Power Relations in the International Monetary Fund: A Study of the Political Economy of *a priori* Voting Power using the Theory of Simple Games

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Abstract:

In general in organisations whose system of governance involves weighted majority voting, power and voting weight differ. Power indices are a value concept for majority voting games which provide a means of analysing this difference. This paper provides new algorithms for computing the two classical power indices (the Banzhaf index and the Shapley-Shubik index) and applies them to the voting distribution in the two governing bodies of the IMF in each year since its foundation. The focus is both substantive, being an analysis of the political economy of the IMF, and methodological, as a study of the use of the power indices. Power relations are studied with respect to two types of decisions: ordinary decisions requiring a simple majority and decisions requiring a special majority of 80% or 85%. Clear cut results are obtained for the former: among the G5 countries discrepancies between power and voting weight have declined over time with the exception of the United States which continues to have much more power than its weight even though that weight has declined. In the nineteen forties the United Kingdom's power was considerably below its relatively large nominal voting power, similarly to some extent for France. Both power indices give results which are qualitatively comparable. For decisions requiring special majorities, however, few general results emerge because of conflict between the indices. We examine the effect of the size of the majority requirement on the power of the leading members and find that the power of the US declines as the majority requirement increases. This result confirms the warnings of Keynes that the US insistence on retaining a national veto for itself might be counterproductive. We conclude that the special majority requirement creates a distortion in the voting system which can be regarded as a serious lack of transparency. We also examine the effect of the EU countries voting as a block rather than

individually and show that it would be dominant and the US power would fall considerably. We conclude that it is not possible to make a clear choice between the two power indices used but that there is some indication that the Banzhaf index may be more plausible.

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Nontechnical Summary:

This paper reports a study of the voting system provided for in the Articles of Agreement of the IMF. It uses a method of analysis derived from the theory of games to study real voting power - the ability to determine voting outcomes. It uses the method of Voting Power Indices to measure each country's power. The paper is partly concerned with aspects of this little-used methodology as well as the IMF.

It presents a separate analysis for each of the two governing bodies of the IMF, the Board of Governors - where each member country has a seat - and the Executive Board - with a small number of representatives, some appointed directly by the USA, Germany, Japan, France and Britain, and some elected by groups of countries. Both bodies employ systems of weighted voting where a country's votes reflect its quota. In such weighted voting bodies it is well known that a member's actual power to swing a vote has no simple relationship to its actual number of votes (another well-known example is the European Council of Ministers).

Our findings are:

1. Votes do not reflect power.
   In both the Board of Governors and the Executive Board power is more unequally distributed than votes.

2. United States more powerful than is apparent.
   The United States has a greater share of the power than its share of the votes; all other members have less power than their votes.

   A separate analysis has been done for each year of the IMF' existence. A similar pattern has emerged for each year although the dominance of the USA has declined in quantitative terms. In the first year, 1946, the USA had 33 percent of the votes but our measure of its voting power was over 43 percent: it could theoretically swing 40 percent of possible votes from losing to winning. By 1996 its share of the votes had fallen to 18 percent and its voting power to over 21 percent. This shows that there is a sort of "hidden weighting" which has benefitted the United States at the expense of the other members.

3. UK less powerful than previously thought.
   The United Kingdom was the principal loser in the early years. In 1946 it had the second-largest number of votes, with over 15 percent. Nevertheless its voting power was considerably less than this, at most 12 percent. This discrepancy has virtually disappeared over time as the United Kingdom's share of the votes has declined and as voting has become more dispersed.

4. Voting "paradoxes" have occurred.
   We have investigated whether any "voting paradoxes" - where a member's share of the votes has increased but its share of power fallen or vice versa - and found that some have occurred in the cases of the UK, France and other countries but their magnitude has been small.

5. Keynes was right to oppose the US insistence on special majorities.
   An important part of the analysis was to examine the effect of different majorities on power relations. The most important decisions require special majorities - originally four fifths but

later increased to 85%. They were introduced at the insistence of the USA so that it would have a single-country veto. We have looked at the power distribution for different majority requirements and found that the introduction of special majorities was short-sighted both from the point of view of the effectiveness of the IMF and that of the USA within the IMF, as Keynes warned in 1943. It makes it difficult for decisions to be taken and gives an effective veto to a small group of other countries which might oppose US plans. The effect of a large majority requirement is to make the power distribution much more equal and therefore is not consonant with the inequality of voting designed into the IMF.

6. Effect of a European Union Block.
   We have looked at the possible scenario of the fifteen EU countries voting as a single unit rather than separately. The combined EU vote in 1996 was 28 percent and therefore it would be dominant with almost 40 percent of the voting power. It would gain power at the expense of the USA.

7. Merits of different power indices.
   The study uses two different power indices in parallel: the Banzhaf index and the Shapley-Shubik index. They purport to be both measures of the same thing but differ in their details. Our purpose in this study was to compare their results and if possible draw conclusions about their relative merits for future work. On the basis of the discussion in the paper we suggest there may be some grounds for preferring the Banzhaf index.

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I. Introduction

The governance of the IMF is of continuing interest not only because of the
obvious importance of the organisation to the management of the world economy
in an era of increasing economic globalisation. It is also a worthwhile subject of
research because the IMF is one of a small number of international organisations
(the World Bank, the European Union Council of Ministers and regional
development banks are others) that operate on the basis of the weighted voting
power of their members (in contrast for example to the United Nations). This
inequality of voting power among members of the IMF reflects the inequality of
their quotas and therefore their respective financial contributions. Inequality of
power among members is therefore a fundamental feature of the design of the
system of governance. However, this raises the important question of precisely
what is meant by unequal power. If power is defined as the ability to influence
voting outcomes, to make a difference to decisions taken in this way, then it is
not always true that there is an exact correspondence between power and nominal
voting strength. Indeed it has been known for a long time (e.g. Banzhaf (1965),
Coleman(1971), Lucas (1983)) that it is generally the case that in a body which

uses weighted voting, there is no simple relationship between power and voting weight[2].

In many weighted voting bodies power is much more unequally distributed than voting weight so that the nominal data can give a misleading picture. Members with very large voting weight can possess a disproportionately greater voting power - there is in a sense an extra "invisible weighting" - and other members proportionately less. Moreover, voting "paradoxes" can and do arise, particularly when there are changes to the membership, the distribution of votes or the decision rule (e.g. Brams (1975), Brams and Affuso (1976), also Felsenthal and Machover (1995)). Cases have been shown to occur where a change in voting weight has led to a change in power in the opposite direction. Similarly it is possible for individual voters to have no power at all despite possessing an apparently significant number of votes. (A good example of this is Luxembourg in the EC Council of Ministers before 1973, whose one vote was never able to make any difference given the votes of the other countries.).

It is therefore of intrinsic interest to consider the voting system in the IMF from this point of view, by analysing the distribution of *a priori* voting power[3]. Despite its importance and interest relatively little research has been done on this

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[2] In the discussion about the IMF voting system the term voting power is used to denote the number of votes (or fraction of the total) cammanded by a member country. Since in this paper we are making a fundamental distinction between this and actual power as defined, we will use the term voting weight instead from now on.

aspect, in contrast to other voting bodies. We are aware of only two studies which have adopted a similar approach to the one employed here: those by Dreyer and Schotter (1980) and recently Strand et.al. (1997).

The design of the voting system for the IMF was naturally an important focus of attention in the discussions surrounding the original Bretton Woods conference in which sharp differences emerged between the British and the Americans. The United States was particularly concerned to retain a national veto for itself in any voting system which applied to the most important decisions while the British preference was for simple majority voting. The American position was criticised at the time by Keynes who pointed out that a voting system which allowed one country a veto could equally give a veto to a small group of smaller countries. We investigate this issue and are able to examine the implications of this and find evidence that Keynes was right.

A secondary purpose of the present study is a methodological one of examining the application of different indices used as measures of voting power. Of the two classical indices used here neither has yet been shown to be generally superior for real world analysis although they both possess equally plausible theoretical foundations. In applications they often give results which are in agreement to a good degree although in some cases they conflict considerably. It

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[3] It cannot be overemphasised that this is fundamentaly different from attempting to draw conclusions from analyses of *observed* voting behaviour.

is hoped that as more applied work is done, and more experience gained, the better able to make judgements on this question we will become. One issue which is evident from the literature on power indices has been the difficulty of calculating them in empirical applications with real data rather than simple examples. This is because the voting power of each individual member depends on not only its own voting weight but the complete configuration of the weights of all the members which means this type of analysis requires a substantial amount of computing resources. In this study we employ new algorithms for computing the indices.

Questions we investigate in this paper are:

(1) How does the voting power of individual countries compare with their nominal votes and how have these relations evolved over time?

 (2) In particular, to what extent is the degree of inequality in the distribution of votes reflected in the distribution of voting power. Is voting power more equally or more unequally distributed than voting weight, and how has the inequality changed over time as new members have joined and quotas increased?

(3) Have voting paradoxes arisen?

(4) Are there important differences in the distribution of voting power between the two main IMF decision-making bodies, the Board of Governors and the Executive Board?

(5) Different types of decisions use different decision rules, some requiring a special majority larger than a simple majority. We investigate the effect of different decision rules on the distribution of power.

(6) How do the results given by the two measures of power compare?

The analysis in this paper is entirely in terms of formal voting power and the formal constitution as laid down in the Articles of Agreement and its amendments; we take the allocation of voting weight among the members at face value. It is commonplace however to note that the organisation is in practice controlled by the United States and the advanced industrial nations since their combined voting weight gives them a majority over the developing countries. The analysis presented here is not primarily concerned with such questions about the power of informal groupings of countries, although the methodology can obviously and usefully be employed to evaluate and compare the voting power of

groupings which did not have an actual majority.[4] We carry out one such

simulation by examining the implications of the countries of the European Union

forming a single grouping rather than voting independently.

We also note at this stage that in practice actual votes are rarely taken in

meetings of the IMF.[5] Indeed they are deliberately avoided, especially by the

Executive Board, in order to avoid the element of confrontation which attends a

contested vote, with decisions being taken after arriving at a consensus. It might

therefore be suggested that an analysis of voting power is beside the point if all

decisions are taken by consensus. However, formal voting procedures may

fundamentally influence the de facto decision making process; power

relationships are fundamentally determined by relative voting strength and the

fact that votes are not taken in meetings is a reflection that these are well

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[4] For example it would be possible to use this approach to comment on the criticisms made by developing countries that the distribution of voting power has been too heavily weighted towards developed countries. This is a consequence of a fundamental aspect of the design of the IMF that dominant voting power should be in the hands of creditor nations who provide the resources. Our analysis should be able to illuminate the extent to which this aim is fulfilled in practice or whether the "invisible weighting" of the United States and other large creditors skews the power distribution even further away from the debtor countries.

This kind of analysis could be enriched by considering the power of individual countries which are members of such groupings in terms of a two-stage process (a composed game) whereby voting is considered first within the grouping then the grouping votes as a block within the organisation. This would be feasible as an extension of the methodology used here but is beyond the scope of the present study.

[5] There is a parallel between voting and power distributions among the countries which are members of IMF and those among shareholders of a public company (the Board of Governors corresponding to the shareholders' meeting) where analyses of power and control are conducted without regard for actual voting patterns, but are in terms of potential votes which could theoretically be taken. There is an important difference however in that weighted voting is a central feature of the day-to-day operations of the IMF, in the Executive Directors as well as the Board of Governors

understood and determine the basis on which a consensus is reached, a framework within which bargaining takes place. It is also well known that, as in other international organisations, the members of the IMF attach considerable importance to questions surrounding the voting power of different members (Zamora (1980)).

In the next section we give a formal description of the constitution of the IMF. This is followed in section III by a description of the determination of voting weights. The measurement of voting power by means of power indices is described in section IV and algorithms for computing them are set out in section V. The data is described in section VI, the results presented in section VII and conclusions in section VII.

II. The Institutional Framework

According to the Articles of Agreement, the IMF has two decision-making bodies: the Board of Governors and the Executive Board[6]. All powers of the IMF are vested in the Board of Governors, which may delegate to the Executive Board authority to exercise all except certain specified reserved powers. The Executive Board is responsible for the general operations of the IMF. The Board of

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[6] See Gold (1972).

Governors has delegated the maximum powers that the Articles permit to the Executive Board.[7]

The powers exercised by the Board of Governors and expressly reserved to it by the Articles refer to matters of a fundamental or political nature or which may have a profound economic impact. The reserved powers of the Board of Governors include the power to: admit new members; require a member to withdraw; approve a revision of quotas; determine the extent of delegation of powers to the Executive Board; determine the distribution of the net income of the Fund; determine the remuneration of Executive Directors; determine the number of Executive Directors to be elected; decide appeals against decisions of the Executive Board; make arrangements to co-operate with other international organisations; decide to liquidate the Fund. Some powers are vested in the Executive Board and the exercise of them does not depend on delegation from the Board of Governors, such as the election of the Managing Director and the power to suspend or terminate suspension of certain provisions. As well as exercising the powers which are either vested in them or delegated to them, an important role of the Executive Board is to make recommendations to the Board of Governors about decisions which the latter is to take.

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[7] Since 1974 the effective policy making body has been the informal Interim Committee which consists of ministers of the same countries which have seats on the Executive Board and meets twice per year. However in this study we are concerned with *a priori* voting power rather than the practice of policy making.

The Board of Governors comprises one governor (or one alternate governor in the absence of the governor) appointed by each member country, usually the minister of finance or the governor of the central bank. It meets once per year; the Articles provide for the possibility of additional meetings and for a vote to be taken without a meeting if the Executive Board judge it necessary. Governors vote in person and there is no proxy voting. Each governor is entitled to cast the number of votes of his country and may not cast fewer; his choice is to cast the total votes or to abstain. Decisions, in the form of resolutions, are taken by a majority of the votes cast except where the Articles require a Special Majority for a particular decision. In all cases a quorum for a meeting is a majority of the governors exercising not less than two thirds of the total voting weight of the members.

The Executive Directors, who are generally officials or diplomats rather than politicians, are appointed or elected by members of the Board of Governors. Like the Board of Governors, the Executive Board, which functions in continuous session, employs a system of weighted voting and therefore it is appropriate to consider its composition and analyse the distribution of voting power among the executive directors separately from the Board of Governors. The Articles provide for not fewer than twelve executive directors, five of whom are appointed by the five members having the largest quotas, and, in the original Articles, two elected by the American republics not entitled to make an appointment and five elected

by the other members. (Two additional directors are to be appointed by the two members whose currencies have been sold by the Fund in the largest amounts in the previous two years if they are not already entitled to appoint.) As the number of members has increased the Articles have been amended to increase the number of elected directors and in the 1994 election there were 19 elected directors, three representing single-country electorates, China, Russia and Saudi Arabia and sixteen elected by groups of countries.

Elections of executive directors are held every two years. There is a minimum and a maximum percentage of the eligible votes that a nominee must receive in order to be elected. In practice the minimum percentage results in most cases in directors requiring the votes of more than one member in order to be elected. The principle behind the requirement of a minimum percentage vote is to encourage the formation of coalitions of members with common interests who elect directors to represent them while the requirement of a maximum percentage prevents too great disparities in the voting strength of individual elected directors.

An executive director casts the votes of those members who voted for him as a unit and cannot split the vote. With the exception of the (Latin) American republics, the Articles do not associate executive directors with predetermined regional or other groupings; the groupings are assumed to be formed endogenously as part of the election process. However the operation of the

system over the years appears to have led to the formation of several such groupings which have each tended to elect an executive director from the same country over several elections and appear relatively stable. Members which combine to form groupings engage in negotiations among themselves through channels which are outside the IMF.[8] However the process of election works, some member countries (in particular Australia, Brazil, Belgium, Canada, China, India, Italy, Netherlands) have provided executive directors practically continuously since 1946.

The basic decision rule is that, except as specifically provided, all decisions are made by a simple majority of the votes cast. This applies to both the Board of Governors and the Executive Board and most of the decisions of the Fund. (The rule is in terms to votes cast rather than total votes but this distinction is ignored in this study.) For certain categories of decision, however, special majorities, defined in terms of proportions of the total voting weight, whether the votes are cast or not, are required. These tend to be the most important types of decision where a degree of consensus is required to make them effective.

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[8] It might therefore be considered appropriate to study the power relationships in the Executive Board in terms of a two-stage voting process: first members vote by weighted voting within their group, second their elected director votes their combined weight as a unit in the Executive Board. This two-stage approach could have the effect of giving a greatly increased voting power to certain individual members of the Executive Board whose voting weight in the Board of Governors was small. For example in 1994 Belgium had 2.1% of the votes in the Board of Governors, but was in a relatively dominant position, with over 40% of the votes, in a grouping whose combined weight in the Executive Board was 5.1%. This approach would require the groups to be well determined, to continue to exist regardless of the outcome of the first stage, which cannot be assumed in all cases, and an assumption that there is a similar process within each grouping, which cannot be assumed either. It is an interesting question

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The original Articles provided for a range of different special majority requirements, as fractions of the total voting weight, from a simple majority to four fifths, for different categories of decision but this was amended (after the Second Amendment) effectively to two: 70 percent and 85 percent. After 1969 a large number of types of decisions now require a special majority of 85 percent. The introduction of this higher figure effectively allowed the United States to keep its veto while reducing its contribution, but at the same time it meant that groups of other countries, if they formed a bloc, such as the EU or the developing countries could also have a veto. The adoption of a higher special majority requirement was a move in the direction of unanimity and resulted in substantially greater equality of power, as we show later.

The question of the size of majority required for a decision was a point of disagreement between the British and American founders of the IMF. Keynes was little interested in decision rules based on precise formulae and advocated that all decisions be taken by simple majority, in contrast to the Americans who insisted on designing voting rules which guaranteed them a veto, with 33 percent of the votes. Keynes addressed the question in his maiden speech to the House of Lords (1943a): ". . . the requirement in the American plan for a four fifths majority will be found, if the paper is read carefully, to relate not to all matters by any means, but only to a few major issues. Whether on second thoughts any one

which is not considered here and remains for future work.

would wish to allow a negative veto to any small group remains to be seen. For example, the American proposals might allow the gold-producing countries to prevent the United States from increasing the gold value of the dollar, even in circumstances where the deluge of gold was obviously becoming excessive; and in some ways, by reason of their greater rigidity, the American proposals would involve a somewhat greater surrender of national sovereignty than do our own." He also wrote (1943b): "I disagree strongly, on non-economic grounds, of the individual country veto-power unless it is granted to all countries regardless of their quotas . . . . the 80 percent majority rule would limit the power of the US with respect to changes it may desire in an existing status as much as it would increase its power to stop undesired changes."

III. Voting Weights

Every member of the IMF has a quota expressed in US dollars which is its subscription to the resources of the organisation and also determines its voting weight. The votes allotted to a member are equal to a basic two hundred and fifty plus one vote for each hundred thousand dollars of quota. Thus voting weight varies linearly according to the size of the quota rather than proportionately. This is one important difference with a business corporation where votes are strictly proportional to contributions to equity capital. The existence of the 250 "basic" votes which every member has independently of its quota reflect concerns expressed at the Bretton Woods conference. It was felt that the radical move (in

an international organisation) of adopting a system of weighted voting for the IMF, where the weights reflected economic and financial factors, should be tempered by the political consideration of the traditional equality of states in international law. To have allocated votes in proportion to quotas would have meant too close a similarity with a business corporation and might have given too high a degree of control to a small group of member countries.

Table 1. "Basic" Votes as a Percentage of Total Votes

	Number of members	Total basic votes	Total votes	Percentage basic votes
1946	39	9750	84475	11.54
1956	58	14500	101280	14.32
1966	103	25750	217657	11.83
1976	128	32000	324114	9.87
1986	149	37250	930300	4.00
1996	179	44750	1493331	3.00

The Articles do not provide for an increase in basic votes when quotas increase and for most countries their voting weight has become more closely proportional to their quota as the latter has increased over the years. The proportion of total votes represented by the combined basic votes has accordingly fallen substantially over the years (see Table 1) from a high of over 14% in 1956 to 3% in 1996. This decline has been in spite of the increase in the number of members, many of which are developing countries with very small quotas.

However for the great majority of members their basic votes have become insignificant. For example for Belgium the share of its total voting weight represented by basic votes has fallen from 10% in 1946 to 0.7% in 1996, for Mexico it has fallen from 21.7% to 1.4% over the same period.

IV. The Measurement of Voting Power

In a weighted voting body the formal power a member exercises is not, in general, the same as its numerical voting strength; this is a general feature of weighted voting and does not depend on the nature of informal relations among the members, such as groupings or parties or their preferences. We can examine this relationship by employing the method of voting power indices based on a simple abstract definition of power.

The approach is in principle very simple. Power is defined in such a way that it can be thought of as being shared among the members. Given a voting rule in terms of the majority requirement for winning a vote, the winning quota, such as a simple majority or a larger special majority, then, if a member's votes exceed the quota, it obviously has all the power; otherwise power is divided among the members on the basis of their ability to influence decisions. This requires a precise definition to lead to a quantitative measure.

The two classical power indices which are used here (and described in, for example, Owen (1995) and Straffin (1994)) are both based on the idea that a

member's power is its capacity to change a vote from losing to winning by using its votes. In general its share of the power is defined as the number of times it can do this expressed as a proportion of the total number of times that any member can do it. Thus the approach does not take any account of the extent to which the voting body itself is able to take decisions; members of a voting body may be powerless because the body itself is incapable of making decisions. The method is confined to the analysis of power relationships among the members relative to those decisions which are made at least in principle.

Although the two classical power indices employ the same basic concept of a swing whereby a member, by joining a coalition changes it from losing to winning, they are mathematically distinct since they employ different coalition models. The Banzhaf index (Banzhaf (1965)) is based on considering coalitions as combinations of members in the sense of a list arranged in no particular order; they might be arranged alphabetically, or in any other arbitrary order - the ordering is irrelevant to the coalition. A member's power index is then the number of such coalitions it can swing from losing to winning, expressed as a proportion of all such swings when all members are considered.

The Shapley-Shubik index (Shapley and Shubik (1954)) on the other hand, counts coalitions on the basis not only of swings, but also the order in which members are listed. Thus, given that a member is able to swing a vote, the index

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takes into account the number of orderings of both the members of the winning coalition and the members not in the coalition - a re-ordering of the same members is counted as a different swing. The index is defined by expressing this number as a proportion of the number of orderings of all members.

For example, consider a voting body with 10 members. Suppose we wish to measure the power of member i. Let us consider the effect of the size of the coalition on the measurement of power according to the two indices. Consider two swings, coalitions which are losing until i joins, one of 4 members and one of 7 members. The Banzhaf index treats these two swings equally: each counts as one swing. The Shapley-Shubik index, however, attaches very different importance to them. For the coalition of size 4, the number of orderings of its members and the remaining 5 members is $4!5! = 2,880$. The contribution of this swing to the index is then $2880/10! = 2880/3628800 = 0.000793$. For the other coalition, however, its contribution to the power index is $7!2!/10! = 10,080/3628800 = 0.002778$. Thus the number of members in a swing coalition has a considerable importance to the computation of the Shapley-Shubik index, cases where the winning coalition and its complement are relatively equal being given much less weight.

Despite being so different in the way they count swings, the two indices have given results which have not been very different in some applications.

However they have given results which have differed considerably in others. There is no clear guidance from the literature on the relative merits of the two indices. While political scientists and lawyers have tended to prefer the Banzhaf index and criticised the coalition model underlying the Shapley-Shubik index, mathematicians and game theorists have tended to the reverse preference ordering since the Shapley-Shubik index has been shown to uniquely satisfy certain axioms. Both indices give rise to a power distribution among the members of the legislature. In addition, however, the Shapley-Shubik index can be interpreted directly as the probability of swing given its coalition model, so that power can be thought of as the probability that each member swings the vote, whereas the Banzhaf index cannot. Evaluating the probability of a swing for each player within the Banzhaf coalition model gives indices which do not sum to unity and therefore must be normalised to give a power distribution[9].

One factor which has to some extent limited the study of power indices in empirical applications has been the difficulty of computing them when the number of members of the voting body is moderate or large. In this study we employ algorithms which enable the approach to be used to study the two bodies of the IMF. Indices are computed for the Board of Governors, which has a large number of members, between 39 in 1946 and 179 in 1996, using a modified

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[9] Authors who have adressed the question of the respective merits of the indices from various angles include Brams (1975), Leech (1990), Roth (1977), Strafffin (1977), Felsenthal and Machover (1995).

multilinear extensions algorithm. Power relations within the Executive Board are studied using a direct enumeration algorithm. These are described below.

V. Computation of the Voting Power Indices

We characterise a voting body by the number of its members, n, their voting weights, $w_1, w_2, \ldots, w_i, \ldots, w_n$ and a decision rule in terms of a quota[10]., q In the present case the weights are proportions so that $\sum w_i = 1$. The quota is the size of majority required for a decision; since any voting body usually has a decision rule which is intended to produce a clear decision, as here, we require q 1/2.

The power index of member i is the value for player i of the corresponding simple game $\{q; w_1, \ldots, w_n\}$. The set of all players is denoted N. For member i, a swing is a coalition represented by a set, S , S $\subseteq$ N, such that i $\notin$ S, where

$$\sum_{j \in S} w_j < q \quad \text{and} \quad \sum_{j \in S} w_j + w_i \quad q. \tag{1}$$

The Banzhaf power index, denoted β_i , is the ratio of the number of swings for member i, $\eta_i = \sum_S 1$, to the total number of swings for all members,

$$\beta_i = \eta_i / \sum \eta_i . \tag{2}$$

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[10] This is here used as a technical term and should not be confused with IMF quotas.

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The Shapley-Shubik index is defined in terms of the number of orderings of members for each swing. Letting $\theta_i = \sum_s s!(n-s-1)!$,where s is the number of members of the set S, and the summation is taken over swings, the index is defined as $\gamma_i = \theta_i/n!$. More usually the expression is written:

$$\gamma_i = \sum_s \frac{s!\,(n-s-1)!}{n!}. \qquad\qquad (3)$$

Two algorithms are used to calculate the β_i 's and the γ_i 's in this study: (1) Direct Enumeration, and (2) a Modified MLE Approximation. The details of these two algorithms are described fully in Leech (1998). The Direct Enumeration algorithm finds every subset $M \subseteq N$ and tests for a swing for each member. This algorithm evaluates the power index directly by counting swings but has the disadvantage that it is compute-intensive. The time taken to test all subsets of N increases exponentially with n, since the number of subsets is 2^n, computation time approximately doubling every time n increases by 1. However it has been found to work well for moderate-sized games such as the IMF Executive Board, for which n 24. For games much larger than this the computing time required would be prohibitive. An advantage of this algorithm is that it can be generalised to compute values of general n-person games, such as Shapley values, since it does not rely on exploiting the special features of Simple Games. For the Board of Governors we use an approximation method which we have called the Modified Multilinear Extension method. This is a development of an approach

due to Owen (1972) who addressed the question of the difficulty of computing values for large games and proposed a method of approximation based on the Multilinear Extension and the use of the central limit theorem. Our algorithm is a combination of this with the direct enumeration approach and is considerably more accurate than Owen's MLE approximation method which was unsatisfactory because of the large voting weight of the United States.

VI. The Data

Separate data sets have been used to analyse power relations in the Board of Governors and the Executive Board. The former consists of percentage voting weights for each member and every year; the latter consists of the voting weights held by each executive director each year.

The voting weights for the Board of Governors for the first twenty six years were taken from Gold (1972) which lists all the voting weights for every member country and every year. The weights for later years (and for the Executive Board in all years) have been calculated using figures taken from an appendix to the IMF Annual Report for each year. (An example - slightly abridged - is in the Appendix to this paper.) The figures presented in the Annual Reports describe the way in which the voting weights of the individual executive directors have been determined through the voting system and therefore which member countries are represented by each director. In almost every year certain

countries have been excluded because they did not participate in elections of directors; China and South Africa have not infrequently been in this group. Also certain new members have often not participated in the elections or been unable to do so because they joined the organisation too late. Such countries have been members and therefore had a vote in the Board of Governors but their voting weight has not been represented by an executive director.

In order to make the figures complete it has therefore been necessary to adjust the reported figures by including these countries. There is therefore a difference between the data for the two bodies in that the percentage voting weights for the Board of Governors sum to 100 while the total for the Executive Board is often less than that. We have used these figures to make a separate analysis of voting power in the Board of Governors for each year by the Modified MLE Approximation method. This method was employed because of the large number of members which made the exact Direct Enumeration Algorithm infeasible. The Direct Enumeration Algorithm was used to analyse the Executive Board however because the number of members of that body has always been small enough for it to be computationally feasible.

VII. Results

A separate analysis has been carried out for each voting body for each year of the history of the IMF. This approach gives a power distribution for any year

which is of interest as well as enabling us to trace its evolution over time.

Separate analyses are given for the two levels of the majority quota required for

ordinary decisions (where a 50% majority is required) and special majorities

(where the required quota was 80% until 1969 and 85% in later years)[11]. We

present results for both 1946 and 1996 - taken as the two extreme years in terms

of the number of members and the concentration of voting weight - in numerical

form and then the evolution of voting power over time graphically. We have also

examined the effect of the size of special majority over its whole theoretical range

between 50% and 100% for the Executive Boards of 1946 and 1996.

VII(a) Ordinary Decisions

Table 2 presents the results for 1946 for ordinary decisions for both the

Executive Board and the Board of Governors. For each body the table lists the

voting weight and both power indices for each member. The figures for the five

largest members which appointed executive directors are presented side-by-side

for comparison but the other executive directors were all elected by groups of

countries. Where a group had an apparently dominant member (of which the

elected director was a national) we have identified that country in parentheses,

although we cannot necessarily assume that the figures refer to the voting weight

or power of the country. These figures are presented side-by-side with those for

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[11] The First Amendment to the Articles, raising the special majority to 85%, became effective in July 1969 but since the data point each year is April 30, we have taken it as being in force from 1970

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the country in the Board of Governors. The other directors, representing groups without any apparently dominant country are denoted D9 etc and in these cases there is no implication of a link between them and the member country alongside it in the Board of Governors.

In both bodies the two indices are in broad agreement about the qualitative nature of the power distribution but differ in the quantitative values of the indices. Both indices give the USA a lot more power than its weight and all the other members less power than weight. The only slight exception to this is the executive director with the least weight, which we have labelled D12[12], has more power than weight, and the same power as a group with a larger weight led by Belgium. The USA had about 33 percent of the voting weight and either 49 or 43 percent of the voting power (depending on the index) in the Executive Board. In the Board of Governors the power discrepancy is slightly larger, the USA's power index being either 54 or 44 percent.

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onwards.
[12] In fact representing a group consisting of Egypt, Ethiopia, Greece, Iran, Iraq,Philippines.

Table 2: Power Indices for 1946 (Ordinary Decisions, q=50%)

	Executive Board				Board of Governors		
	Votes	Bz Index	SS-Index		Votes	Bz Index	SS-Index
USA	33.22	48.77	43.18	USA	33.14	54.27	44.32
UK	15.86	8.59	12.51	UK	15.83	5.63	12.58
France	6.58	5.11	5.54	France	5.67	4.37	4.90
India	5.09	4.21	4.33	India	5.08	3.97	4.35
China	6.88	5.44	6.00	China	6.87	4.97	6.07
(Canada)	4.79	3.87	3.94	Canada	3.88	3.07	3.27
(Netherland)	5.09	4.21	4.33	Netherlands	3.58	2.84	3.00
(Belgium)	3.72	3.54	3.48	Belgium	2.99	2.39	2.49
D9	5.48	4.66	4.94	Brazil	2.09	1.68	1.73
D10	5.23	4.43	4.63	Czechoslov.	1.79	1.44	1.48
D11	4.61	3.65	3.64	Poland	1.79	1.44	1.48
D12	3.46	3.54	3.48	South Africa	1.49	1.20	1.23
Sum	100	100	100	Mexico	1.37	1.10	1.13
Exec.Dirs	12			Denmark	1.11	0.89	0.92
				Yugoslavia	1.02	0.82	0.84
				Chile	0.9	0.72	0.74
				Colombia	0.9	0.72	0.74
				Cuba	0.9	0.72	0.74
				Norway	0.9	0.72	0.74
				Egypt	0.84	0.68	0.69
				Greece	0.78	0.63	0.64
				Iran	0.6	0.48	0.50
				Peru	0.6	0.48	0.50
				Philippines	0.48	0.39	0.40
				Uruguay	0.48	0.39	0.40
				Bolivia	0.42	0.34	0.35
				Luxembourg	0.42	0.34	0.35
				Iraq	0.39	0.31	0.32
				Ethiopia	0.37	0.30	0.31
				Costa Rica	0.36	0.29	0.30
				DominicanR	0.36	0.29	0.30
				Ecuador	0.36	0.29	0.30
				Guatemala	0.36	0.29	0.30
				El Salvador	0.33	0.27	0.27
				Honduras	0.33	0.27	0.27
				Nicaragua	0.32	0.26	0.26
				Paraguay	0.32	0.26	0.26
				Iceland	0.31	0.25	0.26
				Panama	0.3	0.24	0.25
				Sum	100	100	99.97
				Members	39		

By contrast the picture for the United Kingdom is quite different despite the country's large voting weight in 1946 - almost 16% - half the votes of the United States and much more than any other member. The indices both give the UK a substantial power deficiency: in the Executive Board, it only had either 8.59% or 12.51% of the power; it had even less power in the Board of Governors, the Banzhaf index giving it less than 6%. The figures for France are less extreme although it still had less power than voting weight in both bodies[13]. Comparing the indices for the countries which dominated their groupings in the Executive Board (Canada, Netherlands and Belgium) in both bodies suggests that their executive directors had considerably more power than their governors in the respective bodies. The effect comes largely through the concentration of voting weight, however, rather than being especially a property of the power indices.

Table 3 is the equivalent analysis for 1996[14]. The general picture is the same in qualitative terms as it was in 1946, with the United States having an excess of power over weight in both bodies and every other member country having a power deficiency. Apart from that for the USA which remains substantial, the difference between power and weight is generally fairly small. The USA had almost 18% of the votes and 21% or 23% of the power in the

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[13] There is a large discrepancy between the voting weight of France in the two bodies, but these figures are correct according to Gold (1972) and the IMF Annual Report for 1946.

[14] Using the data in the Appendix. Because of the large numbers of countries not all of them have been listed.

Executive Board and 27% or 21% in the Board of Governors. While the total

power is a lot less than in the past, the power discrepancy is still substantial in

relation to the voting weight. The effect of the voting system in the Executive

Board appears to have had a substantial effect in enhancing the power of some

countries, in particular Australia, Belgium and Netherlands.

Table 3. Power Indices for 1996 (Ordinary Decisions, q=50%)

	Executive Directors				Board of Governors		
	Votes	Bz Index	SS-Index		Votes	Bz Index	SS-Index
USA	17.78	23.28	21.33	USA	17.78	27.00	21.02
UK	4.98	4.71	4.91	UK	4.98	4.36	4.91
France	4.98	4.71	4.91	France	4.98	4.36	4.91
Germany	5.54	5.24	5.51	Germany	5.54	4.80	5.49
Japan	5.54	5.24	5.51	Japan	5.54	4.80	5.49
China	2.28	2.16	2.20	China	2.28	2.04	2.19
Saudi Arabia	3.45	3.27	3.34	SaudiArabi	3.45	3.07	3.35
(India)	2.58	2.44	2.49	India	2.06	1.84	1.97
(Canada)	3.72	3.52	3.61	Canada	2.91	2.60	2.81
(Netherland)	4.88	4.62	4.81	Netherland	2.32	2.07	2.23
(Belgium)	5.09	4.82	5.03	Belgium	2.09	1.87	2.00
LatinAm(Brazil)	2.63	2.49	2.54	Brazil	1.47	1.32	1.40
LatinAm(Mexico)	4.31	4.08	4.21				
(Italy)	4.02	3.81	3.91	Italy	3.09	2.75	2.99
(Australia)	3.29	3.12	3.19	Australia	1.58	1.42	1.51
Scandinavia	3.47	3.29	3.36	...	...	...	...
LatinAm(Arg.)	2.14	2.02	2.06	...	...	...	...
Africa(exBritish)	2.5	2.37	2.41	...	...	...	...
Africa(exFrench)	1.34	1.27	1.28	...	...	...	...
D20	3.19	3.02	3.09	...	...	...	...
D21	2.9	2.75	2.80	...	...	...	...
D22	2.8	2.65	2.70	...	...	...	...
D23	2.76	2.61	2.66	...	...	...	...
D24	2.65	2.51	2.56	...	...	...	...
Sum	98.82	100	100.43	Sum	100	100	100.006
Exec Dirs	24			Members	179		

Note: Votes do not sum to 100 in the Executive Directors,
because members who did not cast their votes were not
represented.

VII(b) Special Majorities

Tables 4 and 5 show the analyses for decisions requiring special majorities in the same two years. These decisions concern more fundamental matters and therefore these results might be considered as containing more information about the true distribution of power. They are much more difficult to interpret because the two power indices used disagree qualitatively in many cases and also there are differences in the pattern in the two years. In general we would expect the effect of a higher majority requirement to be to make the distribution of power more equal[15]. That has undoubtedly been the effect on the 1946 data for the Executive Board as measured by the Banzhaf index which indicates considerably less power for the United States and more power for all the other directors. The power of the USA drops to 16.9% (compared with 48.8% with respect to ordinary decisions) and that of the UK is almost the same[16]. The power distribution given by the Shapley-Shubik index, by contrast, is somewhat different, giving less power than weight to every director except the UK; these effects however are moderate. The pattern in the Board of Governors is similar to that for the Executive Board for both indices.

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[15] Theoretically, increasing the quota - ultimately to unity - should make all the indices converge on the same value $1/39 = 2.56\%$ in the 1946 Board of Governors, 0.56% in the 1996 Board of Governors, etc.

[16] It is interesting that the power index for the UK is now almost equal to that of the USA. This is not surprising since if the quota had been 85% then the two countries would have had the same power, swings for a member with a weight of 33.22% being the same as swings for one with 15.86%.

The results for 1996 in Table 5 are similarly inconclusive. The power

distributions given by the two indices are conflicting in qualitative terms: that for

the Banzhaf index being a lot more equal than the distribution of voting weight,

while that for the Shapley-Shubik index appears to be slightly more unequal. The

pattern is similar in both bodies. The main overall conclusion from this analysis is

therefore that, at high levels of the majority quota, it matters greatly which index

is used. It is necessary therefore to try to form a view on the relative merits of the

indices.

Table 4. Power Indices for 1946 (Special Majorities, q=80%)

	Executive Board				Board of Governors		
	Votes	Bz Index	SS-Index		Votes	Bz Index	SS-Index
USA	33.22	16.90	31.94	USA	33.14	13.391	32.10
UK	15.86	16.52	21.04	UK	15.83	13.389	21.06
France	6.58	9.03	6.26	France	5.67	8.25	5.32
India	5.09	6.71	4.90	India	5.08	7.34	4.78
China	6.88	9.94	6.79	China	6.87	10.01	6.50
(Canada)	4.79	6.06	4.52	Canada	3.88	5.57	3.66
(Netherland)	5.09	6.71	4.90	Netherlands	3.58	5.13	3.36
(Belgium)	3.72	4.26	2.63	Belgium	2.99	4.26	2.78
D9	5.48	7.23	5.20	Brazil	2.09	2.98	1.90
D10	5.23	7.10	5.13	Czechoslov.	1.79	2.55	1.61
D11	4.61	5.68	4.29	Poland	1.79	2.55	1.61
D12	3.46	3.87	2.40	South Africa	1.49	2.12	1.33
Sum	100	100	100.00	Mexico	1.37	1.95	1.22
Exec. Dirs.	12			Denmark	1.11	1.58	0.98
				Yugoslavia	1.02	1.45	0.90
				Chile	0.9	1.28	0.79
				Colombia	0.9	1.28	0.79
				Cuba	0.9	1.28	0.79
				Norway	0.9	1.28	0.79
				Egypt	0.84	1.19	0.74
				Greece	0.78	1.11	0.69
				Iran	0.6	0.85	0.53
				Peru	0.6	0.85	0.53
				Philippines	0.48	0.68	0.42
				Uruguay	0.48	0.68	0.42
				Bolivia	0.42	0.60	0.37
				Luxembourg	0.42	0.60	0.37
				Iraq	0.39	0.55	0.34
				Ethiopia	0.37	0.53	0.32
				Costa Rica	0.36	0.51	0.31
				Dominican R.	0.36	0.51	0.31
				Ecuador	0.36	0.51	0.31
				Guatemala	0.36	0.51	0.31
				El Salvador	0.33	0.47	0.29
				Honduras	0.33	0.47	0.29
				Nicaragua	0.32	0.45	0.28
				Paraguay	0.32	0.45	0.28
				Iceland	0.31	0.44	0.27
				Panama	0.3	0.43	0.26
				Sum	100	100	99.95
				Members	39		

Note: Votes do not precisely agree in the two bodies because of the omission of
the votes of Denmark from the Executive Board.

Table 5. Power Indices for 1996 (Special Majorities, q=85%)

	Executive Board				Board of Governors		
	Votes	Bz Index	SS-Index		Votes	Bz Index	SS-Index
USA	17.78	6.20	18.60	USA	17.78	3.12	19.06
UK	4.98	5.25	5.04	UK	4.98	3.11	5.14
France	4.98	5.25	5.04	France	4.98	3.11	5.14
Germany	5.54	5.51	5.72	Germ	5.54	3.11	5.78
Japan	5.54	5.51	5.72	Japan	5.54	3.11	5.78
China	2.28	2.96	2.21	China	2.28	2.70	2.24
Saudi	3.45	4.18	3.52	SaudiAr	3.45	3.03	3.46
(India)	2.58	3.29	2.49	India	2.06	2.57	2.02
(Can)	3.72	4.44	3.81	Canada	2.91	2.93	2.89
(Neth)	4.88	5.19	4.96	Netherland	2.32	2.72	2.28
(Belg)	5.09	5.31	5.17	Belgium	2.09	2.59	2.05
LA(Brazil)	2.63	3.35	2.56	Brazil	1.47	2.06	1.42
LA(Mex.)	4.31	4.85	4.37				
(Ital)	4.02	4.66	4.17	Italy	3.09	2.97	3.08
(Aus)	3.29	4.01	3.24	Austral	1.58	2.19	1.53
Scan	3.47	4.20	3.55	...	...	...	...
LA(Arg.)	2.14	2.79	2.06	...	...	...	...
Af(Br.)	2.5	3.20	2.42	...	...	...	...
Af(Fr.)	1.34	1.87	1.34	...	...	...	...
D1	3.19	3.92	3.10	...	...	...	...
D2	2.9	3.66	2.87	...	...	...	...
D3	2.8	3.53	2.75	...	...	...	...
D4	2.76	3.50	2.72	...	...	...	...
D5	2.65	3.37	2.59	...	...	...	...
Sum	98.82	100	100	Sum	100	100.00	100.00
Ex.Dirs.	24			Members	179		

Note: Votes do not sum to 100 in the Executive Directors, because

VII(c) Historical Trends

Figures 1 to 12 show historical trends in power as measured by the indices for the G5 countries. Similar graphs could be drawn for any member country. Besides the values of the two power indices we also show the corresponding power ratios; for each index this is the ratio of the index to the voting weight. A power ratio provide a simple indication of the extent to which power differs from

weight, the power discrepancy. It can be used to indicate transparency of the voting system: where power and weight are equal the power ratio is equal to unity. (In our case the power ratios have been normalised to 100 so that the power discrepancy is expressed as a percentage). Figures 1 to 6 show the results for ordinary decisions and Figures 7 to 12 those requiring special majorities.

Figure 1 shows the results for the United States: 1(a) shows the voting weight and two power ratios for every year in the history of the organisation for the Executive Board, 1(b) the corresponding power ratios, 1(c) the weights and indices for the Board of Governors, and 1(d) the corresponding power ratios. We see that power has been consistently above weight in every year. As the weight has declined there has been a corresponding decline in power. The power ratios have fallen slowly but they are still both considerably above 100. The Shapley-Shubik index gives a positive power discrepancy of at least 20% (which is down from over 30% in the 1940s), and the effect according to the Banzhaf index is even greater. The results are broadly similar for both bodies although there is generally a greater power discrepancy in the Board of Governors than in the Executive Board. There is no evidence of any voting paradoxes.

Figure 2 shows the results for the UK. There was initially in the 1940s a great excess of weight over power in both bodies: there was a power discrepancy of either almost 20% or 40% in the Executive Board and up to almost 60% in the Board of Governors. This is perhaps somewhat surprising in view of the very large voting weight of the UK at that time. Over time the power discrepancy has declined: the Shapley-Shubik index has been virtually equal to weight since the mid 1960s, although the Banzhaf index has continued to indicate a power deficit (substantial in the Board of Governors). There have been some voting paradoxes, although small in magnitude. For example in 1947 voting weight rose but power according to both indices fell in both bodies. In 1966 weight increased and the

two indices went in opposite directions in the Board of Governors, the Banzhaf index falling and the Shapley-Shubik index rising. A number of other similar changes have occurred but they are all small in magnitude.

Figure 3 shows the trends for France. The power indices track the voting weight, at first falling until the mid 1960s then rising slightly. There has always been a small negative power discrepancy in both bodies. Small voting paradoxes occurred at several points according to one or the other index: 1947, 1957, 1976, 1981 (in the Executive Board). Figure 4 shows the results for Germany which joined the IMF in 1953 and had its own executive director from 1955. The general pattern is one of fluctuating voting weight with the power indices following fairly closely, with a relatively small power discrepancy tending to get smaller over time. Some small voting paradoxes occurred in the Executive Board in 1959, 1970 and 1976. Figure 5 shows the results for Japan, which joined in 1953 and has had its own director since 1971. The results show power to have been only slightly less than weight in every year. Figure 6 shows the power ratios for each index and each body. The diagram brings out the contrast between the power of the United States and the other G5 countries. Although the power ratios for the latter have converged on something only slightly below 100, that of the former has remained well above. This effect is produced by both the indices; only its size is always greater according to the Banzhaf index than the Shapley-Shubik index.

The results shown in Figures 1 to 6 are clear. However they are all for ordinary decisions requiring a majority of 50% of the votes. The results are much less clear when the analysis is repeated for decisions requiring special majorities. Figures 7 to 12 show historical trends for the two power indices for these cases. The majorities assumed are of 80% up to 1969 and 85% thereafter Figure 7 shows the US results. The Shapley-Shubik power ratio fluctuates around 100 in a

range of $\pm$ 20% while the Banzhaf index give a large power discrepancy. Both power ratios fell in 1970 after the special majority was increased to 85%.

Figure 8 shows the picture for the UK with conflicting results according to the two indices. Generally the Shapley-Shubik index gives a positive power discrepancy and the Banzhaf index a negative one but these are quantitatively small effects. Figure 9 shows France where again there is a tendency for the indices to conflict. The Banzhaf index gives a positive power discrepancy. Similar results are found for Germany and Japan in the next two figures. All power ratios, however with the exception of that for the Banzhaf index for the United States have converged to close to 100 (Figure 12).

VII(d) The Effect of the Majority Requirement

The results we have described show the sensitivity of the power indices to not only the choice of index used but also the majority quota, q. It is therefore of considerable importance to investigate further the effect of variation in the latter on the results. We know that increasing the value of q makes the power distribution more equal until in the limit when q = 100% all power indices are equal for all members. How quickly this convergence takes place is an interesting question whose answer might help us to understand the comparative behaviour of the different indices. This experiment may also provide evidence on the question of the best majority requirement to build into the voting rules which occupied the founders of the IMF and thereby a test the assertions of Keynes quoted earlier in section II.

Figure 13 Shows the results of the exact analysis of the Executive Board for the years 1946 and 1996. Power indices have been computed for values of q between 50 and 100, at intervals of 5%. Figures 13(a) and 13(b) show the resulting graphs for the Banzhaf and Shapley-Shubik indices using the 1946 data

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and 13(c) and 13(d) those for the 1996 data. Table 6 shows the same information for the United States and the second largest member, the UK in 1946 and Germany in 1996. The results for 1946 show that Keynes was right as far as the power of the United States was concerned in that its power diminished strongly with q for large values of q. For smaller values however the picture as before depends on the index; the Shapley-Shubik index indicates that US power does not vary monotonically with q, but increases at first to a maximum at q=70 before falling steadily. On the other hand both graphs for the United Kingdom show an increase to a maximum then a decrease. For the other executive directors the relationship is relatively flat with little sensitivity of power to q except close to the limit. The results for the 1996 Executive Board show broadly similar results for the United States as for 1946 except that the relationship is now monotone for both indices; the graphs for the other directors show little sensitivity of the power index to q. Comparing the results for the two indices we see that the Banzhaf index appears to be the more sensitive to the value of q. The Shapley-Shubik index tends to show relatively little change as q increases until quite close to the limit when it converges more rapidly.

Table 6. The Effect of Changing the Majority Requirement

	1946			
	USA	UK	USA	UK
Votes:	33.22	15.86	33.22	15.86
Majority q:	Banzhaf Index		SS Index	
50	48.77	8.59	43.18	12.51
55	45.93	10.61	43.40	12.78
60	39.65	15.09	43.48	12.99
65	32.71	19.10	43.70	13.91
70	26.45	20.51	45.38	15.53
75	21.14	19.21	38.68	18.08
80	16.90	16.52	31.94	21.04
85	13.73	13.73	25.25	25.25
90	11.36	11.36	18.79	18.79
95	9.62	9.62	11.36	11.36
100	8.33	8.33	8.33	8.33
	1996			
	USA	Germany	USA	Germany
Votes:	17.78	5.54	17.78	5.54
Majority q:	Banzhaf Index		SS Index	
50	23.28	5.24	21.33	5.51
55	21.26	5.41	21.24	5.47
60	17.48	5.69	21.15	5.48
65	13.81	5.92	21.01	5.52
70	10.94	6.01	20.91	5.50
75	8.86	5.96	20.80	5.44
80	7.34	5.80	20.31	5.50
85	6.20	5.51	18.60	5.72
90	5.32	5.11	12.75	6.23
95	4.57	4.57	6.92	6.92
100	4.17	4.17	4.17	4.17

VII(e) The Effect of the European Union Voting as a Single Block

Another use of the methodology of power indices is to analyse the effects of possible fundamental changes in the voting system. One obvious scenario which it is interesting to consider here is that of the European Union becoming a single member of the IMF replacing its fifteen individual member countries. The

EU would then be the largest member of the IMF with 28.84% of the voting weight.

It would be difficult to examine the implications of this change for power relations within the Executive Board because in order to do that it would be necessary not just to aggregate the votes of the EU countries into a single large block, but also to specify implications for the votes cast by the elected directors. Rather than attempt to do that we have therefore done the analysis on the Board of Governors by simply elimating the individual members of the EU and replacing them by a single aggregated block vote. The results of the analysis using the data for 1996 and a majority quota of 50% are shown in Table 7. The table shows the voting weights and power indices for all members who would then have at least 1% of the votes. It also shows the effective redistribution of power inherent in this voting allocation, the difference between the respective power index and voting weight and also the change in the values of the power indices from the situation existing before the combination of the EU votes. The results are also displayed graphically in Figure 14.

From Table 7 and Figure 14(a) the effect of the EU countries forming a block is dramatic but its details, again, depend on the index used. The power of the EU is considerably greater than its nominal 28.84%, both indices giving a value of almost 40%. Both indices agree that much of this is at the expense of the United States and Japan. The Banzhaf index gives the power of the United States as under 6%, compared with a voting weight of 17.78% a very large power discrepancy; however the effect obtained using the Shapley-Shubik index is much smaller, though also negative. Both indices also give a very small negative power discrepancy for Japan and very small effects for all other members. There is a different pattern of results for the two indices. The general pattern of power discrepancies given by the Banzhaf index is one where the EU and the smaller

members gains at the expense of the two large but not largest members, USA and Japan. That obtained using the Shapley-Shubik index on the other hand is that every member loses except the largest, the EU.

In Table 7 we also give the changes in the power indices resulting from the combination of the EU countries: these are the differences in the power indices compared with those for the analysis reported in Table 3 where the fifteen EU states are treated separately. These changes are shown graphically in Figure 14(b) and (c). The United States loses a lot of power: 21% according to the Banzhaf index (but much less, 5% according to the Shapley-Shubik index). The effects on other members are small in absolute terms, though some of them are non-negligible in comparison with voting weight. The direction of the change is different for the two indices. According to the Banzhaf index the effect is to restore either partially or completely the previous power deficiency for these countries; according to the Shapley-Shubik index the effect is to reduce their power to less than it had been before.

We can therefore conclude that the main effect of the formation of an EU voting block would be to significantly reallocate power away from the USA in favour of the EU but we cannot draw a clear conclusion about its precise effects on the other members, except that they will be small.

Table 7: Effect of an EU Block Vote, Board Governors,1996

	Weight	Bz Index	SS Index	Differ-ence Bz	Differ-ence SS	Change * Bz	Change* SS
EU	28.84	39.68	36.04	10.84	7.20		
USA	17.78	5.92	15.97	-11.86	-1.81	-21.08	-5.05
Japan	5.54	5.22	5.28	-0.32	-0.26	0.42	-0.21
Saudi Arabia	3.45	3.56	3.17	0.11	-0.28	0.49	-0.18
Russia	2.91	3.00	2.65	0.09	-0.26	0.40	-0.16
Canada	2.91	3.00	2.65	0.09	-0.26	0.40	-0.16
China	2.28	2.35	2.06	0.07	-0.22	0.31	-0.13
India	2.06	2.12	1.86	0.06	-0.20	0.28	-0.11
Switzerland	1.67	1.72	1.50	0.05	-0.17	0.24	-0.10
Australia	1.58	1.63	1.42	0.05	-0.16	0.21	-0.09
Brazil	1.47	1.52	1.32	0.05	-0.15	0.20	-0.08
Venezuela	1.32	1.36	1.18	0.04	-0.14	0.19	-0.08
Mexico	1.19	1.23	1.06	0.04	-0.13	0.17	-0.07
Argentina	1.05	1.08	0.94	0.03	-0.11	0.15	-0.06
Indonesia	1.02	1.05	0.91	0.03	-0.11	0.14	-0.06
…	…	…	…	…	…		

$q = 50\%$. Results shown for countries with more that 1% voting weight.*Change in power index compared with Table 3.

VIII. Conclusions

In Section I we set out the questions we sought to answer in this study: substantive ones about the distribution of voting power and methodological ones about the empirical measurement of power. We claim to have removed some of the obstacles in applying the power indices approach due to difficulties of computation (or the lack of available and appropriate software). Whatever other conclusions we reach we claim that problems of computation which have stood in the way of wider application of the approach have been overcome to an extent and have used algorithms which appear to be very effective.

We proposed to answer six questions. The first and second were to compare the voting power of individual countries with their nominal votes and to study the relative inequality of power and votes over time. We have been able to

give a partial answer in that our results are qualitatively unambiguous for all the countries we have looked at when we considered ordinary decisions requiring a 50% majority: the power of the USA has been much greater than its voting weight in every year and every other country has had correspondingly less power than its voting weight. Over time this discrepancy has fallen for the other G5 countries while remaining large for the USA. Interestingly there was a large power deficit for the United Kingdom in the 1940s, its big block of votes not entitling it to as much power as the financial contribution represented by its quota might warrant[17].

While these results are interesting, it is perhaps more relevant to look at voting power with respect to the more fundamental decisions which require a special majority, such as increasing the size of the fund, adjustments to members' quotas, the admission of new members and constitutional revisions. Unfortunately the results obtained for this case are difficult to interpret, the two power indices used often disagreeing. There is a need to resolve the ambiguity between the different power indices. We would suggest that by applying them to a wider range of voting bodies and historical periods it may be possible to gain insight into their respective performance.

We were also interested in whether voting paradoxes have occurred. At the most general level we sought to find cases where a redistribution or the accession of new members led a member country's votes to change while its voting power changed in the opposite direction. A number of such cases were found, although the quantitative effects were often quite small.

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[17] Although it remained of course the second most powerful member because power indices are monotonic functions of weight.

We made separate analyses of the two decision-making bodies, the Executive Board and the Board of Governors. Generally we found power to have been more unequal in the latter than in the former among the countries which appointed their own executive directors. Among the other countries which were not able to appoint their own directors and took part in the election process, a number have in fact provided executive directors continuously. These directors all cast much larger block votes in the Executive Board than their own country does in the Board of Governors because they were elected by a group of countries and cast the votes of all those who voted for them. How this process can be analysed and in particular whether it can be regarded as a composed game must be left for further research.

We investigated the effect of different decision rules on the distribution of power in the Executive Board and found that a larger majority requirement would have tended to reduce the power of the United States, contrary to common assumption. This result is however consistent with the warnings of Keynes who argued that a voting system which had a majority requirement so high as to ensure a built-in American veto would not necessarily enhance that country's power to influence decisions within the organisation. On the other hand retaining a veto for one country on the basis of a high special majority requirement reduces the ability of the organisation itself to take decisions.

This is perhaps the most interesting result of the analysis. It suggests that retaining special majorities creates a distortion in the voting system which in itself constitutes a lack of transparency. The effect is twofold: it reduces the likelihood that the IMF voting bodies will be able to take a necessary decision because of the high number of votes required for a majority, leading to rigidity; and it also means that the inequality in the distribution of power is substantially different from that which is theoretically built into the voting system: it is much closer to

being egalitarian than intended. We therefore reach the conclusion that the special majorities are a short sighted feature and contribute little to the effective running of the organisation.

We have also done a limited amount of scenario analysis, using the method of power indices to examine the implications of one rather obvious change which might be contemplated: the combination of the fifteen countries of the European Union into a single voting block. Our results show clearly that it would be in a dominant position and that the United States would suffer a large reduction in power below its voting weight.

The main question of methodological interest (apart from the feasibility of calculating them in practice) is to compare the performance of the two classical power indices, the Banzhaf index and the Shapley-Shubik index. The Banzhaf index has been used in more applications largely due to its comparative ease of computation. In this study this has not been an issue. The results indicate that the Banzhaf index tends to be more sensitive to the data than the Shapley-Shubik index and often gives more extreme results. How to relate this to the concept of power in practical contexts is not obvious however. It is possible that by such studies as this, of real institutions for which there exists independent knowledge based on experience, light will be cast on the respective ability of the indices to reflect power relations. From this study we would suggest that the results obtained in section VII(e) for the effect of the majority quota on power relations indicate greater plausibility for the Banzhaf index, the Shapley-Shubik index giving a seemingly too slow convergence to equality. This, together with the discussion of the indices' respective treatment of coalitions in section II leads us to a preference for the Banzhaf results.

Figure 1: Historical Trends (Ordinary Decisions) USA

(a) Power Indices, Executive Directors
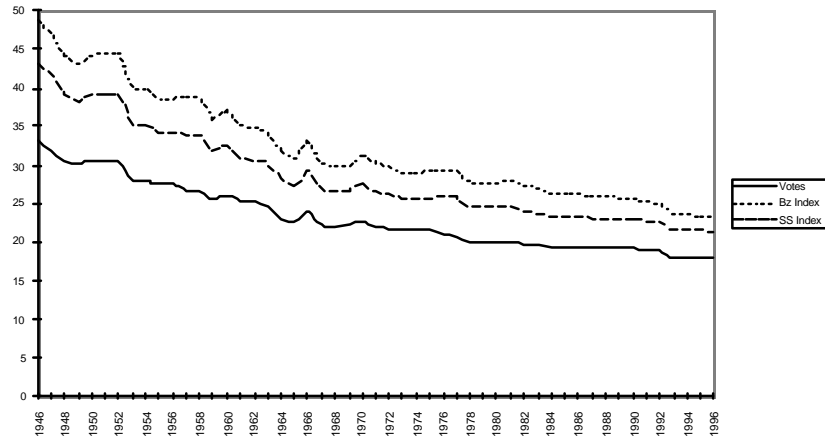
(b) Power Ratios, Executive Directors
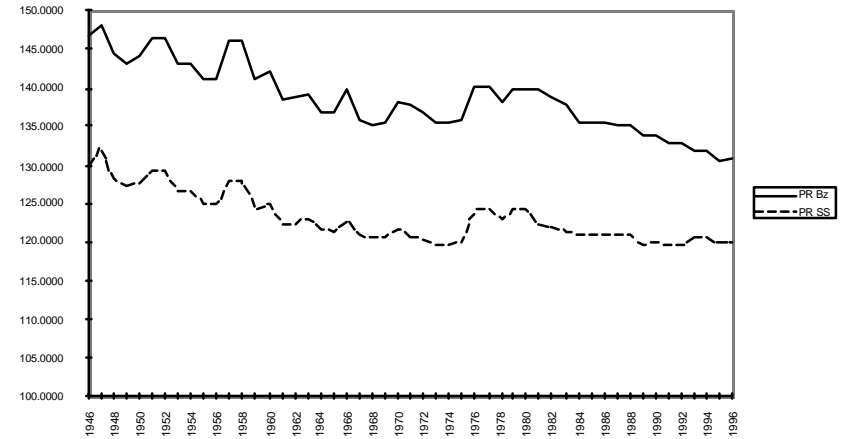
(c) Power Indices, Governors
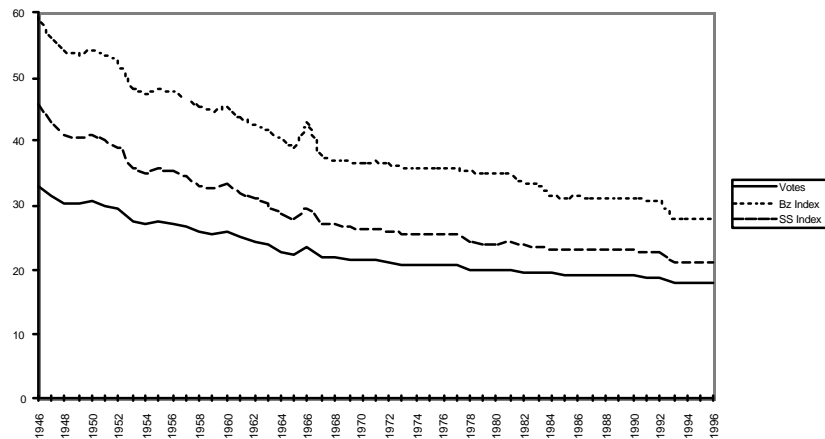
(d) Power Ratios, Governors
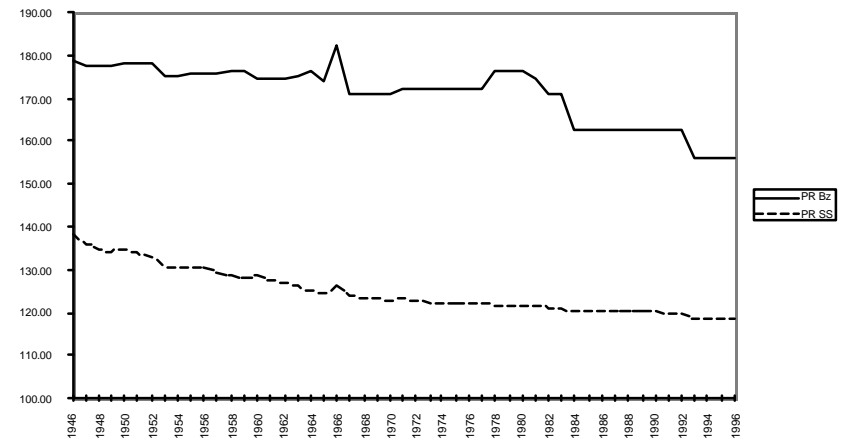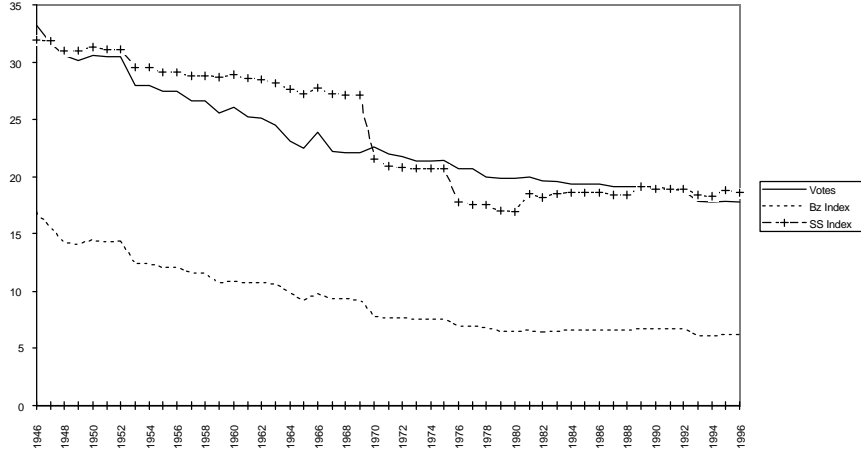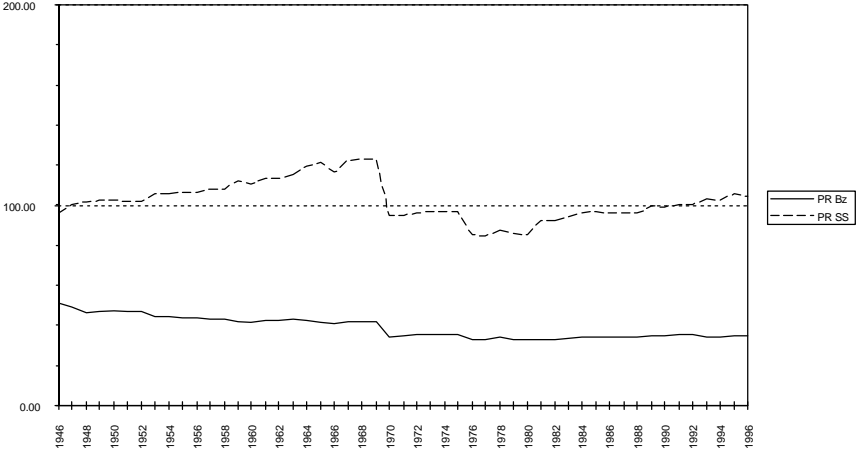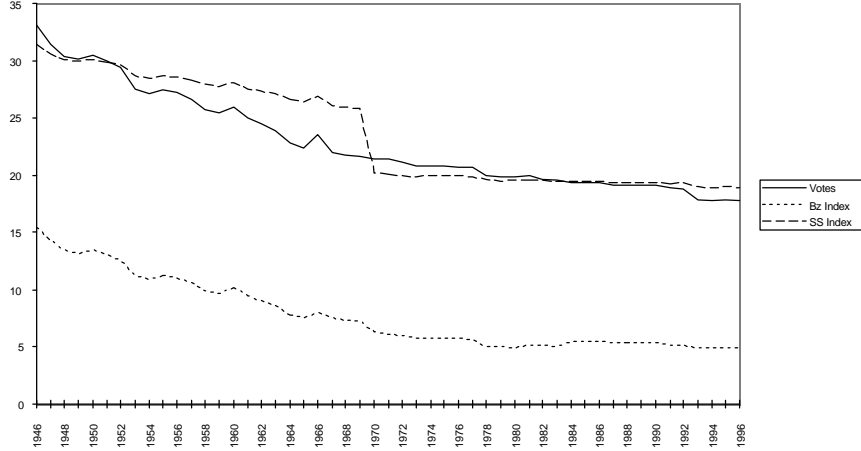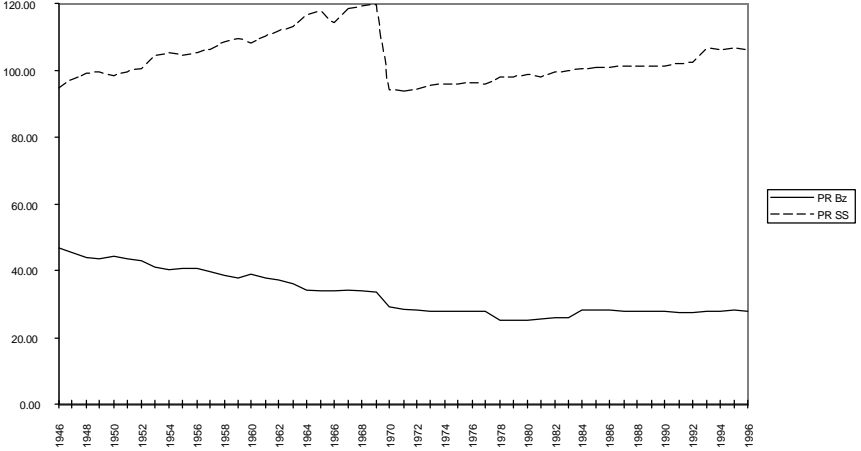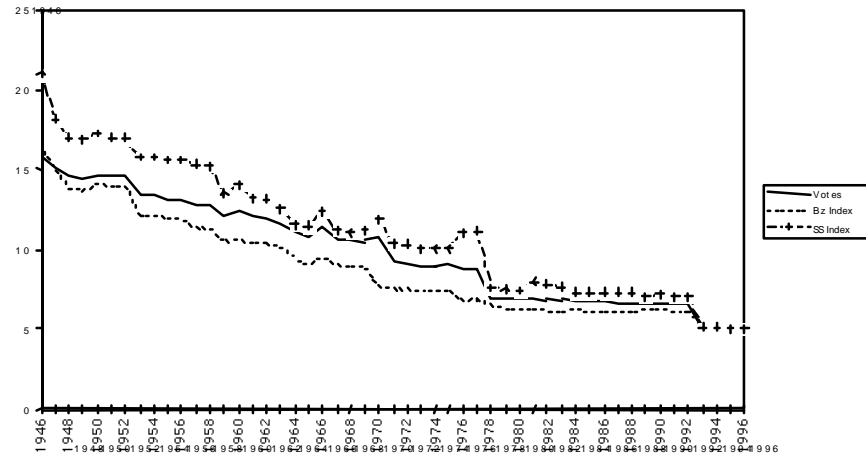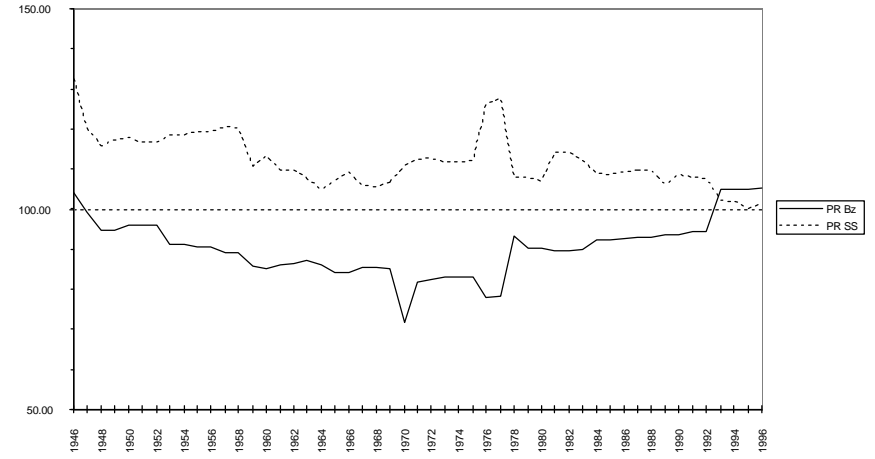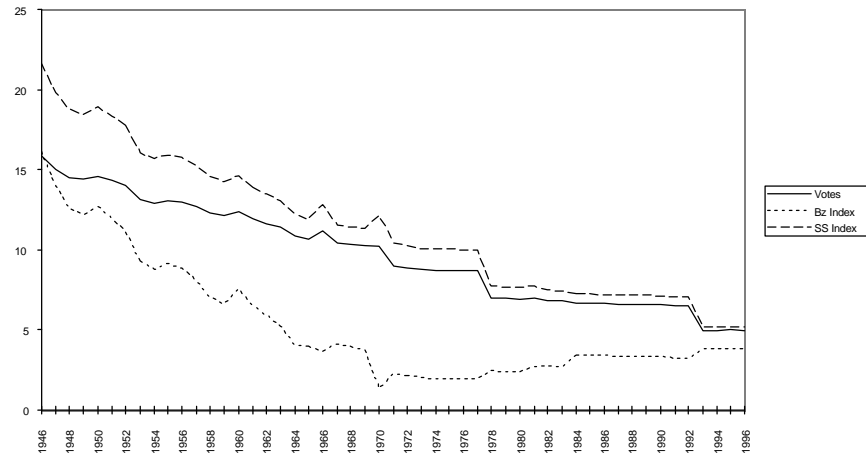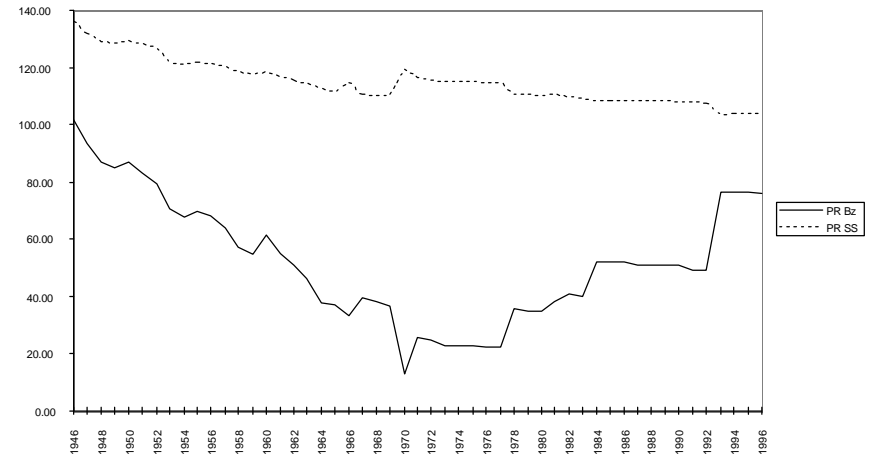
Figure 2: Historical Trends (Ordinary Decisions) UK

(a) Power Indices, Executive Directors
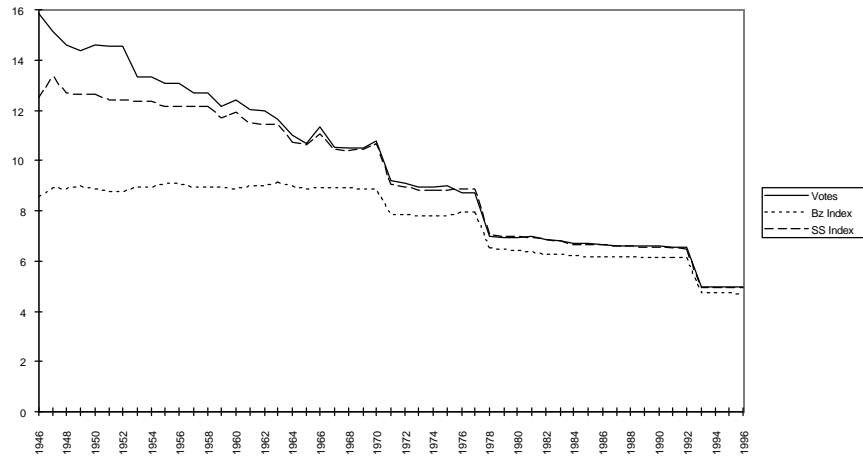
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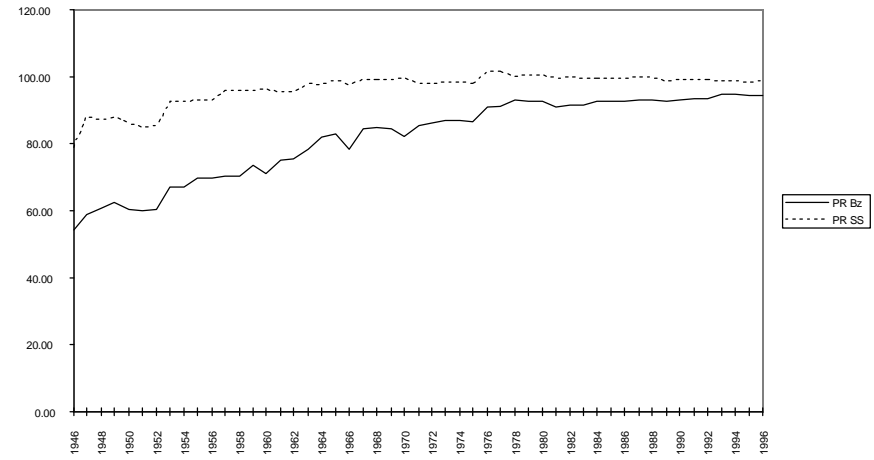
(c) Power Indices, Governors
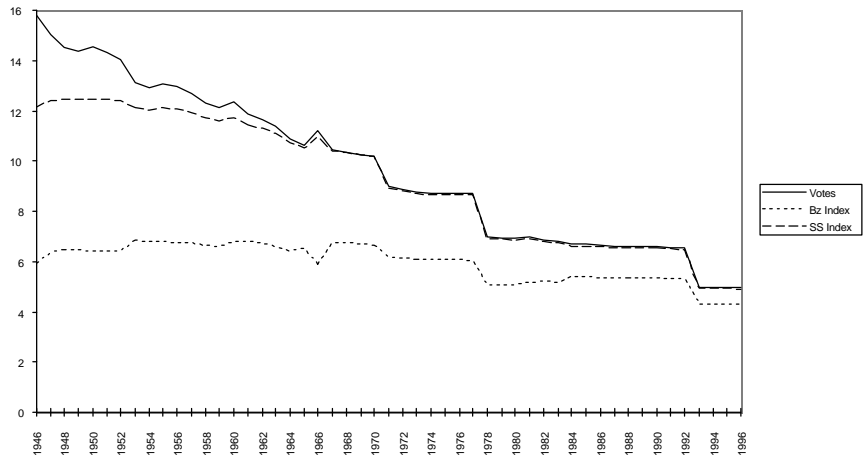
(d) Power Ratios, Governors
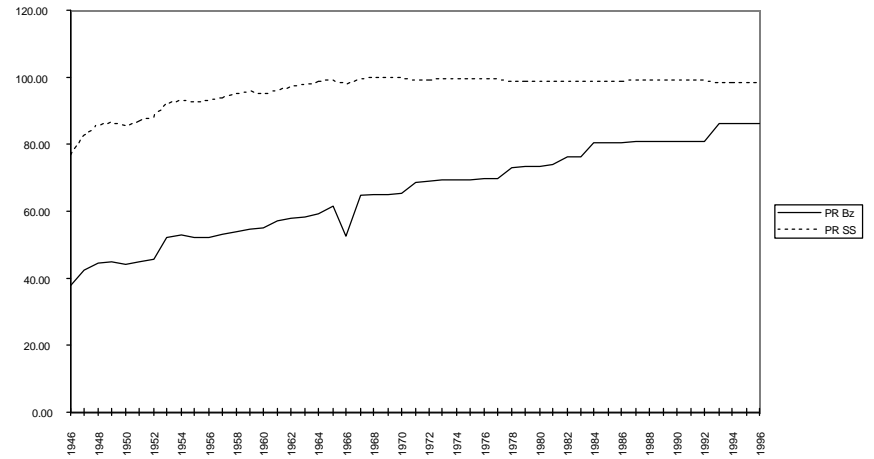
Figure 3: Historical Trends (Ordinary Decisions) France

(a) Power Indices, Executive Directors
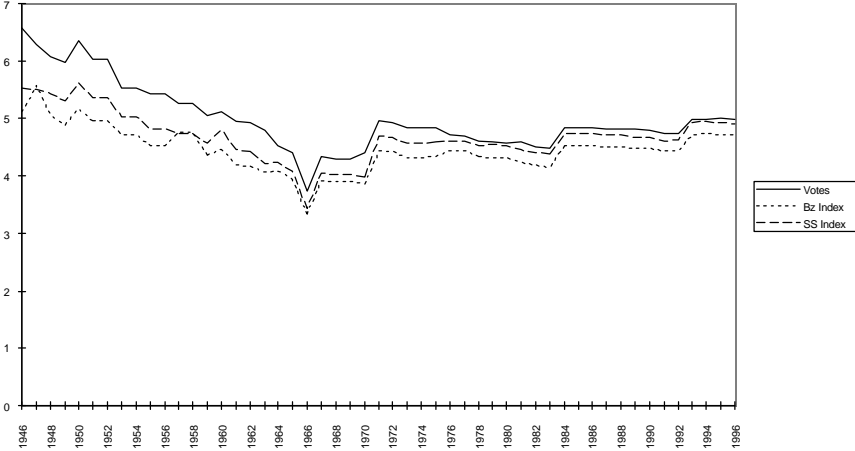
(b) Power Ratios, Executive Directors

(c) Power Indices, Governors

(d) Power Ratios, Governors

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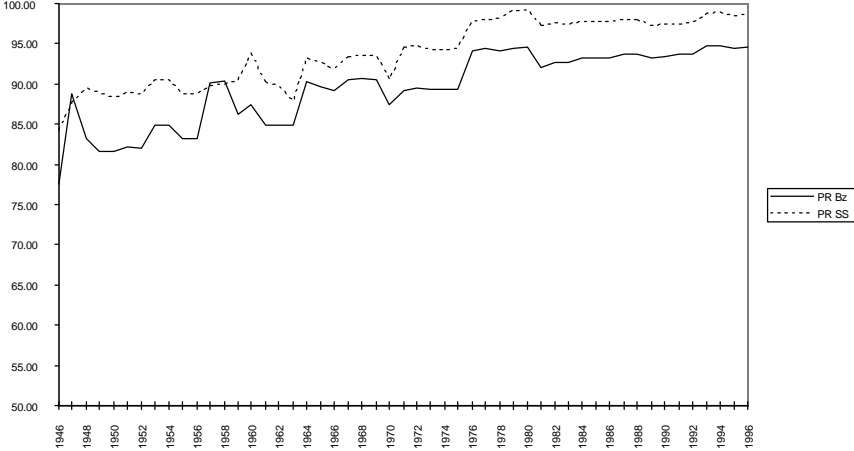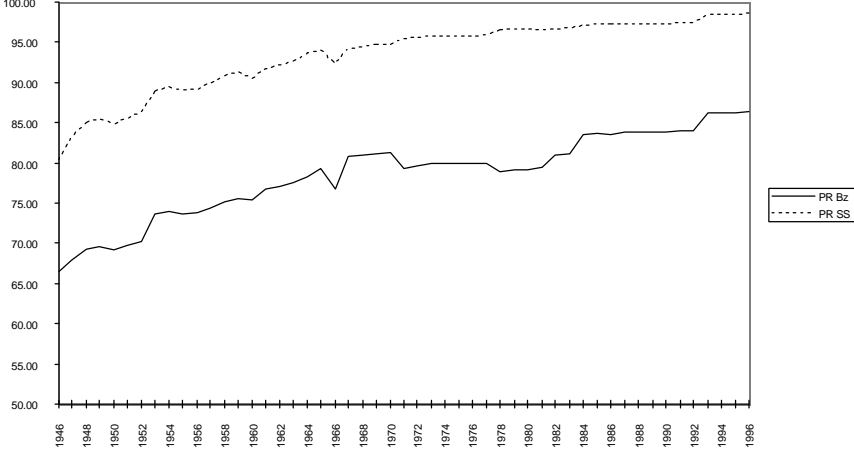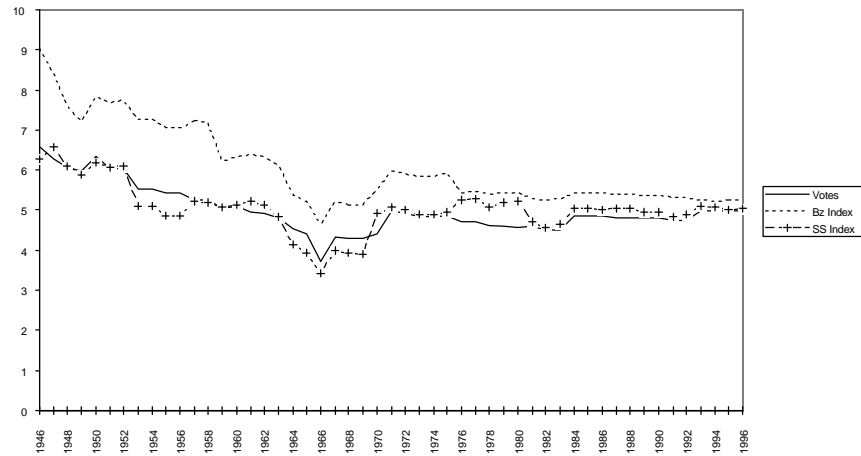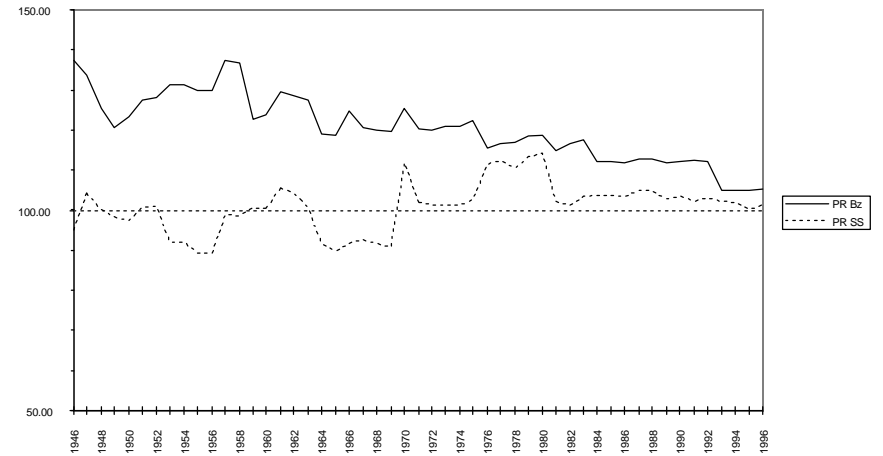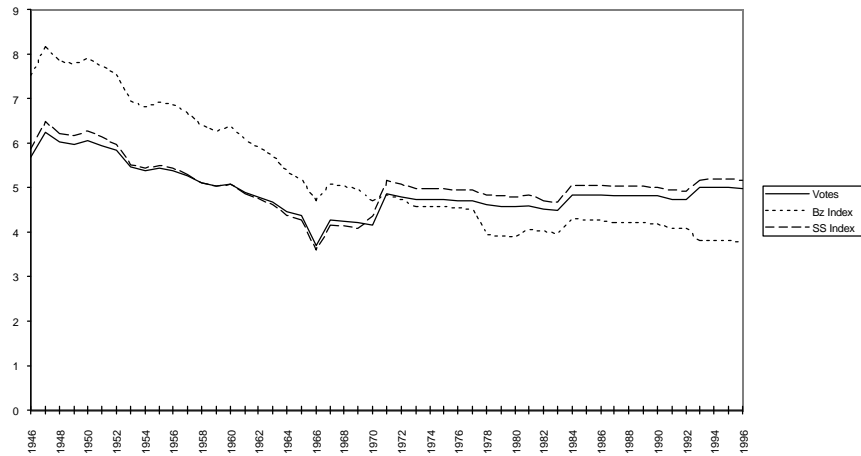
Fig. 4: Historical Trends (Ordinary Decisions) Germany

(a) Power Indices, Executive Directors
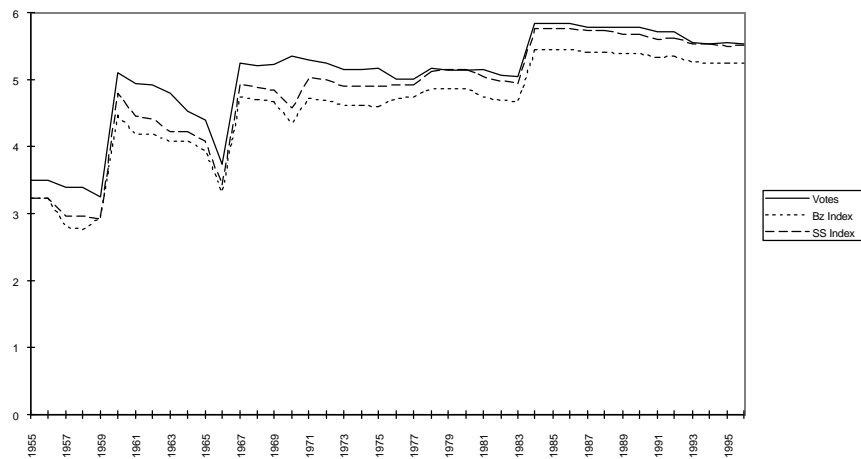
(b) Power Ratios, Executive Directors

(c) Power Indices, Governors

(d) Power Ratios, Governors

46
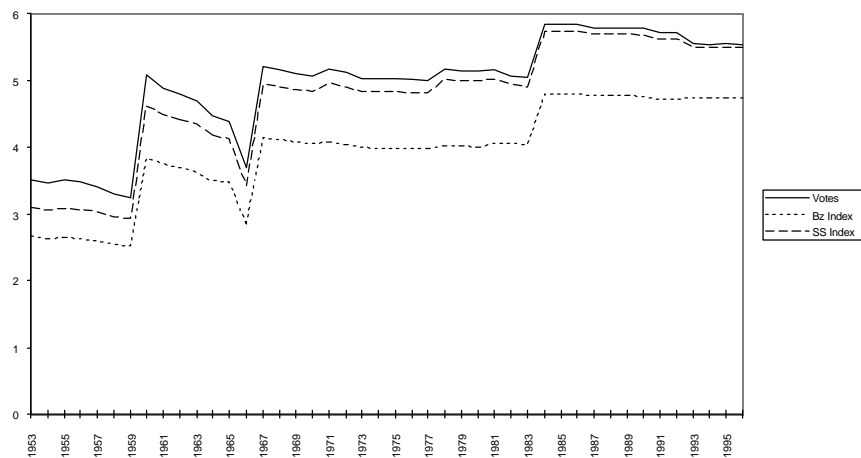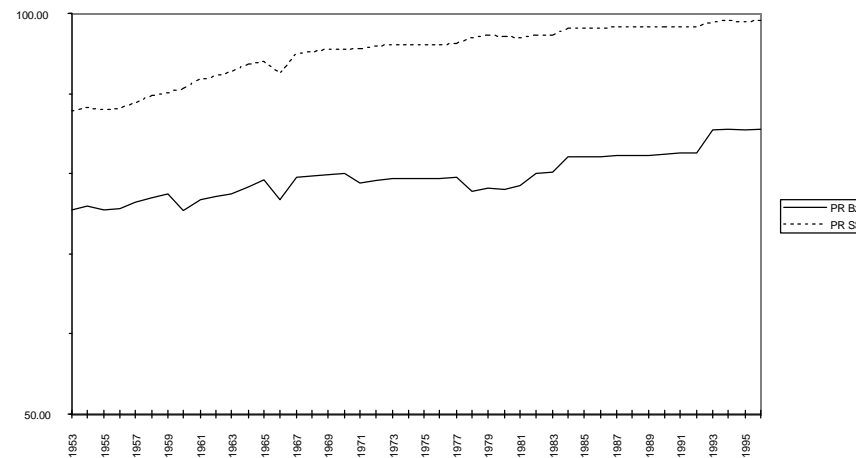
Figure 5: Historical Trends (Ordinary Decisions) Japan

(a) Power Indices, Executive Directors
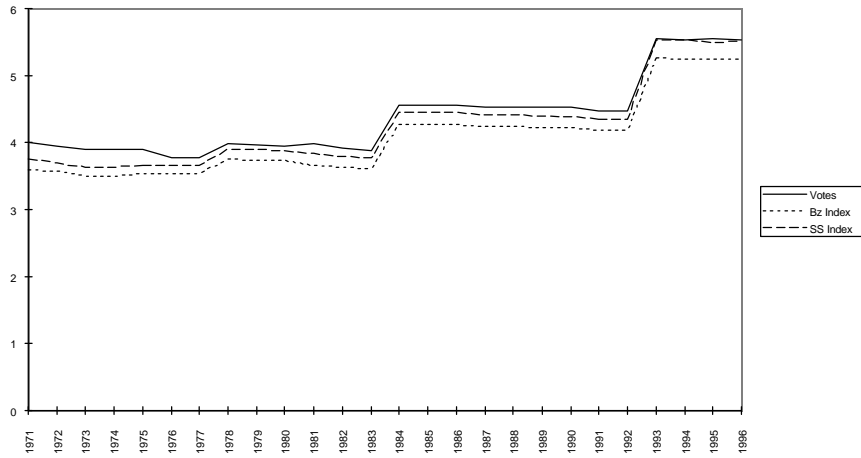
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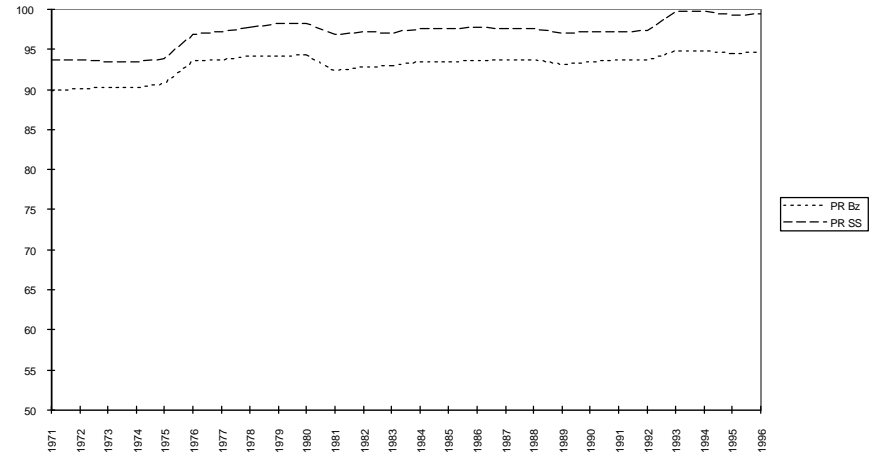
(c) Power Indices, Governors

(d) Power Ratios, Governors
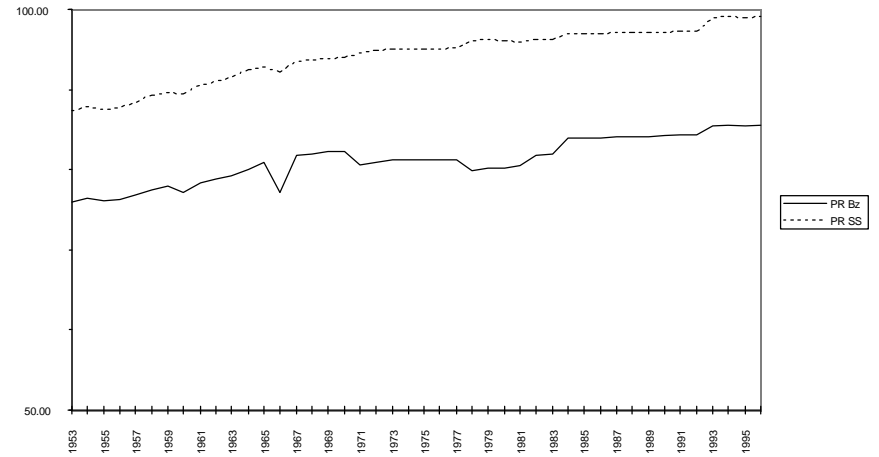
Fig. 6 Trends in Power Ratios (Ordinary Decisions)

(a) Bz Index, Executive Directors
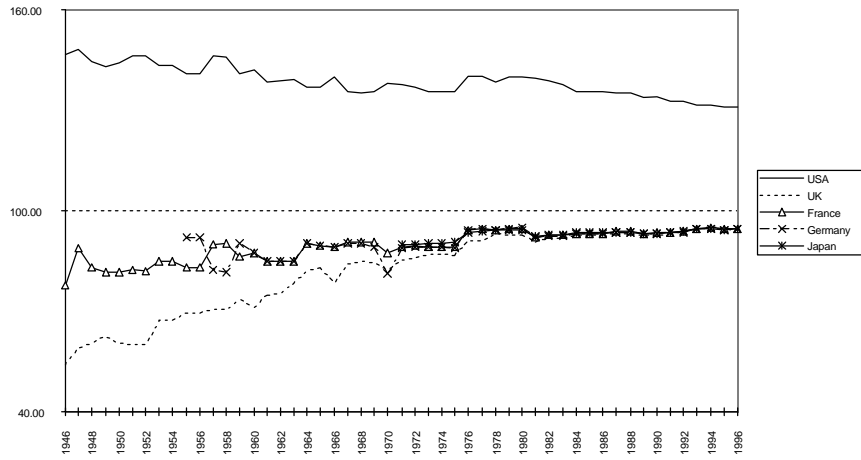
(b) SS Index, Executive Directors
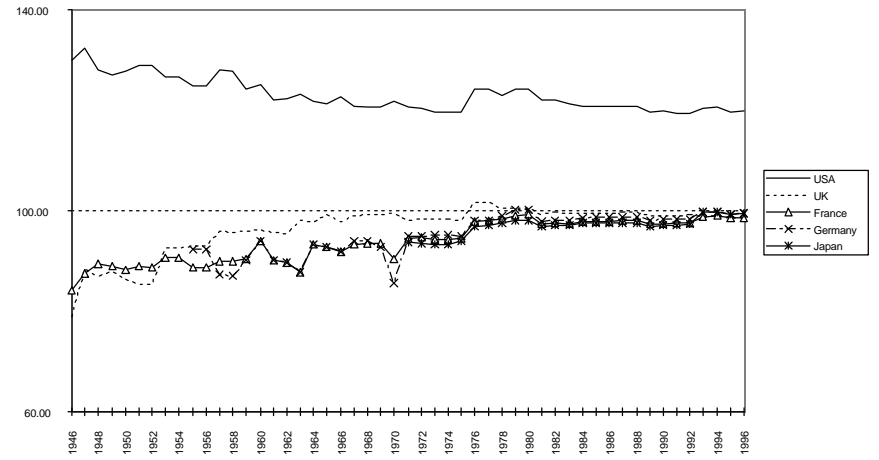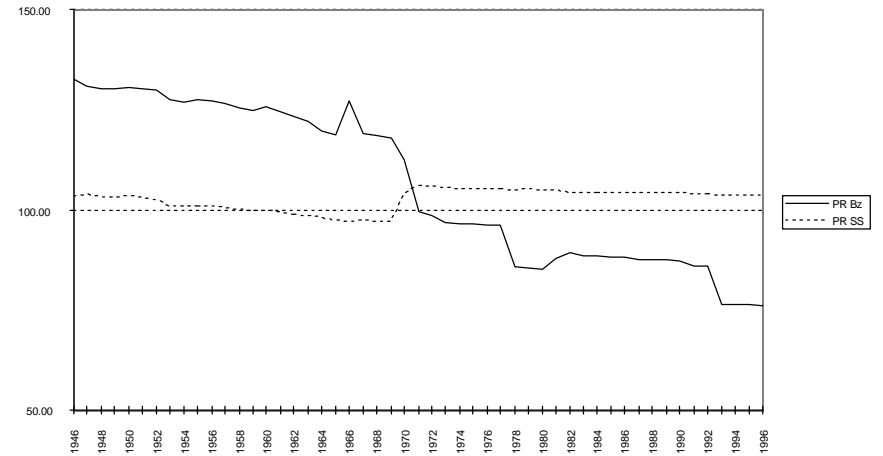

(c) Bz Index, Governors
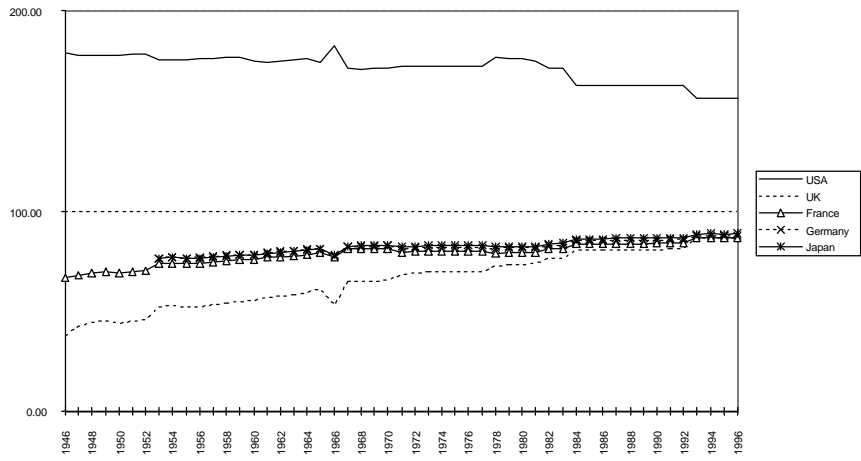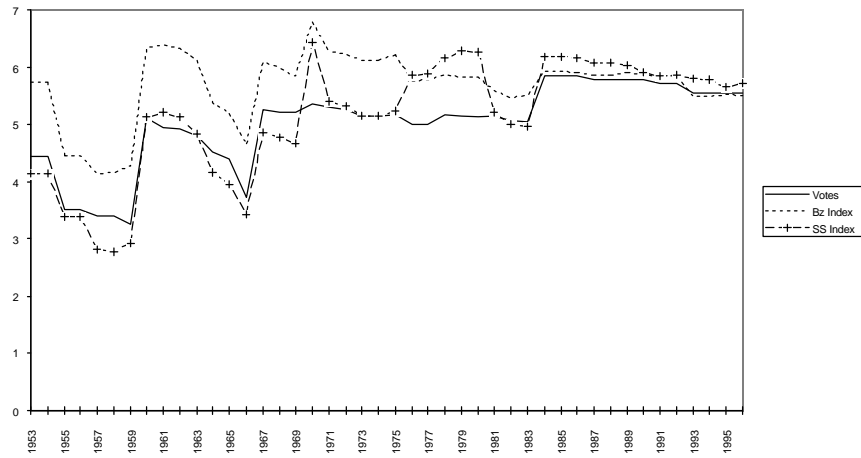

(d) SS Index, Governors
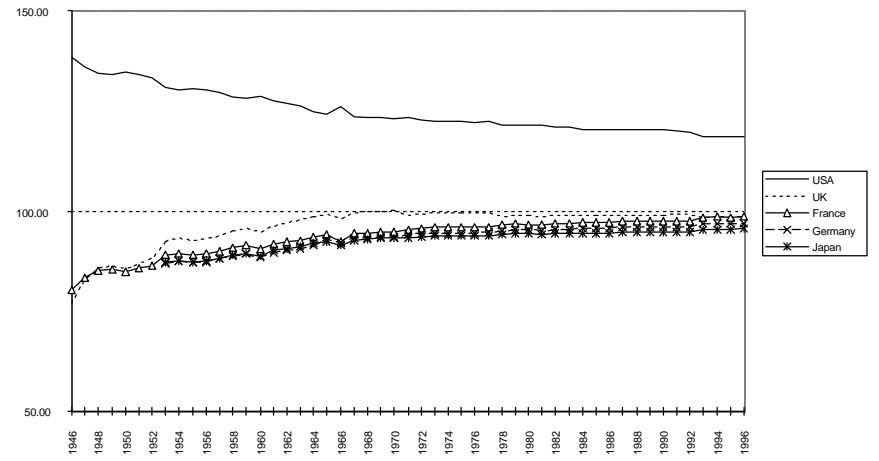
Figure 7: Historical Trends (Special Majorities) USA

(a) Power Indices, Executive Directors

(b) Power Ratios, Executive Directors

(c) Power Indices, Governors

(d) Power Ratios, Governors
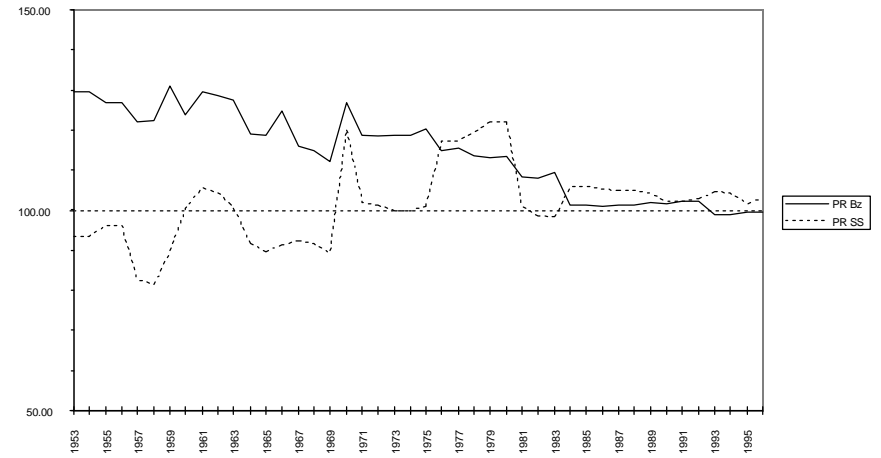

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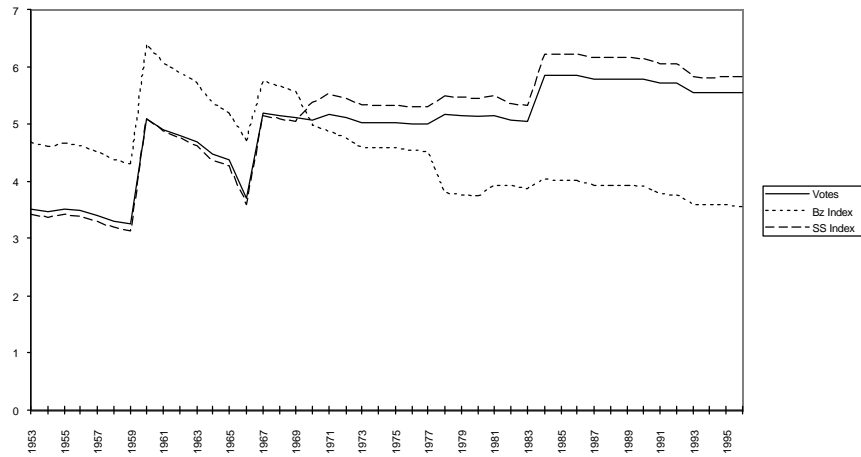# Figure 8: Historical Trends (Special Majorities) UK

(a) Power Indices, Executive Directors

(b) Power Ratios, Executive Directors

(c) Power Indices, Governors

(d) Power Ratios, Governors

50

Fig. 9: Historical Trends (Special Majorities) France

(a) Power Indices, Executive Directors

(b) Power Ratios, Executive Directors

(c) Power Ratios, Governors

(d) Power Indices, Governors

51

Fig. 10: Historical Trends (Special Majorities) Germany

(a) Power Indices, Executive Directors

(b) Power Ratios, Executive Directors

(c) Power Indices, Governors
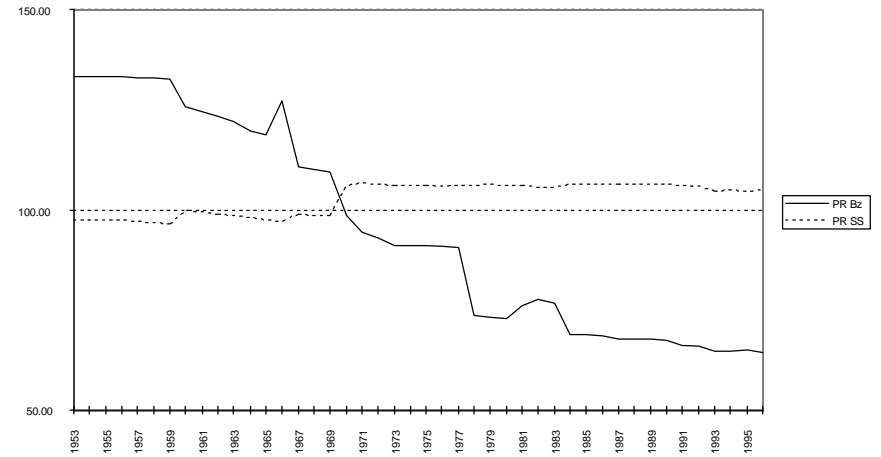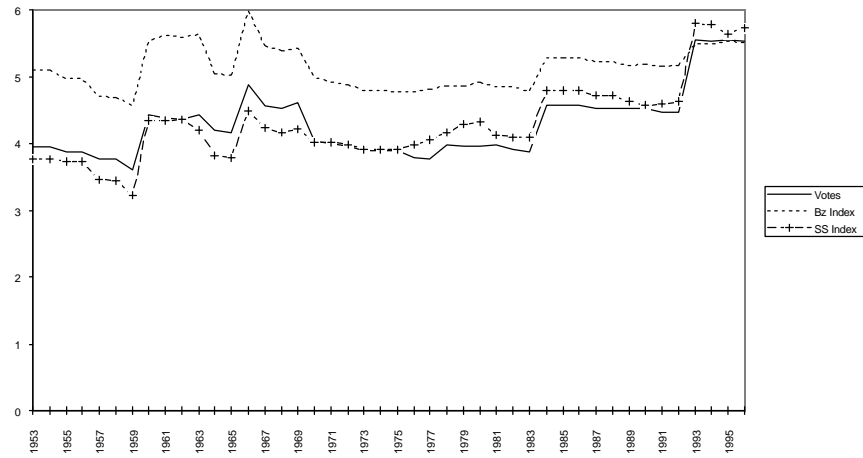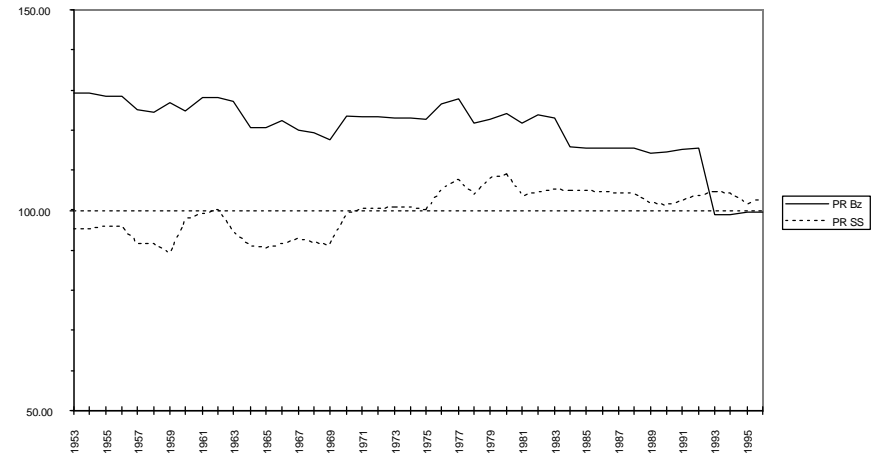

(d) Power Ratios, Governors

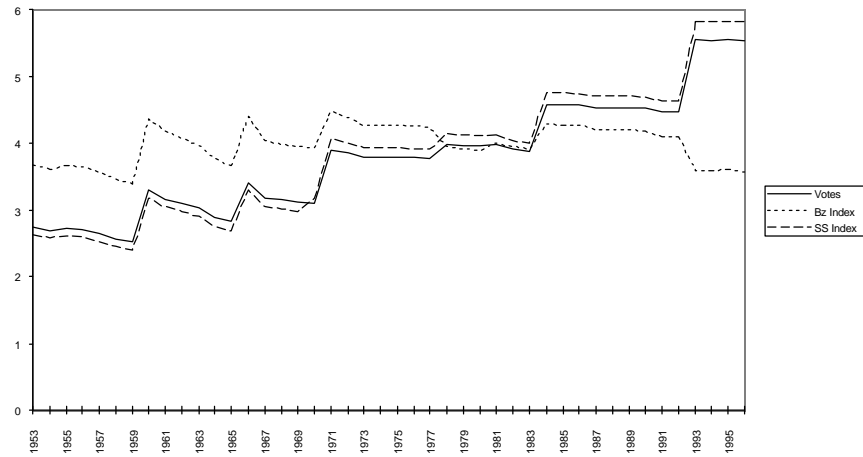# Fig. 11: Historical Trends (Special Majorities) Japan

(a) Power Indices, Executive Directors

(b) Power Ratios, Executive Directors

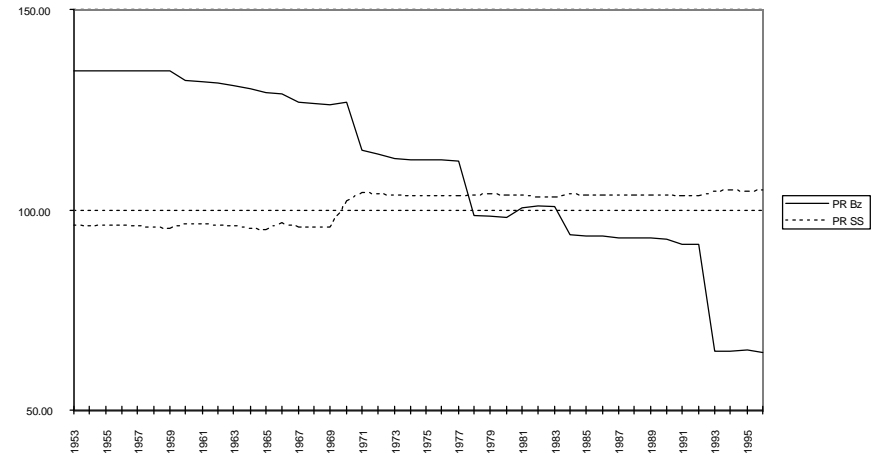(c) Power Indices, Governors

(d) Power Ratios, Governors

Figure 12: Trends in Power Ratios (Special Majorities)

(a) Bz Index, Executive Directors
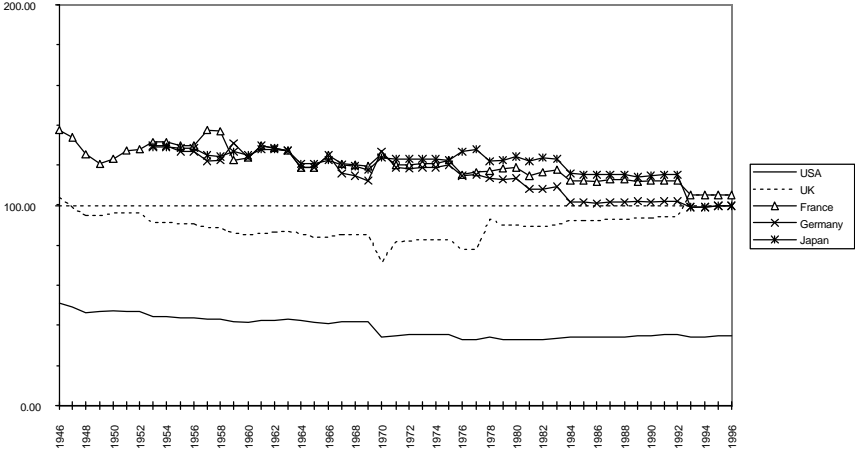
(b) SS Index, Executive Directors
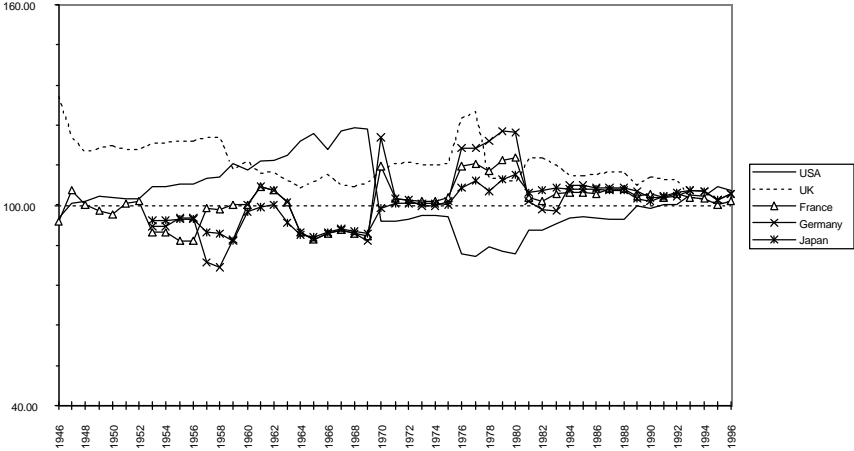
(c) Bz Index, Governors
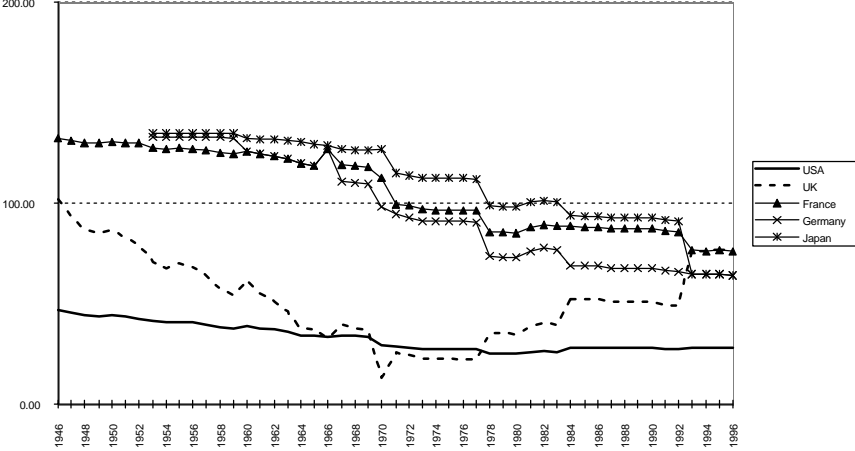
(d) SS Index, Governors

Figure 13: Effect of Variation in Majority Quota

(a) Bz Index 1946

(b) SS Index, 1946

(c) Bz Index, 1996

(d) SS Index, 1996
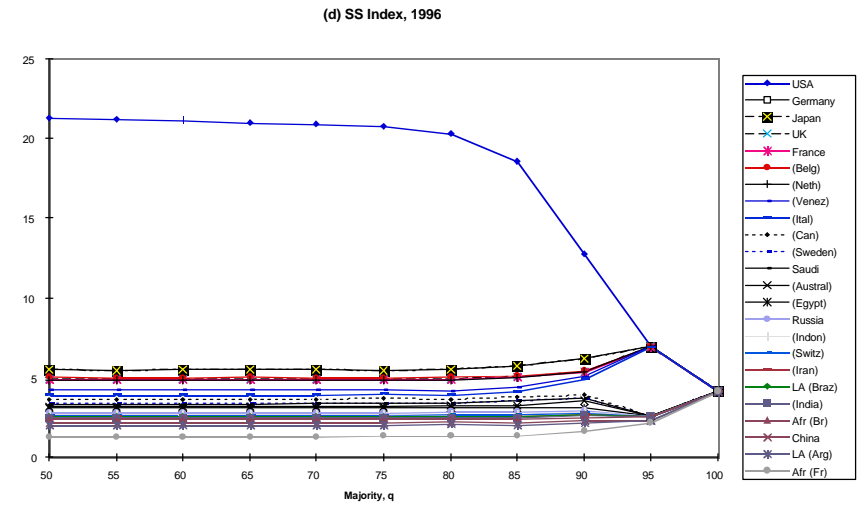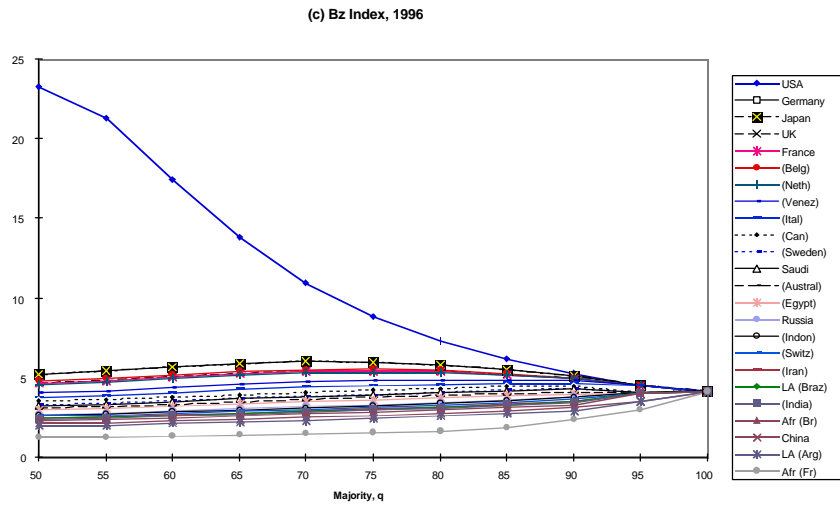
Figure 14 Effect of a European Union Voting Block

(a) Power Indices with EU Block,
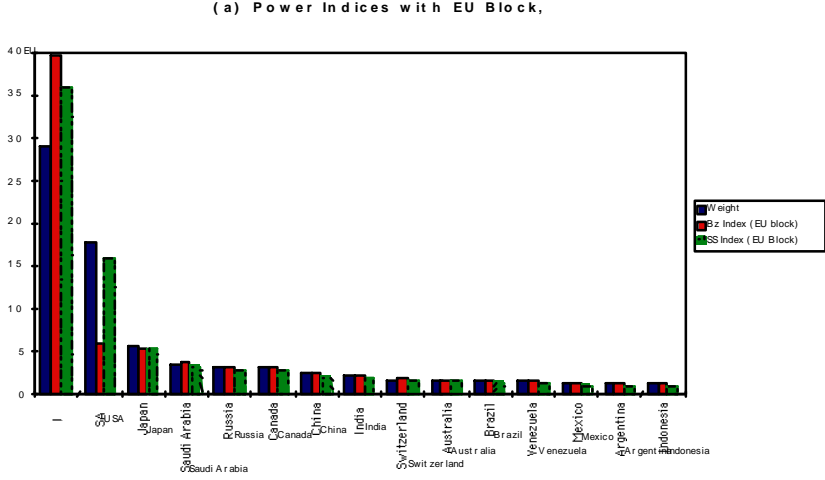
(c) Changes in Power Indices, SS Index
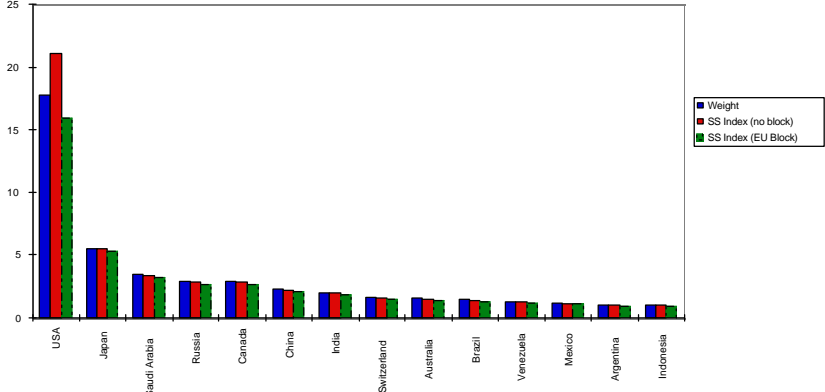
(b) Changes in Power Indices, Bz Index

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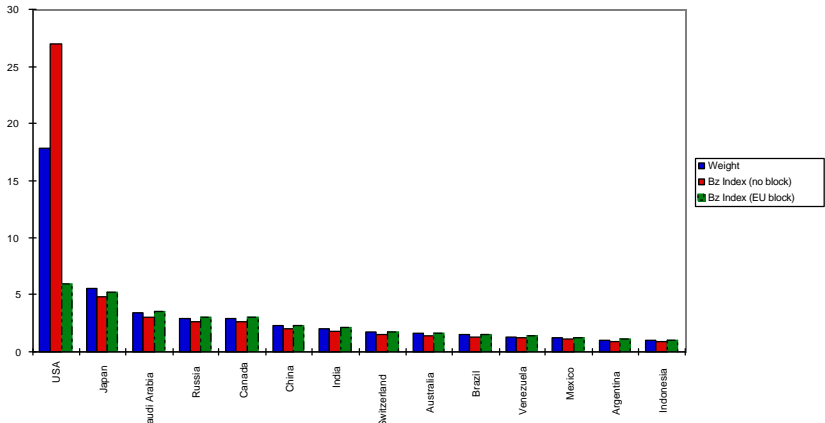
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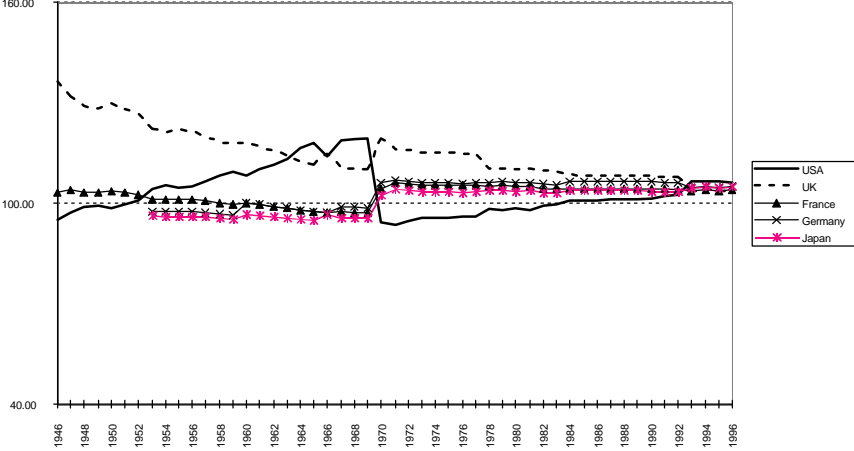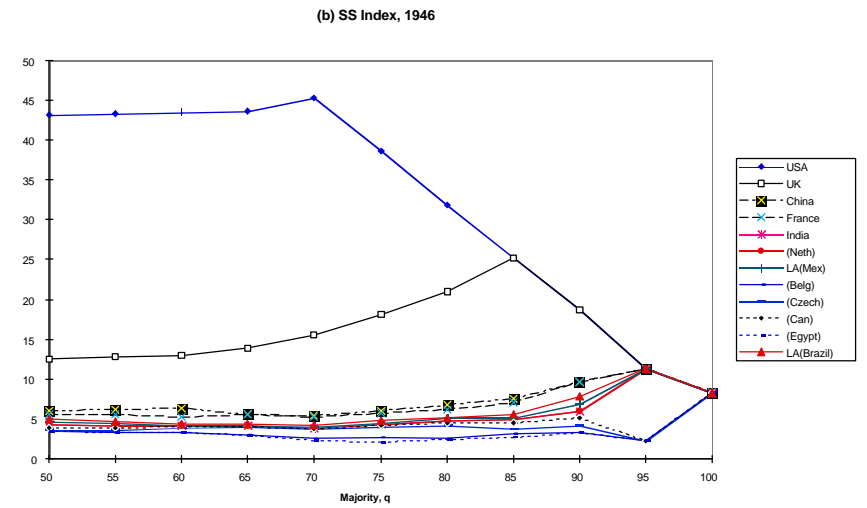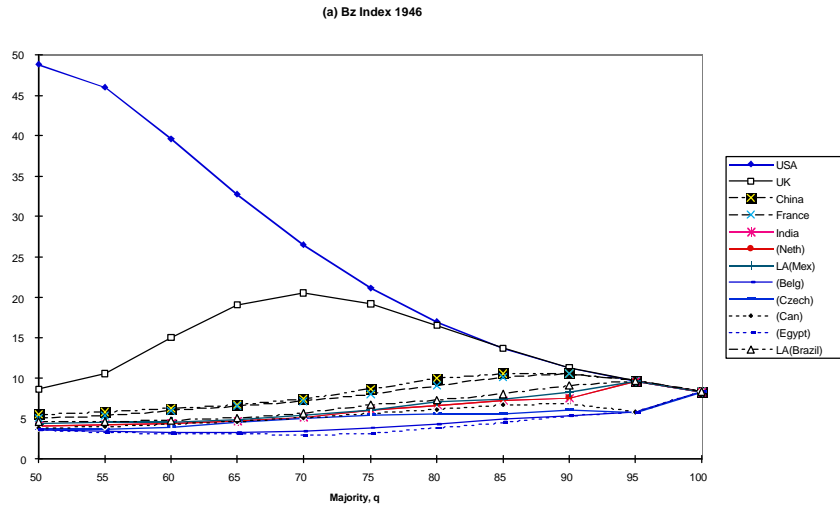
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Dreyer, Jacob S and Andrew Schotter (1980), "Power Relationships in the IMF: the Consequences of Quota Changes," Review of Economics and Statistics, vol. 62, no. 1, pp .97-106.

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Appendix

IMF EXECUTIVE DIRECTORS AND VOTING POWER
ON APRIL 30, 1996[18]

Director[19] (*Alternate*)	Casting Votes of	Votes by Country	Total Votes[20]	Percent of Fund Total[21]
Appointed:				
United States (*United States*)	United States	265,518	265,518	17.78
Germany (*Germany*)	Germany	82,665	82,665	5.54
Japan (*Japan*)	Japan	82,668	82,665	5.54
France (*France*)	France	74,396	74,396	4.98
United Kingdom (*United Kingdom*)	United Kingdom	74,396	74,396	4.98
Elected:				
Belgium (*Austria*)	Austria	12,133		
	Belarus	3,054		
	Belgium	31,273		
	Czech Republic	6,146		
	Hungary	7,798		
	Kazakhstan	2,725		
	Luxembourg	1,605		
	Slovak Republic	2,824		
	Slovenia	1,755		
	Turkey	6,670	75,983	5.09
Netherlands (*Canada*)	Armenia	925		
	Bulgaria	4,899		
	Croatia	2,866		
	Cyprus	1,250		
	Georgia	1,360		
	Israel	6,912		
	Macedonia, former Yugoslav Republic of	746		
	Moldova	1,150		
	Netherlands	34,692		
	Romania	7,791		
	Ukraine	10,223	72,814	4.88
Venezuela (*Spain*)	Costa Rica	1,440		
	El Salvador	1,506		
	Guatemala	1,788		
	Honduras	1,200		
	Mexico	17,783		
	Nicaragua	1,211		

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[18] From the IMF Annual Report 1996.

[19] Nationality of Director or Alternate.

[20] Voting power varies on certain matters pertaining to the General Department with use of the Fund's resources in that Department.

[21] Percentages of total votes (1,493,331) in the General Department and the Special Drawing rights Department.

	Spain	19,604		
	Venezuela	19,763	64,295	4.31
Italy	Albania	603		
(*Greece*)	Greece	6,126		
	Italy	46,157		
	Malta	925		
	Portugal	5,826		
	San Marino	350	59,987	4.02
Canada	Antigua and Barbuda	335		
(*Ireland*)	Bahamas, The	1,199		
	Barbados	739		
	Bruce	385		
	Canada	43,453		
	Dominica	310		
	Grenada	335		
	Ireland	5,500		
	Jamaica	2,259		
	St. Kitts and Nevis	315		
	St. Lucia	360		
	St. Vincent and the			
	Grenadines	310	58,800	3.72
Sweden	Denmark	10,949		
(*Denmark*)	Estonia	715		
	Finland	8,868		
	Iceland	1,103		
	Latvia	1,165		
	Lithuania	1485		
	Norway	11,196		
	Sweden	16,390	51,771	3.47
Saudi Arabia	Saudi Arabia	51,556	51,556	3.45
(*Saudi Arabia*)				
Australia	Australia	23,582		
(*Korea*)	Kiribati	290		
	Korea	8,246		
	Marshall Islands	275		
	Micronesia,			
	Federated States of	285		
	Mongolia	621		
	New Zealand	6,751		
	Papua New Guinea	1,203		
	Philippines	6,584		
	Seychelles	310		
	Solomon Islands	325		
	Vanuatu	375		
	Western Samoa	335	49,182	3.29
Egypt	Bahrain	1,078		
(*Bahrain*)	Egypt	7,034		
	Iraq	5,290		
	Jordan	1,467		
	Kuwait	10,202		
	Lebanon	1,710		
	Libya	8,426		
	Maldives	305		
	Oman	1,444		

	Qatar	2,155		
	Syrian Arab Republic	2,349		
	United Arab Emirates	4,171		
	Yemen, Republic of	2,015	47,646	3.19
Russia (*Russia*)	Russia	43,381	43,381	2.90
Indonesia (*Malaysia*)	Cambodia	900		
	Fiji	761		
	Indonesia	15,226		
	Lao People's Democratic Republic	641		
	Malaysia	8,577		
	Myanmar	2,099		
	Nepal	770		
	Singapore	3,826		
	Thailand	5,989		
	Tonga	300		
	Vietnam	2,666	41,755	2.80
Switzerland (*Poland*)	Azerbaijan	1,420		
	Kyrgyz Republic	895		
	Poland	10,135		
	Switzerland	24,954		
	Tajikistan	850		
	Turkmenistan	730		
	Uzbekistan	2,145	41,229	2.76
Iran (*Morocco*)	Afghanistan, Islamic State of	1,454		
	Algeria	9,394		
	Ghana	2,990		
	Iran, Islamic Republic of	11,035		
	Morocco	4,527		
	Pakistan	7,832		
	Tunisia	2,310	39,542	2.65
Brazil (*Colombia*)	Brazil	21,958		
	Colombia	5,863		
	Dominican Republic	1,838		
	Ecuador	2,442		
	Guyana	922		
	Haiti	857		
	Panama	1,746		
	Suriname	926		
	Trinidad and Tobago	2,718	39470	2.63
India (*Sri Lanka*)	Bangladesh	4,175		
	Bhutan	295		
	India	30,805		
	Sri Lanka	3,286	38,561	2.58
Swaziland (*Zimbabwe*)	Angola	2,323		
	Botswana	616		
	Burundi	822		
	Eritrea	365		
	Albania	1,233		
	Gambia, The	479		
	Kenya	2,144		

	Lesotho	489		
	Liberia	963		
	Malawi	759		
	Mozambique	1,090		
	Namibia	1,246		
	Nigeria	13,066		
	Sierra Leone	1,022		
	Swaziland	615		
	Tanzania	1,719		
	Uganda	1,589		
	Zambia	3,885		
	Zimbabwe	2,863	37,388	2.50
China (*China*)	China	34,102	34,102	2.28
Peru (*Argentina*)	Argentina	18,621		
	Bolivia	1,512		
	Chile	6,467		
	Paraguay	971		
	Peru	4,911		
	Uruguay	2,503	31,985	2.14
Cote d'Ivoire (*Gabon*)	Benin	703		
	Burkina Paso	692		
	Cameroon	1,601		
	Cape Verde	320		
	Central African Republic	662		
	Chad	663		
	Comoros	315		
	Congo	829		
	Cote d'Ivoire	2,632		
	Djibouti	368		
	Equatorial Guinea	493		
	Gabon	1,383		
	Guinea	1,037		
	Guinea-Bissau	355		
	Madagascar	1,154		
	Mali	939		
	Mauritania	725		
	Maldives	983		
	Niger	733		
	Rwanda	848		
	Sao Tome and Principe	305		
	Senegal	1,439		
	Togo	793	19,936	1.34
			1,475,5333[22][23]	98.81[24]

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[22] This total does not include the votes of Bosnia and Herzegovina, Brunei Darussalam, Somalia, and South Africa, which did not participate in the 1994 Regular Election of Executive Directors. The combined votes of those members total 17,808-1.19 percent of those in the General Department and Special Drawing Rights Department.

[23] This total does not include the votes of Sudan and Zaire, which were suspended effective August 9, 1993 and June 2, 1994, respectively, pursuant to Article XXVI, Section 2(b) of the Articles of Agreement.

[24] This figure may differ from the sum of the percentages shown for individual Directors because of rounding.