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Bounding CPT and Lorentz symmetry violations through ultra-high-energy cosmic rays

C A Escobar[1] **and R Potting**[2]

[1] Instituto de Física, Universidad Nacional Autónoma de México, Apdo. Postal 20-364, Ciudad de México 01000, México
[2] CENTRA and Department of Physics, FCT, University of the Algarve, Campus de Gambelas, 8005-139 Faro, Portugal

E-mail: `rpotting@ualg.pt`

Abstract. We review recent work on CPT and Lorentz violation in the context of the Standard-Model Extension. In particular, we show that, when CPT and Lorentz violation is present in the kinetic terms of any particle in the gauge boson or the lepton sector, this will generally lead to proton decay at sufficiently high energy. Using observational data from ultra-high energy cosmic rays, this has allowed to derive new bounds on the corresponding CPT and Lorentz-violation parameters.

1. Introduction

Lorentz symmetry is a fundamental ingredient of both quantum field theory and General Relativity. Any possible deviation for Lorentz invariance to hold exactly in nature would be a matter of great importance. Suggestions that Lorentz invariance violation (LV) might be originate at the Planck scale come from a number of candidate theories for quantum gravity that have been shown to involve LV as a possible effect. For instance, spontaneous breaking of Lorentz symmetry can come about in theories with Lorentz-invariant dynamics, such as in string field theory [1] or loop quantum gravity [2]. On the other hand, there are theories that violate Lorentz invariance at a fundamental level, such as noncommutative geometry [3], Hořava-Lifshitz gravity [4] or spacetime foam [5].

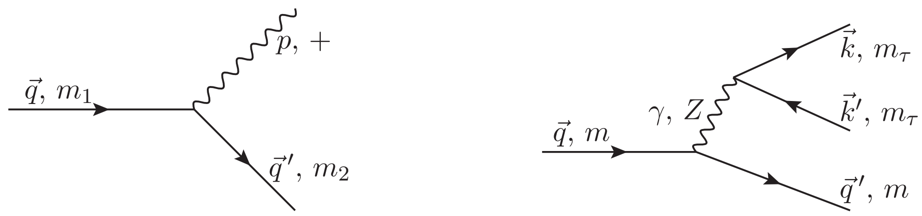
An important development has been the systematic classification of LV effects in the context of low-energy effective field theories, in particular the Standard-Model Extension (SME) [6]. The SME Lagrangian of the matter sector contains all Lorentz-violating gauge-invariant effective operators that can be built using conventional Standard-Model fields, coupled to vector and tensor coefficients that parametrize the LV. In the past two decades in particular, numerous experiments have been performed in order to determine observational constraints on the SME coefficients [7].

LV effects are usually considered to be small deviations to certain physical processes. However, another possibility is that processes which normally are forbidden become allowed in the presence of LV. Here we will consider the case of the Cherenkov-like emission of particles by an incoming fermion. If the latter maintains its identity in the process, or, at least, if the outgoing fermion has the same mass, such a process is obviously disallowed by energy-momentum conservation. However, as we will see, in the presence of a tiny suitable LV term in the Lagrangian, such an emission process becomes allowed if the momentum of the incoming fermion is sufficiently high. What we will explore in the following is the application of this idea to particles that are observed to be stable at very high momentum. Namely, such an observation precludes the possibility that Cherenkov-like emission, with the associated loss of energy of the particle, could ever occur. As a consequence, we can then definitely conclude the absence of any LV process that would turn such an emission possible at the observed energy of the particle.

The strongest bounds on LV can be obtained this way by considering fermions with the highest possible momenta that have ever been observed, namely protons in Ultra-High Energy Cosmic Rays (UHECRs). Here we will report on recent work involving LV coefficients in the kinetic sector of the electroweak gauge bosons [8], as well as in the lepton sector [9]. We will see that in the presence of LV, threshold values exist for the momentum of the protons above which Cherenkov-like emission of such gauge bosons, or of leptons, acquires a nonzero rate. The observation of UHECR protons up till certain ultra-high energies allows us to place bounds on these LV parameters.

2. Cherenkov-like emission of W bosons

We will first consider the process indicated in Fig. 1(a) in which a fermion with a mass m_1 decays to a W boson in the mode $\lambda = +$ and a Dirac fermion with a mass m_2. For simplicity, we will assume here that $m_1 = m_2$. The Dirac fermions are taken to satisfy regular Lorentz-symmetric



(a) Emission of a W-boson

(b) Emission of a tau-antitau pair

Figure 1. Two LV emission processes by an incoming ultra-high-energy fermion

dispersion relations. On the other hand, the quadratic part of the Lagrangian of the W boson field is taken to include a LV and CPT-violating term parametrized by a four-vector k_2^μ:

$$\mathcal{L}_W = -\frac{1}{4}W^*_{\mu\nu}W^{\mu\nu} - \frac{1}{2}m_W^2 W^*_\mu W^\mu + \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}k_2^\mu\left(W^\nu W^{*\rho\sigma} + W^{*\nu}W^{\rho\sigma}\right) . \tag{1}$$

Here $W^{\mu\nu} = \partial^\mu W^\nu - \partial^\nu W^\mu$. The k_2 term arises upon electroweak symmetry breaking from the electroweak gauge sector of the minimal SME (mSME) (consisting of the power-counting-renormalizable terms in the SME) parametrized by the four-vectors k_1 and k_2 [6]

$$\epsilon^{\mu\nu\rho\sigma}k_{1\mu}B_\nu B_{\rho\sigma} + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}k_{2\mu}\left[A^a_\nu A^a_{\rho\sigma} + \frac{2}{3}g_2\epsilon^{abc}A^a_\nu A^b_\rho A^c_\sigma\right] . \tag{2}$$

The W_μ field in Eq. (1) is defined by $W_\mu = \frac{1}{\sqrt{2}}\left(A^1_\mu - iA^2_\mu\right)$ as usual. It can be shown that (1) can be consistently quantized, as long as the components of k_2 are smaller than of the order of m_W, which we will assume to be the case [10].

Lagrangian (1) implies the dispersion relations

$$\Lambda_0(p) = p^2 - m_W^2 = 0, \qquad \Lambda_\pm(p) = p^2 - m_W^2 \pm 2\sqrt{(p\cdot k_2)^2 - p^2k_2^2} = 0 \ , \tag{3}$$

Journal of Physics: Conference Series **1586** (2020) 012015 doi:10.1088/1742-6596/1586/1/012015

for the three helicities of the W boson. As it turns out, the $\lambda = 0$ and $\lambda = -$ gauge-boson polarization modes are timelike for any momentum. On the other hand, it follows from (3) that the gauge-boson momentum is spacelike for the $\lambda = +$ mode whenever

$$(p \cdot k_2)^2 > \frac{1}{4} m_W^4 \ . \tag{4}$$

As we will see below, W bosons with spacelike momenta are a necessary condition for the desired Cherenkov-like processes we will be interested in. For this reason, we will only consider W bosons in the + polarization mode.

2.1. Emission by an elementary Dirac fermion

We will first consider the process indicated in Fig. 1(a) in which the incoming and outgoing fermions are elementary Dirac fermions with masses $m_1 = m_2$ which we take to satisfy regular non-LV dispersion relations.

The expression for the differential decay rate is

$$d\Gamma = \frac{1}{2q^0} \frac{d^3p}{(2\pi)^3} \frac{1}{\Lambda'_+(p)} \frac{d^3q'}{(2\pi)^3} \frac{1}{2q'^0} \left(\frac{1}{2} \sum_{\text{spins}} |\mathcal{M}|^2 \right) (2\pi)^4 \delta^4(q - p - q') \ . \tag{5}$$

It is not hard to show that the condition of energy-momentum conservation imposed by the delta function in (5) corresponds exactly to the condition (4) for p^μ to be spacelike. The squared matrix element $|\mathcal{M}|^2$ is summed (averaged) over the final (initial) fermion spin. The unconventional factor $\Lambda'_+(p) = \frac{\partial \Lambda_+(p)}{\partial p^0}$, in the denominator defines a positive definite normalization in which the phase space and the matrix element are separately observer Lorentz invariant [10], i.e., invariant under simultaneous Lorentz transformations of the momenta and the LV coefficient k_2^μ. Note that for a more conventional phase space normalization this would no longer be the case.

The matrix element corresponding to the process denoted in Fig. 1(a) is

$$i\mathcal{M} = \frac{ig_2}{2\sqrt{2}} \bar{u}(q') \gamma^\mu (1 - \gamma^5) u(q) e_\mu^{(+)*}(p) \ , \tag{6}$$

where $u(q)$ and $u(q')$ are usual Dirac spinors. One can write down similar expressions for the case of anti-particles. The four-vector $e_\mu^{(+)}(p)$ is the gauge-boson polarization vector that corresponds to the dispersion relation $\Lambda_+(p) = 0$.

It can be shown from formula (5) that in order to have a nonzero decay rate, the incoming fermion momentum has to exceed a threshold value:

$$|\vec{q}| > |\vec{q}|_{\text{th}}, \qquad \text{where} \qquad |\vec{q}|_{\text{th}} \approx \frac{M(M + 2m_2)}{2|\kappa|} \ , \tag{7}$$

where κ is a quantity of the order of the components of k_2: $\kappa \sim \pm |k_2^\mu|$ [8]. The gauge bosons are emitted in a very narrow forward cone around the direction of the incoming fermion, $\cos\theta_{pq} = 1 + \mathcal{O}\left(\frac{\kappa^2}{M^2}\right)$, where θ_{pq} is the angle between $\vec{p}$ and $\vec{q}$.

Integrating expression (5) over all W-boson momenta yields the total decay rate:

$$\Gamma = \frac{g^2 |\kappa|}{64\pi} G(a) \theta(a - 1). \tag{8}$$

where

$$G(a) = \ln a - 1 + \frac{1}{a} + \mathcal{O}\left(\frac{m}{M}, \frac{\kappa^2}{M^2} \right) . \tag{9}$$

Here a is defined as the ratio of $|\vec{q}|$ to its threshold value, i.e., $a = |\vec{q}|/|\vec{q}|_{\rm th}$.

A fermion that couples to a CPT-violating W boson will emit W bosons if it has an energy above threshold, each of which carries an energy of at least $|\vec{q}|_{\rm th}$. From Eq. (8) it follows that the decay rate is of the order of 10^{-15} s if $\mathcal{O}(\kappa) = 10^{-7}$ GeV, corresponding to a bound we will find below. It also follows that it will approximately take a time of order $a \times 10^{-15}$ s for all fermions in a decay cascade to fall below threshold for such values of κ.

2.2. Cherenkov emission by a proton

In case the incoming fermion is a proton, the emission of a W boson will provoke a break-up, since the typical momentum transfer lies in the range of the W-boson mass, which is well within the energy range of deep inelastic scattering. The proton decay rate is then of the form

$$\Gamma = \frac{1}{2q^0} \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{4\pi}{\Lambda'_+(p)} e_\mu^{(+)}(p) e_\nu^{(+)*}(p) W^{\mu\nu} \ , \tag{10}$$

where $W^{\mu\nu}$ is the hadronic part, given by

$$W^{\mu\nu} = \frac{1}{8\pi} \sum_\sigma \langle q,\sigma | J^\nu(-p) \sum\!\!\!\!\!\!\!\int_X |X\rangle\langle X| J^\mu(p) |q,\sigma\rangle \ . \tag{11}$$

Here $|q,\sigma\rangle$ is a proton state with momentum q and spin σ and $J^\mu(p)$ is the hadronic current. $\sum\!\!\!\!\!\!\int_X$ indicates a sum over all hadronic final states X along with the corresponding integrations over phase space. $W^{\mu\nu}$ can be evaluated in the parton model (see, e.g., Ref. [12]).

The final result for the proton decay rate is

$$\Gamma = \frac{g^2|\kappa|}{64\pi} \sum_q \int_0^1 dx\, (f_q(x) + \bar{f}_q(x)) G_q(ax)\theta(ax-1) \ . \tag{12}$$

Here the functions $f_q(x)$ and $\bar{f}_q(x)$ are the parton distributions functions (PDFs) for the quarks and antiquarks of flavor q, respectively. They represent the chance of finding a quark with momentum fraction x inside the proton. The function $G_q(ax)$ in Eq. (12) is, up to the corrections of $\mathcal{O}(m/M, \kappa^2/M^2)$, equal to $G(ax)$ in Eq. (9). The integral over x in Eq. (12) can be carried out numerically using fits for the PDFs [8].

We conclude that the threshold value for W emission by a proton to occur is still given by formula (7), while the decay rate will depend on the details of the PDFs, but its order of magnitude value is the same as for the elementary fermion case (at least for values of a not very close to 1).

2.3. Limits from ultra-high-energy cosmic rays

We can use the fact that a proton with an energy above threshold will disintegrate to use astrophysical data to limit k_2^μ. More precisely, such a proton cannot reach Earth if its mean free path L is much smaller than the distance from its source to Earth. Since many ultra-high-energy cosmic ray particles (UHECR) with energies above $57\,\mathrm{EeV} \equiv |\vec{q}|_{\rm obs}$ have been observed, more or less from all directions [13], it follows that $|\kappa| \lesssim \frac{M^2}{|\vec{q}|_{obs}} \approx 1.1 \times 10^{-7}\,\mathrm{GeV} \equiv |\kappa|_0$. We see from (12) that the mean lifetime of protons (in the Earth's frame) t_p is proportional to $|\kappa|^{-1}$. A conservative (large) estimate gives a mean free path of $L \simeq ct_p \sim (\hbar c/|\kappa|_0) \times 10^{15} \sim 10^3\,\mathrm{km}$. It is clear that protons with an energy above this threshold will not be able to reach Earth from any viable UHECR source. This allows us to conclude that

$$|k_2^\mu| < 1.1 \times 10^{-7}\,\mathrm{GeV} \ . \tag{13}$$

Journal of Physics: Conference Series **1586** (2020) 012015 doi:10.1088/1742-6596/1586/1/012015

As it turns out, a bound can be obtained as well on the LV coefficient k_1 in (2) through the ultra-tight bound which exists on the components of the combination $k_{AF} = 2\cos^2\theta_w k_1 + \sin^2\theta_w k_2$ which parametrizes the Carroll-Field-Jackiw term in the photon sector [11]. One finds that

$$|k_1^\mu| < 1.7 \times 10^{-8}\,\text{GeV}\,. \tag{14}$$

3. Cherenkov-like emission of a tau-antitau pair

Next we consider the case of emission of a tau-antitau pair by an incoming fermion through a virtual photon or Z boson, see Fig. 1(b). We will assume that the incoming (= outgoing) fermion has mass m, with non-LV dispersion relation, while the quadratic Lagrangian for the tau and antitau includes a Lorentz- and CPT-violating contribution from the mSME:

$$\mathcal{L}_\tau = \bar{\psi}\left(i\gamma^\mu(\partial_\mu + \gamma^5 b_\mu) - m_\tau\right)\psi\,. \tag{15}$$

As it turns out, the dispersion relations for the spin components following from Eq. (15) are

$$\Lambda_\pm(k) = k^2 - m_\tau^2 - b^2 \pm 2\sqrt{(k\cdot b)^2 - k^2 b^2} = 0\,. \tag{16}$$

Note that these are exactly analogous to the LV dispersion relations we considered for the W boson in (3).

The emission can be mediated by either a photon or a Z boson. Here we will just consider the photon process. Adding the contribution of the intermediate Z boson gives a small correction, which we will ignore here (for details, see [9]).

3.1. Emission by an elementary Dirac fermion

Let us consider again first the process for an incoming elementary Dirac fermion. Analyzing energy-momentum conservation for the emission process shows it is only possible if the incoming fermion has a momentum exceeding the threshold value

$$|\vec{q}_{th}| = \frac{m_\tau(m_\tau + m)}{\xi_{b,q}}\,, \qquad \xi_{b,q} = \left|b_0 - |\vec{b}|\cos\theta_{bq}\right| \tag{17}$$

where θ_{bq} is the angle between the vectors $\vec{b}$ and $\vec{q}$. At threshold, $\vec{k}$, $\vec{k}\,'$, $\vec{q}$ and $\vec{q}\,'$ are all collinear, while $|\vec{k}| = |\vec{k}\,'| = \frac{m_\tau}{m+2m_\tau}|\vec{q}|$ and $|\vec{q}\,'| = \frac{m}{m+2m_\tau}|\vec{q}|$. The differential decay rate can be written as

$$d\Gamma = \frac{1}{2q^0}\frac{d^3k}{(2\pi)^3}\frac{1}{\Lambda_+'(k)}\frac{d^3k'}{(2\pi)^3}\frac{1}{\Lambda_+'(k')}\frac{8\pi e^4}{(k+k')^4}\,\text{Tr}[\bar{u}_\tau(k')\gamma^\mu v_\tau(k)\bar{v}_\tau(k)\gamma^\nu u_\tau(k')]W_{\mu\nu}\,. \tag{18}$$

where

$$W_{\mu\nu} = \frac{1}{16\pi}\frac{d^3q'}{(2\pi)^3}\frac{1}{2q_0'}\sum_{\text{spins}}\text{Tr}[\bar{u}(q')\gamma_\mu u(q)\bar{u}(q)\gamma_\nu u(q')](2\pi)^4\delta(q - q' - k - k')\,. \tag{19}$$

Note that the tau-lepton spinors in Eq. (18) satisfy a modified Dirac equation, corresponding to Lagrangian (15).

While evaluating the decay rate analytically from Eq. (18) is not feasible, it can be worked out in the limiting cases just above threshold and for momenta far above threshold. It follows

$$\Gamma = \frac{e^4|\xi_{b,q}|}{2\pi^2}H(a)\theta(a-1) \tag{20}$$

where

$$H(a) = \begin{cases} \dfrac{2m_\tau^2}{\sqrt{m(m+m_\tau)^3}}(a-1)^2 & \text{if } a - 1 \ll 1; \\ A_1 + A_2\ln a + \ldots & \text{if } a \gg 1\,. \end{cases} \tag{21}$$

Here A_1 and A_2 are constants of order 1 that depend on m and m_τ.

3.2. Cherenkov emission by a proton

If the incoming fermion is a proton of momentum q, expression (19) for $W_{\mu\nu}$ will take the form of Eq. (11), with the substitution $J^\mu(q) \to J^\mu(k+k')$. It can be shown that for ultra-high momenta, $|\vec{q}| \gg |\vec{q}|_{th}$, the momentum transfer in the proton's rest frame is tiny, so that the rate formula for the elementary fermion applies. However, for small $|\vec{q}|$, the momentum transfer in the proton's rest frame is large, leading to a desintegration of the proton. In that case, we are in the deep inelastic scattering regime. The appropriate treatment is then to use a parton model calculation, just like in section 2.2. The threshold value for the tau-antitau emission to occur will be given by formula (17). The decay rate will depend on the details of the PDFs, but its order of magnitude value is the same as for the elementary fermion case.

3.3. Limits from ultra-high-energy cosmic rays

Just like in section 2.3, a bound on the LV parameter can be extracted by using that protons have been observed in UHECRs of energies above 57 EeV. In this case, this allows us to derive the upper bound

$$\xi_{b,q} \approx |b^\mu| < \frac{m_\tau(m_\tau + m)}{|\vec{q}|_{obs}} \approx 8.5 \times 10^{-11}\,\text{GeV}\,. \tag{22}$$

One can show that a conservative estimate for the mean free path of the protons becomes, in this case, $L \simeq 3 \times 10^9$ km. Once more, this value is much below the distance to any viable UHECR source.

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