Sustainable Group Sizes for Multi-Agent Search-and-Patrol Teams

Musad Haque, Magnus Egerstedt, and Clyde Martin

***Abstract*— We identify sustainable sizes for a multi-agent system that consists of two classes of agents: one class is responsible for searching an area; the other for providing perimeter security for that area. In this context, sustainability means the ability of the system to accomplish the task while balancing shared resources. Bio-inspired rules based on the pride structures of African lions are developed to determine the sustainability of a group size.**

I. INTRODUCTION

Problems such as formation control, consensus, and containment have been exhaustively studied in the field of multi-agent systems (for a representative sample, see [1], [2], [3], [4]). These problems typically involve issues pertaining to multi-agent classifications such as heterogeneity, distributiveness, communication, etc. However, the size of a multi-agent system is rarely addressed outside of the multi-robot foraging literature.

Foraging, as understood in the multi-robot context, involves agents that search and contain objects in the environment, with applications found for example in search and rescue scenarios. For pure foraging tasks, the issue of sustainable sizes for a multi-agent system has been previously addressed (for a representative sample, see [5], [6], [7]) and the idea is to let the number of agents be selected based on social foraging theory, in which the performance of a group increases with size until a critical number is reached [6].

This idea is applicable only if *all* the agents are performing the same task. The novelty of this work is that we determine sustainable sizes for a heterogeneous multi-agent system that consists of a foraging team together with a team providing boundary protection by patrolling the perimeter of the foraging area. Our setup can be applied to a network of unmanned vehicles, e.g. UAVs, that consists of a team of searchers and another team that maintains air supremacy for the searchers. To find sustainable sizes for our engineered system, we draw inspiration from natural systems; in particular, we consider the social structures of African lions.

A pride of African lions contains up to 9 males and 18 females [8]. If the size of the pride grows too large, members often break away and factors such as food availability and the ability to ward off intruders influence the size of a pride [9]. Females are usually in charge of foraging for food, while males are responsible for territorial defense. With this biological system as our inspiration, we analyze sustainable sizes for a multi-agent system by producing a simple, yet expressive model that can be applied to engineering applications such as search-and-patrol using teams of autonomous vehicles.

The rest of the paper is organized as follows: Section II describes how connections are made between the search-and-patrol task and the social structures of African lion prides. In Section III we present our model of the pride that is used to determine a set of sustainable pride sizes and in Section IV, we introduce a notion of utility for a group and identify the optimal group size. Finally, conclusions are presented in Section V.

II. BIOINSPIRATION

African Lions, *Panthera leo*, live in well-defined social structures known as prides. Typically, these prides consist of $1-3$ adult males and $2-9$ adult females along with their dependent cubs [10]. Males that attach themselves to a group of lionesses, also known as the resident males, gain significant reproductive advantages over solitary males [12]; as a result, resident males must frequently defend their lionesses from non-resident males [11]. If a group of males successfully take over a pride, by defeating the original resident males, it first ejects the original group of males and then kills their cubs [11]. They breed new cubs with the lionesses to start their own pride; thus, territorial defense is an important task for males to protect their cubs from infanticidal males and in turn, increase genetic fitness [10].

When lionesses have a low success rate of catching a certain prey, they utilize a highly-coordinated group hunting technique where lionesses in the "wing" positions will entrap their prey by driving them towards the lionesses in "center" positions [11], as shown in Fig. 1(a). Prey caught by females are shared by the entire pride, with males being the first to "claim their share" [8].

Too many females reduce the ability to coordinate and catch prey [13], whereas, too many males result in frequent in-fighting to gain access to females [8]. We take such factors into account, along with other parameters such as food availability to develop our model of the pride. However, recall that our goal is not biomimicry, i.e. to replicate all aspects of the natural system; instead, our goal is to extract characteristics and draw on biology for engineered applications. However, it is important for our model to be rich; in fact, we will show that when we apply biological field data, such as the encounter rate with prey, our sustainable group sizes closely resemble actual pride sizes.

A potential application of our work is determining the sustainable sizes for the US Navy suppression of enemy

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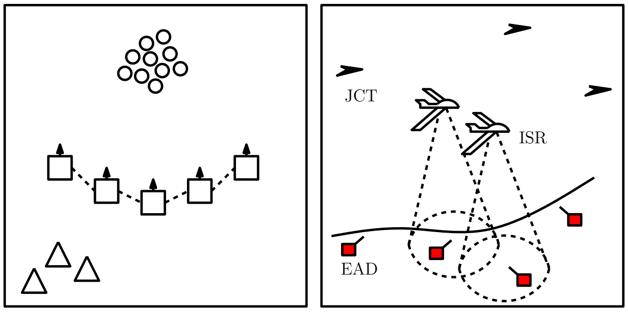


(a) (b)

Fig. 1. In (a), the pride structure is shown; males (triangles) patrol the pride area, whereas female lions (squares) are in charge of foraging for prey (circles). A search-and-patrol application is shown where combat teams (labeled JCT) provide security for surveillance teams (labeled ISR) that are searching for vehicles (labeled EAD).

air defenses (SEAD) missions conducted by teams of autonomous vehicles. This mission involves gathering intelligence on the enemy's air defenses (e.g., nature, location, etc.) [14]. The intelligence gathering aspect of the mission is to be carried out by a team of UAVs known as the Intelligence, Reconnaissance, and Surveillance (ISR) team and another team known as the Joint Combat Team (JCT) maintains air supremacy for the ISR team, as shown in Fig. 1(b). Thus, with engineering applications at the back-end of bio-inspired work, we require the developed model to be transferable as well as expressive.

Our main contribution in this paper is the following: for a given number of males and females, we determine whether the group is sustainable and by sustainable, we mean that the females can forage sufficient prey to feed the entire pride and at the same time, there are an adequate number of males to patrol the territory.

III. PRIDE MODEL

According to our notion of sustainability, a sustainable group size should successfully forage enough food for the entire group and at the same time provide territorial defense. We will present metrics for both these tasks and provide simulations for different values of the parameters used in this model, such as the number of females a male will consort at a time.

A. Foraging

Assume that a pride consisting of M males and F females requires a minimum of $P_{min}(M, F)$ of food to survive. If a lion and a lioness needs an average of p^M and p^F kg/hr of food, respectively, then the minimum energy intake required by the pride is $P_{min}(M, F) = Mp^M + Fp^F$ kg/hr (typically, $p^M = 2.5$ and $p^F = 2.2$ according to [9]) .

We will assume that only lionesses are responsible for foraging and hunting prey. Lionesses are capable of hunting small prey like warthogs (*Phacochoerus aethiopicus*) and wildebeest (*Connochaetes taurinus*) on their own. However, their success rate is low with larger prey, such as zebra (*Equus burchelli*) and buffalo (*Syncerus caffer*); in fact, hunting a large prey like a buffalo requires males to join the hunting group as well [11], [15]. We assume that the pride is specifically hunting zebras, a medium sized prey that is large enough to require a group of females, but small enough to exclude the participation of males.

The success rate of capturing prey increases with group size if the success rate of a lone hunter is low [13]; however, a foraging group too large becomes conspicuous to prey and takes away its ability to stalk prey [8]. The type of prey lions hunt can out run them [8] and this is why "stalking" prey is important to a successful hunt.

Let $Pr(F) \in [0,\ 1]$ be the probability of an encountered prey being captured when there are F females in the pride. $Pr(F)$, inspired by the group performance curve in social foraging theory [16], is quadratic in F with maximum value at $F = F'$. More specifically,

$$Pr(F) = \begin{cases} \frac{F(2F'-F)-(2F'-1)}{(F')^2-2F'+1} & \text{if } 1 < F < 2F'-1, \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where F' is the optimal number of hunters. This is a simple formulation that captures the following key ideas: a single lioness cannot capture a medium sized prey, the amount of prey caught generally increases with the number of hunter, and yet too many hunters takes away their ability to remain inconspicuous to their prey (typically $3 - 8$ lionesses are observed in the hunt according to [8]).

Table 5 in [9] lists the encounter rate with zebras for Serengeti lions during both prey scarcity and abundance. The encounter rate, λ, is measured in a $2000\ km^2$ area and has the unit $herd/hr$. For that area, $\lambda = 0.008\ herd/hr$ represents a scarce zebra density and $\lambda = 0.245\ herd/hr$ was considered an abundant zebra density. The number of individuals in a herd and the average weight of an individual prey is also provided in [9] and with these data, we can calculate λ in units of $kg/km^2/hr$. With this formulation, the encounter rate with zebra would become $\lambda \in [0.8, 24.6]\ kg/km^2/hr$.

We assume that the foraging area is a circle with radius $R(M, F)$ and thus, for a given number of females, F, and encounter rate, λ, the expected amount of prey captured is

$$P_{cap}(F) = Pr(F)\lambda\pi R(M, F)^2, \tag{2}$$

measured in kg/hr. If we assume that the lionesses select the size of the foraging area such that $P_{cap} = P_{min}$, then the radius of the foraging area is given by

$$R(M, F) = \sqrt{\frac{P_{min}(M, F)}{Pr(F)\lambda\pi}}, \tag{3}$$

measured in km and a size is sustainable from a food availability point of view if $0 < R(M, F) < \infty$. Even if a pride is sustainable energy-wise, it might still be defense-wise unsustainable. Next, we look at the factors that determine the ability for males to protect the pride from potential intruders.

B. Territory defense

We described how the lionesses specify the radius of the foraging area, $R(M, F)$, to meet the energy demand of

$P_{min}(M, F)$. If we assume that the foraging area set by the females is indeed the pride area, then males need to defend a circle with radius $R(M, F)$ from intruders. Furthermore, we will also assume that the males reside in the boundary of this circle, equidistant from each other.

Lions communicate through roars, that can be heard about $8\ km$ away during territorial advertisement and when intimidating intruders [8]. With this notion of a limited communication range of a lion, we can define the minimum number of males needed to patrol the pride area, M^d_{min}, as follows:

$$M^d_{min}(R) = \frac{2\pi R(M, F)}{\Delta},$$

where Δ is the maximum allowable arc length between two males that prevents intruders from entering into the pride area and the subscript "d" is used to denote defense. Note that our notion of territory defense does not depend on actual confrontation with intruders, rather it depends on the ability of the males to "plug holes" in the boundary of the pride area.

In a pride, males are also competing with each other to mate with females and we model this "constant competition" [8], or in-fighting, by assuming that each male consorts k females and each non-consorting male is involved in a fight with a consorting male. According to [17], there can be "serious fights" between consorting and non-consorting males and in our model, we regard such in-fighting among resident males as a distraction from their primary role of patrolling the pride area.

We let the number of males patrolling the pride be given by the function $G(M, F)$, and only require that $G(M, F) = M$ when $M \leq F/k$ and that $G(M, F)$ is decreasing with increasing $M - F/k$. One formulation that satisfies both of these requirements is

$$G(M, F) = \begin{cases} M & \text{if } M \leq \frac{F}{k}, \\ M - 2(M - \frac{F}{k}) & \text{otherwise.} \end{cases}$$

A pride size is sustainable from a territory defense perspective if $G(M, F) \geq M^d_{min}(R)$. Finally, a group size, which we denote by the ordered pair (M, F), is considered sustainable if it is both energy-wise and defense-wise sustainable. More precisely, we have the following definition:

Definition 3.1: (Sustainable set): The set $\mathcal{S} = \{(M, F) \mid 0 < R(M, F) < \infty,\ G(M, F) \geq M^d_{min}(R)\}$ denotes the set of sustainable group sizes.

Within the sustainable set of prides, a smaller radius is more desirable by the lionesses since a smaller area reduces encounters with intruders, which will in turn ensure more safety to their cubs [8]. Also, within the set of sustainable prides, more patrolling males will guarantee more "cushion" from non-resident males; thus, it is likely that the biological system itself has a preference on the size of the group, but since we intend the artificial systems to draw from nature (and not the other way around), in the next section, we address this idea of assigning values to group sizes within the sustainable set in the context of engineered systems.

Simulations based on this model are shown in Figs. 2-4. In Fig. 2, the effects of varying the number of females each male consorts, k, on the sustainable sizes are shown. As k increases, each male guards more females and this increases the number of non-consorting males in the pride. Thus, in-fighting increases with k, which distracts more males from patrolling the territory and makes the pride more susceptible to intruders. As a result, the set of sustainable prides decreases as k increases, when λ, F', Δ are all held constant.

In Fig. 3, the effects of varying the optimal number of foragers, F', is shown. As F' increases, from (1) we notice that the support of the function $Pr(F)$ increases; thus, the set of sustainable group sizes increases too. The encounter rate with prey, λ, is varied in Fig. 4. For a large λ, the foraging radius is small and as a result, the foragers can meet the energy intake for larger prides compared to those under smaller values of λ. Our group sizes mostly consist of $1 - 3$ males $2 - 9$ females, using field data recorded in [8], [9], [13] for λ, F', k, and Δ. Thus, our simple mathematical model is in fact expressive enough to capture the underlying structures of lion prides.

In the next section, we assign a utility to each sustainable group size and consequently, identify an optimal group size.

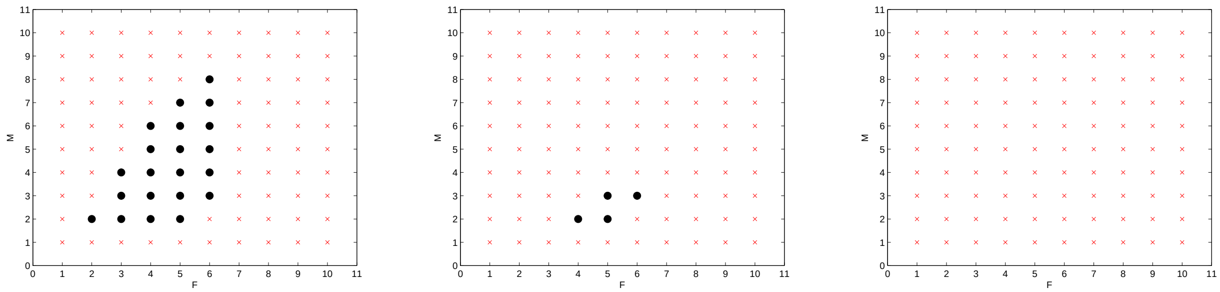
IV. OPTIMAL GROUP SIZE

In the previous section, for a given group size (M, F) we developed a method to characterize it based on parameters like the communication range of males, Δ. For engineered systems, it may also be useful to know the optimal group size, ($M^\star$, $F^\star$), from the set of sustainable sizes. We define a utility function based on our multi-agent search-and-patrol task as follows:

$$U(R, G) = \omega \frac{1}{R(M, F)^2} + (1 - \omega) G(M, F), \tag{4}$$

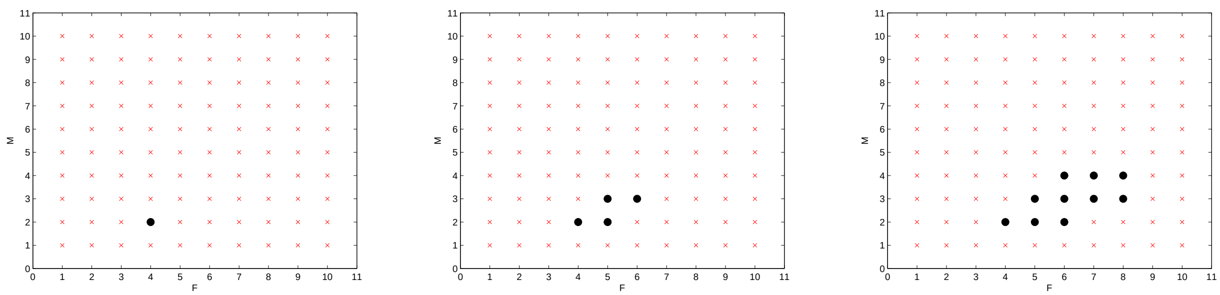
where $\omega \in [0, 1]$. With this particular choice of a utility function, when $\omega = 1$, the optimal size minimizes the radius of the foraging area for lionesses and for a group of UAVs, this corresponds to the size that minimizes the radar footprint of the joint ISR and JCT fleet described in Fig. 1(b). Also, $\omega = 0$, could correspond to the scenario that requires the JCT team to provide the maximum possible security to the ISR team. Optimal group sizes for different values of ω are shown in Fig. 5.

Recall that $\mathcal{S}$ is the set of ordered pairs that represent the sustainable group sizes. Given $\mathcal{S}$, we define $\mathcal{M} = \{M \mid (M, F) \in \mathcal{S}\}$, i.e. the projection of $\mathcal{S}$ onto the first coordinates (males) and define $\mathcal{F} = \{F \mid (M, F) \in \mathcal{S}\}$, i.e. the projection of $\mathcal{S}$ onto the second coordinates (females). With this notation, we are now ready to present the results of this paper:



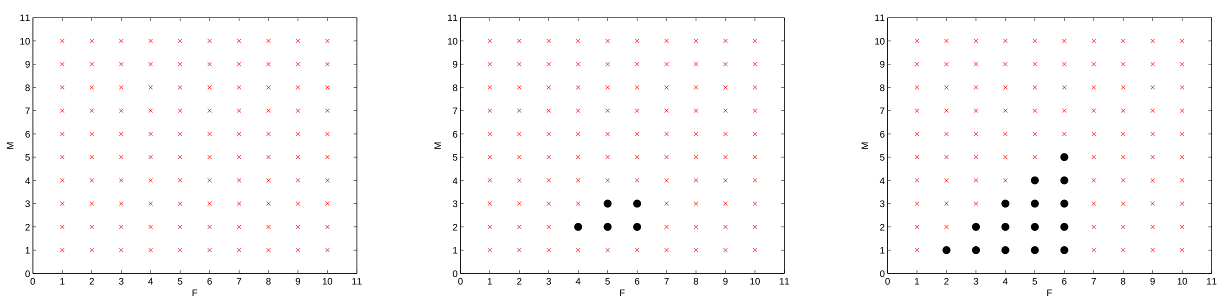
(a) $k = 1$ (b) $k = 2$ (c) $k = 3$

Fig. 2. Sustainable group sizes for different values of the number of females a male can consort at a time, k, are denoted with an dot, while unsustainable group sizes are denoted by a cross. In (a)-(c), the values of other parameters are as follows: $\lambda = 6$, $F' = 4$, and $\Delta = 10$.



(a) $F' = 3$ (b) $F' = 4$ (c) $F' = 5$

Fig. 3. Sustainable group sizes for different values of the optimal number of foragers, F', are denoted with an dot, while unsustainable group sizes are denoted by a cross. In (a)-(c), the values of other parameters are as follows: $\lambda = 6$, $k = 2$, and $\Delta = 10$.



(a) $\lambda = 1$ (b) $\lambda = 8$ (c) $\lambda = 24$

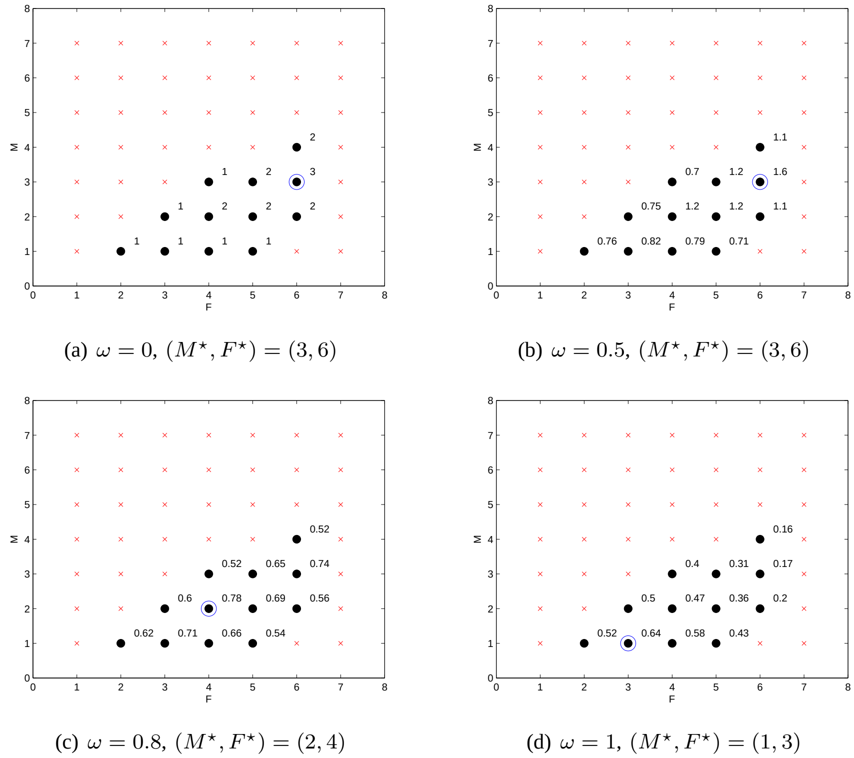
Fig. 4. Sustainable group sizes for different values of the encounter rate with prey, λ, are denoted with an dot, while unsustainable group sizes are denoted by a cross. In (a)-(c), the values of other parameters are as follows: $F' = 4$, $k = 2$, and $\Delta = 10$.

Lemma 4.1: (Maximum security): If $\omega = 0$ in the utility function given by (4) and we denote F_{max} as the largest element in $\mathcal{F}$, then the optimal group size is $(\frac{F_{max}}{k},\ F_{max})$

Proof: If $\omega = 0$ in (4), then the utility function is given by $U(R, G) = G(M, F)$. For a given size $(M, F) \in \mathcal{S}$, the function $G(M, F)$ is is maximized when $M = \frac{F}{k}$, as shown in Fig. 6. As a result, $U(R, G)$ is maximized over $\mathcal{S}$ when $F = F_{max}$ and $M = \frac{F_{max}}{k}$, where F_{max} is the maximum element in $\mathcal{F}$. ■

Lemma 4.2: (Minimum footprint): If $\omega = 1$ in the utility function given by (4), and we denote M_{min} as the smallest element in $\mathcal{M}$, and let $\tilde{F}$ minimize $\frac{P_{min}(M_{min},F)}{P_{cap}(F)}$, then the optimal group size is $(M_{min}, \tilde{F})$.

Proof: If $\omega = 1$ in (4), then the utility function is given by $U(R, G) = \frac{1}{R(M,F)^2}$. For two sustainable sizes (M_1, F) and (M_2, F), it is obvious that $R(M_1, F) < R(M_2, F)$ if $M_1 < M_2$. Thus, for the optimal group size $(M^\star, F^\star)$, $M^\star = M_{min}$, where M_{min} is the smallest element in $\mathcal{M}$ and from the definition of $P_{cap}(F)$ in (2), $U(R, G)$ is minimized over $\mathcal{S}$ when $F^\star$ minimizes $\frac{P_{min}(M_{min},F)}{P_{cap}(F)}$ $\forall\ F \in \mathcal{F}$. ■



(a) $\omega = 0$, $(M^\star, F^\star) = (3, 6)$

(b) $\omega = 0.5$, $(M^\star, F^\star) = (3, 6)$

(c) $\omega = 0.8$, $(M^\star, F^\star) = (2, 4)$

(d) $\omega = 1$, $(M^\star, F^\star) = (1, 3)$

Fig. 5. The optimal group size (ring) is shown for different values of ω. The sustainable group sizes are denoted with a dot, while unsustainable group sizes are denoted by a cross. In (a)-(d), $\lambda = 15$, $k = 2$, $F' = 4$, and $\Delta = 10$.

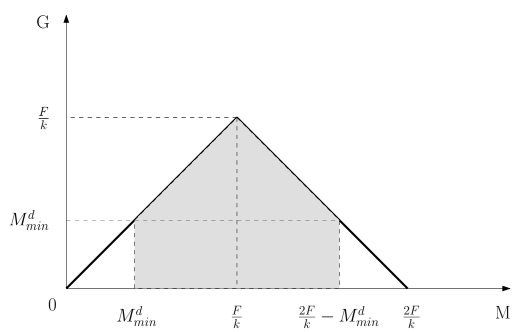


Fig. 6. A plot of the function, $G(M, F)$, which represents the number of patrolling males is shown.

V. CONCLUSIONS

African Lions, *P*anthera leo, live in well-defined social structures known as prides, that can contain up to 9 males, 18 females, and their dependent cubs. Factors such as food availability and territory defense affect the size of a pride and we take these factors into consideration to identify sustainable sizes for a multi-agent system that consists of two classes of agents: one class is responsible for searching an area; the other for providing perimeter security for that area. With engineering applications as a possible back-end of our bio-inspired work, we developed a simple mathematical model of the pride. Moreover, set parameters such as the encounter rate with prey, average energy intake of adults, etc. to biological field data, our sustainable group sizes closely resembled actual pride sizes.

VI. ACKNOWLEDGMENTS

This work was supported by The Office of Naval Research through the MURI, Heterogenous Unmanned Network Teams (HUNT).

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