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The Power of a Double Take: Doubling Back to Show Growth in a Pandemic

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Synopsis

The COVID-19 pandemic inspired reflections on the importance and challenges of teaching exponential growth, such as doubling, to varied audiences.

Keywords: college, COVID, edutainment, elementary school, exponential growth, mathematics for social sciences, public outreach, STEM, television.

1. Motivation

Before the current pandemic, most people did not see exponential growth as relevant to their lives. Graphic artist Frank Mariani created a COVID-19 cartoon in March 2020 that went "viral", thanks to a social media share by American astrophysicist and science communicator Neil deGrasse Tyson and a later post on the American Mathematical Society website.[1] The cartoon depicts an algebra teacher at the chalkboard drawing graphs (showing exponential growth as well as "flatten the curve" trajectories) while one student flippantly says to another "Like we'll ever use this stuff."

In early 2020, many Americans seemed surprised by the power of exponential growth, wondering why they should worry about COVID-19 spread when the number of US cases still seemed small. They might have heard on the news then that the number of cases roughly doubled every three days (a more

[1] One can still see this cartoon on the top right corner of http://www.ams.org/education/teaching-resources, last accessed on July 23, 2021.

precise estimate is 2.68 days before widespread mitigation efforts [6]), but didn't seem to realize that meant, for example, that the number of cases was less than 1% of what it would be in just three more weeks (since $2^7 > 100$).

This lack of intuition about doubling reminded me of when I tried teaching that topic on a children's educational television show. As I double back nine years to reflect on the experience, I see many lessons that have continued to inform my teaching (see Section 3). While learning trajectories for exponential growth have been studied for eighth-grade pre-algebra students [2], an even bigger challenge for this show is that its target and on-stage audiences were students in grades 1-3! But as my son taught me when he was about that age [3], if I really understand a concept, I should be able to teach *anyone*. That challenge of reaching an elementary school audience ultimately feels not unlike the challenge of reaching a general audience given how many adults have had little formal mathematics coursework and recent exposure to it and given how many of them may have long-standing anxiety or poor attitudes towards mathematics. And educating a general audience about exponential growth such as doubling acquires life-or-death importance in a pandemic.

2. Televised Episode

2.1. The Program

In mid-September 2011, El Paso PBS-affiliate KCOS-TV began airing the first of four seasons of a locally-produced children's educational show, *Blast Beyond*. The show has a spaceship motif inhabited by an on-stage audience of spacesuit-wearing "cadets" aged 6 to 9 from a local school, enthusiastic host Captain Rob (Robert Bettes, Chief Meteorologist on CBS-afilliate KDBC-TV), puppet characters (Cosmo the engineer, Ivan the chef, and Debvar the security chief), a crackerjack 3-piece rock band in sunglasses and jumpsuits, and walkie-talkie wielding Commander Alba at "Mission Control" giving instructions for games. The 303 half-hour episodes (most with STEM themes) were recorded to air twice each weekday to a broadcast area of 2 million people spanning parts of Mexico, New Mexico, and Texas.

I participated as the featured guest on six different episodes of this show.[2] During each of these six episodes on different mathematics/statistics topics, I presented hands-on mini-lessons and also contributed all lyrics, most games, and some banter for segments when I was off-stage.

2.2. The Episode

The episode on doubling was filmed on January 19, 2012, with 17 first- and second-graders from the El Paso Jewish Academy, and it was first aired on January 30. Like other episodes of this program, the time was broken into segments of a few minutes each, alternating among games, mini-lessons, banter, and bits of music. The episode can be viewed at `https://youtu.be/ycTc0tIpkcI`, but in case the link becomes inactive, I now describe its trajectory.

After initial banter, Captain Rob implemented a warmup activity I adapted from [11], having the students identify doubles in pictures of various things from real life: bicycle ($1+1$ wheels), car ($2+2$ wheels), ant ($3+3$ legs), one's hands ($5+5$ fingers), and egg carton ($6+6$ eggs).

After the band played a short song about doubling (with its final section in "double time"), I had the students generate the doubling sequence 1-2-4-8-16 and performed a related mental magic trick for three different students chosen. This (easily Googled) trick uses base 2 where the magician asks a person to privately think of a number between 1 and 63 and say which of six printed 32-number cards include that number, after which the magician promptly announces the secret number. For the show, I adapted that trick to a set of 5 cards to predict a person's secret number chosen from 1 to 31, which conveniently works for the day of a month and makes the trick easier to implement with such young participants. Each student selected all 16-number cards (from a set of five) that included the day (1-31) of the month they were born and then at a glance I announced each student's day!

When I was brought back onstage later in the episode, I posed and (after students made predictions) answered two questions:

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[2] Readers can access these episodes at `https://www.youtube.com/playlist?list=PLdfaHtJTO4LJZfSgu2Ztvn3aI6R8B1giR`, last accessed on July 23, 2021.

1. Suppose your parents gave you an allowance for a 31-day month like January so that on Day 1 you get 1 penny, and then on Day 2 you get 2 pennies, on Day 3 you get 4 pennies, etc. If the doubling pattern continues, how much money will you get on the last day of the month?

   [A student volunteered 75 pennies as an answer, nowhere near the correct answer of $(\$0.01)(2^{30}) = \$10,737,418.24$! This also calls to mind a *FoxTrot* syndicated comic [1] where a math teacher offers students one second of homework the first week of school with the amount doubling each week over the 36-week school year.]

2. As we see, every time a piece of paper is folded, the thickness of paper in one's hand doubles, so how thick would a piece of typing paper be if you could fold it in half 42 times?

   [Since a 500-sheet ream of paper is 2 inches thick, a piece of paper's thickness is $1/250$ of an inch, which when multiplied by 2^{42} exceeds the distance between the Earth and its moon [5]! Folding 14 times yields a thickness equal to a typical height of an adult woman, and folding about 100 times yields the diameter of the observable universe! Of course, folding paper 100, 42, or even 14 times is physically impossible, as students will attest by watching a video such as `https://youtu.be/AfPDvhKvaa0` or by trying it out themselves.]

My closing activity involved a human number line demonstration where we lined up the 17 students shoulder-to-shoulder and passed out papers so that the students held the consecutive numbers 1 through 17. Starting with the number 1, I had students represent linear growth by continuing to add 2 so that the students with paper signs corresponding to 1, 3, 5, 7, 9, 11, 13, 15, 17 hold them up to show constant gaps along the number line. Then they lowered their papers and we modeled doubling (exponential) growth by starting with 1 and continuing to multiply by 2, resulting in students raising their number sheet if they had 1, 2, 4, 8, or 16, which showed gaps getting larger.

3. Reflections

The allowance and paper folding questions leveraged striking examples [10] or counterintuitive examples (see Appendix C of [4]) to engage by making people do a double take. Examples such as typing paper thickness have done double duty in other areas of my adult education outreach work. During media interviews about the lottery,[3] I try to concretize the tiny probability of winning (a share of) a jackpot. For example, the current probability of one ticket matching all numbers in a Mega Millions drawing can be modeled as the chance of picking a particular sheet of typing paper from a stack 19.1 miles high!

As I describe in [8, page 66], "aspects of such engaging education ... also enhance the work university professors like us more commonly do, including: identifying a compelling 'hook' to generate initial engagement for a lesson (whether the lesson will be 5 or 50 minutes long), identifying the main point or essence of a lesson/concept (which helps greatly when one is interviewed by TV or newspaper reporters), identifying ways to make a lesson interactive (i.e., through building in questions asked to the cadets, or questions asked by Captain Rob, having demos, etc.), developing more confidence in having a 'plan' for a lesson but also in being able to make on-the-fly adjustments when all the moving parts or background knowledge of the audience is not quite what you thought it would be."

During the pandemic, some politicians doubled down on denial, hoping the populace wouldn't fully recognize its danger. But educators and researchers have a special duty to accurately assess the situation and figure out the most effective way to convey it. In particular, researchers found that there are ways to frame facts about a pandemic to mitigate how much people underestimate exponential growth [9]. I came across a randomized experiment/survey of $n = 459$ Swiss college students in non-STEM fields where students had less bias when exponential growth was communicated or framed in terms of time for cases to double (instead of in terms of growth rates) and time until a threshold number of cases is reached (instead of number of cases).

[3] Readers can see the author's lottery literacy page at `https://larrylesser.com/lottery/`, last accessed on July 23, 2021.

The implications of this study have the potential to make people more likely to take preventative measures such as wearing masks, social distancing, working from home, etc. Effects of how exponential growth is framed also appear in the context of financial literacy [7].

One of my Spring 2020 classes was one of the twenty sections of Math for Social Sciences, a departmentally coordinated precalculus course for non-science majors that included material on exponential growth well before the pandemic made us pivot to online meetings. The curriculum included how the formula $Q(t) = Q_e e^{kt}$ could be used to show that doubling time was $\ln(2)/k$. However, the examples used to explore the material frequently lacked an authentic quantity of data (such as world population growth modeled from population values from only two different years) and/or had numbers just made up about bacteria culture growth, auto thefts, etc. Whenever I next teach that material, I will take time to give it more authenticity from the pandemic growth we have experienced. And if or when there is another pandemic, I will also use what I have learned to communicate more readily and effectively its underlying mathematics to broader audiences.

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