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An Interview with the (Other) Corona Graph

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Synopsis

The following is an interview, conducted in April 2021, with a graph product known as the corona graph. *Disclaimer: The views expressed herein are not ours. Actually, this entire interview is fiction, other than the mathematics, of course.*

Keywords: corona graph

Interviewer: [to audience] Today we're excited to sit down with a famous corona graph— but probably not the one you're thinking of. [To Corona Graph (**CG**):] It's quite an honor to talk with you today. But first, for those out there who might not know much about you, why don't you tell us who you are?

CG: Thank you for having me. So, right, as you mentioned in your introduction, I'm that "other" [air quotes] corona graph. I'm a graph, or to some, a network. I'm actually a special combination of two graphs.

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Interviewer: A special combination?

CG: Yep, a graph product. Take any two graphs you want. Call one G, and one H. Then however many vertices you have in G, let's call that n, take n copies of your graph H. Then you'll take each vertex of G, and join it to every vertex in one of those copies of H. And, that's pretty much who I am.

Interviewer: Fascinating. I never would have guessed.

CG: Yeah, well, depending on how I'm drawn, it's kind of hard to see sometimes.

Interviewer: Certainly. I understand that you normally don't do interviews. What changed your mind today?

CG: [Nods and pauses] Well, I ... first, I just want to acknowledge this pandemic, and how devastating it's been on humans. Mortality is a hard concept for me to grasp, but I can see that things have been rough for humankind.

Interviewer: I think people will appreciate that. How have you done this year?

CG: [Sighs] I don't want to complain, because my struggles have not compared to you humans, honestly, but ... I can't say it's been easy.

Interviewer: Tell us more about that.

CG: Well, with a name like *Corona Graph*, every time I introduce myself or get ID'ed, everyone just goes silent. I've been shunned most of the year, and the looks I've gotten when people hear my name ...

Interviewer: I imagine that it can't be easy having a name like yours right now.

CG: Honestly, I thought about changing it, but it's who I am, you know? I've tried going by just $G \odot H$, or G, but it doesn't feel right, except among friends.

Interviewer: If I may ask, how on earth did you and this virus end up with the same name?

CG: I wish I could say it is a coincidence. But I want people to understand that we're not directly related at all. "Corona" means crown, and I believe coronaviruses got their name because they looked like crowns under a microscope. When my first graph, G, is a cycle, and my second graph, H, is a single vertex, I also look like a crown. And many folks say they can see the resemblance. [Shows picture from Figure 1.]

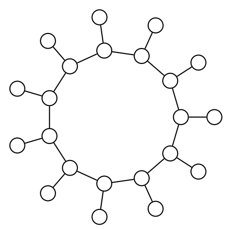

Figure 1: Corona graph $G \odot H$ for $G = C_{10}$ and $H = K_1$.

Interviewer: I see.

CG: Luckily, as a graph I can be drawn in any way, so I just move around some of my vertices. It's a good disguise. You should try it sometime, Bob.

Interviewer: Smart. But who's Bob?

CG: Aren't you Bob? I thought all interviewers are named "Bob."

Interviewer: Umm, no. I'm actually a woman, and "Bob" is more typically a name for men.

CG: Oh, interesting! Human gender is still a mystery to me. Names, too. Only special graphs like me have permanent names. The rest, we just make up a temporary name, like: "Let G be a graph." Or: "Let Bob be an interviewer." Sorry about that.

Interviewer: No worries. This might sensitive, but have you Googled yourself recently?

CG: [waves a vertex dismissively] Eh, that doesn't bother me. It was a bit of a shock at first, seeing that I didn't even make the top 10 hits when I googled my own name, but the anonymity is a bit of relief. It's the judgment and looks that hurt.

Interviewer: That makes sense. So tell us a little bit about your early life.

CG: I'd be happy to, though as a mathematical object, I've always existed, really. But in modern mathematical history, mathematicians known as Frucht and Harary introduced me to humans in 1970 [2]. So I guess you can credit them.

Interviewer: How was your relationship with them when you were young?

CG: Well, for a long time I resented that they called me a "new and simple operation" because I like to think I'm anything but simple, and I'm certainly not new, but now I understand what they meant. As I've aged, I've grown to appreciate the fact that when they introduced me, they noted that I'm isomorphic to the wreath product of two groups.

Interviewer: Because you enjoy group theory?

CG: Nah, group theory is fine, but I really like the wreath thing. It's festive.

Interviewer: I see. So tell me a little bit about your personal life now. Any family? Kids?

CG: Ah, well, I'm actually a graph family all by myself, so that gets a little bit complicated.

Interviewer: Interesting. Anyone special in your life? Romantic interests?

CG: Again, the whole mathematical object thing makes that a bit impossible. But I do have to admit, I've always had a thing for Cartesian products.

Interviewer: Why is that?

CG: Well, we have a lot in common. But one intriguing difference is that Cartesian products are commutative, which I find really exotic.

Interviewer: You aren't commutative?

CG: Oh heavens, no! Suppose G is a cycle with ten vertices, and H is K_1. Then $G \odot H$ has 20 vertices, and looks kind of like a sun or crown, like I showed you earlier, but $H \odot G$ has 11 vertices, and looks like a wheel, like this [shows Figure 2].

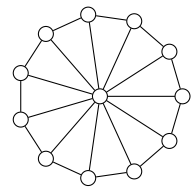

Figure 2: Corona graph $H \odot G$ for $G = C_{10}$ and $H = K_1$.

Interviewer: Ah, thank you for explaining. Now, some of our listeners will want to know more about you. We'd like to read you questions some of our listeners sent in. Would that be OK?

CG: Sure, that sounds fine to me.

Interviewer: Great. This first one is from Alex, who's five years old and from Florida. He writes, "Dear Corona Graph, My favorite color is yellow. What's your favorite color? And how big are you?"

CG: [Smiling] Well, Alex, I really like all colors. To me, one color may as well be another, and sometimes, I even use numbers to represent colors. But I digress. So maybe you'd like to know about my chromatic number— [to Interviewer:] I forget how human time works sometimes. Do five-year-olds know about proper colorings? Should I explain it?

Interviewer: Umm … I think explaining it would be the best bet.

CG: I would love to. OK, so in a proper vertex coloring, I put a color on each of my vertices. It's beautiful! But there's a special rule. If two vertices have an edge joining them, then their colors must be different. So if I color one vertex yellow, none of its neighbors can be yellow. But the big question is: what's the smallest number of different colors I would need to color myself in and not break that rule?

Interviewer: Ah, is that the chromatic number?

CG: That's right, we usually say $\chi(G)$ for the chromatic number of G. Now for me, it depends on the chromatic numbers of the two graphs who make me: G and H. If $\chi(G)$ is bigger, this is cool: then my chromatic number $\chi(G \odot H)$ is just the same as $\chi(G)$! But if $\chi(H)$ is at least as big as $\chi(G)$, then things change. In that case, my chromatic number is $\chi(H) + 1$. So I need just one more color than H does. Pretty efficient, eh?

Interviewer: Hmm, I guess so ... how about Alex's other question, about how big you are?

CG: Ah, yes. Well, Alex, since all vertices in each of my copies of H are adjacent to a vertex in G, my diameter — you know, the largest distance between any pair of vertices— is just the diameter of G plus 2. Oh wait! I can say more about colors. Some folks have worked on total colorings of me, which color everything, not just my vertices. In [4] for example, they found that I'm of Type 1— is this getting too racy? I don't want any parents calling in with complaints ...

Interviewer: I think we're OK on that front, but let's go to another question. Sam, age 9, from New York writes, "Dear Corona Graph, I'm nine years old. You're cool, but I think Strong Products are cooler. How do you rate yourself compared to other graphs?"

CG: [frowns] Hmm, the strong product, really? Kind of comparing apples and oranges ... [inaudible grumbling] OK, so maybe Sam means to ask about rankings. Can I talk a little bit about rankings?

Interviewer: Of course.

CG: So in a vertex ranking we use numbers 1, 2, 3, and so forth. We want to put a number on each vertex, but have a rule: if two vertices have the same number, then any path between them must have a higher number somewhere on it.

Interviewer: Ah, interesting. Is there a rank number?

CG: Why yes, there is: $\chi_r(G)$! And it's the fewest numbers you can use to make a ranking of G.

Interviewer: Similar to the chromatic thing, I guess.

CG: That's right! They're related, but that's for another show.

Interviewer: So tell us about your rank number.

CG: I'd be honored. OK, so to get my rank number, you just add up the rank number of G and the rank number of H. Crazy, right? But I can prove it. Just put an optimal ranking using labels 1 through $\chi_r(H)$ on my copies of H.

Then, do an optimal ranking of G, but instead of using labels 1 through $\chi_r(G)$, you add $\chi_r(H)$ to each of those labels, so you use $\chi_r(H)+1, \chi_r(H)+2, \ldots$. This is a ranking because each of the pieces is a ranking, and if you take a path from any copy of H to any other, the vertices in G between it all have higher labels.

Interviewer: But how do you know you can't do better than $\chi_r(G)+\chi_r(H)$? You've only shown us that you can find a ranking like that ...

CG: Great question! Hey, are you a mathematician?

Interviewer: Well, it's sort of a prerequisite for interviewing a graph, so ...

CG: Ah, good point. You're absolutely right! Anyway ... the vertices in a copy of H are at a distance at most two, so in any ranking the label of a vertex in G can be made higher than the labels of the vertices of H that are adjacent to G. This would mean that the best we could do would be to use labels 1, 2, ..., $\chi_r(H)$ in copies of H, and $\chi_r(G)$ more labels for G. End proof!

Interviewer: This has been truly fascinating, but we're running out of time, so I'll just ask one more question. This is from Maddie, age 8, from Alaska. She wants to know what makes you think you're so important.

CG: Another question from a kid, and a tough one!

Interviewer: You had no idea you were so famous with children, did you?

CG: Or perhaps infamous? [laughing] Well, Maddie, I've been featured all over the place. You know zero forcing, Maddie, that relates to combinatorial matrix theory? So in [1] some folks found a bound on my zero forcing number in terms of the zero forcing numbers of G and H. And it's not just about math, either. In [3] some other folks point out that when my G and H have certain properties, I'm of biochemical interest, you know. And in [5], some other people study some of my properties when $G = H$, and also describe how I could be used to investigate real world complex networks.

Interviewer: Impressive! But, unfortunately, that's all the ti—

CG: Not too shabby, huh? I could go on for hours. These papers are just the tip of the iceberg. If you search on `MathSciNet`[3] for papers with "corona" and "graph" in the title, there were 181 of them last I checked. And most of them are about me, not about that oth—

Interviewer: [Clears throat, clapping loudly] Corona Graph, thank you so much for coming today! It was a pleasure to meet you.

CG: And you as well! Sorry, I get a little self-involved sometimes, but this has been a lot of fun, Bob— sorry, what was your name, anyway?

Interviewer: It's not that important. I have a name, and it's unique. And it's not Bob.

CG: [laughing] Good one! Interacting with humans is one of my favorite hobbies, honestly. I hope that you all can overcome this virus soon, and get back to the good stuff. Like graph theory. And celebrating important holidays, like π Day.

Interviewer: We hope so too. All the best to you.

[Exchange of an elbow-vertex bump].

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[3] "`MathSciNet` is an electronic publication offering access to a carefully maintained and easily searchable database of reviews, abstracts and bibliographic information for much of the mathematical sciences literature." See https://mathscinet.ams.org/mathscinet/help/about.html?version=2 for more.

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