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Abstract: In this paper, a topology defragmentation method is developed for co-placements of Phasor Measurement Units (PMUs) and their communication infrastructure (CI). Electric power networks are defragmented into sets of branches and a realistic cost model, based on reports from industry, is developed. Instead of considering channel limitations, a more practical consideration of the presence of Dual-Use Line Relays and PMUs with different channel capacities is used to obtain a least-cost solution for specified levels of observability. Formulations are proposed to address budget limitations, to maximise benefits from add-on application, and to enhance application-sensitive deployments. The approach is demonstrated on a number of IEEE test networks and results reflect practical situations where optimal solutions (especially PMU buses and the capacity of deployed PMUs) depend on the availability of equipment, existing devices and the level of observability specified. Solutions obtained specify the bus, branch, PMU type, and PDC to connect to and are optimal without the need for algorithmic parameter tuning. It is established, among other conclusions, that placement results should transcend beyond mere statements of the number of installations, but on the specifics of the deployments at the buses, branches and storage locations.

1 Introduction

Utilities are often faced with the challenge of balancing the benefits of improved monitoring and the costs of investing in the monitoring devices in such a way that the investment costs do not outweigh the benefits in the long or short term [1]. A cost/benefit analysis would often be needed to determine whether the observability of the whole grid or of only a selected number of buses and lines are desired. Whichever the case may be, in order to develop a viable PMU placement solution, it is necessary to consider the key variables that affect installation costs[2]. Chief among these is the cost of laying CIs, which require considerable investments especially for standalone PMU units. In [3–8] PMU installation cost reduction strategies were proposed through simultaneous placements of the measurement devices and their communication links. In particular, [4] considered the costs of switches and routers as well as the length of the transmission media in their formulations. More recently, using a practical cyber-security network, [7] described, in addition, the use of repeaters in order to minimise propagation delay. Most of these works use mainly meta-heuristic methods (e.g Genetic Algorithms, GA [4, 7], or the modified form of GAs [3]) to obtain the co-placement solutions. Although meta-heuristic methods can provide multiple optimal solutions over a wide range of functions, they are computationally inefficient. Moreover, their performance depends on tuning of several algorithmic parameters, and hence global optimality may not be guaranteed.

For co-placements of CI and PMU, some methods such as [8] have discussed the placement of a Phasor Data Concentrator (PDC) at every PMU location. This would imply that every connection to the regional or central PDCs would require a separate CI. In modern electricity networks, several entities own parts of the network and may wish to protect access to data on their local network by having their own PDCs. In this context, we examine a more cost-effective approach in which PMUs are connected to the nearest local PDCs. Each local PDC can be connected to a regional PDC or to a set of cooperative PDCs. Each regional PDC is then connected to a central PDC [9]. Figure 1 shows an example of this hierarchical structure.

A PMU channel capacity can be defined as the maximum number of simultaneous measurements of voltage and current phasors which can be obtained from the device. In practice, PMUs may lack the capacity to measure all current phasors along the branches connected to the buses they are installed at. This can lead to a lack of observability of the unmeasured branches, or to an increase in the total number of installed PMUs. Although the effects of PMU channel limitations have been examined in some works([11–15]), the majority of them did not consider the presence of multiple channel capacities in their setups. While such a non-homogeneous mix was examined in [11], only two different capacities were considered at a time in the results. However, this paper considers the presence of non-homogeneous channel capacities to drive down costs, minimise channel wastage in order to reflect more practical scenarios. In the context of this paper, the non-homogeneous mix includes DULRs and PMUs with higher channel capacities.

A Dual-Use Line Relay (DULR) is capable of measuring voltage phasors at either ends of the buses to which it is connected and the current phasor along a single connecting branch between the buses. The data reporting rate of the DULR is about 30 samples/s which is the same as that of a traditional PMU. The possibility of driving down the installation costs by using, exclusively, DULRs with extensible PMU capabilities has been explored in [2]. A multi-objective minimisation of the costs of DULRs, CIs, standalone PMUs, PDCs, and substation shut-down was also developed in [10]. Although both of these works([2] and [10]) used ILP which guarantee optimality under respective situations, only an exclusive use of DULRs was considered in [2]. Since phasor measurement is only an added functionality of the DULR, careful calibration is often needed before they can be used.

The conventional approach to placement has been on the minimisation of costs leading to the specification of buses for placements of measurement devices. A more qualitative strategy would be to specify, in addition, the actual branches for the installation, the size or capacities of the measurement devices to install, and the PDC to connect the device to for storage, data analysis or data preprocessing. This paper focuses on the development of a topology defragmentation approach which gives these qualitative solutions. Furthermore, this work demonstrates the cost savings that may be obtained from modelling the effect of equipment availability, the enhancement of the prospect of adopting WAMS by accounting

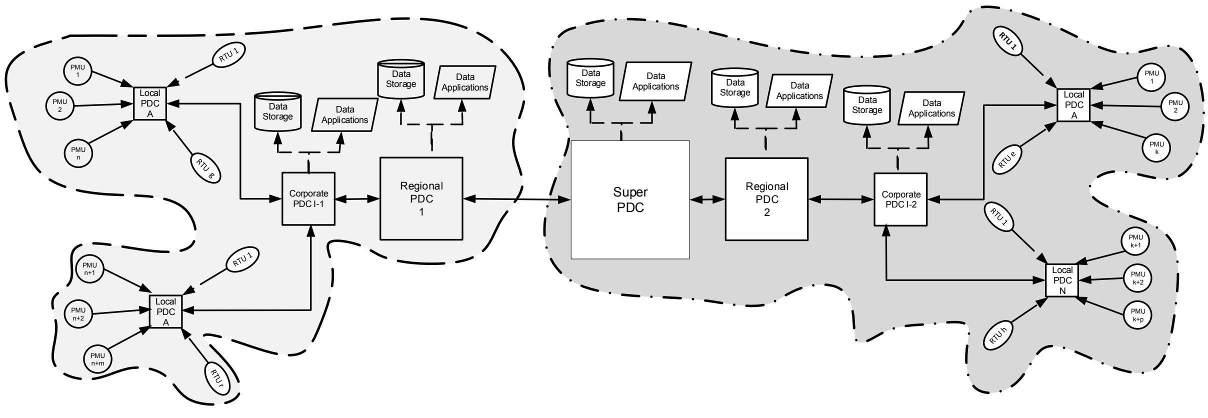

Fig. 1: A hierarchical PDC structure, (as in [9])

for financial limitation, describing the execution of the approach for application-sensitive deployments, and modelling the benefits of extra incentives that vendors add to their measurement devices in the form of measurement-based application. This may help utilities to consider the costs of WAMS in the light of the benefits theyhave to offer with a more viable knowledge on how they may proceed with investment in measurement devices within their budgets.

In this context, this paper provides the following contributions:

1. Develops a practical but flexible PMU installation cost model and then obtains an optimal PMU-CI co-placement solution using this model. Previous co-placement approaches ([3–7, 10]) did not consider non-homogeneous channel capacities and corresponding variability in channel costs.
2. Proposes a novel implementation of a topology defragmentation method to solve the co-placement problem using the PDC hierarchy suggested in [9] (Figure 1). This approach guarantees optimality without the need for parameter tuning, in contrast to the meta-heuristic algorithms [3, 4, 7]. This approach essentially extends [11] to the co-placement problem with the consideration of more non-homogeneous capacities and channel minimisation.
3. Computes, using the proposed approach, the actual channel capacity of measurement device to be installed by considering the presence of a non-homogeneous mix of channel capacities. This computation is different from methods which consider only channel limitation [12–15] and those that did not consider the limitation ([18]).
4. Develop budget-constrained and an adjusted cost formulations (considering add-on application benefits) for the placement and co-placement problems.
5. Proposes method to reduce cost through multiple placements at a single bus.

2 Coplacement Formulation using Topology Defragmentation

This paper formulates the co-placement problem using an extended form of the Set Cover Problem (SCP) approach described in [11]. The formulated problem has been solved using Integer Linear Programming (ILP) to compute an Optimal PMU Placement (OPP) solution. The method is applicable to any transmission or distribution network with a given topology, and is illustrated below.

For every branch connected to bus j, it is possible to form a set S_j of each branch connected to bus j in turn. It follows that if bus j has more than one branch, the number of formed subsets S_j would be more than one. Consequently, we identify each subset of set S_j as S_{jr} using r as the counter. For example, in Figure 2, which shows the WSCC 9-Bus tests system, the number of branches connected to Bus 6 is greater than one, therefore the various subsets $S_{61} = \{6, 7\}, S_{62} = \{6, 5\}$, and $S_{63} = \{6, 3\}$. Similarly, for Bus 1, $S_{11} = \{1, 4\}$, but since there is only one branch connected to Bus 1, $S_{12} = S_{13} = \{\}$. Notice that each subset S_{jr} contains j, meaning that j is the dominant element in all sets formed from branch connections to it. As a definition, we refer to the dominant element j as the *centre* of all subsets formed from it. For subsets involving taking 2 branches at a time, we extend the subscripted indices and define a new subset S_{jlr} with l identifying the number of branches in the set S_{jlr}. In this manner, the previous subsets formed for Bus 6 for single-branch connections ($l = 1$) would be $S_{611} = \{6, 7\}, S_{612} = \{6, 5\}$, and $S_{613} = \{6, 3\}$, and the subsets for $l = 2$ would be $S_{621} = \{6, 3, 5\}, S_{622} = \{6, 3, 7\}$, and $S_{623} = \{6, 5, 7\}$. If E_j is the number of branches which are connected to

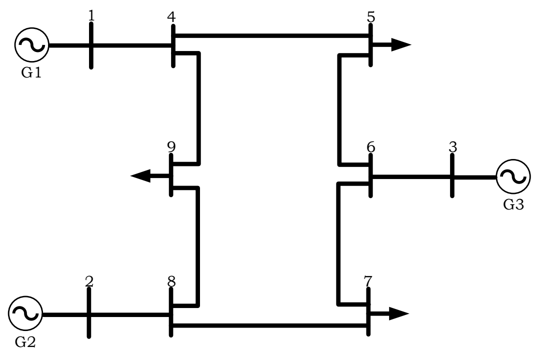

Fig. 2: WSCC 9-Bus Test System

the bus j, then the total number number of subsets for each set of a bus j at a given l is,

$$r_{jl} = \begin{cases} \frac{E_j!}{(E_j-l)!l!} & \text{if } l \leq E_j \\ 0, & \text{if } l > E_j \end{cases} \tag{1}$$

This process may be repeated for values of $l \leq \Delta(E)$, where $\Delta(E)$ is the maximum number of branches in the network. The steps above effectively collate the sets of connected branches at all buses from the minimum number of connection $l = 1$ to a maximum number, $l = L$. If $L < \Delta(E)$, then there is at least one bus at which a PMU may not be able to measure all current phasors along the branches connected to that bus without support from another PMU. This is the case of a non-homogeneous limited channel capacity, and $L = \Delta(E)$ is the case with non-homogeneous multichannel capacity. Since $l = 1, 2, \ldots, L$, defines the number of branches in a subset S_{jlr}, the various values of l may be seen as the mixes of PMUs of l current channel capacities available as possible candidates for placements, and L the maximum current channel capacity in the non-homogeneous mix. Furthermore, since the number of elements in each set S_{jlr} is $l + 1$, each PMU in the non-homogeneous mix is assumed to have a total capacity of $l + 1$. This enables the simultaneous measurements of one voltage phasor at the bus j and l number of current phasors along the branches indicated by elements in the subset S_{jlr} at which j is a centre. The total number of subsets, $r^s = \sum_{j=1}^{n} \sum_{l=1}^{L} r_{jl}$.

The Containment Matrix The bus and connected branches of any network can be described by the well-known connectivity matrix, a_{ij} whose entries i, j is 1 if $i = j$ or if Bus i is connected to Bus j and 0 otherwise. The subsets S_{jlr} describe a defragmentation of the network into sets of branches. Each subset can be defined by a parameter b_{ijlr} as,

$$b_{ijlr} = 1 \text{ if } (i = j) \cap (i, j \in S_{jlr}) \text{ or if} (i \in S_{jlr}) \cap (j \in S_{jlr}).$$

If this condition is not true, then $b_{ijlr} = 0$.

The parameter b_{ijlr} over all the subsets in the network can be contained in a four-dimensional containment matrix $\mathbf{B}$ of dimension $n \times n \times L \times r^j$. However, in the two-dimensional space, the containment matrix is an $n \times n$ array in $L \times r^s$ places. The formation of the containment matrix is illustrated in Figure 3 for $l = 1$ given that $L = 1$. Since the maximum number of connections in the network is 3 and $L < 3$, this is a non-homogeneous limited channel case, specifically the exclusive deployment of DULRs, as in [2].

The subsets $S_{j11} = \{\{1, 4\}, \{2, 8\}, \{3, 6\}, \{4, 9\}, \{5, 4\}, \{6, 7\}, \{7, 6\}, \{8, 9\}, \{9, 4\}\}$. The subsets $S_{j12} = \{\{\}, \{\}, \{\}, \{4, 5\}, \{5, 6\}, \{6, 5\}, \{7, 8\}, \{8, 7\}, \{9, 8\}\}$, and $S_{j13} = \{\{\}, \{\}, \{\}, \{4, 1\}, \{\}, \{6, 3\}, \{\}, \{8, 2\}, \{\}\}$. For each element in S_{jlr}, an entry is made in the $j-$th column of the $jlr-$th matrix b_{ij} as shown in Figure 3. As an illustration, for the subset $S_{911} = \{9, 4\}$, the entries $b_{4,9} = 1$ and $b_{9,9} = 1$ at $l = 1, r = 1$. Similarly, the entries $b_{6,3} = 1$ and $b_{6,6} = 1$ for the subset S_{613} at $l = 1, r = 3$. Notice that in Figure 3, $b_{j,j} = 1$ when j is the centre of the subset S_{jlr}. Furthermore, for any given l and r, the entries $b_{i,j} = 0$ when S_{jlr} is a null set.

Following from the descriptions above, it is possible to form a subset S^d_{jlr} which represents the option of purchasing a PMU with $l + 1$ channels, installing it on a bus j and then connecting that PMU to a local PDC d. All subsets of S_{jlr} are formed for each local PDC d. For D total number of PDCs, the overall co-placement containment matrix (with a corresponding increase in dimension to $n \times n \times L \times r^s \times D$ from $n \times n \times L \times r^s$) is then formed using the process described previously.

3 Modelling PMU Installation Costs

Since the subsets S^d_{jlr} represent PMUs of different capacities which can be connected to a number of PDCs, there is a cost C^d_{jlr} associated with their installations. In this section, we develop a realistic cost model (2) based on reports in [2, 13, 16, 17]. The model (2) estimates the total costs of instrument transformers, active and passive communication components, labour, procurements, calibration, testing, and security, and considers equipment which are already available at the utilities. PMUs connect to lines and buses through instrument transformers. However, they cost about 6 times the value of a PMU. Therefore, if they are effectively available, installation costs can be reduced substantially. Indeed, utilities have reported that installation cost was halved when this was the case [16].

It becomes necessary to define an *effectiveness* of the availability to mean that the components or equipment are available and fit for the purpose of PMU installation. For instance, with respect to the scenarios outlined in [2] regarding 1) the availability of the instrument transformer but not for PMU use out of security concerns 2) the availability but not for PMU use due to performance concerns, and 3) the availability but not for PMU use in order to keep existing circuits separate, we define an *effective availability* as the scenario where the instrument transformers are available for PMU use without aforementioned concerns. For co-placement problems, the effectively available communication lines are those which have enough bandwidth for the purpose at hand.

$$\begin{aligned} C^d_{jlr} = {} & \underbrace{\overbrace{\rho^{\text{Pt}} C^{\text{Pt}}}^{\text{CT}} + \overbrace{\sum_{p \in S_{jlr}} \rho_p^{\text{Ct}} C^{\text{Ct}}}^{\text{PT}}}_{\text{Costs of instrument transformers}} + \underbrace{\overbrace{\overbrace{\eta \left(C^{\text{pcom},d}_{\text{perkm}} (m^d_j - \eta^d_j) \right)}^{\text{comm. line}} + \overbrace{C^{\text{pcom},d}_{\text{perrep}} \left(\text{floor} \left(\frac{m^d_j}{100} \right) - \lambda_{\text{rep}} \right)}^{\text{repeaters}}}^{\text{passive}} + \overbrace{\overbrace{\gamma_s C^{\text{acom}}_{\text{swi}}}^{\text{switches}} + \overbrace{\gamma_r C^{\text{acom}}_{\text{rou}}}^{\text{routers}} + \overbrace{\gamma_g C^{\text{acom}}_{\text{gps}}}^{\text{GPS}}}^{\text{active}}}_{\text{costs of communication infrastructure (CI)}} \\ & + \underbrace{\overbrace{\zeta_{\text{w}} C^{\text{wage}}}^{\text{wages}} + \overbrace{\zeta_{\text{lc}} C^{\text{lpkm},d} m^d_j}^{\text{laying comm. line}} + \overbrace{\zeta_{\text{f}} C^{\text{fld},d}_{jlr}}^{\text{field expenses}} + \overbrace{\zeta_{\text{tv}} C^{\text{trav},d}_{jlr,\text{perkm}} m^d_j}^{\text{travel expenses}} + \overbrace{\zeta^{\text{train}}_{\text{tn}} C^{\text{train},d}_{jlr}}^{\text{training}}}_{\text{Labour costs}} + \underbrace{C^{\text{mproc},dv}_{jlr}}_{\text{Procurement cost}} + \underbrace{\kappa_c C^{\text{calibr}}_{jlr} + \kappa_t C^{\text{test}}_{jlr} + \kappa_s C^{\text{security}}_{jlr}}_{\text{costs of calibration, testing, and security}} \end{aligned} \quad (2)$$

The parameters of cost model (2) are defined in Tables 1 and 2. The cost per kilometre of laying a communication line to a local PDC d is dependent on the substation's communication requirements for synchrophasor connection, especially in terms of the available bandwidth. When transmitting over a long distance, it is necessary to install repeaters at every 100km in order to avoid signal degradation [7]. This is modelled in the 4th terms of (2), in addition to the consideration of any effectively available number of units. The 5th, 6th, and 7th terms of (2) model the costs of unavailable active communication equipment such as switches, routers, and Global Positioning Systems (GPS).

The labour costs include all general expenses incurred by labour while commuting around installation sites as well as those actually incurred on the sites [16]. On the whole, labour costs are a function of the distance of the PMU site to the PDC. The various costs of labour are modelled in the 8th-12th terms of (2). Each of these terms correspond, respectively, to the labour wages specifically for when external workforces are used, where a communication line is needed to connect to a local PDC d, field expenses incurred from installing the PMU at a bus j and from laying the communication lines to the local PDC, travel expenses for specialised crews, and training expenses when labour need to be trained to carry out installations. In practice, the higher the cost $C^{\text{trav},d}_{jlr,\text{perkm}} m^d_j$ of specialised crew travelling from one installation location to another, the lower the cost $C^{\text{train},d}_{jlr}$ of training the crew[16], and vice-versa.

The unit cost of a PMU has been the main focus of many placement literature. However, depending on the availability of the ancillary equipment and sensors needed for the PMU installation, this is merely around 5-30% of the total installation costs [16].

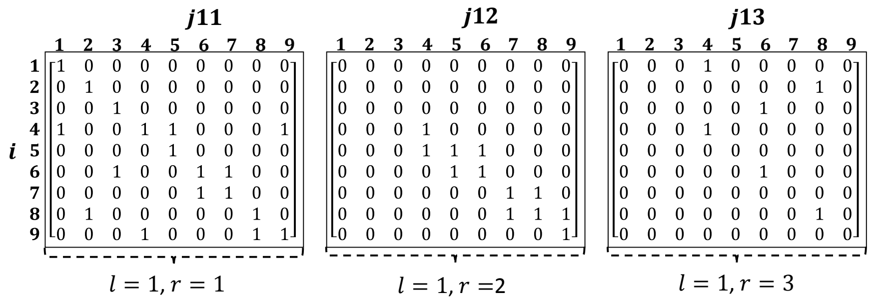

Fig. 3: Containment Matrix illustration for the WSCC 9-Bus network at $l = 1$ and $L = 1$

Table 1 Description of the availability indices of Equation (2)

Index	Particulars	Value
$\rho_c, \rho_p,$	Instrument Transformers	1, if NOT *effectively* available, 0, otherwise
η	Communication Line	1, if a viable communication line to PDC d is NOT available, 0, Otherwise
$\gamma_s, \gamma_r, \gamma_g$	Switches, routers, and GPS (respectively)	1, if NOT available 0, otherwise
$\zeta_{\text{w}}, \zeta_{\text{lc}}, \zeta_{\text{f}}$ $\zeta_{\text{tv}}, \zeta_{\text{tn}}$	Wages, laying communication lines, field , travel, and training expenses (respectively)	1, if considered 0, otherwise
$\kappa_c, \kappa_t, \kappa_s$	Calibration, testing, and security, (respectively)	1, if considered, 0, otherwise
λ_{rep}	Number of already, available repeaters	Number of units available
η_j^d	Length of comm., line available	*Effective* km length of communication line available

Table 2 Description of Costs and other parameters of Equation (2)

Parameter	Description
$C^{\text{Pt}}, C^{\text{Ct}}$	Cost of one three-phase potential, one 3-phase current transformers respectively.
$C^{\text{pcom},d}_{\text{perkm}}$	Cost, per km, of laying a communication line from bus j to PDC d.
$C^{\text{pcom},d}_{\text{perrep}}$	Cost of one repeater unit.
m_j^d	Shortest distance of bus j to PDC d in km.
$C^{\text{acom}}_{\text{rou}}, C^{\text{acom}}_{\text{swi}}, C^{\text{acom}}_{\text{gps}}$	Cost of active communication equipment: switches, routers, and GPS respectively
$C^{\text{wage}}, C^{\text{lpkm},d}, C^{\text{fld},d}_{jlr},$ $C^{\text{trav},d}_{jlr,\text{perkm}}, C^{\text{train},d}_{jlr}$	Labour costs: Wages, per km labour cost of laying communication line, field, travel, and training expenses, respectively
$C^{\text{mproc},d}_{jlr}$	Cost of procurement.
$C^{\text{calibr}}_{jlr}, C^{\text{test}}_{jlr}, C^{\text{security}}_{jlr}$	Costs of calibration, testing, and security.

The last three terms of (2) model the costs of calibration, testing, and security of the PMU, effectively considering the costs of the measures which have to be taken before the PMU is fully ready for use. These include but are not limited to the calibration of measurement devices, software upgrade of existing DULRs to activate their PMU functions, testing the PMU devices in readiness for installations. Although some utilities do not consider PMUs as critical cyber-security assets yet, PMU data must be protected from dangerous cyber attacks.

Each cost vector C^d_{jlr} in (2) across all sets S^d_{jlr} can be combined as elements in the vector $\mathbf{C} \in \mathbb{R}^{(n \times L \times r^s \times D) \times 1}$.

4 The Optimisation Formulation

The placement or co-placement problem is posed as the formulation (3). The solution is an OPP whose multidimensional vector outputs give decisions on the bus j on which the PMU is to be installed, the number of PMUs to be placed on the bus, the sets of branches to be monitored, the number of PMU channels to be used at the bus, and the local PDC to connect to.

$$\underset{\mathbf{x}}{\text{minimize}}\ \mathbf{C}^T \mathbf{x} \tag{3a}$$

$$\text{subject to}\ \mathbf{Bx} \geq \mathbf{M} \tag{3b}$$

$$\mathbf{Hx} \leq \mathbf{h} \tag{3c}$$

$$\mathbf{Fx} = \mathbf{f} \tag{3d}$$

$$\mathbf{Gx} = \mathbf{g} \tag{3e}$$

$$\mathbf{x} \in \{\mathbf{0}, \mathbf{1}\} \tag{3f}$$

where $\mathbf{x} \in \mathbb{R}^{(n \times L \times r^s \times D) \times 1}$ is the decision variable vector. The matrices $\mathbf{H}, \mathbf{F}$, and $\mathbf{G}$ are selection matrices which have the same dimensions as matrix $\mathbf{B}$. If only PMU placements are desired, the vectors $\mathbf{X}$ and $\mathbf{C}$ have the dimension $(n \times L \times r^s) \times 1$ and $\mathbf{B}, \mathbf{H}, \mathbf{F}, \mathbf{G} \in \mathbb{R}^{n \times (n \times L \times r^s)}$ are 4-D matrices that can also be viewed in 2-D (as described in Section 2).

Constraint (3c) stipulates that the total number of PMUs to be deployed at each bus should not exceed the corresponding elements of the vector $\mathbf{h} \in \mathbb{R}^{n \times 1}$. The matrix $\mathbf{H}$ has the same dimension as $\mathbf{B}$, but contains identity matrix of dimensions $n \times n$ in $L \times r^s \times D$ places.

Constraint (3d) accounts for the presence of pre-existing flow and PMU measurements. In a similar manner to $\mathbf{H}$, $\mathbf{F}$ is a selection matrix and has a value of 1 only at the diagonal of its $n \times n$ matrices for which a placement corresponding to an element of vector $\mathbf{x}$ already exists. The $i-$th element of the vector $\mathbf{f} \in \mathbb{R}^{n \times 1}$ contains the number of directly placed pre-existing measurement for the bus i.

The constraint (3e) prohibits the placement of certain PMUs at some specific buses. For example, it is reasonable to require that procured measurement devices which are to be installed at that substation must have burdens which are not higher than the circuit capacities of the effectively available instrument transformers, and to prohibit all connections of PMUs which are known to have a higher burden to that bus. In a more general sense, the formulation (3e) can be applied to prohibited buses, PDCs, or substations for which the devices may not be installed at, supplied from, or connected to

IET Research Journals, pp. 1–11

respectively. For each of such buses, the selection matrix $\mathbf{G}$ has a value of 1 only at the diagonal corresponding to the decision variable of the prohibited PMU(s), and the corresponding $i-$th element of the vector $\mathbf{g} \in \mathbb{R}^{n\times 1}$ is zero for the $i-$th bus.

The constraint (3b) describes the network and compels each bus to an observability level which must be at least greater than the elements of the corresponding vector $\mathbf{M}$. The vector $\mathbf{M} \in \mathbb{R}^{n\times 1}$ in (3b) specifies the extent of network bus observability. For the specification of $\mathbf{M}$, one of the following scenarios is possible.

4.0.1 Full observability: When full observability of all network buses is desired, the vector $\mathbf{M} = \mathbf{R} + 1$, with the element of the vector $\mathbf{R}_i$ containing the level of redundancies which may vary across each bus i. However, even with $R(i) = 0$ for all or a selected number of buses, observability is still guaranteed from at least one PMU.

4.0.2 Application-sensitive selective observability: Selective observability is defined here as the situation where some, not all, of the network buses are monitored. This is particularly the case in application-sensitive placements, for instance, in secondary voltage control or model validation, where PMU monitoring data on pre-identified pilot buses or generator buses are critical. For the selective observability case, $\mathbf{M} = \mathbf{K}\Upsilon$, where the elements of Υ are non-zero only at the selected candidate buses. However, the formulation of constraint (3b) in this way does not compel direct PMU placements at these selected buses, and therefore does not preclude the observability of non-selected buses. Elements of the parameter vector $\mathbf{K}$ determine the level of redundancies at the selected buses.

4.1 Budget-constrained observability

The level of observability desired may be limited by the amount of investment the utilities are willing or able to invest at a time. The extent of budget released for PMU deployment by the utility may not exceed the benefits offered by improved grid monitoring in the long term [1]. Consequently, only a finite number of PMUs can be deployed in a period for budget-constrained placements. This may often result in unplanned PMU deployments on as-needs-be basis, which may be costlier in the long run as the measurement devices may not be optimally deployed. In order to prevent this, a two-step process is proposed.

The first involves a solution of the OPP (3) to obtain the candidate PMU buses which would give full-observability in the long term, and the second step involves the maximisation of observability across selected buses with a stipulation, in addition to the budget constraint, that only the candidate solutions in the first step, which yield the observability of the selected buses, are deployed. The second step may then be written as,

$$\underset{\mathbf{u}}{\text{maximize}}\ \bar{\mathbf{W}}^T\mathbf{u} \tag{4a}$$

$$\text{subject to } \mathbf{B}\hat{\mathbf{M}}\mathbf{x} \geq \mathbf{u} \qquad \forall i \in I \tag{4b}$$

$$\hat{\mathbf{M}}\mathbf{C}\mathbf{x} \leq \text{Budget} \tag{4c}$$

$$\mathbf{x}, \mathbf{u} \in \{\mathbf{0}, \mathbf{1}\} \tag{4d}$$

In (4), the weights $\bar{\mathbf{W}} \in \mathbb{R}^{n\times 1}$ are positive integers used for the prioritisation of observability of the selected bus respectively, with the value of $i-$th element of $\bar{\mathbf{W}}$ proportional to the criticality of bus i observability. $\bar{u} \in \mathbb{R}^{n\times 1}$ is the observability index vector. Note that $\hat{\mathbf{M}}$ is a selection matrix which extracts only the candidate solutions obtained in the first step from the vector $\mathbf{x}$, and has the same dimension as $\mathbf{B}$.

4.2 Placement to maximise benefits of add-on applications and to minimise installation costs

The IEEE C37.118.1-2011 specifies that PMUs must be capable of measuring voltage and current phasor as well as the rate of change of frequency (ROCOF). In addition to these standard requirements, many vendors have also developed a number of off-the-shelf applications which are available without additional costs to utilities. These extraneous applications include, but are not limited to,

1. Oscillation detection,
2. Prony analysis,
3. Islanding detection,
4. State estimation,
5. Voltage control,
6. Stability indication.

In practice, it may happen that although the DULRs with added PMU functionalities may be much cheaper than standalone PMUs, they may not have the same expanse of applications that can be obtained from the standalone units. Utilities may wish maximise the range of applications they may obtain with good juxtaposition of the added application values and economy. Following from this, the new, more practical objective would be to maximise the benefits offered by these devices in the form of added off-the-shelf functionalities and to minimise the costs of procuring these devices. If an adjusted cost,

$$\bar{C}^d_{jlr} = C^d_{jlr} - \bar{B}^d_{jlr} \tag{5}$$

and

$$\bar{B}^d_{jlr} = \sum_{\varrho=1}^{n^l_{\text{app}}} \mu^l_{\varrho}\bar{B}_{\varrho} \tag{6}$$

From Section 4, C^d_{jlr} is the total cost of the availability and cost models of Section 3. $\bar{B}_{\varrho}$ is the benefit, relative to cost, of purchasing a PMU which has an add-on application ϱ. n_{app} is the total number of applications under consideration. The benefit $\bar{B}_{\varrho}$ may be evaluated in practical terms as the cost that would have been incurred if the application ϱ were to be built in-house, or purchased separately. $\bar{B}_{\varrho}$ may also be evaluated in terms of the relative importance to the application needs at the time of placement. For instance, if improved observability for voltage control is the PMU deployment goal, add-on capabilities such as voltage instability prediction and state estimation may be assigned higher benefits, relative to those of other applications of interest. μ^l_{ϱ} is a binary parameter indicating whether or not a particular application is offered as an add-on for the PMU of channel capacity $l + 1$, and $\bar{B}_{\varrho}$ is the benefit, in monetary unit, of the add-on as evaluated by the utility. In essence, $\bar{B}^d_{jlr}$ is the total benefit, in monetary unit, of all the applications offered for a PMU of capacity $l + 1$. The adjusted cost $\bar{C}^d_{jlr}$ and the total benefits of applications $\bar{B}^d_{jlr}$ can be written in the vectorised forms $\bar{\mathbf{C}}$ and $\bar{\mathbf{B}}$, in a similar manner to the vectorisation of C^d_{jlr}, such that (5) can be written in the vector form,

$$\bar{\mathbf{C}} = \mathbf{C} - \bar{\mathbf{B}} \tag{7}$$

The OPP (3) may then be modified to incorporate (5), resulting in (8).

$$\underset{\mathbf{x}}{\text{minimize}}\ \bar{\mathbf{C}}^T\mathbf{x} \tag{8a}$$

$$\text{subject to } \mathbf{B}\mathbf{x} \geq \mathbf{M} \tag{8b}$$

$$\mathbf{H}\mathbf{x} \leq \mathbf{h} \tag{8c}$$

$$\mathbf{F}\mathbf{x} = \mathbf{f} \tag{8d}$$

$$\mathbf{G}\mathbf{x} = \mathbf{g} \tag{8e}$$

$$\mathbf{x} \in \{\mathbf{0}, \mathbf{1}\} \tag{8f}$$

With the adjusted costs (8a) (from (7)), the OPP (8) seeks to maximise the benefits obtained from add-on applications on the one hand while minimising their installation costs on the other. The choice of n_{app} may differ in general for various types of PMUs, and the added functionalities would usually be a function of PMU capacities. For instance, with DULRs, the additional functionality are their primary capability, namely, to serve as relays. Standalone units may have facilities for more additional functionalities, proportional to the units' sizes. Therefore, for DULRs, one may choose $n_{app} = 1$ while $n_{app} \geq 2$ for standalone units.

4.3 Other Considerations

Zero-injection buses can be easily added to constraint (3b) as,

$$\mathbf{Bx} + \mathbf{AYz} \geq \mathbf{M} \quad (9)$$

and specify an extra constraint, $\mathbf{Y}^T\mathbf{A}^T = \mathbf{z}$. Where $\mathbf{A} \in \mathbb{R}^{n\times n}$ is the matrix form of a_{ij}(Section (2)). $\mathbf{Y}$ is a matrix form of the auxiliary variable y_{ij} originally described in [18], and $\mathbf{z}$ is the vector of zero-injection buses whose elements are 1 when the bus corresponding to the element is a ZIB and 0 if it is not. Details of the formulation of y_{ij} can be found in [18].

In addition, it is possible to model the changes in the system topology due the loss of line by modifying a_{ij} as specified in [18] and then forming the containment matrix, $\mathbf{B}$.

4.4 Implementation

The formation of subset S_{jlr} is automated in MATLAB, and the formulation (3) was implemented in a commercial optimisation software known as the Advanced Interactive Multidimensional Modeling System (AIMMS) using CPLEX 12.8 as solver. However, the formulation of (3) lends well to implementation on non-commercial optimisation packages.

5 Results and Discussions

In order to demonstrate results from the discussion so far, relatively realistic PMU and associated PMU placement cost values, expressed in monetary units and as may be found in literature, are used. The results from ZIBs will not be included at this time. However, this does not detract from the veracity of the approach and reported results may be regarded as the optimal solutions when no ZIBs are considered.

In sum, Figure 4 describes how the various software were used in the implementation of the proposed algorithm.

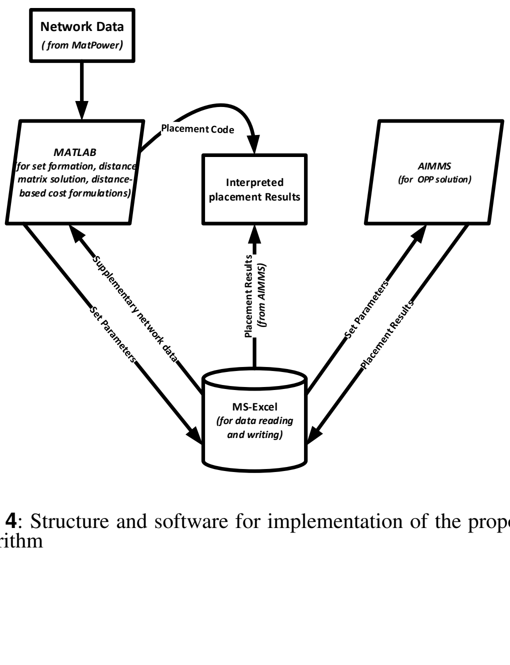

Fig. 4: Structure and software for implementation of the proposed algorithm

5.1 Base Case

First, we discuss the base placements of PMU without the consideration of CI co-placements. Consequently, all aspects of cost model (2) which relate to CIs are ignored. This means that only the network connectivity information and channel-influenced unit costs of the PMU are used as parameters. We assume that each PMU of channel capacity, l has an associated cost c_l which is $\alpha(l-1) \times c_1$, with c_1= 1p.u, as suggested in [8]. We show the feasibility of (3) on various test networks, starting with the example Figure 2 and choose $\alpha = 0.1$. In addition, we assume that there no measurement redundancy, pre-existing measurements, or prohibited, and specify that not more than one PMU must be installed at a bus, in order to address the requirements of constraints (3b), (3d), (3e) and (3c).

Table 3 shows the base case placement results of (3) for the example Figure 2 at various values of maximum current channel capacity, L, following from the description in Section 2. Note that since DULRs can only measure a single branch current in addition to the voltage phasor at the bus, the case with $L = 1$ represents the exclusive deployments of DULRs for monitoring, while $L = 2$, with relatively more non-homogeneous mixes available for deployments, allows for the placements of DULRs and 3-channel PMUs. $L = 1$ and $L = 2$ are the limited channel cases since $\Delta(E) = 3$, while $L = 3$ is the non-homogeneous multichannel case. The solution in Table 3 shows that fewer measurement devices are deployed when higher channel capacities are present in the non-homogeneous mix, using a cost differential factor $\alpha = 0.1$. In addition, the table shows the bus and actual branches to be monitored, as well as the capacity of the PMU to be deployed on that bus. The result obtained is the same as those of [3, 8, 15, 18]. However, this specification makes the proposed solution superior to those in the aforementioned works, and unique from[11] in the determination of several and specific capacities, as well as channel minimisation. The inclusion of channel minimisation in the objective of OOP (3) eliminates channel wastage. As can be observed, across all values of L in the table, each bus is observed only once. This prevents *accidental* redundant placements and gives tighter control over the selection of buses where redundancy is desired.

Table 3 Base Case: Non-homogeneous [Limited and Multichannel] Placement for the IEEE 9-Bus Network, $\mathbf{R} = \mathbf{0}$

	Branches					
	$L=1$	$L=2$		$L=3$		
Bus	l_1	l_1	l_2	l_1	l_2	l_3
1	1–4	1–4		1–4		
2	2–8	2–8				
3	3–6	3–6				
4	4–9		4–5–9			
5	5–4					
6	6–7				6–3–5	
7		7–6				
8						8–2–7–9
9						
Total	6	4	1	1	1	1
	6	**5**		**3**		

The performance of (3) is also demonstrated for relatively larger networks; the IEEE 57-Bus, 118-Bus, and 300-Bus test systems. The bar charts and embedded plots of Figure 5 show the different channel mixes that were deployed for the test networks at different values of the maximum channel capacities, L. It can be seen that the number of deployed PMUs converges from certain values of $L \leq \Delta(E)$. This shows that at the value of α, the presence of higher capacities in the homogeneous mix does not necessarily affect placement solutions. However, if the differences in capacity-based cost are appreciable, the solution favours DULR or lower-capacity PMU placements even when higher capacities are present in the non-homogeneous mix.

IET Research Journals, pp. 1–11

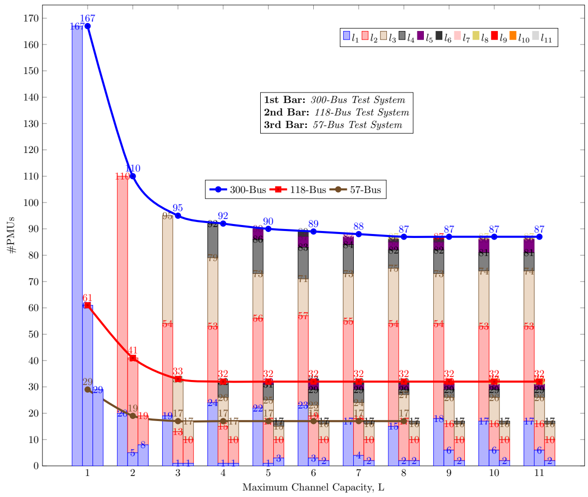

Fig. 5: Base Case: Stacked charts and superimposed plots showing placement comparisons using 3 IEEE Test Cases (i.e. the 300-Bus, 118-Bus, and 57-Bus Networks) as case studies at $\mathbf{R} = \mathbf{0}$, $\mathbf{h} = \mathbf{1}$. The stacks show the channel capacities deployed for each network at different values of L.

For instance, suppose $\alpha \geq 1$ for the 9-Bus system of Figure 2, only DULRs are deployed even at $L = 3$.

The sensitivity of solutions to (3a) parameters $\mathbf{h}$ and $\mathbf{R}$ at full grid observability are shown in Table 7. The parameters were modulated at values of L in which solution was found to have converged for each network at $\mathbf{h} = \mathbf{1}, \mathbf{R} = \mathbf{0}$. All buses in the respective are assumed to have the same values $\mathbf{h} = \mathbf{1}, \mathbf{R} = \mathbf{0}$ respectively. As expected, increasing the level of redundancy resulted in an increase the number of deployments. However, the problem is infeasible when double redundancies are specified for $\mathbf{h} = \mathbf{1}$, since there is no combination of PMUs that can give the expected level of redundancy with a restricted number of individual bus deployment. On the other hand, the problem is again feasible by setting $\mathbf{h} = \mathbf{2}$ for $\mathbf{R} = \mathbf{2}$.

5.2 CI Co-Placement

Next, we examine the performance of proposed formulation (3) for co-placements of PMUs with CIs, especially for known but multiple PDC locations, to reflect situations where multiple utilities have their own PDCs. For this, the cost model (2) and the moderately large IEEE 118-bus test system of Figure 6 are used as case study. Distances between the network buses are obtained from [21]. As shown in Figure 6, the network is divided into 3 regions, and all buses separated by a transformer are assumed to form a substation location with PDCs. These are at 8-5, 26-25, 30-17, 38-37, 63-59, 64-61, 65-66, 68-69, and 81-80, are labelled alphabetically as PDCs A-I respectively. For the other parameters of model (2), the following are assumed:

1. A kilometre length of communication line, cyber-security setup, commissioning, a unit switch, and a unit repeater devices are each assumed to cost 0.667 units and a GPS device 0.033 unit.
2. Instrument transformers are sold at 3 unit per piece and
3. Base wages are 0.667 unit and 0.083 unit is charged for each additional channel installed.
4. communication lines are laid at 0.05 unit per km and allowances for training and travel are 0.083 per bus and 0.017 units respectively.

5.2.1 Full observability: The PMU-CI co-placement result for full observability(Section 4.0.1) are shown in Table 8 for $L = 9$. The results includes, in addition to specifications given in Table 7, the local PDCs which the PMUs may connect to. Since there are no PDCs in Region 3, the most of the PMUs in that region connect to

Table 4 Assumed Scenario for Pre-existing measurements

Bus	Device	Capacity	Branch	PDC
10	DULR	2	10-9	A
12	PMU	4	12-7,12-14, 12-117	B
49	PMU	8	49-42,49-45, 49-48, 49-50, 49-54, 49-66,49-69,	G
73	DULR	2	73-71	C

Table 5 Total Costs and Placements for Table 4 Scenario

	$\mathbf{h} = \mathbf{1}$		$\mathbf{h} = \mathbf{2}$	
	$\mathbf{R} = \mathbf{0}$	$\mathbf{R} = \mathbf{1}$	$\mathbf{R} = \mathbf{0}$	$\mathbf{R} = \mathbf{1}$
Total Costs (monetary unit)	963.4	2071.6	963.4	1873
Total Placements	33	67	33	64

the PDC I. In consequence, PDC I has the highest number of connections across all combinations of $\mathbf{h} = \mathbf{2}$ and $\mathbf{R} = \mathbf{1}$. With $\mathbf{R} = \mathbf{0}$, the number of deployments is the same as those obtained when only the nominal costs are used. However, the total costs of deployments decreases when more than one PMU can be deployed at a bus for an increased level of redundancy. Indeed, at $\mathbf{h} = \mathbf{2}, \mathbf{R} = \mathbf{0}$, the number of PMUs reduces from 32 (obtained in [2, 4, 11] to 31 although there was no corresponding decrease in the total cost of deployment. At $\mathbf{h} = \mathbf{1}, \mathbf{R} = \mathbf{1}$, total placements (69) was higher than at $\mathbf{h} = \mathbf{2}, \mathbf{R} = \mathbf{1}$ (65). This means that the number of placements allowed at each bus may influence the total number of deployments. With respect to this, the proposed approach installed fewer PMUs compared to [2, 4, 11].

5.2.2 Co-placements with pre-existing measurements: We illustrate the performance of formulation (3) when a number of flow or phasor measurement devices are already installed and full observability is required. In order to achieve this, Constraint (3d) requires that the system be specified as in Table 4, which contains the various pre-installed capacities of PMUs. Compared to full observability installation, unplanned pre-existing measurements may cost more if the already-installed devices were not installed at optimal locations. However, the total number of placements at increased redundancy especially for $\mathbf{h} = \mathbf{2}$ may be less than that obtained without existing measurements, depending on the channel capacity of the existing devices.

5.2.3 Application-sensitive co-placements: Secondary voltage control: Table (6) shows a co-placement when only the observability of selected buses of the 118-Bus network are required for use in a secondary voltage control application. For this purpose, these are the set of buses with the highest short-circuit capacities in an area, known as *pilot* buses. These are selected from [22] as 2, 19, 39, 56, 68 88, 103, and 114. The bus, actual branch(es), and PDC locations can be found in the table, and the PMU channel capacities can be inferred from the number of connected branches. It can be observed that as described for constraint (3b) in Section 4.0.2, some of the selected buses may not be candidate solutions, but are observable through placements from other buses. In addition, for selective placements such as this, the deployments of DULRs is encouraged, especially when no redundancy is specified. For $\mathbf{K} = 1$, some cost savings can be achieved at $\mathbf{h} = 2$. Compared to placement for secondary voltage control in [23], the proposed approach uses realistic costs and a linear objective.

5.3 Comparison of placement costs with equipment availability and add-on incentives

In the following, the adjusted cost formulation (8), in which the benefits of add-on applications are considered, is compared to the

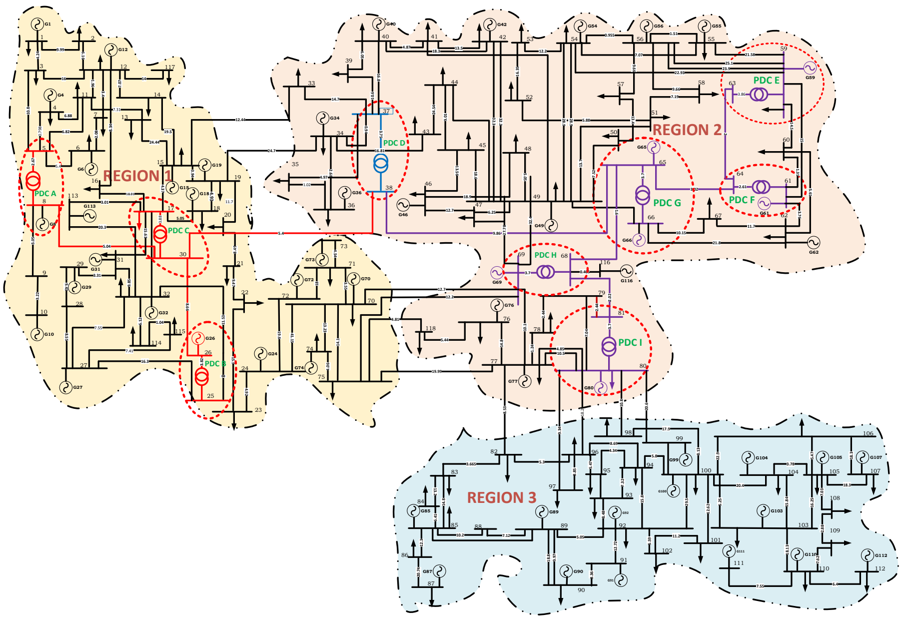

Fig. 6: 3-Region IEEE 118-Bus Test System with local PDC locations indicated in red broken ellipses

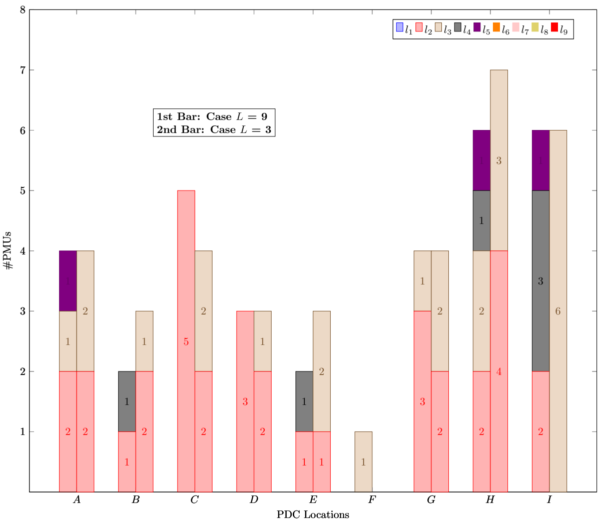

Fig. 7: PMU-PDC Co-Placement: PMU Placements by Channel Mixes and PDC Locations using the IEEE 118-Bus Test System, *$L = 9$ (Non-homogeneous Multichannel Case* and *$L = 3$ (Non-homogeneous Limited Channel Case)*

formulation (3). From a utility's point of view, add-on applications offered on top of primary PMU functionalities may have benefits which justify an investment in the measurement device. These benefits may be quantified from a knowledge of the cost of developing the application in-house, or more subjectively, from the relative importance of the applications on offer. Table 9 shows the assumptions on the benefits obtained of 4 applications **a**, **b**, **c**, and **d**. Clearly, the application **d** is the most important to the utility and **a** is the least beneficial.

From a commercial point of view, it is reasonable to assume that a wider range of applications would be available on larger units, as shown in Table 10. From the table, it can be seen that devices with 5 or more channel capacities have the highest number of applications

Table 6 Co-Placement solutions for Selected Pilot Bus Observability

$\mathbf{h} = 1$, $\mathbf{K} = 1$	$\mathbf{h} = 1$, $\mathbf{K} = 1$	$\mathbf{h} = 2$, $\mathbf{K} = 2$
1-2 [A] 32-114 [B] 34-19, 37-39 [D] 59-56 [E] 69-70, 85-88[H] 81-68-80, 100-103 [I]	1-2, 2-1 [A] 32-114, 115-114 [B] 15-14 [C] 34-19, 37-39, 39-37[D] 55-56, 59-56 [E] 69-68-70,75-70, 85-88, 88-89[H] 80-79, 81-68-80, 100-103, 103-100 [I]	1-2, 2-1 [A] 32-114 [B] 15-19, 32-114 [C] 34-19, 37-39, 37-34-39 [D] 59-56, 56-54-55 [E] 69-70, 69-68-70, 85-88 [H] 80-79, 81-68-80, 85-88, 100-103, 100-98-103 [I]
TP = 9 **TC = 169 units**	**TP = 18** **TC = 388 units**	**TP = 18** **TC= 353.84 units**

on offer, and DULRs have the lowest number of incentives. In practice, the add-ons offered on the measurement devices would differ from vendor to vendor.

The bar chart in Figure 8 compares the placement costs resulting from the formulations (3) and (8). This is summarised as comparison of costs without add-on inceibtives (OPP (3)), and with the same and different benefits across all applications (OPP (8)). A uniform benefit of 1 monetary unit is assumed for all applications in the same-benefit scenario.

Furthermore, the indices of the cost model (2) are modulated such that instrument transformers are assumed to be available at all buses, and placement results are compared under the scenrios of availability and non-availability of instrument transformers for both formulations (3) and (8). The following comparisons are made for full observability, no measurement redundancy $\mathbf{R} = \mathbf{0}$ and with only a single maximum allowable placement at buses $\mathbf{h} = \mathbf{1}$.

Figure 8 shows that the availability of instrument transformer reduces placement costs by around 26.1% in the base case formulation (3). Add-on incentives may motivate investments into measurement devices under the assumed scenario with the adjusted cost formulation (8) having costs which are lower than those obtained without the incentives. The highest savings on placement costs are

IET Research Journals, pp. 1–11

Table 7 Base Case: Placement solutions at different $\mathbf{R}$ and $\mathbf{h}$

	$\mathbf{h=1}$			$\mathbf{h=2}$		
Network	$\mathbf{R=0}$	$\mathbf{R=1}$	$\mathbf{R=2}$	$\mathbf{R=0}$	$\mathbf{R=1}$	$\mathbf{R=2}$
9-Bus $(L=3)$	$1\ l_1$ $6\ l_2$ $8\ l_3$	$1\ l_1\ 4\ l_3$ $2\ l_1\ 6\ l_3$ $3\ l_1\ 8\ l_3$	n/a	$1\ l_1$ $6\ l_2$ $8\ l_3$	$4\ l_2$ (2) $6\ l_2$ (2) $8\ l_2$ (2)	$1\ l_1, 4\ l_2, 4\ l_3$ $2\ l_1, 6\ l_2, 6\ l_3$ $3\ l_1, 8\ l_2, 8\ l_3$
14-Bus $(L=4)$	$2\ l_3$ $6\ l_4$ $8\ l_1$ $9\ l_2$	$2\ l_3, 4\ l_4$ $5\ l_1, 6\ l_3$ $7\ l_1, 8\ l_1$ $9\ l_2, 10\ l_1$ $13\ l_3$	n/a	$2\ l_3$ $6\ l_4$ $8\ l_1$ $9\ l_2$	$2\ l_3, 2\ l_4,$ $6\ l_3\ 7\ l_1,$ $8\ l_1\ 9\ l_3,$ $10\ l_2\ 13\ l_3$	$1\ l_1, 2\ l_3, 2\ l_4,$ $4\ l_2, 6\ l_3, 7\ l_2,$ $7\ l_3, 8\ l_1\ 9\ l_2,$ $11\ l_1, 11\ l_2, 13\ l_2$ $13\ l_3$
30-Bus $(L=3)$	$3\ l_1, 5\ l_1, 6\ l_3,$ $10\ l_3, 11\ l_1, 12\ l_3,$ $19\ l_2, 23\ l_1\ 25\ l_2,$ $27\ l_3$	$2\ l_3, 3\ l_1, 4\ l_1,$ $6\ l_3, 7\ l_1, 9\ l_3,$ $10\ l_3, 11\ l_1\ 12\ l_3,$ $13\ l_1, 15\ l_3, 16\ l_1,$ $19\ l_3, 20\ l_1, 22\ l_2\ 23\ l_1,$ $25\ l_3, 26\ l_1, 27\ l_3, 28\ l_1,$ $29\ l_1,$	n/a	$3\ l_1, 5\ l_2, 6\ l_1,$ $10\ l_3, 11\ l_1, 12\ l_3,$ $15\ l_3, 20\ l_1\ 25\ l_2,$ $27\ l_3$	$2\ l_3, 3\ l_1, 4\ l_1, 5\ l_2,$ $6\ l_3, 8\ l_1, 9\ l_1,$ $10\ l_3$(2), $11\ l_1$, $12\ l_3$(2), $15\ l_3, 19\ l_1, 20\ l_1,$ $24\ l_2, 25\ l_1, 25\ l_3, 27\ l_3,$ $29\ l_1$	$1\ l_1, 2\ l_3, 3\ l_1,$ $4\ l_2, 5\ l_2, 6\ l_3$(2), $7\ l_1, 9\ l_1, 9\ l_2,$ $10\ l_3$(2), $11\ l_1\ 12\ l_3$ (2), $13\ l_2, 15\ l_2, 15\ l_3,$ $17\ l_1, 18\ l_1, 19\ l_1, 19\ l_2,$ $22\ l_1, 24\ l_2, 25\ l_2, 25\ l_3,$ $26\ l_1, 28\ l_1, 29\ l_1, 29\ l_2,$ $30\ l_1$

Solutions show bus numbers and the PMU current channel capacities. Numbers in parentheses are frequencies of placements. Actual branches deployed are not shown.

Table 8 118-Bus Test Network: Co-Placement solutions at different $\mathbf{R}$, $\mathbf{h}$ using Detailed Cost Model (2)

$\mathbf{h=1}$		$\mathbf{h=2}$	
$\mathbf{R=0}$	$\mathbf{R=1}$	$\mathbf{R=0}$	$\mathbf{R=1}$
3-l_1, 5-l_3,[A]-[2]	1-l_2,3-l_1, 5-l_4, 6-l_1 [A]-[4]	3-l_1, 5-l_3 [A]-[2]	1-l_1, 3-l_1, 5-l_3(2) [A]-[4]
23-l_3, 26-l_2,115-l_2,[B]-[3]	22-l_3, 24-l_1, 26-l_2, 27-l_4, 32-l_3, 115-l_2 [B]-[6]	23-l_3, 26-l_2,115-l_2 [B]-[3]	23-l_3, 23-l_4, 25-l_2, 26-l_1, 115-l_1, 115-l_2 [B]-[6]
9-l_2,12-l_5,15-l_1, 17-l_2, 20-l_2, 29-l_2 [C]-[6]	9-l_2,10-l_1, 11-l_3,12-l_5,17-l_3, 19-l_3, 21-l_2, 29-l_2,30-l_2,31-l_1, 117-l_1 [C]-[12]	9-l_2,12-l_5,15-l_2, 17-l_2, 20-l_1, 29-l_2 [C]-[6]	9-l_1, 9-l_2,12-l_4, 15-l_2 (2), 17-l_3, 17-l_4,20-l_1, 20-l_2, 29-l_1, 29-l_2 [C]-[12]
34-l_2,37-l_3,40-l_2 [D]-[3]	34-l_3,36-l_1,37-l_3, 40-l_3, 41-l_2, 43-l_1 [D]-[6]	34-l_2,37-l_4,40-l_2 [D]-[3]	34-l_2, 34-l_3, 37-l_2,37-l_4, 40-l_2 [D]-[5]
56-l_4, 63-l_1,[E]-[2]	54-l_1,56-l_5,59-l_3, 63-l_1,[E]-[4]	56-l_4, 63-l_2 [E]-[2]	56-l_3, 56-l_4, 59-l_4(2), 63-l_2 [E]-[5]
62-l_4 [F]-[1]	61-l_3, 62-l_4 [F]-[2]	62-l_3 [F]-[1]	61-l_3 [F]-[1]
45-l_2,49-l_6,52-l_2 [G]-[3]	45-l_3,48-l_1,49-l_6,50-l_1, 51-l_2, 52-l_1, 66-l_2 [G]-[7]	45-l_2,49-l_6,52-l_2 [G]-[3]	45-l_2,45-l_3,49-l_4(2), 52-l_1, 52-l_2, 66-l_2(2) [G]-[8]
68-l_4, 71-l_1, 75-l_4, 77-l_1, 85-l_4, 86-l_1,[H]-[6]	68-l_3, 69-l_2, 69-l_2, 70-l_3, 71-l_2, 73-l_1, 75-l_2,77-l_3,83-l_2, 85-l_3, 86-l_2,89-l_1, 89-l_2, 116-l_1, 118-l_2 [H]-[14]	68-l_4, 71-l_3, 75-l_3, 77-l_3,85-l_4, 86-l_1 [H]-[5]	68-l_1, 68-l_3, 71-l_2, 71-l_3, 75-l_2, 75-l_4,77-l_3, 77-l_3, 85-l_4, 86-l_1 [H]-[10]
80-l_4 91-l_2, 94-l_3, 101-l_2, 105-l_4, 110-l_4 [I]-[6]	79-l_2, 80-l_5, 91-l_1, 92-l_4, 94-l_4, 96-l_3, 100-l_5, 101-l_2, 105-l_4, 106-l_1, 108-l_2, 110-l_4, 111-l_1, 112-l_1 [I]-[14]	80-l_4 91-l_2, 94-l_3,101-l_2, 105-l_5, 110-l_3 [I]-[6]	80-l_5(2), 85-l_4, 86-l_1,91-l_1, 91-l_2, 94-l_3, 94-l_4,101-l_1, 101-l_2, 105-l_4,105-l_5, 110-l_3, 110-l_4 [I]-[14]
Total Placements = 32 **Total Cost = 911 units**	**Total Placements = 69** **Total Cost = 2016 units**	**Total Placements = 31** **Total Cost = 911 units**	**Total Placements = 65** **Total Cost = 1826.7 units**

*The result in the table may be interpreted using this example. 54-l_4, $[E]-[1]$ means to install a 5-channel($L=l+1, l=4$) PMU device on Bus 54 and connect to PDC E. The total number of PMUs connected to a a particular PDC are indicated in square parentheses.

*Actual branches deployed are obtained as part of the solution, but are not shown in this table.

Table 9 Assumed benefits of add-ons

Application, ϱ	**a**	**b**	**c**	**d**
Benefit, B_ϱ **(units)**	1	3	3	6

Table 10 Assumed availability μ_ϱ^l

Application	$l=1$	$l=2$	$l=3$	$l\geq4$
a	$\surd$	$\surd$	$\surd$	$\surd$
b	$\times$	$\times$	$\surd$	$\surd$
c	$\times$	$\surd$	$\times$	$\times$
d	$\times$	$\times$	$\times$	$\surd$
n_{app}^l	**1**	**2**	**2**	**3**

made when installation equipment are available and the benefits of applications are considered, as shown in Figure 9.

It can be shown that although the formulations (3) and (8) return optimal solutions for all the scenarios under study, actual placement locations are sensitive to the scenarios under consideration, especially when the availability of instrument transformers or the presence of existing devices was considered. Note that in terms of actual investment in measurement devices, the formulation (3) gives the actual WAMS installation costs, while the solution of OPP (8) puts the result in proper perspective of potential benefits.

6 Conclusion

A topology defragmentation and channel-capacity minimisation approach to PMU co-placements with communication infrastructure has been described in this paper. Using the proposed method, the optimal PMU solutions include the bus, branch(es), PMU channel capacities, and the optimal PDC location to connect to. These are more detailed than can be obtained from existing methods and the performance is demonstrated for a range of IEEE test networks. The

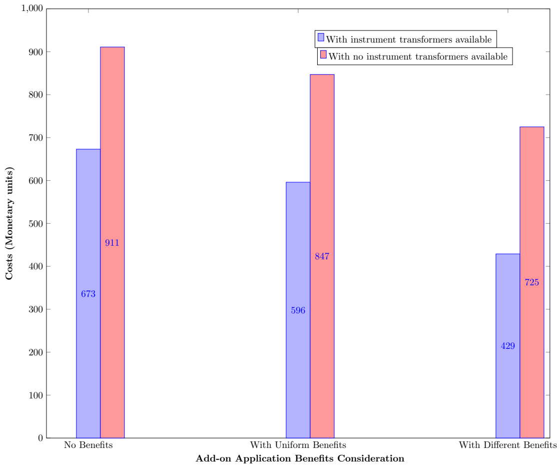

Fig. 8: PMU-PDC Co-Placement: Comparison of placement between formulations (3) and (8) under various scenarios

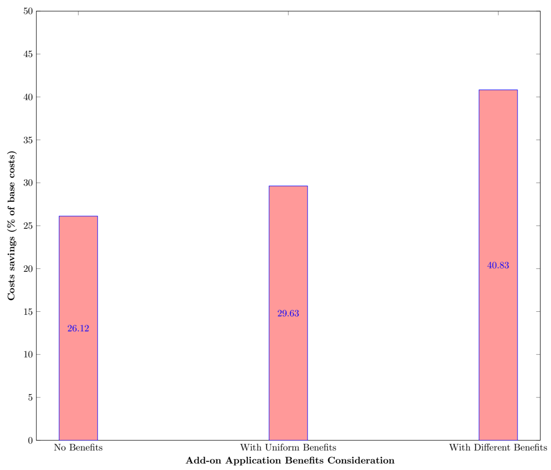

Fig. 9: PMU-PDC Co-Placement: Costs savings with available instrument transformer across scenarios

setup allows the economically-desirable options of deploying multiple PMUs at a single bus, selective deployments at optimal costs for application-sensitive installations, and accounts for budget limitations. These are achieved through novel formulations, including but not limited to, a two-step budget-constrained placement and an adjusted cost formulation which simultaneously minimises costs and maximises benefits derived from a defined range of add-on applications. Worthy of note is that the qualitative placement results obtained in this paper transcend beyond numerical quantification of placed measurement devices, and therefore direct comparison with the number of placements in literature would neither suffice nor do justice to the approach.

In addition to the viability of the novel formulations introduced in this paper, the compelling reason to use the proposed approach is that optimality is guaranteed without the need for algorithmic parameter tuning. This eliminates the need for meta-heuristic algorithms that have been previously applied to the co-placement problem. Furthermore, compared to existing methods, it is more practically applicable as real costs and a range of channel capabilities can be specified for distribution and transmission networks at different levels of observability. Consequently, it can be used for PMU deployments where full grid observability is desired and in applications like generator model validation and secondary voltage control which require data only from only a selected number of buses. Although the formulations for the consideration of zero-injection buses are described in this paper, the effect of these buses are not illustrated in the description of results. It is expected that fewer numbers of placements than those reported, would be obtained in the solutions if these buses are considered in the co-placement problem. However, this does not detract from the veracity of reported solutions, and results may be interpreted as being optimal without the consideration of zero-injection buses.

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9 Appendices

This appendix describes the actual form of the matrices of the optimisation formulation (3) in Section 4.

$$\mathbf{B}_d = \left[\left[\overbrace{\left[\overbrace{\begin{bmatrix} b_{1111} & \dots & b_{1n11} \\ \vdots & \ddots & \vdots \\ b_{n111} & \dots & b_{nn11} \end{bmatrix}}^{r=1} \quad \ddots \quad \overbrace{\begin{bmatrix} b_{111r_s} & \dots & b_{1n1r_s} \\ \vdots & \ddots & \vdots \\ b_{n1i1r_s} & \dots & b_{nn1r_s} \end{bmatrix}}^{r=r_s}\right]}^{l=1} \quad \vdots \quad \overbrace{\left[\overbrace{\begin{bmatrix} b_{11L1} & \dots & b_{1nL1} \\ \vdots & \ddots & \vdots \\ b_{n1L1} & \dots & b_{nnL1} \end{bmatrix}}^{l=1} \quad \ddots \quad \overbrace{\begin{bmatrix} b_{11Lr_s} & \dots & b_{1nLr_s} \\ \vdots & \ddots & \vdots \\ b_{n1Lr_s} & \dots & b_{nnLr_s} \end{bmatrix}}^{r=r_s}\right]}^{l=L}\right]\right] \tag{10}$$

And the containment matrix $\mathbf{B}$ over all n buses, L non-homogeneous channels, and D PDCs may be summarised as,

$$\mathbf{B} = [\mathbf{B}_d \quad \dots \quad \mathbf{B}_D] \tag{11}$$

The elements b_{ijlr} of each $\mathbf{B}_d$ take on binary values as described in Section 2.

The process of formulating $\mathbf{B}$ in (11) holds true for matrices $\mathbf{H}, \mathbf{F}$ and $\mathbf{G}$, with the description of their respective elements retained. For instance, $\mathbf{H}$ can formed as,

$$\mathbf{H}_d = \left[\left[\left[\overbrace{\left[\overbrace{[I_{n\times n}]}^{r=1} \quad \ddots \quad \overbrace{[I_{n\times n}]}^{r=r_s}\right]}^{l=1} \quad \ddots \quad \overbrace{\left[\overbrace{[I_{n\times n}]}^{l=1} \quad \ddots \quad \overbrace{[I_{n\times n}]}^{r=r_s}\right]}^{l=L}\right] \quad \vdots \quad \left[\overbrace{\left[\overbrace{[I_{n\times n}]}^{r=1} \quad \ddots \quad \overbrace{[I_{n\times n}]}^{r=r_s}\right]}^{l=1} \quad \vdots \quad \overbrace{\left[\overbrace{[I_{n\times n}]}^{l=1} \quad \ddots \quad \overbrace{[I_{n\times n}]}^{r=r_s}\right]}^{l=L}\right]\right]\right] \tag{12}$$

$$\mathbf{H} = [\mathbf{H}_d \quad \dots \quad \mathbf{H}_D] \tag{13}$$

The decision vector is formed as,

$$\mathbf{X}_d = [[x_{111} \quad \cdots \quad x_{n11}] \quad \dots \quad [x_{11r_s} \quad \cdots \quad x_{n1r_s}] \quad \dots \quad [x_{nL1} \quad \cdots \quad x_{nL1}] \quad \dots \quad [x_{nLr_s} \quad \cdots \quad x_{n1r_s}] \quad]^T \tag{14}$$

$$\mathbf{X} = \left[\mathbf{X}_d^T \quad \dots \quad \mathbf{X}_D^T\right]^T \tag{15}$$

The cost vector can be described as,

$$\mathbf{C}_d = [[C_{111} \quad \dots \quad C_{n11}] \quad \dots \quad [C_{11r_s} \quad \dots \quad C_{n1r_s}] \quad \dots \quad [C_{niL1} \quad \dots \quad C_{nL1}] \quad \dots \quad [C_{nLr_s} \quad \dots \quad C_{n1r_s}] \quad]^T \tag{16}$$

$$\mathbf{C} = \left[\mathbf{C}_d^T \quad \dots \quad \mathbf{C}_D^T\right]^T; \tag{17}$$