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Fantasy at the Service of Mathematics

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Synopsis

This article aims to introduce the reader to a book published in 2016 under the title "Amazing Tales from the Magic Wood and Famous Problems of Mathematics" by Elli Shor and Clara Ziskin. The book offers an original method of presenting mathematical facts and history through a fantasy narrative. The book's two authors, Clara Ziskin and Alla Shmukler (Elli Shor), together with consultant psychologist Esther Williams, share here several excerpts taken from the first part of the book as well as related illustrations and mathematical riddles, so that the reader can form an informed impression of the book, its structure, and its nature.

Keywords: mathematics, fantasy, history, psychology, education.

On the front cover of the book *Amazing Tales from The Magic Wood and Famous Problems of Mathematics*, published in 2016 by the London publisher Austin Macauley, there is a slightly open gate, behind which is seen something fulsome, replete of meaning, which attracts the eye and makes one to take a second look at it. This is an invitation to visit two parallel worlds presented in this book.

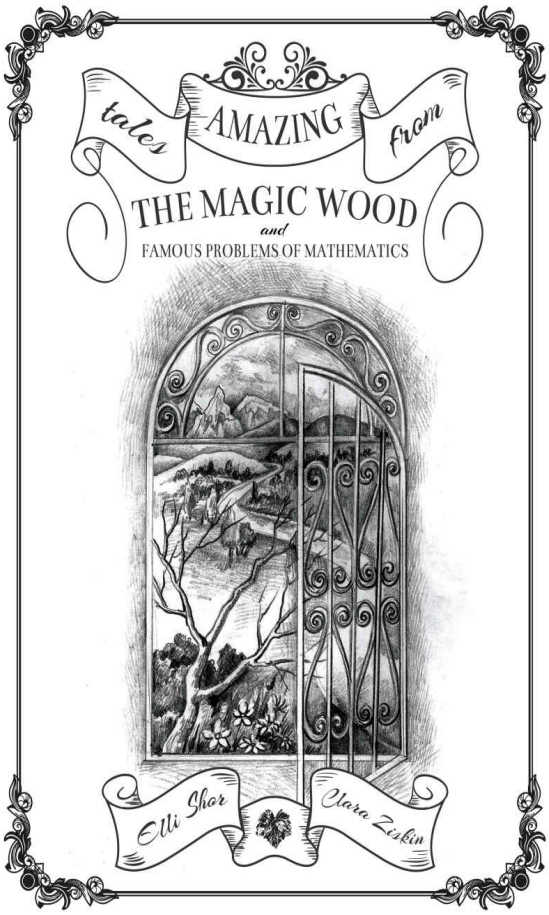


Figure 1: Front cover of *Amazing Tales from The Magic Wood and Famous Problems of Mathematics.*

The first is the fantastical world of the Magic Wood, an illustration of Isaiah's prophecy. There is no violence in this world, the food is only vegetarian, and death is only of natural causes. There are no predators and no prey. Among the fulsome, varied landscape replete with hills, rivulets, woodlands and meadows, a central lake surrounded by growths of bamboo—in the midst of all this splendour there live wolves and lions peacefully side-by-side with rabbits and squirrels. This world exists outside of objective time and its own time is marked by the changing seasons which treat the inhabitants of the Magic Wood most kindly, showing only their good features.

That said, the inhabitants themselves are far from ideal; they can be sly and deceptive. Their lives are far from easy and are visited by illnesses, anxieties, interests, and every-day problems. Such is the picture of the Magic Wood in the first part of the book.

The second world, depicted in the second half of the book, is the real world of Mathematics that exists in real time. It is the habitat of Mathematics and the guarantor of its ongoing development. Miraculously, however, time has no power over Mathematics that can preserve the newness and importance of its theorems, theories and discoveries through the millennia of its history.

The relationship between the two worlds is maintained by means of riddles that accompany the tale of the Magic Wood and that serve as the backdrop or discrete instances of problems made famous in the history of Mathematics.

To illustrate this let us examine in full two out of the ten Magic Wood tales, each accompanied by its riddle, numbered as they are in the book.

Tale No. 1: The Seven Mice

> The mice of Magic Wood are such sweet creatures: they are covered with shiny black fur, their slightly crossed eyes sparkle, and their long tails can curl up into tight little coils that allow them to jump up into the trees, should the need arise. Most of the time, however, the mice of Magic Wood can be found underground, where each mouse family is cozy and safe in its own little dwelling.
>
> The homes of the mice are quite spacious and comfortable. Little steps dug into the earth lead to the entrances of the underground homes, and above each abode, a little chimney pokes out of the ground: you see, it is very chilly under the ground all year round, and the mice's little ovens are always working.
>
> There is one particular mouse family that lives in such a home near the foot of one of the southern hills of Magic Wood. Each of its members has a little birthmark in the shape of a star, and so they are known as the Star family.

The Star family home has a large, comfortable living room with a built–in fireplace. There is a snug little bedroom for the parents, a good–sized room where the seven sons sleep, and, most importantly, there is a storeroom where food and fuel are stored.

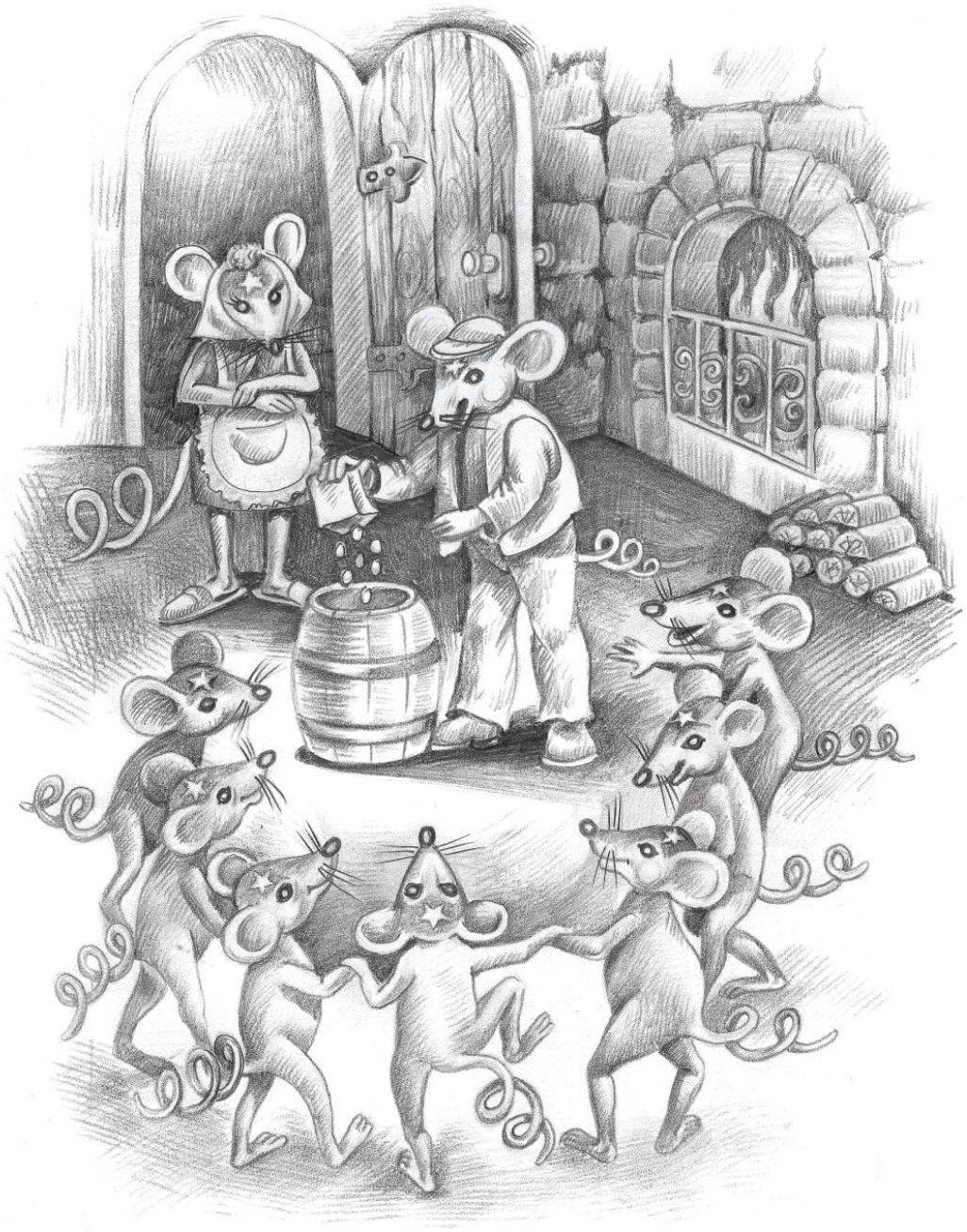
One evening, towards the end of the summer, when the family was gathered around the fireplace, Papa Star started to speak:

"Dear children. As you know, our barrel of seeds has been empty since the end of last winter. Since these seeds are our main source of food during the long days of winter, it is vital that we begin refilling the barrel right away so that we will be prepared when winter returns. And since now is the time when the grain stalks in the fields are ripe and bursting, now you must go and gather seeds."

And he continued. "Mama Star has sewn for each one of you a wee, little sack to hang around your neck. The sack holds exactly seven seeds. Starting Monday morning, each one of you will go out to the fields, bring back a sack–full of seeds, and empty it into the barrel in the storeroom. This way, at the end of the week, we will have . . . ah. . . we'll have a barrel full to the brim with seeds."

You must have noticed that Papa Star hesitated a bit at the end of his speech. He wanted to say precisely how many seeds would be in the barrel by the end of the week, but, alas, he couldn't calculate the number. Do you know why? The reason for this is simple: the mice in Magic Wood can only count up to ten, and a full barrel holds much more than ten seeds. Even so, through previous experience, Papa Star knew that if each of his sons would gather seven seeds for seven days, the barrel would be full to the top. Using this knowledge, the father was able to get himself out of an embarrassing situation.

Suddenly, Mama Star interrupted: "But Papa! You have forgotten something very important! We are forbidden to gather seeds on the seventh day of the week! According to the rules of Magic Wood, this day is a holiday to be used only for fun and relaxation."



"Oh! Bless my soul, my dear!" said Papa Star. "You're absolutely right! How could I have forgotten such an important detail?" And then he started to consider how to remedy the situation. He thought and thought, and finally said: "Children! Listen! You will not be able to proceed with your present task the following week, because then you'll have another task that I have already prepared for you. But the problem can be solved in another way. This is what you will have to do. On each of the six days, in addition to the seeds in your sack, you will each bring an extra seed between your teeth. Now you'll be putting eight seeds into the barrel every day for six days. On the seventh day we will have our day of rest. Then the next day, that is the first one of the next week, I myself will go to the field with a sack, fill it up, and put those seeds into the barrel. That way the barrel will be full."

Everybody thought that Papa's plan was ingenious and would work very well. When night fell, the Star family left the living room to go to bed. They all cuddled under their quilts, happy and content, dreaming wonderful dreams about the delicious seeds they would have in the winter.

Here is the riddle that accompanies the story.

Was Papa Star correct in his calculations of how to fill the barrel, and how did he come to this conclusion when he can count only to 10?

Truly, dear reader, how did he?

Tale No. 4: The Gift of the Goldfish

Right in the center of Magic Wood there is a large lake which is associated with many tales from the Magic Wood. One such tale is the legend of Goldfish—a fish that could fulfill any wish. He is named this name because of his special color: a pure gold that is quite rare for fishes in nature. He lives at the bottom of Great Lake, and, according to the legend, if one calls to him exactly at midnight on the night of a full moon, he will appear above the waters to listen to wishes.

Pretty Alice, Mr. Old Fox's daughter, was mesmerized by this legend. You see, Alice had a special wish, deep in her heart. Read on and you will learn what it was.

Alice and her father lived in a little house that stood at the edge of Great Lake Valley, right near the edge of the wood. The house was divided into two sections: the residential section, which looked out onto the lake, and the business section—where Mr. Old Fox had his store— which faced the wood. And what a wonderful store it was! One could find there all sorts of amazing, special items, and all the residents of Magic Wood loved this store. Mr. Old Fox was wise and cunning, and, like most of his kind, had a great talent for persuasion. Almost no one who visited the store left without some interesting purchase.

In contrast to her father and the rest of the foxes, Alice was quite naïve. She trusted everyone and lacked cunning. Her friends made fun of her and called her "silly and simple." Sometimes, she would fall victim to some prank that they would play on her. Even Mr. Old Fox complained: "Oh, Alice! When the time comes, how will I be able to leave you this store?"

Miserable Alice was hurt by her father's words and decided to ask Goldfish's help. And so, one balmy night at the end of the summer, on a night when the moon was full, Alice came to the lake exactly at midnight and began to call out to Goldfish. All of a sudden the surface of the lake began to swell. At first, few ripples appeared on the surface, but then they were followed by

larger and stronger waves. Finally, on the crest of the largest wave, Goldfish appeared.

"What do you want, little fox?" asked the fish. "What worries you and what is your wish?"

Alice answered, "Dearest Goldfish, I wish to be sensible and cunning like my father and friends."

Alice thought she heard a soft chuckle from the fish, but perhaps it was the rustle of the bulrushes.

Goldfish said, "Sweet little Alice. You are already quite sensible. You ask for cunning, but I'll grant you a quality that is much more precious—wisdom."

Goldfish flipped his tail and a small box made of bulrushes appeared before Alice.

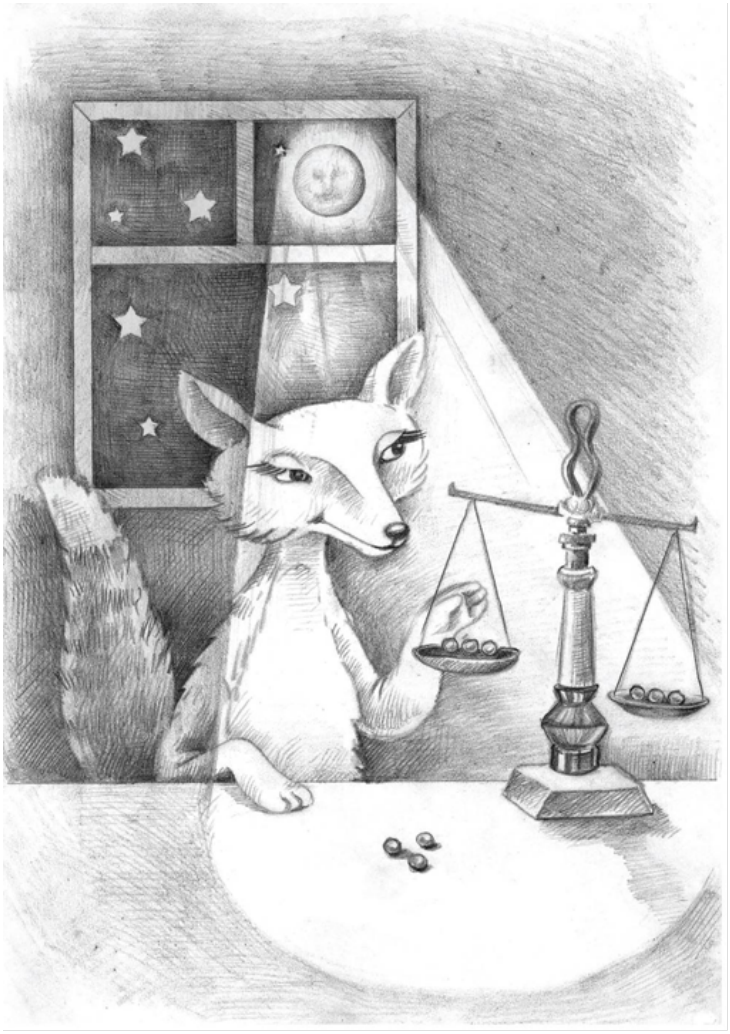
"In this box are nine pearls that appear identical. I got them from the Pearl Oyster, my loyal friend. I have put on one of the pearls a special spell to make it into the Pearl of Wisdom. True wisdom will be granted to anyone who will—alone and without help—successfully discover this pearl amongst the others!"

"But how can I pick out a special pearl amongst nine identical pearls?" Alice asked apprehensively.

"That's your problem. You must be clever! But I will tell you these two things: One, all the pearls are identical in weight, except the Pearl of Wisdom, which is lighter. Two, its existence is transient: in a short while, it will vanish into thin air. So don't dawdle! You must hurry to discover which one it is." And with these words, Goldfish dove off the crest of the wave and disappeared into the depths of the lake.

Clutching the little box, Alice hurried home. Already on her way, she believed that she had come up with a way to discover the Pearl of Wisdom from amongst the other identical pearls. A soon as she got home, she went into the store and grabbed the little balance scale with two pans that her father used to weigh

those small, dainty, precious objects that his customers adored buying.



She took the nine pearls out of the little box, placed them on the table, and divided them into three groups—three pearls to a group. Then, as quick as a wink —indeed, with only two weighings— she had discovered which of the pearls was the lightest.

She had the Pearl of Wisdom in her hand!

> Suddenly, a brilliant beam of light emerged from it—the light of wisdom. As the light kissed Alice on her ears, eyes and forehead, the whole world around her seemed to change: now she felt that she could hear, see and understand many things that she had never even thought about before. This was an amazing feeling, pleasant yet scary at once, for, as it is said, "in much wisdom is much grief: and he who increases knowledge increases sorrow."[1]
>
> It didn't take long until Alice regained her composure and looked at the table. There were only eight pearls left. The Pearl of Wisdom had vanished!
>
> The Pearl of Wisdom had disappeared, but its magic certainly had not, because once wisdom is gained, it is never lost.
>
> "Simple" Alice was now "Wise" Alice. From the pearls that remained, Alice made herself a pretty necklace that she never took off. And when anyone asks her how her amazing transformation from simple to wise occurred, she would smile, touch her necklace, and say, "I simply managed to quickly solve one interesting weighty problem."

The riddle accompanying the story of Alice the Fox who had gained wisdom thanks to the Pearl of Wisdom, comes naturally as a question:

> **How it is possible, using only two weighings on a balance scale, to detect amongst nine identical-looking items one, lighter than the others, identical in weight?**

Sure, dear reader, you have already found the answer to this question, haven't you?

[1] Ecclesiastes 1:18

Interlude

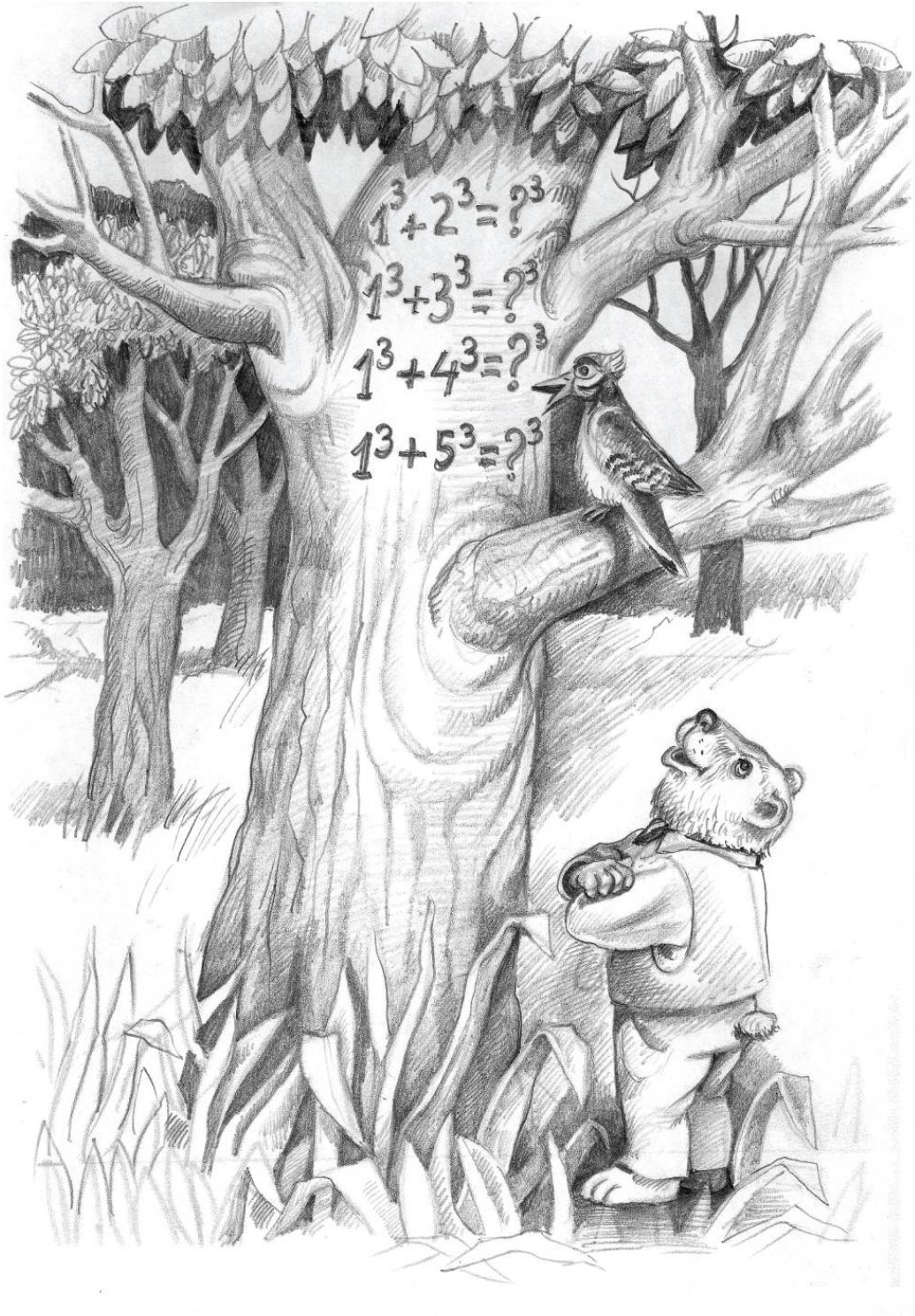
The two tales above allow to understand what kind of tales are to be found in the first part of the book. The first tale is a reinterpretation of one of the problems in the oldest written mathematical document, known as the *Rhind Mathematical Papyrus*. The first essay in the second half of the book tells about the location and content of this document. The second tale is a discrete instance of the well-known problem of a fake coin, formulated in the riddle that accompanies the tale. The fourth essay in the second part of the book tells about the development of the problem, its various versions and their solutions.

In the next two sections we describe two more of the tales from the book *Amazing Tales from the Magic Wood and Famous Problems of Mathematics*.

Tale 6: The Bear and The Three Cubes

In the theory of numbers there are several problems whose formulation is understandable to anyone with basic knowledge of mathematics and which professional mathematicians failed to solve to this day, or have been solved after a long period of hard work. The best known of these problems, related to the so-called 'Fermat's Last Theorem', was investigated both by mathematicians and laypersons for three and a half centuries, till its solution in the late 20^{th} century.

The dramatic story of Fermat's Last Theorem with all its challenges and participants is told in the sixth essay of *Amazing Tales from the Magic Wood and Famous Problems of Mathematics*. At the same time in the parallel world of the Magic Wood, Fermon, one crafty bear, uses the impossibility of finding a nontrivial solution to the equation $x^3 + y^3 = z^3$ using natural numbers in order to obtain a free barrel of honey. How he manages this is told in the story "The Bear and The Three Cubes" in the first part of the book. See an accompanying illustration on the next page.


$1^3+2^3=?^3$
$1^3+3^3=?^3$
$1^3+4^3=?^3$
$1^3+5^3=?^3$

Tale 10: Ants that Tell the Truth and Ants that Lie

In the 1930s the mathematical world was shaken up by Kurt Gödel's discoveries in the field of mathematical logic. Gödel, the Austrian mathematician and logician, proved that in each mathematical theory based on a sufficiently wide system of axioms there will always be a statement that can be neither proved nor disproved with the help of these axioms. The tenth essay in the second part of the book, 'Gödel's Theorem and its Popularization', is dedicated to this issue. Among other arguments it claims that one can decide upon the truth or falsity of such inescapable and indeterminate statements by means of exercising the free will principle to make a definite choice between two opportunities. The necessity of such choice-making comes to the fore in the world of the Magic Wood for Anita, the intrepid traveller.

A young, pretty and clever ant, she is assisted in her explorations of the most complex anthills of the Magic Wood by her ability to negotiate successfully a variety of logical puzzles and conundrums.

Anita has a personal problem: she has not yet succeeded in finding a worthy life-partner. While visiting a certain ant colony it seemed to her that she had finally met just the candidate for this position in the person of a young and attractive male ant Antek. To check her choice Anita addresses a list of suggestions and instructions on her subject composed by the Magic Wood's Council of Judges for the Ant Community. However, it turns out that this list does not provide a unambiguous answer in her case. Anita then decides to discover it by herself. Precisely how? That can be discovered from the last paragraph of the tenth story, "Ants That Tell The Truth and Ants That Lie", the final tale in the Magic Wood cycle.

General Discussion

As we mentioned before, the book *Amazing Tales from the Magic Wood and Famous Problems of Mathematics* is made up of two separate parts, connected in a specific way, and yet lending themselves to being read separately.

The first part is written in the fantasy genre, consisting of ten distinct tales, and is accessible to all self-directed readers of any age. The second part is written in the style of popular essays dealing with the history of mathematics, and is accessible to readers with a basic knowledge of the language of mathematics.

We have already described, with more or less detail, four of the tales that make up the heart of *Amazing Tales from the Magic Wood and Famous Problems of Mathematics*. Here are the names of the remaining six, together with parallel essays from the second part:

2. Deer and Doe Build A Shelter // Dido's Problem

3. Rabbit Games // The Josephus Flavius Problem

5. In Mrs Squirrel's Shop // Bachet's Weights Problem

7. Friends Take A Walk // The Problem of the Seven Bridges of Konigsberg
8. The Annual Spring Pageant of the Migrating Birds // Graeco-Latin Squares and Euler's Thirty-Six Officers Problem
9. The Lion Barber // Russell's Country Barber Paradox

The authors' goal is that, reading the two parts side-by-side, in parallel, readers come to perceive mathematics through the medium of fantasy and see it anew, as a living and dynamic field of human knowledge, one that has place for both reality and imagination. One can expect that the development of such perception will help overcome commonly held distaste towards and fear of mathematics. We feel that tenets of developmental psychology justify our expectations.

In addition to pursuing a therapeutic goal the book pursues the goals of education and enlightenment. The authors write in the Afterword:

> And so, dear readers, our journey to some corner of mathematics' majestic palace is complete. If this visit has aroused in you the desire for more, whether short jaunts or longer treks into the wide world of mathematics, we have achieved our goal. Every meeting with math, every exploration of it on any level, develops thinking skills and teaches us to concentrate on what is essential, filter out the insignificant and make correct conclusions. Such skills are needed by all, in daily life and in business.

We shall finish with the quote of Blaise Pascal opening the book:

> Mathematics is such a serious discipline that one should not miss any opportunity to make it a little entertaining.

We hope that the balance of the entertaining and the earnest attained in the book *Amazing Tales of the Magic Wood and Famous Problems of Mathematics* will grant its readers both pleasure and profit.