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Upper-Bound Cost Analysis of a Market-Based Algorithm Applied to the Initial Formation Problem

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***Abstract*— In this paper, an analysis of a market-based approach applied to the Initial Formation Problem is presented. This problem tries to determine which mobile sensor should go to each position of a desired formation in order to minimize an objective. In our case, this objective is the global distance traveled by all the mobile sensors. In this analysis, a bound on the efficiency for the market-based algorithm is calculated and it is shown that the relative difference as compared with the optimal solution increases with the logarithm of the total number of mobile sensors. The theoretical results are validated with numerous simulations.**

I. INTRODUCTION

Mobile sensor networks have been shown to be a powerful tool for enabling a number of environmental monitoring activities such as monitoring of seismic activity, monitoring of civil and engineering infrastructures, and monitoring of biological agents throughout a region [14]. In most sensor network applications, individual sensor agents collect information about their neighboring agents using peer-to-peer communication. Unfortunately, as the size of the network increases, bandwidth limitations and the absence of feasible communication channels severely limits the possibility of conveying and using global information. As such, the utilization of decentralized techniques for forming new sensor topologies and configurations is a highly desired quality of mobile sensor networks. Establishment of these sensor configurations involves determining how to allocate sensor positions to mobile sensor agents in order to achieve a desired topology.

This Initial Formation Problem [1], [13] can be cast as a task allocation problem which can be stated as follows:

Given a desired topology expressed by a number of positions, $\{P_1, P_2, ...P_N\}$*, a group of mobile sensors* $\{S_1, S_2, ..., S_M\}$*, a function* $C(P_i, S_j)$ *that specifies the cost of going to the position* P_i *by mobile sensor* S_j *and considering that the number of positions must be less or equal than the number of mobile sensors, i.e.,* $N \leq M$*. Find the assignment that allocates one position per mobile node and minimizes the global cost defined as* $\sum_{j=1}^{M} C(P_i, S_j)$*, where* i *is the position assigned to mobile sensor* j*.*

If the number of mobile sensors is bigger than the number of positions, the problem remains the same but there will be some mobile sensors that will not be allocated with any position. Therefore, all the mobile sensors take part on the allocation mechanism, but only the same number of mobile sensors as positions will have a task to execute. The other mobile sensors will be idle until new tasks are introduced or generated dynamically. Also, these mobile sensors could be used to take care of positions that cannot be finished since one or more mobile sensors could fail in the middle of a execution.

This problem can also be viewed as a classical job assignment problem where mobile sensors are workers and the desired positions are the different jobs. The classic job assignment problem can be solved using centralized solutions such as the Hungarian method [9], but this type of solution requires a total knowledge of the world and has all the disadvantages related to centralized systems: low fault tolerant, computational complexity and slow response for dynamic changes in the environment.

In the last few years, different approaches have been used to solve the multirobot task allocation problem [4] in a distributed way with local information. So far, one of the most successful has been the market-based approach [2], [3], [6], [16] since it offers a good compromise between the communication requirements and the quality of the solution by using negotiations to allocate the different positions. This negotiation is typically implemented by using some variant of the *Contract Net Protocol* [15], [17], where two roles are played dynamically by mobile sensors: auctioneer and bidders. The auctioneer is the mobile sensor in charge of announcing the desired positions and selecting the best bid from all the bids received from the bidders. The best bid is considered the one with the lowest cost.

In order to use a market-based algorithm to solve the Initial Formation Problem, we must reformulate it as a task allocation problem where the tasks are waypoint tasks that coincide with the positions of the formation. For that reason, the cost used in the bids is a quantity that reflects how much it will cost the mobile sensor to go to a certain waypoint, such as the euclidean distance or the traversability index [8]. Also, it is important to point out that one mobile sensor can only be allocated one task, since the final objective is to assign one position of the formation to each mobile sensor.

Although the efficiency of the market-based algorithms have been proven in numerous simulations and some real implementations [5], [7], none of these works has obtained a theoretical bound on the real efficiency of these algorithms. As far as we know, the only work that obtains a bound for a task allocation algorithm based on auctions is [11] but it supposes that all the mobile sensors know all the desired

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positions from the beginning. Therefore, their decentralized implementation differs from the classical market-based approach and computes the bids using the global information of the desired positions plus the local information of the mobile sensor. Our algorithm goes a step forward and it uses the basic market-based approach where the bids are calculated just using the local information of the mobile node, i.e., its position and the desired position to be allocated. Also, for the algorithm explained in this paper, we prove that the upper bound of the global cost gets worse when the number of mobile sensors increases. From the best of our knowledge, this is the first time that it is proven that a market-based algorithm only obtains efficient (close to the optimal) solutions when the number of mobile sensors is relatively small.

The paper is organized as follows. In the next section, a basic market-based algorithm that solves the Initial Formation Problem will be explained. In Section III a mathematical analysis of the performance will be presented supposing that the costs of the different desired positions are uniformly distributed. The bound calculated in this section will then be validated with various simulations in Section IV. Finally, conclusions and future work are provided in Section V.

II. MARKET-BASED APPROACH FOR THE INITIAL FORMATION PROBLEM

A. Description of the algorithm

A market-based algorithm has been used to solve the Initial Formation Problem. As usual in algorithms based on the Contract Net Protocol, two roles are played dynamically by mobile sensors: auctioneer and bidders. The auctioneer is the agent in charge of announcing the desired positions and selecting the best bid from all the received bids. In our case the best bid is the one with the lowest cost and the cost is equal to the distance from the mobile sensor to the desired position. The complete algorithm is explained in Algorithm 1. On the other hand, the bidder role is explained in Algorithm 2. The basic idea is that each mobile sensor must have only one desired position, so it will keep the position with the lowest cost. If it wins a new position that has a lower cost than the one already won, it will sell the old position to the mobile sensor with the best bid but worse than its own bid. The best bid worse than the mobile sensor's bid is selected in order to avoid infinite loops in the negotiation. This scenario could happen when two mobile sensors have the best bids for at least three positions.

For formation initialization, a slight modification is instituted for the market-based structure. At the beginning, positions are introduced by a human operator, such as a scientist, using a monitoring center or a planner that generates the desired positions. Therefore, in our system there are two types of agents: mobile nodes and monitoring center, and two types of roles: auctioneer and bidders. Both types of agents can play both roles. However, the monitoring center plays the auctioneer role at the beginning and after all the positions are introduced, it switches to the bidder role with a constant bid equal to infinite for all positions in order to assure that it will never win a desired position after the auction starts, or if we want to minimize the communication messages, it will not bid to any announced task. It is important to point out that the monitoring center need not be unique, i.e, the same algorithm works with distributed insertion of tasks. Also, tasks can be generated dynamically by mobile sensors, and therefore, there is no firm requirement for existence of a monitoring center. It is rather just an implementation detail.

Algorithm 1 Auctioneer algorithm

```
if announcement-position list is not empty then
  announce desired position
  while timer is running do
    receive bids
  end while
  calculate best bid worse than the mobile sensor's bid
  send desired position to best bidder
  delete desired position from announcement-position list
end if
```

Algorithm 2 Bidder algorithm

```
a new message is received
if new message is a desired position announcement then
  calculate bid (distance to the position)
  if mobile sensor has won already a position then
    if cost of won position is bigger than received position then
      send bid to the auctioneer
    end if
  else
    send bid to the auctioneer
  end if
else if new message is a desired position award then
  if the mobile sensor has already won a desired position
  then
    if cost of the new position < cost of the won one
    then
      introduce old position in announcement-position
      list and delete it from won-positions list
      introduce won position in the won-positions list
    else
      introduce won position in the announcement-
      position list
    end if
  else
    introduce won position in the won-positions list
  end if
end if
```

B. Matrix-scan algorithm for the assignment problem

The matrix-scan is a heuristic algorithm to solve the assignment problem. When this problem is expressed in a matrix form where each element is the cost associated with the respective worker and job, the algorithm works as follows:

- The smallest element of the entire matrix is selected.
- The row and column associated to this element are deleted and therefore the order of the matrix is reduced by one.
- The matrix is searched again for the smallest element and the process is repeated until a matrix of order one is reached.
- The selected elements are the solution of the assignment problem.

C. Basic market-based algorithm as a matrix-scan algorithm

The use of reallocations in the basic market-based algorithm ensures that it will obtain the same solutions as the matrix-scan algorithm. This fact is illustrated with the following example:

- The initial positions of the mobile sensors and the desired positions are the ones show in Figure 1.
- Supposing that the columns represent the desired positions and the rows represents the mobile sensors, the matrix that models this specific problem is:

$$\begin{pmatrix} 30.0 & 41.23 & 20.0 \\ 50.0 & 10.0 & 44.72 \\ 80.0 & 72.11 & 30.0 \end{pmatrix}$$

- Following the algorithm steps, the smallest element of the matrix is selected. This element is 10.0 which assigns the mobile sensor B with the position number 2. The row and column of the selected element is deleted and the following matrix is obtained:

$$\begin{pmatrix} 30.0 & 20.0 \\ 80.0 & 30.0 \end{pmatrix}$$

- Again the smallest element of the new matrix is selected. This element is 20.0 and, therefore, the mobile sensor A is assigned to the position number 3.
- Finally, the last assignment is made such that mobile sensor C is assigned to position number 1.

As can be observed in Figures 1 and 2, the solution obtained with the basic market-based approach is exactly the same as the one obtained by the matrix-scan method. Thus, the same bound can be applied to both algorithms.

III. ANALYSIS OF THE ALGORITHM

A. Assumption

The analysis of the algorithm will be based on a probabilistic approach [10]. The main assumption for this analysis is which distribution is used to model the costs. In this section, a theoretical bound for the matrix-scan algorithm, and therefore, for the basic market-based algorithm is found supposing that the costs, i.e., the distance among the mobile sensors and the desired positions, are uniformly distributed.

Thus, it can be stated that our proof assures that:

- The elements in the cost matrix are uniformly distributed in $[0, D]$, where D is the maximal distance between every mobile sensor and desired position.

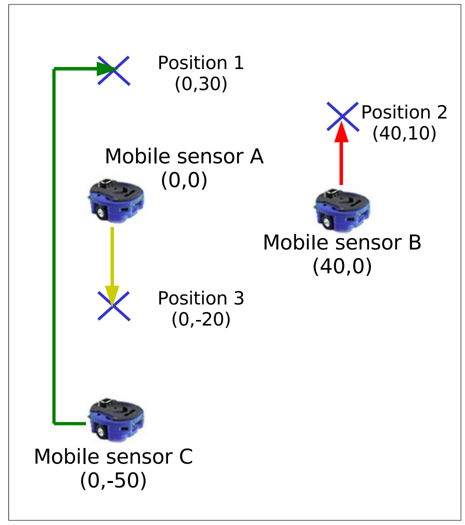

Fig. 1. Initial position of the mobile sensors and the desired positions, and also, the final assignment obtained with the basic market-based algorithm.

B. Theoretical bound

If we model our assignment problem with a matrix as done in Section II-C, it is supposed that all the elements d_{ij} are uniformly distributed in the matrix within the range $[0, D]$. However, it is more interesting to have all the elements (c_{ij}) between $[0, 1]$, so all the elements of the matrix will be divided by D.

For the Initial Formation Problem, we minimize the global distance traveled by all the mobile sensors. However, it is usually easier to solve the maximal assignment problem than the minimal one, and also, it is known that both problems are equivalent. If $e_{ij} = (1-c_{ij})$, solving the maximal assignment problem for the matrix with elements e_{ij} is equivalent to solve the minimal assignment problem for the matrix c_{ij}. Thus, the relation between our original cost matrix (d_{ij}) and the one used to calculate the bound (e_{ij}) is:

$$d_{ij} \in [0, D], \tag{1}$$

$$c_{ij} = \frac{d_{ij}}{D} \in [0, 1], \tag{2}$$

$$e_{ij} = 1 - cij = 1 - \frac{d_{ij}}{D} \in [0, 1]. \tag{3}$$

Let the expected value given by the matrix-scan method be denoted by $M(n)$ for a matrix of order n. This value can be considered as the sum of the costs for the allocated positions in our particular problem. Also, it can be supposed that the maximum value in the matrix is x, where $0 \leq x \leq 1$. The remaining elements after that maximum element is selected, are then independently and uniformly distributed in $[0, x]$. Therefore, the expected value of the matrix-scan method on a matrix of order $n-1$ is $xM(n-1)$, considering that if every element of the $n-1$ matrix is divided by x, a $n-1$ matrix of elements in the range $[0, 1]$ is again obtained. So, given that the maximum value is x, then $M(n) = x + xM(n-1)$ where $n^2 x^{n^2-1} dx$ is the probability that x is the maximum value. Thus,

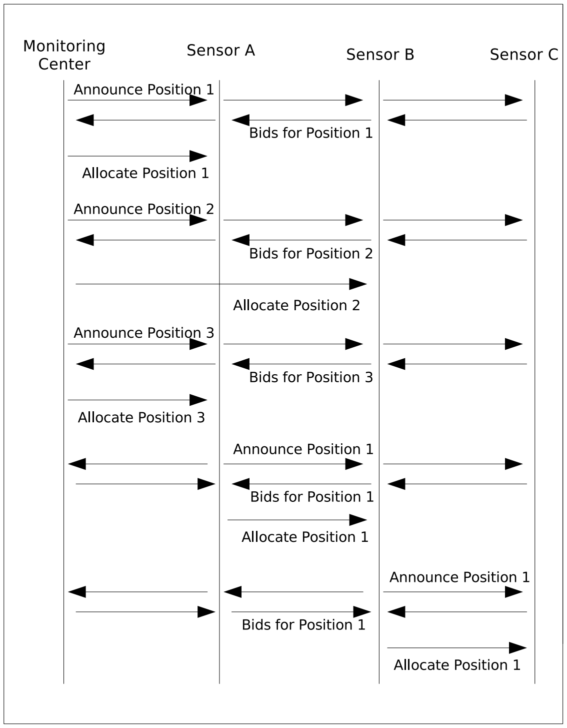

Fig. 2. Messages exchanged among the different mobile sensors using the basic market-based algorithm. The initial positions of the mobile sensors and the positions of the formations are the same as Figure 1.

$$\begin{aligned} M(n) &= \int_0^1 [x + xM(n-1)] n^2 x^{n^2-1}\, dx \\ &= \frac{n^2}{n^2+1}[1 + M(n-1)]. \end{aligned} \qquad (4)$$

A lower bound for $M(n)$ is found, using the following recursive relation:

$$\begin{aligned} M(n) &= \frac{n^2}{n^2+1}[M(n-1)+1] \\ &= \frac{1}{1+\frac{1}{n^2}}[M(n-1)+1]. \end{aligned} \qquad (5)$$

Since $\dfrac{1}{1+\frac{1}{n^2}} = 1 - \dfrac{1}{n^2} + \left(\dfrac{1}{n^2}\right)^2 - \ldots$, then

$$M(n) > \left[1 - \frac{1}{n^2}\right][M(n-1)+1] \qquad (6)$$

or

$$M(n) - M(n-1) > 1 - \frac{M(n-1)}{n^2} - \frac{1}{n^2}; \qquad (7)$$

and since each element that contributes to $M(n-1) \in [0,1]$, $M(n-1) \leq n-1$,

$$M(n) - M(n-1) > 1 - \frac{n-1}{n^2} - \frac{1}{n^2} = 1 - \frac{1}{n}. \qquad (8)$$

Taking the sum of both sides, with n going from 1 to T, gives the result

$$\sum_{n=1}^{T} [M(n) - M(n-1)] > \sum_{n=1}^{T} \left(1 - \frac{1}{n}\right), \qquad (9)$$

which is equal to

$$M(T) - M(0) > T - \sum_{n=1}^{T} \frac{1}{n}. \qquad (10)$$

As $M(0) = 0$ and $\gamma^1 > \sum_{n=1}^{T} \left(\frac{1}{n}\right) - \ln T$,

$$M(n) > n - \gamma - \ln n. \qquad (11)$$

Now, $M(n) = M_e(n) = x_1 + x_2 + \ldots + x_n$ where x_n is the maximum value of the matrix of order n. Therefore, the sum of the cost for the minimization problem is $m_c(n) = 1 - x_1 + 1 - x_2 + \ldots + 1 - x_n = n - M_e(n)$. Due to the fact that the values are normalized within the interval $[0,1]$, $m_d(n) = D(n - M(n))$. As a result, an upper bound for our algorithm has been found:

$$m_d(n) < D \cdot (\gamma + \ln n). \qquad (12)$$

The relative error in comparison with the optimal solution has the following upper bound:

$$\frac{C - C^*}{C^*} = \frac{C}{C^*} - 1 = \frac{m_d(n)}{C^*} - 1 < \frac{D \cdot (\gamma + \ln n)}{C^*} - 1, \qquad (13)$$

where C^* is sum of the costs for the optimal solution, C the sum for the solution obtained with our market-based approach and D is the maximum possible cost.

A simple lower bound for the optimal solution is

$$C^* \geq n \cdot d, \qquad (14)$$

where d is the smallest cost among all the mobile sensors and desired positions. Thus,

$$\frac{C - C^*}{C^*} \leq \frac{D \cdot (\gamma + \ln n)}{n \cdot d} - 1. \qquad (15)$$

This bound can be narrowed using the following result from [12]:

$$c^* \geqslant 1 + 1/e + O(n^{-1+\varepsilon}), \qquad (16)$$

where c^* is the optimal solution when the costs are uniformly distributed between $[0,1]$.

However, it is important to point out that only when the number of mobile sensors is large enough the term $O(n^{-1+\varepsilon})$ can be dismissed. Thus,

$$c^* \geqslant 1.368. \qquad (17)$$

[1] γ is the Euler-Mascheroni constant which is defined as $\gamma = \lim_{n\to\infty} \left[\sum_{k=1}^{n} \frac{1}{k} - \ln n\right]$ and $\gamma \simeq 0.577$

Since it has been supposed that the costs are uniformly distributed within the range $[0, D]$, a lower bound for the optimal solution is:

$$C^* \geqslant D \cdot 1.368. \quad (18)$$

Finally, using the lower bound for the optimal solution from Equation 18, the following improved bound is obtained:

$$\frac{C - C^*}{C^*} < \frac{D \cdot (\gamma + \ln n)}{D \cdot 1.368} - 1 = \frac{\gamma + \ln n}{1.368} - 1. \quad (19)$$

It can be observed that the relative difference with the optimal solution is proportional to $\ln n$ which means that the efficiency of this algorithm gets worse when the number of mobile nodes and desired positions increases. Thus, the market-based algorithm described in this paper only obtains efficient solutions close to the optimal when the number of mobile sensors is relatively small.

IV. VALIDATION THROUGH SIMULATIONS

A multi-mobile sensor simulator has been used to test the decentralized algorithms discussed in this paper. The simulator is based on an architecture designed for heterogeneous mobile sensors and divided into three layers [18]. The highest layer is independent from the type of mobile sensor and is aware of the existence of other sensors. Thus, the task allocation algorithm is implemented in this layer. The other two layers are used to execute the different tasks allocated to the mobile sensor and make easier the creation of new algorithms by using a modular and component-based architecture.

The basic market-based algorithm has been tested using uniformly distributed costs between 0 and 500 in a virtual world of 353.55x353.55 meters. The simulations were run using different numbers of mobile sensors and desired positions ranged from 4 up to 20, and for every case one hundred simulations were run. Firstly, some simulations have been performed using the matrix-scan algorithm as a equivalent of the original distributed algorithm, obtaining the results shown in Figure 3. It can be seen that the global cost calculated from the experimental results are always smaller than the theoretical bound calculated in the previous section. On the other hand, the same simulations have been run using the multi-mobile sensor simulator and the distributed algorithm, the results are shown in Table I where, in each cell, the mean of the global cost and the error in percentage in comparison with the optimal solution are presented. The optimal solution has been calculated using the Hungarian method [9]. As can be seen in this table, the experimental results coincide with the theoretical ones since the error, as compared with the optimal solution increases with the number of mobile sensors. This fact can be observed in more detail in Figure 4, where the global cost obtained from these simulations is compared with the bound from Equation 12. As was expected the experimental global cost is always smaller than the theoretical bound and the results are very similar to the ones obtained using the matrix-scan method shown in Figure 3. Finally, in Figure 5 both the theoretical and experimental percentage error in comparison with the optimal solution are compared. In this figure, there is more difference between the theoretical and experimental results since we are using for the theoretical error a conservative upper bound for the optimal solution, and in the experimental results, we are using the exact optimal solution. In summary, the experimental results shown in this section validate the theoretical bound that comes from the analysis of the market-based algorithm (see Section III).

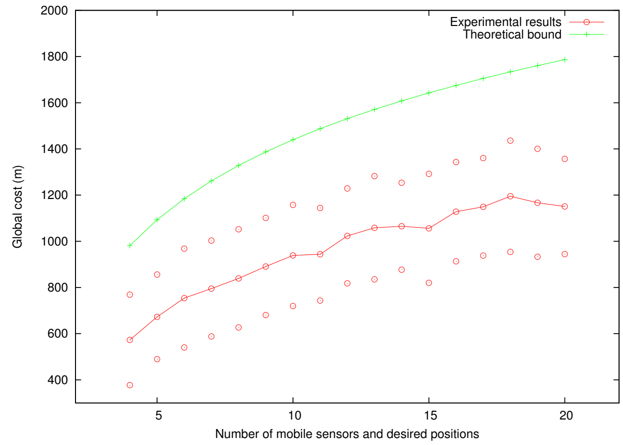

Fig. 3. Comparison between the theoretical and experimental global costs, i.e., the distance traveled by all the mobile sensors. The experimental results are calculated using the matrix-scan algorithm over 100 missions per case where the costs of going to the desired positions for each mobile sensor are uniformly distributed between 0 and 500. The circles represent the standard deviation of the global cost from the experimental results.

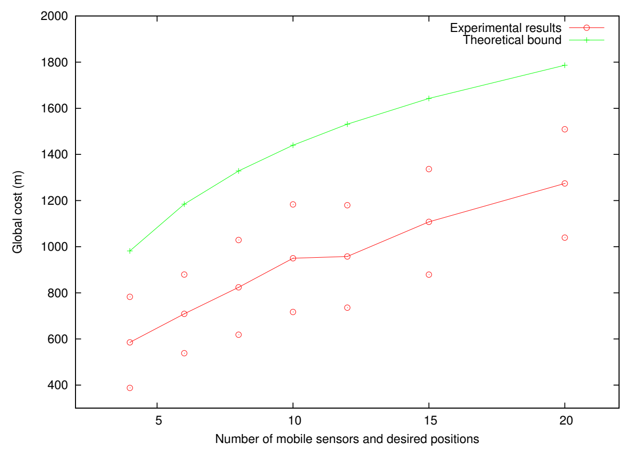

Fig. 4. Comparison between the theoretical and experimental global costs, i.e., the distance traveled by all the mobile sensors. The experimental results are calculated using distributed basic market-based algorithm over 100 missions per case where the costs of going to the desired positions for each mobile sensor are uniformly distributed between 0 and 500. The circles represent the standard deviation of the global cost from the experimental results.

V. CONCLUSIONS AND FUTURE WORK

A. Conclusions

The Initial Formation Problem has been stated and described in terms of the task allocation problem. In order

# of Positions & Mobile Sensors	*Market-Based*	*Optimum*
4	585.24 (12.09%)	521.23
6	708.86 (20.88%)	584.75
8	823.74 (28.15%)	642.78
10	949.96 (35.22%)	702.55
12	957.65 (41.57%)	676.43
15	1107.78 (56.53%)	707.68
20	1274.13 (66.96%)	763.12

TABLE I

RESULTS COMPUTED FOR FORMATIONS WITH DIFFERENT NUMBER OF MOBILE SENSORS AND DESIRED POINTS OVER 100 SIMULATIONS PER EACH CASE. IN EACH CELL THE MEAN OF THE GLOBAL COST (IN METERS) AND THE ERROR IN PERCENTAGE WITH THE OPTIMAL SOLUTION ARE PRESENTED.

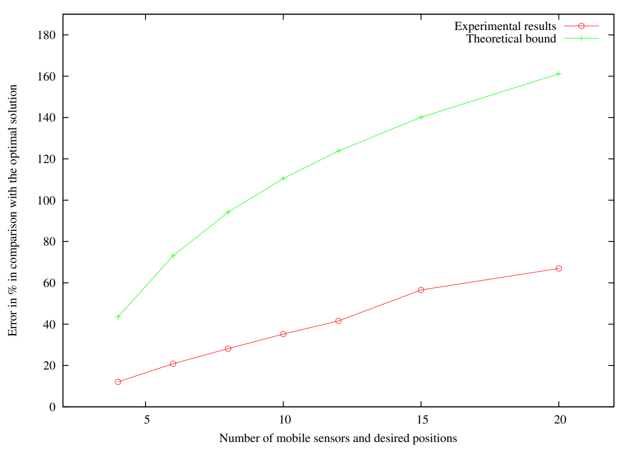

Fig. 5. Comparison between the theoretical and experimental errors in percentage in comparison with the optimal solution. The experimental error is calculated over 100 missions per case where the costs of going to the desired positions for each mobile sensor are uniformly distributed between 0 and 500. The theoretical error is calculated from Equation 19.

to solve it, a market-based algorithm has been developed. This algorithm obtains the same results as the matrix-scan algorithm used for the job assignment problem. Although the market-based algorithm does not obtain an optimal solution that minimizes the global cost, it is known that these types of algorithms obtain efficient solutions. This has usually been demonstrated by means of numerous simulations without a theoretical proof that supports it. However, based on the analysis of the market-based algorithm explained in this paper, we have demonstrated that the efficiency of this algorithm gets worse when the number of mobile sensors increases. Therefore, it can be stated that this algorithm only obtains solutions close to the optimal (efficient solutions) when the number of mobile sensors is relatively small. Finally, the theoretical bound calculated for this algorithm has been validated with numerous simulations.

B. Future work

Future work includes the extension of the analysis to other algorithms used for the Initial Formation Problem. Also, we are interested in finding a theoretical bound that avoids the assumption that costs must be uniformly distributed or that accepts other types of distributions for the costs. Finally, the general task allocation problem will be considered and considerations made on how the upper-bound analysis could be adapted to solve it.

VI. ACKNOWLEDGMENTS

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