

PROJECT ADMINISTRATION DATA SHEET

ORIGINAL REVISION NO. _____

Project No. (R6022-0A0)
E-25-637 GTRC/~~OMX~~ DATE 9/18/85

Project Director: Dr. Benson H. Tongue School/~~ME~~ ME

Sponsor: U. S. Army Research Office
Research Triangle Park, NC

Type Agreement: ~~P-22557-56~~ DAAG 29-85-K-0207

Award Period: From 9/15/85 To 6/14/87 (Performance) 11/14/88 (Reports)

Sponsor Amount: This Change Total to Date
Estimated: \$ 75,000 \$ 75,000
Funded: \$ 75,000 \$ 75,000

Cost Sharing Amount: \$ 14,715 Cost Sharing No: E-25-330

Title: Nonlinear Rotocraft Analysis-Experimental and Analytical

ADMINISTRATIVE DATA

OCA Contact William F. Brown X4820

1) Sponsor Technical Contact:

Dr. Gary L. Anderson
Engineering Sciences Div.
Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

2) Sponsor Admin/Contractual Matters:

Mr. T. A. Bryant
ONR RR
Georgia Tech

Defense Priority Rating: None Shown Military Security Classification: unclassified
(or) Company/Industrial Proprietary: N/A

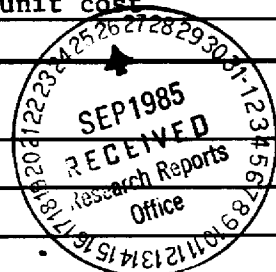
RESTRICTIONS

See Attached Gov't Supplemental Information Sheet for Additional Requirements.

Travel: Foreign travel must have prior approval - Contact OCA in each case. Domestic travel requires sponsor approval where total will exceed greater of \$500 or 125% of approved proposal budget category.

Equipment: Title vests with GIT; Requires prior approval to purchase any equipment not in the proposal budget, any ADPE item, and equipment item with unit cost \$10,000 or more.

COMMENTS:



COPIES TO: SPONSOR'S I. D. NO. 02.102.001.86.002

Project Director Administrative Network Procurement/GTRI Supply Services Research Security Services GTRC 12/9/85

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Date 4/13/89

Project No. E-25-637

Center No. R6022-OA0

Project Director B. H. Tongue

School/Lab ME

Sponsor ARMY

Contract/Grant No. DAAG29-85-K-0207

GTRC XX GIT

Prime Contract No.

Title Nonlinear Rotocraft Analysis - Experimental and Analytical

Effective Completion Date 9/14/88

(Performance) 1/14/89

(Reports)

Closeout Actions Required:

- None
- X Final Invoice or Copy of Last Invoice
- Final Report of Inventions and/or Subcontracts
- Government Property Inventory & Related Certificate
- Classified Material Certificate
- Release and Assignment
- Other

Includes Subproject No(s).

Subproject Under Main Project No.

Continues Project No.

Continued by Project No.

Distribution:

- X Project Director
- X Administrative Network
- X Accounting
- X Procurement/GTRI Supply Services
- X Research Property Management
- Research Security Services

- X GTRC
- X Project File
- 2 Contract Support Division (OCA)
- Other



GEORGIA TECH 1885-1985

DESIGNING TOMORROW TODAY

GEORGE W. WOODRUFF
SCHOOL OF MECHANICAL ENGINEERING

December 19, 1985

Dr. Gary L. Anderson
Chief, Structures and
Dynamics Branch
Engineering Sciences Division
U. S. Army Research Office
P. O. Box 12211
Research Triangle Park, NC 27709-2211

Dear Gary,

I have enclosed a copy of my first progress report. I hope it is acceptable. If anything else is needed, I will be happy to furnish it.

Best wishes for Christmas,

Benson

BHT/kf

Enclosure

Georgia Institute of Technology
Atlanta, Georgia 30332-0405

Semi Annual Progress Report

Title: Nonlinear Rotorcraft Analysis - Experimental and Analytical

Institution: School of Mechanical Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332

Period Covered: 9/15/85 - 12/15/85

Project Number: P-22557-EG

Contract Number: DAAG29-85-K-0207

Author: Benson H. Tongue

Personnel Supported: Benson H. Tongue, George T. Flowers

The early phases of the proposed work have been initiated. A comprehensive literature search began and is ongoing. Most of the background work in the field has been collected and reviewed.

Design work for a suitable rotorcraft model has begun. An appropriate landing gear model (including desired degrees of freedom and spring, damper characteristics) is being developed.

Coding of a multi-degree-of-freedom landing gear coupled to an articulated rotor has begun. Suitable transformations to allow a reduction in the number of blade coordinates in spite of geometric and physical nonlinearities are being sought.

An extremely important academic hurdle for the student working on the project is the Ph.D. qualifying examination, which took place during the last two weeks of November. It was deemed critical that the student pass this exam on his first sitting, as a repeat of the exam six months later. During this period the student would not be able to devote all his energies to the project owing to worry over the exam and preparation time. Therefore, the importance of the exam was stressed during the preceding month or so.

It is a pleasure to report that the student passed all areas of the exam and indeed distinguished himself by his performance.

PROGRESS REPORT

TWENTY COPIES REQUIRED

1. ARO PROPOSAL NUMBER: 22557-EG
2. PERIOD COVERED BY REPORT: 1 July 1986 - 31 December 1986
3. TITLE OF PROPOSAL: Nonlinear Rotorcraft Analysis -
Experimental & Analytical
4. CONTRACT OR GRANT NUMBER: DAAG29-85-K-0207
5. NAME OF INSTITUTION: Georgia Institute of Technology
6. AUTHORS OF REPORT: Benson H. Tongue-----
7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS REPORTING PERIOD, INCLUDING JOURNAL REFERENCES:
 - 1.) Tongue, B. H. and Flowers, G. T. "Nonlinear Rotorcraft Analysis," submitted to AIAA Journal
 - 2.) Tongue, B. H., "Non-Linear Rotorcraft Response," submitted to AFOSR/ARO Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms
8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:
 - Benson H. Tongue - Principal Investigator
 - George T. Flowers - Graduate Research Assistant - pursuing Ph.D.
 - Philip M. FitzSimons - Graduate Research Assistant - pursuing Ph.D.
 - Michael D. Janowski - Graduate Research Assistant - obtained M.S. Thesis title - Analysis of a Simplified Ground Resonance Model
 - Kegin Gu - Graduate Research Assistant - pursuing Ph.D.

Benson H. Tongue
Department of Mechanical Engineering
Georgia Institute of Technology
Atlanta, GA 30332

BRIEF OUTLINE OF RESEARCH FINDINGS

A fully nonlinear limit cycle analysis has been developed and compared with both a restricted nonlinear analysis (the previous state of the art) and full numerical simulations. The fully nonlinear analysis allows an excellent quantitative agreement with numerical simulations. The cost of the limit cycle determination has been reduced by two to three orders of magnitude over numerical integrations. A methodology for analyzing generic rotor systems with an arbitrary number of response harmonics has been developed. A comparison of the restricted nonlinear analysis with the fully nonlinear one shows marked differences at high amplitude oscillations, with the restricted analysis overestimating the actual response amplitude. The current newly developed analysis allows the inclusion of reasonably complex geometric nonlinearities in the limit cycle analysis with little difficulty.

Future work will involve an extensive experimental comparison with the analytically predicted results. The analysis will be extended to the case of hingeless rotors and a full parametric study for a generic helicopter configuration will be undertaken. An analysis of the air resonance phenomenon will also be pursued.

PROGRESS REPORT

TWENTY COPIES REQUIRED

1. ARO PROPOSAL NUMBER: 22557-EG
2. PERIOD COVERED BY REPORT: 1 January 1987 - 30 June 1987
3. TITLE OF PROPOSAL: Nonlinear Rotorcraft Analysis -
Experimental & Analytical
4. CONTRACT OR GRANT NUMBER: DAAG29-85-K-0207
5. NAME OF INSTITUTION: Georgia Institute of Technology
6. AUTHORS OF REPORT: Benson H. Tongue
7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS REPORTING PERIOD, INCLUDING JOURNAL REFERENCES:
 - 1) Flowers, G. T. and Tongue, B. H., "Modeling and Analysis of a Nonlinear Rotorcraft", submitted to Applied Mathematical Modeling, 1987
 - 2) Tongue, B. H. and Flowers, G. T., "Nonlinear Rotorcraft Analysis", submitted to International Journal of Nonlinear Mechanics, 1987
 - 3) Janowski, M. and Tongue, B. H., "Construction and Analysis of a Simplified Growth Resonance Model", to appear in the Journal of Sound and Vibration, 1987
 - 4) Tongue, B. H., "On the Global Analysis of Nonlinear Systems Through Interpolated Cell Mapping", to appear in Physica D., Sept. 1987
 - 5) Tongue, B. H. and Gu, K., "Interpolated Cell Mapping of Nonlinear Systems", submitted to the ASME Journal of Applied Mechanics, 1987
 - 6) Gu, K. and Tongue, B. H. "A Method to Improve the Modal Convergence for Structures with External Forcing" to appear in the ASME Journal of Applied Mechanics, 1987
8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

Benson H. Tongue - Principal Investigator
George T. Flowers - Graduate Research Assistant - pursuing Ph.D.
Kegin Gu - Graduate Research Assistant - pursuing Ph.D.

Benson H. Tongue
Department of Mechanical Engineering
Georgia Institute of Technology
Atlanta, GA 30332

BRIEF OUTLINE OF RESEARCH FINDINGS

The original experimental setup has been revised in light of shortcomings that became apparent during preliminary data acquisition. The main problems were a good deal of noise in the signal and a lack of sufficient control over the system's lumped damping elements. A precision slip ring assembly has been installed and the rotor mounting has been modified. The data produced by the model is now uncontaminated by unwanted noise. Rotary damping elements have been machined both for the fuselage and the blades. Springs have been installed on the model's blades in order to allow a simulation of both an articulated as well as a simplified hingeless blade set. Parameter identification routines have been developed to identify the linear and nonlinear damping and spring characteristics of the system. The resulting parameter coefficients are being used in the analytical model of the craft. Comparisons between the analytical and experimental model are now under way.

A parametric limit cycle study in which the linear blade damping and the hydraulic fuselage damping were varied has been completed and interesting results have been obtained. The fully nonlinear model has demonstrated a surprising degree of agreement with the quasi-nonlinear model that was developed during Year One of the project. It appears that the full nonlinearities alter the quantitative but not the qualitative trends of the limit cycle responses.

The air resonance phase of the research is progressing according to schedule. An appropriate simplified rotorcraft model has been selected and the development of the equations of motion of this model is currently under way. As would be expected, aerodynamics will play a much larger role in the air resonance study than has been the case for ground resonance. Analyses of this analytical model for air resonance behavior and the effect of nonlinearities will be carried out in the coming months.

ABSTRACTS OF SUBMITTED PAPERS

Nonlinear Rotorcraft Analysis

A limit cycle analysis for a fully nonlinear modeling of a helicopter is presented and the results compared to numerical integrations of the equations of motion and to a restricted nonlinear analysis. The model is used to analyze the nonlinear ground resonance behavior of a helicopter. Although a restricted analysis will give a qualitative idea of the system's response, it is shown that a fully nonlinear analysis is necessary to obtain quantitative agreement with numerical integrations if the motions become large. A harmonic balance approach is utilized and symbolic manipulation is used to obtain the coefficients of the various harmonics of the system response. A single harmonic is shown to be sufficient to allow the amplitudes and frequencies of the limit cycle response to be accurately predicted.

Modeling and Analysis of a Nonlinear Rotorcraft

Analyzing a system of nonlinear equations for the presence of limit cycles is often a tedious and time consuming proposition, even for relatively simple systems. The large amount of algebraic manipulation required is extremely difficult to perform accurately and can quickly degenerate into an intractable problem. This paper discusses the development of a mathematical model for a helicopter with nonlinear damping in its landing gear. The use of symbolic manipulation in the derivation of the complex set of equations is stressed. The resulting equations represent an important extension to the type of nonlinear equations that have previously been analyzed and allow a more accurate calculation of the system's response. The techniques used are described in detail and are applicable to the study of generic rotating systems.

Construction and Analysis of a Simplified Ground Resonance Model

It has long been recognized by the rotary wing aircraft community that an instability known as ground resonance can occur which can threaten both the integrity and performance of an aircraft. Although the behavior of linear rotorcraft models have been well established, only recently has the behavior of a vehicle with nonlinear characteristics been studied. A simplified physical model is developed in the present analysis which exhibits many of the same response characteristics as the nonlinear model of a helicopter. A parametric study of this model is undertaken to see how parametric changes affect the steady-state response of the system. Through the use of the model, it is shown that the limit cycle behavior of a nonlinear ground resonance model behaves in a similar manner to a rotating shaft with a setscrew or key.

A Method to Improve the Modal Convergence for Structures with External Forcing

The traditional approach of using free vibration modes in the assumed mode method often leads to an extremely slow convergence rate, especially when discrete interactive forces are involved. By introducing a number of forced modes, significant improvements can be achieved. These forced modes are intrinsic to the structure and the spatial distribution of forces. The motion of the structure can be described exactly by these forced modes and a few free vibration modes provided that certain conditions are satisfied. The forced modes can be viewed as an extension of static modes. The development of a forced mode formulation is outlined and a numerical example is presented.

Interpolated Cell Mapping of Nonlinear Systems

A method is proposed to efficiently determine the basins of attraction of a nonlinear system's different steady state solutions. The phase space of the dynamical system is spacially discretized and the continuous problem in time is converted to an iterative mapping. By means of interpolation procedures, an improvement in the system accuracy over the Simple Cell Mapping technique is achieved. Both basins of attraction for a representative nonlinear system and characteristic system trajectories are generated and compared to exact solutions.

On Obtaining Global Nonlinear System Characteristics Through Interpolated Cell Mapping

Period doubling bifurcations and fractal basin boundaries are investigated by means of Interpolated Cell Mapping (ICM). ICM is a new method to determine the continuous time trajectory of a nonlinear system by spacially discretizing the system and employing an interpolation procedure to estimate the system's response at any arbitrary point in phase space. The parameter values at which period doubling occurs for a Duffing's oscillator is found by both conventional time integration and two mapping techniques and the results compared. The ICM method is shown to very accurately determine the points of bifurcation. The dimension of a fractal basin boundary is determined by ICM and compared to an exact determination. The ICM procedure produces the same fractal dimension determination as the exact analysis but only requires one thousandth of the computation time.

PROGRESS REPORT

TWENTY COPIES REQUIRED

1. ARO PROPOSAL NUMBER: 22557-EG
2. PERIOD COVERED BY REPORT: 1 July 1987 - 31 December 1987
3. TITLE OF PROPOSAL: Nonlinear Rotorcraft Analysis -
Experimental & Analytical
4. CONTRACT OR GRANT NUMBER: DAAG29-85-K-0207
5. NAME OF INSTITUTION: Georgia Institute of Technology
6. AUTHORS OF REPORT: Benson H. Tongue, George T. Flowers
7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER ARO SPONSORSHIP DURING THIS REPORTING PERIOD, INCLUDING JOURNAL REFERENCES:
 - 1.) Tongue, B.H. and Flowers, G.T., "Nonlinear Rotorcraft Analysis," accepted for publication in the International Journal of Nonlinear Mechanics, April, 1988.
 - 2.) Flowers, G.T., "Modeling and Analysis of a Nonlinear Rotorcraft," accepted for publication in Applied Mathematical Modeling, 1988.
 - 3.) Tongue, B.H., "On the Global Analysis of Nonlinear Systems Through Interpolated Cell Mapping," published in Physica 28D, No. 3, October 1987, pp. 401-408.
 - 4.) Tongue, B.H. and Gu, K., "Interpolated Cell Mapping of Nonlinear Systems," accepted for publication in the ASME Journal of Applied Mechanics.
 - 5.) Tongue, B.H. and Flowers, G.T., "Ground Resonance - A nonlinear Analysis," presented at the Second Technical Workshop on Dynamics and Aeroelastic Stability Modeling of Rotorcraft Systems, Boca Raton, November 18-20, 1987.
(Continued on Separate Sheet)
8. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT AND DEGREES AWARDED DURING THIS REPORTING PERIOD:

Benson H. Tongue - Principle Investigator
George T. Flowers - Graduate Research Assistant - pursuing Ph.D.
Keqin Gu - Graduate Research Assistant - pursuing Ph.D.

Benson H. Tongue
Department of Mechanical Engineering
Georgia Institute of Technology
Atlanta, GA 30332

7. LIST OF MANUSCRIPTS SUBMITTED OR PUBLISHED UNDER AND SPONSORSHIP DURING THIS REPORTING PERIOD, INCLUDING JOURNAL REFERENCES (Continued)

- 6. Flowers, G.T. and Tongue, B.H., "Linearization Considerations for a Simple Air Resonance Model," presented at the Second Technical Workshop on Dynamics and Aeroelastic Stability Modeling of Rotorcraft Systems, Boca Raton, November 18-20, 1987.**

BRIEF OUTLINE OF RESEARCH FINDINGS

The period of time under discussion was to have involved studies of both ground and air resonance questions, both of which have been addressed. With regard to the ground resonance work, the analytical work has been progressing in an extremely satisfying manner. A study of the effect of varying linear blade damping on the limit cycle behavior of the rotorcraft model has been completed. Further parameter variation studies are currently being concluded. They include varying fuselage relative stiffnesses, fuselage relative inertias, varying blade stiffnesses for the hingeless blade model, and breakaway forces for fuselage damping.

The experimental ground resonance work has also shown excellent results. A study in which the fuselage inertias were varied has been completed and others are planned. They include varying the relative fuselage stiffnesses and the blade inertias. Identification of the system damping parameters, which has given some difficulties in the past, is proceeding and is yielding some interesting insights into the modeling of nonlinear system parameters.

The air resonance work is proceeding well and is currently ahead of schedule. Using symbolic manipulation, the full dynamical equations of motion for an air resonance model were derived and programmed for numerical integration. This model includes simplified linear aerodynamics. A simplified dynamical model has been developed and numerical integration results compared. These results indicate that the qualitative behavior of the system is well captured by the simplified model. At the present time, work is progressing on further simplifying the aerodynamical model so that the system can be analyzed for interesting dynamical behavior in a straightforward and efficient manner while still retaining physical validity.

to the $n\sigma$ case. This task is 90 percent complete and the results will be documented by May 31, 1986. In task (4) methods for translating 1D test levels into equivalent 3D test levels have been formulated and a program has been developed to display the oscillation path and the shaker head path in 3D color plots. The adequacy of the proposed 3D test will be examined using shock spectra and fatigue cycle count equivalence. This task will be completed by June 30, 1986.

22326-EG Ting W. Lee, State University of New York

"Design of Mechanisms with Prescribed Entire-Motion Characteristics"

This research is concerned with the design of mechanisms with prescribed entire-motion characteristics. In particular, it is intended to apply recent advances in kinematic and dynamic theories, coupled with computer-graphics techniques, to develop design methods capable of predicting the performance characteristics of a representative class of realistic spatial mechanisms and mechanical systems.

The research this past year has focused on two difficult problems. The first concerns the dynamic analysis and design optimization of a robotic positioner. A novel device, the Stewart Platform, is investigated for dynamic force analysis and design optimization in order to use it for a transfer mechanism or a robotic positioner. Kinematic analysis including the effects of constraints, such as ball joints limitations and link-length limits, is performed and the theory is developed into a software for computer graphics simulation. Currently, the work has been on the formulation of a dynamic model of the Platform mechanism. It is expected that stability criteria as well as force analysis will be developed. Useful design charts and design guidelines will be offered.

The second task is concerned with the error analysis of manipulators. A three-dimensional mathematical model of error analysis is developed for the open-loop linkage involving cylindrical and revolute joints. The model is suitable for the error analysis of robot manipulators. It uses the dual number (3x3) screw calculus and involves the formulation of a displacement equation explicit in terms of the tolerance of links and the clearance of joints. The heuristic optimization of Lee and Freudenstein is employed to estimate the extreme tolerances and clearances of a manipulator at the design stage. Two industrial robots, the PUMA 500 and SEIKO 3000, are used to illustrate the theory.

22557-EG Benson H. Tongue, Georgia Institute of Technology

"Nonlinear Rotorcraft Analysis Experimental and Analytical"

Ground resonance, a phenomenon that affects helicopters during take off and landing if the parameters of the helicopter's structure are in a specific regime, is manifested by an oscillatory response that quickly grows to levels that can destroy the vehicle. The cause of this behavior is the coupling between the motion of the helicopter's blades and the motion of the helicopter

fuselage. Air resonance refers to a similar phenomenon that occurs to helicopters in the air. In this case the coupling is entirely internal to the helicopter itself and no ground contact need occur.

The objective of this program is to develop analytical models that allow a nonlinear analysis of the ground and air resonance phenomenon. Linear models can and have been developed which indicate the onset of ground and air resonance for systems initially in equilibrium. These models are unable to predict what the finite amplitude response of the helicopter will be and are unable to determine the existence of high amplitude limit cycles that occur at the same rotor speed that produces a stable response about equilibrium positions. Solutions will be sought that indicate the amplitudes and frequencies of finite amplitude oscillatory responses. This will allow a very cost effective way to predict the behavior of rotorcraft for a wide variety of parameter combinations. An experimental program will be pursued in parallel with the analytical work in order to verify the analysis.

The analytical approaches being used are limit cycle analyses and numerical simulations. The limit cycle technique presumes the existence of a finite amplitude oscillation at a specific frequency. A solution of the equations of motion is then sought that satisfies these conditions. This approach is quite accurate as long as the response can be easily characterized by a limited number of harmonic components. In the simplest case, a single harmonic will be sufficient to characterize the motion of the helicopter. In highly nonlinear situations, an assumption of a continuous frequency spectra may be needed. Numerical simulations are used to verify the accuracy of the limit cycle analyses and also to provide information about the response of the fully nonlinear system for situations that are not amenable to limit cycle analysis.

A single harmonic limit cycle analysis has been derived that applies to a model rotorcraft having two in-plane degrees of freedom and an articulated rotor with lagging blades. The analysis gives the amplitude and frequency response of the system for retained nonlinearities to the second order. Theoretical analysis has indicated that, under a harmonic response, the retention of second order structural nonlinearities will not alter the amplitude-frequency characteristics of a first order model with second order damping nonlinearities. This allows a savings in computation time and cost. Results have been attained which indicate the existence of interesting limit cycle behavior for the nonlinear model, such as multiple solutions and hysteresis, which are not predictable from a linear standpoint.

An experimental apparatus has been assembled and data acquisition equipment is being added to the system. Preliminary tests have demonstrated the feasibility of using the model to simulate limit cycle behavior when in the ground resonance regime. Precise correlations with theoretical predictions will be undertaken in the near future.

**NONLINEAR ROTORCRAFT ANALYSIS -
EXPERIMENTAL AND ANALYTICAL**

FINAL REPORT

**AUTHOR
B. H. Tongue**

6 November 1988

U.S. ARMY RESEARCH OFFICE

**CONTRACT 22557-EG
RESEARCH AGREEMENT NO. DAAG29-85-K-0207**

**GEORGIA INSTITUTE OF TECHNOLOGY
SCHOOL OF MECHANICAL ENGINEERING**

**APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED**

THE VIEWS, OPINIONS, AND/OR FINDINGS CONTAINED IN THIS REPORT ARE THOSE OF THE AUTHOR AND SHOULD NOT BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF THE ARMY POSITION, POLICY OR DECISION, UNLESS SO DESIGNATED BY OTHER DOCUMENTATION.

1. PROBLEM STUDIED

A parallel effort involving both analytical and experimental work was performed to allow a deeper understanding of the limit cycle behavior of rotorcraft and of the implication of this behavior for the safe operation of such rotorcraft. The theoretical development encompassed articulated and hingeless rotors with a great enough number of included degrees of freedom to allow accurate predictions of a simplified rotorcraft's responses. The experimental model allowed the simulation of both hingeless and articulated rotors in ground resonance. The purpose of the above work was to identify the regimes of helicopter operation that can support limit cycling and the range of excitations the system can withstand before entering a limit cycle. Both ground and air resonance behaviors were analytically investigated.

2. MAIN RESULTS

The research program proved to be extremely successful. A sophisticated harmonic balance procedure was applied to the fully nonlinear ground resonance model and was shown to capture all of the essential dynamical behavior of the system. This modeling went substantially beyond any of the nonlinear ground resonance analyses that preceded it. Construction of the equations of motion and the subsequent solution of these equations was greatly aided by an extensive use of symbolic manipulation. The results showed that destructive, high amplitude limit cycles could exist, not only in the linearly unstable regime, but also at rotor speeds for which linear analyses would predict purely stable behavior. Thus one would expect that a strong perturbation to the craft, such as a hard landing, could push the system into a high amplitude limit cycle that would lead to the destruction of the aircraft. These predictions were fully verified by the experimental ground resonance rig. The experimental parameters were entered into the generic analytical model and led to responses that were very closely matched by the behavior of the actual model. Thus the findings were shown not to be simply academic exercises, but responses that were physically realizable. Parametric studies were completed to indicate the response of the system under a widely varying set of physical conditions. The generic methodology of the analyses can easily be applied to a variety of rotor problems.

The air resonance study led to a moderately complicated structural model having aerodynamic interactions. The complexity of the nonlinear response made a limit cycle analysis computationally infeasible, thus leading to a use of more traditional numerical integration. The same sorts of behavior that were predicted for the ground resonance model were shown to be possible in the air resonance model, i.e. limit cycling in a linearly stable region of rotor speeds. The effect of various nonlinearities on the response was assessed, with the conclusion that the most significant nonlinear effects were engendered from discrete nonlinear structural elements. This model was also examined for more complicated nonlinear effects than simple harmonic limit cycles. The most notable outcome of this work was the demonstration that under a harmonic excitation (such as occurs with Higher Harmonic Control) the rotor response can become chaotic. It is believed that this is the first time that the existence of chaos has been indicated in a rotorcraft.

3. PUBLICATIONS AND PRESENTATIONS

1. "Construction and Analysis of a Simplified Nonlinear Ground Resonance Model," M.D. Janowski and B.H. Tongue, *Journal of Sound and Vibration*, **122**, No. 2, 22 April 1988, pp. 233-242.
2. "Nonlinear Rotorcraft Analysis," B.H. Tongue and G.T. Flowers, *International Journal of Nonlinear Mechanics*, **23**, No. 3, 1988 pp. 189-204.
3. "Nonlinear Rotorcraft Analysis using Symbolic Manipulation," G. T. Flowers and B. H. Tongue, *Applied Mathematical Modelling*, **12**, April 1988, pp. 154-160.
4. "Interpolated Cell Mapping of Dynamical Systems," B.H. Tongue and K. Gu, *ASME Journal of Applied Mechanics*, **55**, No 2, June 1988, pp. 461-467.
5. "On Obtaining Global Nonlinear System Characteristics Through Interpolated Cell Mapping," *Physica* **28D**, 1987, pp. 401-408.
6. "An Experimental Study of the Effects of Nonlinearities on Ground Resonance," G.T. Flowers and B.H. Tongue, submitted to the *Journal of Guidance, Control and Dynamics*, 1988.
7. "Nonlinear Air Resonance," G.T. Flowers and B.H. Tongue, in preparation.
8. "On the Existence of Chaotic Responses in a Rotating Blade," B.H. Tongue and G.T. Flowers, in preparation.
9. "Nonlinear Rotorcraft Analysis," presented at the AFOSR/ARO Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, VPI&SU, Blacksburg, VA, March 23-25, 1987.
10. "Ground Resonance - A Nonlinear Analysis," presented at the Second Technical Workshop on Dynamics and Aeroelastic Stability Modeling of Rotorcraft Systems, Boca Raton, November 18-20, 1987.
11. "Linearization Considerations for a Simple Air Resonance Model," presented at the Second Technical Workshop on Dynamics and Aeroelastic Stability Modeling of Rotorcraft Systems, Boca Raton, November 18-20, 1987.
12. "Dynamical Behavior of a Nonlinear Air Resonance Model," to appear in the proceedings of the AIAA/ASME/ASCE/AHS/ASC 30th Structures, Structural Dynamics, and Materials Conference, 1989.

4. PERSONNEL

Principal Investigator:

Benson H. Tongue

Graduate Student:

**Michael Janowski, "Analysis of a Simplified Nonlinear
Ground Resonance Model"**

M.S. Thesis, 1986.

Graduate Student:

**George T. Flowers, "A Study of the Effects of Nonlinearities
on the Behavior of Rotorcraft in Ground and Air Resonance"**

Ph.D. Thesis, 1988.

Graduate Student:

**Keqin Gu, "Determining Attractors, Basins of Attraction
and Trajectory Control of Nonlinear Dynamical Systems"**

Ph.D. Thesis, 1988.

Graduate Student:

Philip FitzSimons, Ph.D. expected 1989.