Multiband superbackscattering via mode superposition in a single dielectric particle

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The superposition of resonances in a subwavelength particle can be used to achieve powerful scattering beyond the single channel limit, and can also determine the directionality of scattered radiation. It has been proposed that by overlapping only modes with equivalent polarity in the far-field, a 'superbackscattering' condition, where the total backscattered power is maximised, can be achieved. This effect can be observed through the simple geometry of a high permittivity, subwavelength sphere with a hollow core, and we demonstrate this experimentally by comparing the radar cross section (RCS) of such structures, attaining a doubling of the RCS compared to a solid particle. Furthermore we show that several sets of modes can be overlapped at once, leading to a multi-band, superbackscattering effect.

Control over the magnitude and directionality of electromagnetic scattering by an object is fundamental to applications as diverse as novel antenna design[1], energy harvesting[2] and radar detection[3]. Subwavelength particles are of particular interest as their strong resonant response enables them to provide a much larger scattering cross section than their physical dimensions[4]–[6]. Although the scattering spectra from a subwavelength object is determined by the well-established Mie theory - where each resonance has a defined order, magnitude and radiation pattern[7], it can be shown that by manipulating the particle structure one can excite modes at significantly different frequencies than might be expected, opening the door to a much wider range of behaviours.

A particularly promising case is the condition of mode superposition, or 'stacking', where a resonator is engineered so that multiple resonances occur degenerately at a single frequency. There is a fundamental limit to the scattering cross section of any single resonance, referred to as the single channel limit, which is $3\lambda^2/2n\pi$ for a 3D resonator, where *n* is the refractive index of the surrounding media[7]. However, there is no fundamental limit to the number of modes that can occur simultaneously in a particle. Therefore manipulating the structure of a scatterer so that several modes occur degenerately can lead to extremely powerful scattering of radiation, including the well-documented 'superscattering' condition, where the particle scatters radiation much more powerfully than the theoretical single channel limit[5], [8]–[11].

As well as the power of scattered radiation, mode superposition can also lead to an increase in directionality of the scattering. Generally it is found that this mode superposition (typically between the magnetic and electric dipoles of a system) produces a Huygens source where backscattering is all but eliminated, which has applications for antireflective coatings[12]–[14] and superdirective antennas[15], [16]. This has recently been experimentally demonstrated in both the microwave[17]–[19] and optical regimes[20], [21]. Whilst nearly all current superscattering works focus on this forward scattering condition, it has also been theoretically posited by Liberal et al, that by careful selection of the modes selected for stacking, a strong enhancement in *backs*cattered radiation of a

single particle (the 'superbackscattering' condition) can be achieved[22]. This has applications for improving the scattering cross section of small, hard to detect objects such as consumer drones[23]–[25].

In this paper we demonstrate through simulation and experiment the realisation of the 'superbackscattering' effect via mode superposition in a spherical dielectric object. Moreover we extend the theoretical discussion to show that this effect can occur through multiple mode combinations, and that it is possible to achieve multiband superbackscattering via careful choice of geometry. We choose a spherical geometry to give the system an isotropic response, independent of the incident angle, and confirm results from numerical simulations with experimental data taken in an anechoic chamber.

The Mie modes of a dielectric particle will each scatter radiation with a direction and phase dependent on the mode family (electric or magnetic), and mode order - as shown in Fig. 1. The superposition of modes with opposing phase in a given direction will lead to destructive interference along this vector. For mode superpositions such as the electric dipole and the electric quadrupole (ED+EQ), or the electric and magnetic dipoles (ED+MD) it can be seen in Fig.1a that due to the polarity of radiation for each mode (shown by coloured arrows), this condition is fulfilled along in the reverse direction. Constructive interference occurs in the forward direction as the phase of the two modes along this vector will be matched. This is known as the first Kerker condition. The second Kerker condition, where all forward scattering is suppressed, can also be achieved, but only far from a resonance, due to the symmetry conditions of the system[7]. Superposition of modes of equivalent polarity in both directions, such as the electric dipole and the magnetic quadrupole (ED+MQ) produces enhanced scattering (and a tighter beam width) in both the forward and reverse directions (Fig. 1b). So whilst the greatest *ratio* of backscattered radiation is achieved through the second Kerker condition, since this is only attainable far from resonance, the greatest total power scattered in the monostatic direction has been shown to occur when following the mode superposition conditions described above[22]. This is referred to as the superbackscattering condition.



Figure 1 : Illustration of scattering patterns from various resonant modes of a dielectric sphere excited by radiation incident along the x-axis and polarised along the z-axis. (a) Shows the effect of superimposing the electric and magnetic dipoles, resulting in solely forward scattering – the first

Kerker condition. (b) Shows the effect of superimposing the electric dipole and magnetic quadrupole, producing enhanced bidirectional scattering – the 'superbackscattering' condition.

Liberal et al[22] and Rezvani[26] have shown that making a spheroidal, air filled cavity in the centre of a high permittivity dielectric sphere would lead to a shifting of the dipolar mode whilst not strongly affecting the resonance of nearby modes. The reasoning behind this can be seen in Fig. 2, where it can be observed that the electric dipole has a strong field component at the centre of the sphere and higher order electric modes also have significant fields towards the centre, so reducing the permittivity here will result in a shift to higher frequencies for these resonances. Magnetic modes on the other hand, have a null at the particle centre, and thus changing the dielectric constant here does not affect them significantly until the radius of the hole is large.



Figure 2 : Simulations of the normalised electric field intensity for the first seven Mie resonances of a 25 nm dielectric sphere with a relative permittivity of $\varepsilon = 10.5$.

Choosing an appropriate diameter cavity will shift the electric modes enough to overlap with the neighbouring magnetic modes, producing the superbackscattering effect. We explore this by simulating fabricable materials in the microwave regime in Fig. 3, where the scattering cross section (SCS) of a 25 mm diameter sphere made of Premix 1050[27] is simulated, with a changing radius of a hollow (air-filled) core. The SCS is normalised to $3\lambda^2/2n\pi$ in order to clearly demonstrate how overlapping the modes allows this limit to be surpassed. The dielectric has a relative permittivity of ε = 10.5 with a loss tangent of δ = 0.001. It can be clearly seen that the electric modes in general are far more strongly affected by small hole diameters than the nearby magnetic modes, and the presence of the hole causes a blueshift in the resonant frequency. This allows these modes to approach and overlap with the neighbouring magnetic mode, in each case a mode that is one order higher, so electric dipole (ED) intercepts the magnetic quadrupole (MQ) etc. When an overlap occurs, the scattering cross section is increased significantly - the simulated SCS of the peak overlaps at 5.1, 6.63 and 8.1 GHz reach a maximum of 2.91, 4.48, and 5.8 times the single channel limit, and so this clearly represents a superscattering effect.

As discussed, lower order electric modes have a greater field component close to the centre of the particle, as can be seen in the electric field plots of Fig. 2, and so these modes show the greatest frequency shift for small hole diameters, with increasing mode orders only achieving a crossover for larger diameters. Nevertheless, due to the lower Q-factor of the lower order modes, it is possible to achieve hole diameters where a superscattering effect is achieved across several bands at once. Fig. 3b shows scattering cross section traces for a solid sphere and hole radii of 4 mm and 5 mm. A hole

radius of 4 mm only achieves a crossover of the ED and MQ, resulting in a modest superscattering effect, whereas the sphere with $r_i=5$ mm shows three overlap points producing a 3 band superscattering effect. As the modes stacked here all have equivalent polarity, this will lead to a significant increase in the backscattering, as explained previously and illustrated in Fig. 4.



Figure 3 : (a) Scattering cross section of a 25 mm diameter sphere made up of premix 1050 normalised to the single channel limit, $3\lambda^2/2n\pi$. Line traces have been added as a visual aid to highlight the position of the modes. (b) Plots showing the scattering cross section of a solid sphere, a sphere with a 4 mm radius cavity, with a single mode crossover between the ED and MQ at 5.1 GHz, and a 5 mm radius hole, with three crossover points at 5.1, 6.63 and 8.1 GHz.

The utility of this effect to enhance backscattering can be demonstrated experimentally by measuring the monostatic radar scattering cross section (RCS). To achieve this, solid and hollow (5 mm radius cavity) spherical samples were milled from premix 1050. A 5mm radius cavity was chosen to give the

largest value of backscatter. These were placed in an anechoic chamber mounted on rohacell 31HF foam, chosen for its extremely low relative permittivity ($\epsilon = 1.04$). They were illuminated by microwave radiation between 2.5-7.5 GHz using an Anritsu MS46122B VNA and a DP240-AB Dualpolarisation horn antenna built by Flann Microwave. The quasi-monostatic RCS is measured using a second antenna, placed near to the first. A time gating function is applied to reduce experimental noise and all measurements are calibrated to a standard of known RCS, in this case a 12 mm radius brass sphere. Fig. 4 displays the results.



Figure 4 : An experimental demonstration of the superbackscattering effect. (a) Shows the simulated (red) and measured (blue) monostatic RCS of a 12.5 mm radius solid sphere constructed of Premix 1050. (Inset : Illustrations of the cross-section of each particle.) (b) Shows the same sphere, with a 5 mm radius cavity in the centre, leading to a superbackscattering effect at 4.35 & 6.37 GHz.

The results show a strong agreement with the simulations, and it can be seen that the RCS around the MQ peak at 5.1 GHz is increased by a factor of 1.5 and that around the MH peak at 6.63 GHz by a factor of 2.05. This latter peak is equivalent to the RCS of a metal sphere 40 times the cross sectional area of the particle and is comparable with the average RCS of many small consumer drones[23]–[25]. This ability to attain such a large RCS for a subwavelength particle has applications in any field where a small object needs to be made more visible to radar, such as drones, small satellites and possibly even small pets for enhanced ease of detection in the radar of autonomous vehicles.

In conclusion, we have experimentally demonstrated the principle of superbackscattering, via the superposition of resonant modes with equivalent polarity in the reverse (monostatic) direction, leading to enhanced backscatter from a subwavelength particle. We demonstrate that such a particle can be structured so that this effect occurs across multiple bands at once, and results from numerical simulations show excellent agreement with simulations. This behaviour could be useful for enhancing the radar cross section of small objects such as consumer drones.

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Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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