.

When considering the two groups as a whole, after attending the workshop, participants demonstrated more alignment with inquiry-based teaching, based on all three categories (perception of its importance, confidence in us-

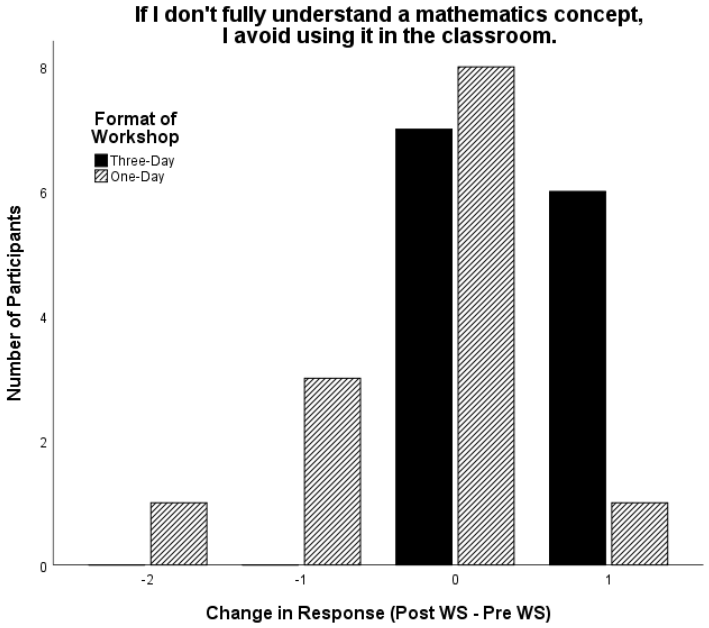

Figure 15. Three-day vs. one-day: If I don't fully understand a mathematics concept, I avoid using it in the classroom.

ing it, and (intent to) implement it). However, when differentiating between the two workshop settings, we see a striking difference in the proportion of participants who changed in such a direction. Overall, we observed 64% of participants increased their perception of the importance of those skills related to inquiry-based teaching (versus 16% who decreased their opinion of the importance of such skills). In this category, the two workshops were comparable (with 66.67% of all three-day participants and 61.5% of one-day participants increasing their opinion regarding the importance, and with 16.67% and 15.38% decreasing their opinion, respectively). Regarding participants' confidence in applying inquiry-based instruction, 80% of all participants increased their confidence (versus 16% who decreased their confidence). In this category, the two workshops differed greatly, with 91.67% of all three-day workshop participants increasing in confidence (and no one decreasing in confidence) versus only 69.23% of one-day participants increasing in confidence (and 30.77% decreasing). Once again, in the implementation of inquiry-based learning, we see a stark difference between the two workshops. Even though 88% of all participants (together) increased their (intended) frequency of implementation (with only 4% decreasing), the three-day workshop had 100% of its participants increase in their overall intended implementation (versus

76.92% for the one-day). Additionally, about 7.69% of one-day participants indicated an overall decrease in intent to implement inquiry-based teaching approaches (versus 0% for the three-day).

4 Discussion

The results of this study indicate that participation in the Math Teachers' Circles workshops (in either format) resulted in more alignment with teaching techniques relevant to inquiry-based instruction. Both the three-day and one-day workshop saw a number of significant changes (16 items for the three-day and 13 for the one-day workshop). For each of the workshops, all but one of those changes was in the direction consistent with inquiry-based instruction. Additionally, when examining overall changes within individuals (as opposed to item-by-item) regarding their perspective of inquiry-based teaching (based on the three criteria: importance, confidence, and implementation), a large majority had more positive than negative changes regarding inquiry-based teaching. This outcome was also observed in the category of importance of inquiry-based instruction (with 66.67% of three-day participants and 61.5% of one-day participants having more changes in alignment with inquiry-based instruction than opposed to it) despite no significant changes in any of the individual items assessing importance. Among both workshops, participants' confidence in using techniques connected to inquiry-based instruction had also increased (with 35.71% of such items for the three-day workshop and 21.43% of such items for the one-day workshop resulting in positive changes - and none resulting in negative changes; and with 80% of all participants indicating a more positive overall change regarding their confidence).

Importantly, for both workshops, the greatest impact was regarding implementation of inquiry-based teaching, with 88% of all participants indicating more inquiry-based techniques with increased (intended) implementation than those techniques with decreased (intended) implementation. Additionally, for each of the workshops, 41.18% of items assessing implementation had positive changes. This suggests both workshop formats can have a sizable impact on teachers' disposition toward inquiry-based instruction, in particular with regard to their intention to apply this style of instruction in their classrooms.

Despite an overall positive shift towards inquiry-based teaching from among both workshops' participants, there were some notable differences between the two groups. First, only three of the survey items had significant changes among both workshops: implementation of "provid[ing] hands-on experiences before introducing new concepts," confidence in "creat[ing] open-ended problems," and implementation in "creat[ing] open-ended problems." Besides these, the

other changes observed differed between the groups. The proportion of implementation items with changes was comparable (as well as the proportion of items with changes in the disposition of the teaching and learning of mathematics), but the three-day workshop had a slightly larger proportion of confidence items with significant increases. Additionally, when comparing changes between the two groups, we found four items with significant differences. In three of those items, the three-day workshop saw more alignment with inquiry-based approaches or ideology than the one-day workshop. The most striking of these being the item "the average math student can, at best, memorize math procedures and apply formulas," where the two workshops saw a change in opposite directions (85.71% of the changes among the three-day participants were toward the disagree side versus 83.33% of the changes among one-day participants were toward the agree side of the scale). Additionally, when examining overall impact among individuals, the differences in the two workshop formats becomes evident. In particular, the proportion of participants with overall gains regarding confidence and implementation of inquiry-based techniques was much greater in the three-day workshop than the one-day workshop (for confidence, 91.67% of three-day and 69.23% of one-day participants had greater gains than losses in confidence; for implementation, 100% of three-day and 76.92% of one-day participants indicated more items with increased implementation than decreased implementation). This suggests that while both workshop formats can have a positive impact on participants' dispositions toward inquiry-based teaching, the three-day workshop seems to have a more pronounced impact.

5 Conclusion and Future Work

This initial study of the effect of Math Teachers' Circle (MTC) workshops on elementary, middle, and high school teachers' dispositions toward inquiry-based learning found overall gains which aligned with the goals of inquiry-based learning (IBL). Participants of both the one-day and three-day workshops reported gains in confidence and planned implementation of IBL, as well as changes in their disposition of the teaching and learning of mathematics (13 and 16, respectively). Almost all changes observed were in a direction aligned with the goals of workshop, but there were a few surprises in the participants' responses. The participants in the three-day workshop moved in the undesired direction with respect to avoiding a mathematics concept if they don't full understand it. Additionally, the one-day workshop participants moved more towards agreeing with the belief that lecturing is the best way to teach

mathematics. However, for all other significant changes except these two, movement was in an affirmative direction.

We note that participants of the three-day workshop indicated more change in the desired direction than the participants in the one-day workshop. While the more resources and time for the three-day workshop did yield more gains with the workshop goals, the relevance of that difference is yet to be determined. When examining item-by-item, the differences in the workshops is not as noticeable (with only slightly more gains for the three-day workshop). However, when examining overall changes in the individuals, the three-day workshop seemed to produce a far more measurable impact. Further research is needed in order to determine whether these gains in the three-day format produce actionable changes in teaching styles of participants.

We plan to extend this initial study to include more Math Teachers' Circle summer workshop participants outside of the Southwest Chicago MTC. We also would like to conduct interviews with participants to see if the workshop had a long-lasting impact on the teachers and to determine whether continued participation in Math Teachers' Circles continues to improve teacher confidence in using and their implementation of inquiry-based learning techniques.

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