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GEORGIA INSTITUTE OF TECHNOLOGY OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION SHEET

Date 11/1/82

Project Title: Missile System Support Locations Optimization

Project No: E-24-646

Project Director: Dr. L. G. Callahan

Sponsor: Redstone Arsenal

Effective Termination Date: 9/30/82

Clearance of Accounting Charges: 11/30/82

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FORM OCA 10:781

E-24-648

Final Summary Report
for the
United States Army Missile Command
Redstone Arsenal
Huntsville, Alabama 35898

"Graceful Degradation of Air Defense Communications: A Probabilistic Model"

Under Contract DAAH01-81-D-A003
Delivery Order 0036 34

Conducted by

The School of Industrial and Systems Engineering
Georgia Institue of Technology

Leslie G. Callahan, Project Director

James M. Burd, Investigator

September 1982

(unclassified)

SUMMARY

The problems of defining a weapons system and obtaining a measure of effectiveness for the component parts in a gracefully degrading situation are not new. These problems have been addressed many times using many different techniques. In the past the ability to evaluate the effectiveness of the various components of a weapons system was not as critical as the ability to measure the overall effectiveness of the weapons system. However, with our increasingly high technology weapons systems, the contributions of the parts are critical to the effectiveness of the whole and these effectivenesses must be measured as a part of the overall evaluation of the weapons system. This is not a trivial task and becomes even more difficult when applied to a gracefully degrading system.

This research was directed at developing and demonstrating a type methodology which can provide a measure of effectiveness for the communications system embedded in the Patriot Air Defense System. The methodology developed is probabilistic in nature and is based on the fact that communications can be modeled as network processes and these processes possess the Markov Property.

Using the Markov Property, it is possible to develop the steady state probabilities for the communications systems and equate these to the percentages of time the system will occupy these states over a long period of time. The stated model can now be modified and the resulting steady state probabilities can be used for a sensitivity analysis of the model parameters, and consequently a basis for procurement and product improvement decisions.

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CHAPTER I

INTRODUCTION

A. Description of the Problem

Within the military community, it has long been a truism that a combat unit is evaluated based upon its ability to move, shoot, and communicate, treating these three functions as separate, mutually exclusive characteristics which are desireable in the unit. With Today's highly complex, sophisticated military technology and increasingly automated battlefield, the unit which is unable to successfully perform any or all of these tasks simultaneously and to incorporate these functions into an integrated weapons system becomes a battlefield liability rather than an asset, unable to perform in a flexible, responsive manner to the rapidly changing situations generated on the modern, highly dynamic battlefield. It is therefore essential that, in order to more definitively evaluate unit capabilities and performance, the integrated weapons systems of today and the future be evaluated not just on their abilities to perform the three main functions of move, shoot, and communicate, but that a greater understanding be gained of the interaction of these three functions. For example, it is no longer sufficient for an artillery battery commander to be able to move his cannons to a piece of real estate, set up and place his first rounds on target. If he is unable to communicate, he does not know what type of mission he is shooting, what type of round to use, and with the longer range of the cannons of today, he cannot see where his rounds are impacting. This unit has become a liability on the battlefield rather than an asset. The problem then is to develop a measuring system whereby a unit's effectiveness is determined as a combination of what it can do, as well as what it cannot do.

B. Objective of the Project

In this project, the objective is to propose a set of indices or measurement standards whereby one of the functions of an integrated weapons system can be evaluated, analyzed and improved based upon the results of the evaluation.

The weapons system to be examined is the Patriot Air Defense Missile System, a new, all weather surface to air guided missile designed to replace Nike Hercules and Improved Hawk in defeating the hostile air threat through the 1980's and beyond [13]. As with all modern air defense weapons systems, the Patriot fire unit is extremely dependent upon communications to remain an effective element of the integrated air defense system. Because of this dependence upon communications to effectively perform its primary mission of defending point or area targets against hostile aircraft, a set of indices which represent the ability of a Patriot fire unit, battalion or group to communicate as part of its primary mission would then become a measurement standard against which one could evaluate the overall effectiveness of a unit.

C. Scope of the Project

This project will be limited in scope to examining the system peculiar communications of a Patriot Air Defense Missile Battalion and its subordinate firing batteries. Excluded from this discussion is the communications link from the Battalion Operations Center to the Group Operations Center. The factors which will be examined in determining the measurement standards for the unit are only those directly applicable to communications system operations and the associated equipment. To limit the scope of the project to a manageable level, such factors as weather, terrain, state of training and changing missions, although important, will be disregarded to establish a clean environment within which the communications system can be examined. The ability of the communications system to perform its mission as measured against the predetermined standards will be evaluated by establishing a fixed model of the Patriot communications system and subjecting this model to graceful system degradation and analyzing the behavior of the system at each level of degradation. Within the context of this project, Graceful Degradation is defined as the stepwise reduction of a system's ability to perform its specified mission, rather than a sudden catastrophic failure which in one step destroys a system's capability. Rather than perform an exhaustive enumeration of all possible combinations and permutations of graceful degradation, a set scenario will be followed to demonstrate how graceful degradation affects system effectiveness and comments will be made as to means of forestalling these effects, thereby maintaining a higher standard of effectiveness.

CHAPTER II

LITERATURE REVIEW

A. Historical Approaches

The concept of viewing units as weapons systems is not new. In 1955, Project WESCOM examined the problem of defining a weapons system, what it is, how it functions, why it functions and how to improve the manner in which it functions [14]. In examining the problem, the specific area in which Project WESCOM was interested was weapons system communications. The concepts proposed by Project WESCOM were that it was no longer sufficient to view the communications of a weapons system as the means by which commanders talked to one another but rather that the entire weapons system must be decomposed into its various sub-units, each of these performing its own specialized functions. The communications system was then seen as a specialized subsystem, functioning as an integral part of the weapons system. It was then observed that it was no longer adequate for a communications system to merely serve as a faithful transmitter and reproducer of input messages. Rather, the communications system must function as an integrated element of the weapons system, tailored to the specific needs of the weapons system. It then proposed that the effectiveness of the communications system could only be measured in terms of its contributions to the effectiveness of the weapons system.

In 1956, George H. Clement presented a paper entitled "Weapons System Philosophy," in which this concept of viewing units and collections of units as weapons systems was further refined and defined [2]. One strong point which was made was the function of scale when viewing a weapons system. At each level of interest, functions and their relative importance vary dependent upon the scope of the project. For example, to the division commander, the functioning of one specific platoon in a given operation is not a major issue; however, to the commander of the company to which that platoon is organic, the functioning of the platoon is of great importance. The thrust of the paper then is to look at each element of the weapons system and evaluate it as it affects the functioning of the weapons system, that it may not be necessary for a function to be absolutely perfect, that in viewing the function with the weapons system as a backdrop, less than perfect is entirely acceptable. Once again, the efficiency of a sub-system is seen as its contribution to the efficiency of the weapons system as a whole.

In a paper published in 1957 entitled "The Flow and Functions of Communications in Weapons Systems," the actual purposes of communications systems as part of a weapons system were examined in detail. Here were specified in precise definitions the tasks and responsibilities which were placed upon communications systems as integrated into a weapons system [3].

As part of an effort aimed at force modernization, in 1969 a paper was presented entitled "Force Structure Planning: Determination of Micro-Weapon Structure." In this paper was presented the need for developing a methodology whereby a micro-structure of weapons systems based upon the functions of surveillance, fire power, communications and mobility could be expressed. This structure would then be used as a means of evaluating weapons systems performing the same general and specific functions. The results of these evaluations would then be used as a basis for analytical comparison when deciding upon weapons systems for procurement.

It is therefore seen that there has long been recognized the need for a means whereby a weapons system could be decomposed into its component parts or functions and that these component parts then be evaluated not just as separate entities but by their contributions to the weapons system as a whole.

B. Modeling Communications

To the systems analyst, when a system must be analyzed, if possible, the system should be viewed by a mathematical representation or model of the system. By developing a mathematical model of the system, the analyst is often better able to see the system in its purest terms, without any fog or confusion introduced by extraneous or incorrect factors and data. In an effort to provide a model for the communications system, the model which can be most readily seen as analogous is the network. The concept of modeling integrated systems as networks has received considerable attention. This is because systems which can be modeled as networks possess a special mathematical structure which make them easier to understand and solve. The study of network flow problems has produced numerous solution techniques and there exist a wide variety of algorithms which are available as computer software packages to solve network problems.

Basically a network flow problem consists of a set of activities or nodes with a system of interconnecting arcs or links[1] The network flow problem then becomes a question of solving for commodities (such as information or material) which are transferred on the arcs from node to node. The nodes can be any activity such as warehouses, data processing centers or other centers of activity. From the above very simplistic definition, it can readily be seen that a communications system can be modeled as a network. Of the many algorithms available for solving the network flow problem, some are too restrictive in their application for solving the communications, air defense network problems. Examples of these constrained algorithms are the Assignment Problem, the Transportation Algorithm and the Transshipment Problem. Each of these algorithms is a very powerful tool in solving network flow problems, but they are restricted to solving problems of a very special structure and hence do not extend to the generalized network problem.

Another algorithm of interest in solving the communications network problem is the construction of a Minimum Spanning Tree (MST). A MST is a set of arcs generated to connect a given node to a set of nodes using the minimum resource on the arcs, be that resource time, dollars, or any other quantifiable measure. This is readily adaptable to the air defense communications network by viewing the Battalion Operations Center (BOC) as the starting node, the batteries as the other nodes in the node set and the communications data links as the connecting arcs in the system. This algorithm is useful because the MST can be continuously updated as the system is gracefully degraded, thereby showing how the system can be reconfigured to meet mission requirements and further indicating first occurrence of failure to obtain a MST, therein showing failure of the communications system to accomplish its mission. This MST problem expands to a Maximal Flow Problem (MFP) by including all possible spanning trees to the problem, thereby generating all possible arcs over which a flow may pass. In a research paper entitled "Graceful Degradation of Air Defense Capabilities" by James Lovell in 1982, it was demonstrated that the effects of graceful degradation upon an air defense network can be measured in terms of costs assigned to target allocations for a gracefully degrading network. As part of the cost function, the probability of communication was included as an integral part, with costs being inversely proportional to the probability of successful communications.

Each of the network algorithms possesses the advantages of ease of application when modeling a communications network, as well as speed and ease of computation using the efficient algorithms coded on a high-speed computer. Unfortunately, none of these techniques are able to provide a measure of efficiency for the Patriot communications systems, due to some system characteristics which are further discussed in Chapter Three.

Therefore, it is necessary to look beyond these standard network representations of communications systems to find a model which is capable of providing a general measure of effectiveness for a communications network.

Because a system's effectiveness cannot be considered as deterministic, where all required data is known and all actions are perfectly predictable, the system should be evaluated based upon the probabilities of specified results using the probabilities of certain actions occurring. This viewpoint, when modeling a communications system, brings about the consideration of probabilistic or stochastic models. Of the numerous stochastic models available, many have as their basis the application of the Markov Property. This Markov Property, as a general statement, is that the future state of a system is dependent only upon the present state, not the past. Therefore, the Markov Property is also known as the Memoryless Property [15]. In a dissertation written by Francis A. Gay entitled "Performance Modelling for Gracefully Degrading Systems," Dr. Gay addressed the issue of evaluating the performance of gracefully degrading systems based upon Markov Processes [4]. The gracefully degrading systems modeled by Dr. Gay are those which are designed to provide a high grade of service by reconfiguring the system and/or reallocating resources when a failure occurs. Two models based upon Markov processes were developed. The one model was a "capacity" model, in which the effect of failures was considered as to the system's capabilities or capacities. The other model was the "workload" model, which considers the system's ability to satisfy its computational demands. The outputs of these models were the proportion of time a system spends in various degraded modes of capacity or the amount of throughput or work performed by a system in degraded modes. This dissertation also demonstrated that the models produced identical results when the systems were modeled as discrete time Markov processes or continuous time Markov processes.

As was stated earlier, there exist many types of models available to represent the communications systems, from deterministic models such as networks to stochastic models, such as the Markov process. It therefore falls to the modeler to determine which model to use or if a new model is to be developed.

C. Communications Survivability

Once it is known how well a communications network works, one of the goals of a model is to take this evaluation and utilize the results to design communications systems which are superior to those which already exist. To do this, it is not sufficient just to know how a system works, it is also necessary to know what actions or events will cause a system to cease working. Those factors which affect a communications network survivability are referred to as system vulnerabilities.

In 1967, Dr. Howard Frank presented a paper entitled "Vulnerability of Communication Networks," in which he explored some of the factors which affect communication survivability [6]. In presenting these factors, Dr. Frank further discussed the difficulties inherent in making quantitative rather than qualitative judgements on system survivability due to the problems in developing exact mathematical representations of the actions and interactions of all the factors which impact on system performance. Dr. Frank demonstrated how the concept of modeling survivability of relatively simple networks is feasible but that extending these concepts to complex systems becomes extremely difficult due to the combinatorics involved. When a system becomes complex, with a high degree of interconnectivity, the sheer number of possible combinations which must be enumerated and investigated becomes enormous. For this reason, true definitive work in representing system survivability is still limited to relatively simple networks.

As a follow-on, Dr. Frank, along with Dr. Ivan Frisch, presented a paper in 1970 entitled "Analysis and Design of Survivable Networks," in which the issue of analyzing networks was specifically addressed [5]. Of particular interest in this paper was the discussion of the problems involved in analyzing networks due to the division of network survivability into nearly disjoint areas: deterministic survivability and probabilistic survivability. In the deterministic case, the attacker is assumed to have complete knowledge about the system being attacked and also uses a deterministic attack strategy. In the probabilistic problem, the adversary may have only partial knowledge about the network or he may employ a randomized attack strategy.

Because attack and network models are operated separately, typical survival criteria for a network can be classified as either deterministic or probabilistic. A network is considered to "survive" an attack if 1) all points can communicate with one another; 2) there are flow paths between specified pairs of points; 3) the number of points in the largest connected section exceeds a specified threshold; or 4) the shortest surviving path between each pair of points is no longer than a specified length. In conducting a deterministic analysis, the objective might be to determine if these criteria are met subject to a known attack. When conducting a probabilistic analysis, the objective might well be determining the probabilities any or all of these criteria will be satisfied.

When the problem involves modeling and analysis of networks in a probabilistic manner, many difficulties are encountered which are not present in deterministic models. Among these are the problems caused by correlation of events, which will effect the expected probabilities of joint events. Another problem is caused when branch or node failures have different probabilities. Because of the combinatorics involved in analyzing these multinomial distributions, the exact computational solution becomes almost impossible to obtain, with the degree of difficulty increasing proportional to the complexity and size of the network. Therefore, when analyzing networks as a probabilistic model, it is usual to assume equal probabilities on the arcs or nodes and treat the random variables as following the binomial distribution. This then simplifies the problem to the point where a solution is obtainable.

In addition to the theoretics presented by Dr. Frank and Dr. Frisch, they also present the concept of using simulations, in particular Monte Carlo Simulations utilizing modern, high-speed computers, to provide repeated runs of the models under investigation utilizing varying degrees of graceful degradation and attack strategies to observe how the system behaves. These results can then be analyzed to obtain an average or expected result of the system.

This idea of applying simulation to network analysis when investigating system survivability was used by Dr. Edith Martin in her dissertation entitled "Operational Survivability in Gracefully Degrading Distributed Processing Systems" [9]. In this work Dr. Martin investigated which factors were present in a model designed to explain the manner in which a distributed processing system functioned as a gracefully degrading network. To do this, Dr. Martin, applying the principles of experimental design, developed an experiment in which sufficient samples of experimental results were obtained to enable factors to be isolated and analyzed. A complex computer simulation of a network representation was then run to obtain the sample cases and then these samples were analyzed using Linear Regression Analysis, a basic analytical tool in operations research. From this analysis, using an initial set of 32 candidate regressors, a model was developed, composed of 23 variables, which functioned best overall as both an explanatory and prediction model. In examining the correlation matrix developed for the model, it is obvious that there is a high degree of multicollinearity in the model. This is not surprising when considering the number of variables present in the model and the variables description. However, as the model appears to explain and predict the behavior of the system, the multicollinearity is not a problem and need not be unduely emphasized when discussing the results of the work.

From this work, Dr. Martin proposed ten results or propositions which serve to explain why some networks survive and gracefully degrade while other networks quickly approach catastrophic failure. Among these results are several which seem applicable when discussing the Patriot communications system. The first of these hypotheses to be considered is that the more nodes in the distributed network, the more likely that performance is satisfactory. In other words, the more nodes available, the more resources available to perform the mission and if one or more are lost, the remainder will continue to perform the mission.

The second of the hypotheses to be considered is that as communications requirements approach total system capabilities, the likelihood of satisfactory performance decreases.

A third hypothesis to consider is that the higher the distributed network connectivity, the greater the probability the network will survive. This basically says that greater connectivity permits more alternative routes as primary arcs are lost.

Another hypothesis of interest is that failure to properly assign the application system to the distributed network initially makes satisfactory gracefully degrading performance difficult. This states that the system must be designed and tailored to meet its needs and not applied blindly. A system which is established after considering the requriements to be placed upon the system will be more efficient than any system not so established.

The final hypothesis considered concerns the higher the level of application system connectivity the poorer the prospects for satisfactory performance. What this states is that as nodes and arcs become more specialized, the possibilities for survival are reduced.

The works just presented represent only a small portion of the research which has been done in the areas of modeling and analyzing communications networks. However, these are sufficient to introduce the concepts which will be utilized as the basis upon which this paper is built.

CHAPTER III

BACKGROUND DISCUSSION

A. Communications Needs

As was indicated previously, the Patriot Air Defense Missile System is extremely dependent upon communications for operational effectiveness. This dependence is in both the area of command and control and the area of operational readiness [12].

1. Operational Readiness

Because the Patriot missile is a command launched, command guided missile during all but the final few seconds of its flight, the Patriot firing battery can be rated fully operationally ready only if all of its system peculiar communications equipment is properly functioning. If there is any loss of communications capability, the system is degraded, to what level is dependent upon which communications circuits are nonoperational. Details as to which circuits perform specific functions are presented later in this chapter.

2. Command and Control

Current U.S. Army tactical doctrine proposes that in air defense operations, control of air defense fires be retained at the highest level possible, consistent with the current tactical situation, while execution of the air defense mission be delegated to the lowest possible level, consistent with command capabilities [4]. This doctrine of centralized control, decentralized execution places enormous demands upon the air defense communications system. The communications system must maintain both voice and data link capability utilizing radios only to enable controlling headquarters to disseminate states and stages of alert, air defense warnings, weapons control orders and the many other standard operating procedures required to control air defense weapons systems in defending area and point targets while precluding the engagement of friendly aircraft. In addition, these communications systems are utilized by the firing units to report their current status and, when engaging allocated targets, to insure proper target correlation with the assigning headquarters prior to launching a missile. As can readily be seen, due to the dependence on communications for command and control, as communications circuits are lost, efficiency is reduced. If radio data links are non-operational, some information and all automatic target allocation capabilities are lost, thereby degrading the capabilities of the system. If some of the voice circuits are lost, system capabilities are further degraded, requiring more and greater adjustments to continue the mission. When all communications with higher headquarters and adjacent units are lost, the unit is forced to operate autonomously, the most inefficient manner of operation for an air defense unit. That is because the unit is forced to function alone, rather than as an element of an integrated system, receiving and passing orders and information, defending and being defended. As can be seen, as the communications of a Patriot fire unit with its higher headquarters and adjacent units are gracefully degraded, the unit operates less efficiently.

B. Patriot System Communications Equipment

1. Battery Level Communications

Within the firing battery, system peculiar communications is of two types. Hard wire is used to provide data link communications between the Equipment Control Station (ECS) and the multifunction phased array radar. It is over this hard wire communications circuit that commands are given to the radar, missile alerts are transmitted and Track via Missile (TVM) guidance commands are relayed to in-flight missiles. This hard wire circuit also serves as the information data link for targets which are being tracked between the radar and the Weapon Control Computer (WCC) in the ECS. The cables which carry the data are extremely durable, cannot be jammed by electronic counter measures (ECM) and cannot be monitored to divulge intelligence to the enemy. They therefore provide a reliable and secure means of communication for the Fire Control Section (FCS). There are two drawbacks to the use of cables for the sole means of data links. The first disadvantage is that equipment dispersal is limited by cable length (itself limited by technical considerations such as signal loss). The second disadvantage is that due to the size, weight, and cost of cables, it is not practical for a unit to carry spare cables with it, thereby making it possible for a unit to be rendered nonoperational should the cables be damaged, lost or destroyed.

The second type of Patriot peculiar communications system used within the firing battery is the Data Link Terminal (DLT), a VHF radio link between the ECS and the Launching Stations (LS). This DLT, which exists from the ECS to each of the LS (normally 9) in the firing battery, is used to transmit launcher data, missile prelaunch data and the firing data between the WCC and the LS for each missile on the launcher. Because more than one LS is available, so long as the VHF radio in the ECS is operational, there can be a graceful degradation of system capabilities due to loss of a LS(s) or loss of communications to an LS(s). Because data transmission is via radio positioning requirements are that line of sight with the DLT antenna on the ECS must be maintained but much greater freedom so far as dispersal of equipment is possible, as this is not governed by cable length. Normal considerations for displacement of LS from the ECS is 90 to 1000 meters. However, along with the greater flexibility of positioning provided by radio arise some disadvantages. Because the data link is via radio, the possibility of jamming must be considered. Although this is slight, it is an area to be considered. Of more serious concern is that with the radar in the FCS and all the radio communications being used, the battery presents an extremely large radio frequency (RF) signature which can be used by the enemy to pinpoint its location for avoidance, suppression or distruction.

In addition to the system peculiar communications equipment discussed previously, the Patriot battery has organic to it numerous radio and wire communications assets to provide internal command and control of unit operations, to include the organic stinger teams. However, none of this equipment is capable of substituting for the system peculiar equipment required for the air defense mission. See Appendix 1 for Battery level systems communication requirements and configurations.

2. Battery-Battalion Intercommunications Equipment

The sole type of communications available for battery-battalion intercommunications under anticipated tactical deployment is radio. Although under special circumstances such as fixed emplacement or limited deployment, wire could be used for communications, under typical deployment Patriot mobility and dispersion preclude the use of wire due to time and distance constraints. Therefore, wire will not be considered when analyzing battery-battalion intercommunications. The systems peculiar communications euqipment which provides battery-battalion command and control communications circuits, both voice and data link, are located in the Information Coordination Central (ICC) at battalion level and ECS at battery level. In addition to the ICC and the ECS's, there are two other major end items which are integral to the battery-battalion communication's network. These are the Communications Relay Set (CRS), of which each battalion has four, and the Antenna Mast Set (AMS), one of which is associated with each ICC, ECS, and CRS in the system. From this point forward, whenever an ICC, CRS, or ECS is mentioned, that reference will include the AMS.

The battalion communications network is designed to provide reliable, flexible communications, easily and quickly adapting to any tactical situation. The major feature of the battalion communications system which promotes this reliability is the multi-routing capability. Each ECS and ICC contain three UHF radio sets, while each CRS has four UHF radio sets. This gives each of these major end items the capability to receive, transmit and retransmit any message on all of their organic UHF radios. This provides the battalion communications network with the capability of forming many possible radio links, each receiving, transmitting and automatically retransmitting any message received. All messages which are received are checked for content and then automatically retransmitted. If the message originated at the receiving unit is addressed to the receiving unit or was previously received, it is not retransmitted. This automatic retransmission of messages over all existing links in the communications network provides several paths over which a unit may receive a message, thereby increasing the probability of the unit receiving the message. On each AMS are located four parabolic directional antennas. For the ICC and the ECS's only three of these antennas are in use, for the CRS's, all are in use. Each of these antennas is remotely controlled from inside the item of equipment with which it is associated and is operated in conjunction with just one UHF radio, communicating with just one particular unit. This feature of directional transmission, combined with multi-routing certain technical aspects of the message format and secure encryption transmission are efforts to reduce the effects of electronic warfare, both interception and interference, from disrupting the Patriot battalion communications network. As stated previously, the heavy dependence upon radios for communication, combined with the radar emissions, provides a large radio frequency signature for the units, exposing them to overt and covert enemy action. This danger is further increased by the dispersal limitation of the AMS from the communications shelters imposed by cable lengths. To provide a backup capability to the AMS, the ICC and ECS's have attached to the communications shelters corner reflector antennas which are useful for short distance transmissions. In addition to other placement considerations when deploying the battalion, line of sight must be maintained from antenna to antenna. One of the prime functions of the CRS's is to act as relay stations when line of sight cannot be maintained between ICC and ECS's.

As a backup to the system peculiar communications equipment, additional voice radio circuits are available utilizing both AM and FM radios. Although there exists no backup data link capability, these backup voice circuits would enable the dissemination of state and stages of alert, weapons control orders and other necessary command and control information. The problems introduced by using these circuits are that all target allocations must be made manually, which is slow, inefficient, and sometimes inaccurate. In addition, this places a heavier volume of traffic on these circuits, which are routinely used for administrative and tactical information transmission. See Appendix 2 for a typical battalion communications network configuration.

C. Battalion-Group/Adjacent Battalion Communications

As the Patriot system is currently configured, only the ICC has the capability of establishing UHF voice and data link communications with Group and adjacent battalions. This is because only the ICC has the requisite equipment to convert messages received from outside the battalion into data which is usable by the Patriot system. There is therefore a sole entry/exit point for system-usable communications such as target allocations. The CRS's are capable of retransmitting the messages between the ICC and Group/adjacent battalions, but only that, the messages cannot be converted into usable format at the CRS's.

Should the systems peculiar communications link from ICC to group fail, there are of course AM and FM radio circuits available to continue communications, but these are limited, as stated previously for the battery-battalion communications.

Being aware that this situation exists, when discussing communications within the battalion, it will be assumed that communications as needed exist between the ICC and group/adjacent battalion. This is because should the ICC become nonoperational, there is no longer a need for a battalion level model of Patriot systems communications.

CHAPTER IV

MODEL DEVELOPMENT

In selecting a model for use in developing a measure of effectiveness for the Patriot communications system, the features of the models presented in Chapter II must be considered. As stated previously, the two most commonly used models are networks and Markov Processes. Each of these modeling techniques has inherent advantages and disadvantages. The major advantages to network models are that there are numerous solution algorithms available and that the models and the solutions have considerable intuitive appeal. It is simple to view a communications system as a network and a solution which provides specified quantities on arcs and nodes is easily grasped. Unfortunately, the major disadvantage of a network model is that it becomes difficult to relate the solution to a measure of efficiency as adapting a scale rating to flow quantities is not an easy task, especially in a continuously degrading situation.

Conversely, the major advantage to a Markov Process model is that the solution is readily adaptable to a fixed scale evaluation scheme as the states of the process can readily function as the scale. The major disadvantages to using a Markov Process model are in defining the states of the process and determining the values of the probabilities to be used. Also, because the Markov Process is an entirely probabilistic model, the difficulties inherent to probabilistic models as discussed in Chapter II are encountered.

Because, as Dr. Gay demonstrated in his dissertation, a model based on Markov Processes illustrates the capability of a system or in a probabilistic sense the capacity of a system, which is a measure of its effectiveness, the Markov Process model will be used in this paper.

A. Markov Processes

Before delineating the specifics of the model as applied to the Patriot communications system, the general properties of the Markov Process to be used will be explained. The particular Markov Process to be used in the model is the Markov Chain (MC). An MC can be well-defined in two cases, the discrete case and the continuous case. The difference in the two cases is that of occurrence of transitions or changes to the system. In the discrete case, transitions occur only at discrete, defined time intervals while for the continuous case, transitions occur continuously, the probabilities of transitions being defined as a Poisson Process, governed by the Poisson Distribution. For this paper, the system will be modeled as a discrete time Markov Chain. Although the requirement that transitions occur at discrete time intervals may seem too restrictive, Dr. Gay demonstrated in his dissertation that, when considering long-run probabilities, for gracefully degrading systems, discrete and continuous MC's are equivalent.

There are several advantages to using discrete time MC's. The first is that there is no need to demonstrate that the failures occurring in the gracefully degrading system are Poisson Processes. Although this could be done, by not using failure probabilities governed by a particular probability distribution the results will be of a nonparametric nature and will be more general in their application. Also, because a discrete MC is being used, the MC contains a special structure which simplifies the calculations in obtaining the long-run probabilities.

The special structure which offers ease of solution is that, by limiting the possible states into which the system can transition to a finite number and having the MC meet several class properties, the long-run or steady state probabilities can be solved for as a set of simultaneous linear equations.

These special properties which the system must meet are that, in addition to a finite number of states, the MC must be irreducible, positive recurrent, and aperiodic. The requirement for irreducibility means that any state must be accessible from any other state. Positive recurrence merely states that the system must be capable of transitioning from any state back to that state in less than an infinite number of transitions. The requirement for aperiodic states that within the MC there is no underlying structure which prevents the system from being in any state after a sufficiently long period of time. That is to say, there is no cycle present in the transition probabilities which prevents the system from transitioning from one state to a given state in a specified number of transitions. If these requirements are met, then the long-run or steady state probabilities for the MC can be obtained. These steady state probabilities define the probability the system is in a particular state after a long period of time or these steady state probabilities can be viewed as the proportion of time a system is in a given state over a long period of time.

The typical representation of a Markov Chain for discrete time processes is that of a square matrix. This square matrix is commonly referred to as the one-step transition matrix. That is because the entries within the matrix represent the probability of a transition from the state as indicated by the row label to the state as indicated by the column label in one step or one transition only. This transition matrix has several properties which it must satisfy. All entries must be greater than or equal to zero (0) but less than or equal to one (1). The sum of the probabilities for each row must equal one (1). If these requirements are not met, the matrix is not stochastic in that the row values would fail to completely sepcify the probability distribution.

From the transition matrix the steady state probabilities are calculated. Because the transient behavior of the MC is of little interest, in that intermediate transient probabilities and rate of convergence to steady state are of no real importance to the model, the steady state probabilities are calculated directly by solving a set of simultaneous linear equations. Once the steady state probabilities have been calculated and their impact analyzed, the system can then be modified in an effort to shift the steady state probabilities as needed to achieve more desireable results. This is done by modifying the underlying structure of the system that determines the transition matrix.

B. Model Definition

As was stated previously, when utilizing a Markov Process to model a network, by proper selection and definition of the possible states of the process, the steady state probabilities represent the proportion of time the system spends in the states over a long period of time. These steady state probabilities can then be viewed as the measures of effectiveness for the system. In this paper, as the communications for the battery and for the battery-battalion are being modeled, two different sets of states must be defined, one for each model.

1. Battery Communications

In modeling the battery level system peculiar communications, it becomes necessary to differentiate between radio and hard wire communications. As stated previously, the hard wire communications are used in the fire control area. The unit maintains no spares on hand for these cables and each cable must be fully operational for the system to be operational. As there is no graceful degradation possible for this portion of the communications system, there is no real need to develop a stochastic model to represent its effectiveness.

This portion of the communications system either works or it doesn't work, with probability as developed from reliability data.

The radio communications, used to provide pre-flight and firing data for the missiles on the launching stations, is a system which can be gracefully degraded and as such is of interest to this paper. This model, because the communications network it represents is extremely basic, is a very simple model to develop and understand. As an introduction to the technique being employed, this is beneficial in that using the model in a simple case illustrates the use of the model in a context which is easily understood. To apply the model, the transition matrix must be derived. To do this, the states of the system must be defined so that probabilities can be assigned in the transition matrix. As was stated previously, by properly defining the states of the system, the states and their associated steady state probabilities can represent the efficiency or effectiveness of the system. The states which identify the capabilities of the system are defined by the user of the model and have meaning only when applied to the definitions. For this model, the states of the system are defined as follows:

State 1: The system is 85% or more operational. This means that communications are operational to seven (7) or eight (8) launchers (for a standard battery).

State 2: The system is 60% or more operational but less than 85%. This means that communications are operational to five (5) or six (6) launchers.

State 3: The system is 25% or more operational but less than 60%. This means that communications exist to two (2), three (3), or four (4) launchers.

State 4: The system is less than 25% operational. This means that communications either do not exist or exist to only one (1) launcher.

Admittedly, the definition of these states is arbitrary on the part of the modeler, but they serve to give a fixed scale value to the capacity or efficiency of the battery. Any disagreement as to definitions for the states could be resolved and the transition matrix generated from the definitions modified accordingly. A sample case using this model and one variation to this model are presented in Chapter V.

2. Battery-Battalion Communications

In defining the model to be used for Battery-Battalion Communications, care must be taken to ensure the model is not overly simplified, as this would provide an unrealistic appraisal of the true nature of system while a model which is too complex would be of little use as the solution would be difficult to obtain and understand. For these reasons, it becomes essential that the model be developed to meet the needs of the user, rather than trying to develop a genralized model which can be used to obtain any desired information. Once again, due to the complexity of the problem, the probabilities must be treated as binomially distributed. For the battery model, this was not unreasonable as unit deployment considerations are such that there should not be that much variation in probabilities of communications from the ECS to any particular LS. When considering the Battery-Battalion model, this assumption becomes suspect as significant differences in range, terrain, weather, and the multitude of other factors which affect the probabilities of communications are not considered. However, it can be argued that of the factors which have been omitted, some would tend to cancel out others so the assumption of a binomial distribution may not be that bad after all.

For the model to be used for the Battery-Battalion Communications System, the states of the system are defined as follows:

State 1: The system is defined as being in state 1 when every battery has all three UHF radios successfully sending and receiving radio transmissions.

State 2: The system is defined as being in state 2 when every battery has at least two UHF radios successfully sending and receiving radio transmissions.

State 3: The system is defined as being in state 3 when every battery has at least one UHF radio successfully sending and receiving radio transmissions.

State 4: The system is defined as being in state 4 when at least one battery has no radios sending or receiving successfully. This means that at least one battery has been forced to operate autonomously due to lack of communications with the battalion or adjacent firing batteries.

As was stated for the Battery model, these definitions of states are arbitrary on the part of the modeler and can be changed to meet the purpose of the study being initiated. However, they do serve the intended purpose in this model of providing a fixed scale against which the ability of the system to perform its assigned mission can be evaluated. A sample case using the model and one variation to this model are presented in Chapter V.

CHAPTER V

MODEL IMPLEMENTATION

In demonstrating how the models of the communications systems would be implemented, it is necessary to use fictional probability data for radio transmission capabilities. This is brought about by the security implications of using real life reliability data in conjunction with a model which represents system capabilities. Therefore, the probabilities to be used in this chapter are for demonstration only and do not represent actual data. To use the model for actual data, all that is required is to substitute actual probabilities for the fictional probabilities used in the samples presented below.

A. Battery Model

As previously stated, the battery model consists of defining the states of the transition matrix for the Markov Chain in terms of the number of operational communications links to the battery's launching stations. For the purpose of illustration, the probability of a transmission being successfully completed between the ECS and the LS is .7, therefore the probability of an unsuccessful transmission between the ECS and teh LS is .3. For the initial case being presented, no repair is possible, therefore, there is no opportunity for reconstitution of the battery after losses are suffered. In Appendix 3, the tree diagrams are shown illustrating the calculations for the various probabilities which are used in the transition matrix. The transition matrix and the sets of simultaneous linear equations are also shown. Finally, the solution to the system is shown. Here, the solution is that when the system reaches steady state after a sufficiently long period of time, the probability that the system will be in states 1, 2, or 3 is zero, while the probability that the system will be in state 4 is 1. These results are no surprise as it is intuitive that any system which is incapable of being repaired, with a positive probability of failure, will eventually fail, becoming totally nonoperational.

As a variation on this and to demonstrate one of the uses of the model, using the same fictional probability data, the transition matrix for the system is calculated with repair for each link possible during a transition step. In defining the probabilities for the repair per transition, it is assumed that the probability of a nonoperational communications circuit becoming operational is .6, while the probability of a nonoperational circuit remaining nonoperational is .4. The transition probabilities are calculated as shown in the tree diagrams in Appendix 4. In addition, Appendix 4 also shows the system of simultaneous linear equations and the steady state probabilities which result from solving these equations. As opposed to the steady state probabilities for the no repair possible model, the repair possible model demonstrates a shift in steady state probabilities from a guaranteed failure to positive probabilities for remaining in states which correspond to higher states of unit capability.

B. Battery-Battalion Model

As stated in Chapter IV, the Battery-Battalion Model consists of defining the states of the transition matrix for the Markov Chain in terms of the number of operational communications links for each battery in the battalion. For the purpose of illustration, the probability of a communications link being operational for the batteries is .7, while the probability of the communications link being nonoperational is .3. For the initial case being presented, it is assumed that there is no possibility of repair or reallocation of assets within the system. Once a communications link becomes nonoperational, it remains nonoperational, precluding any attempts to reconstitute the communications network after losses of capability are suffered. In Appendix 5 the tree diagrams are shown illustrating the calculations to obtain the various probabilities which are placed in the one step transition matrix. The one step transition matrix and the set of simultaneous linear equations from which the steady state probabilities are solved for are also shown. Solving this set of simultaneous linear equations by any of the numerous techniques available yields the solution as shown.

As was the case for the Battery Model, when no repairs are permitted, the steady state probabilities for states 1, 2, and 3 are equal to zero while the steady state probability for state 4 is equal to 1. This means that in a system where the component parts have a positive probability of failure but there is no probability of repair or reconstitution, the system must ultimately fail with probability equal to 1.

As a variation to the Battery-Battalion Model presented, using the same fictional probability data, the system will be modeled where repair of nonoperational communications links are permitted in one transition. The probability of a nonoperational link becoming operational is .6 with the probability of a nonoperational link remaining nonoperational is .4. The transition probabilities are calculated as shown in the tree diagrams in Appendix 6. In addition, Appendix 6 also shows the system of simultaneous linear equations and the steady state probabilities which result form solving these equations.

As shown, when the steady state probabilities are solved for, the system is now capable of remaining in states other than guaranteed failure. This shift in steady state probabilities is a means by which the effect of repairability on a system can be measured.

CHAPTER VI

RESULTS AND CONCLUSIONS

In the material presented herein, it has been shown that it is entirely possible to model air defense communications systems as a successively degrading system. Through the use of a probabilistic model based upon Markov Processes, the ability of this system to perform its assigned functions within the context of the weapons system as a whole can be measured. This ability of the model to provide defined measurement data as to the system's capabilities makes it possible to quantify gracefully degrading systems. By determining the steady state probabilities of the system as defined by the states of the system, it is possible to identify the state or level of capability at which the system is currently operating and furthermore, to determine what proportion of time the system will operate at this state.

In the variations to the two models presented, it was demonstrated that by varying only one system characteristic, the whole nature of the system could be transformed. In the examples presented, by permitting the nonoperational communications links to be repairable with positive probability in one transition, the models were transformed from guaranteed failure models to systems where positive probabilities existed for operating at higher states of the model. It is of additional interest that the model indicated that in excess of 75% of the time, the system would operate as a degraded system.

Having now shown that the system can indeed be modeled probabilistically and measures of effectiveness assigned to the model to quantify system degradations, the question now is, "What else can be done?"

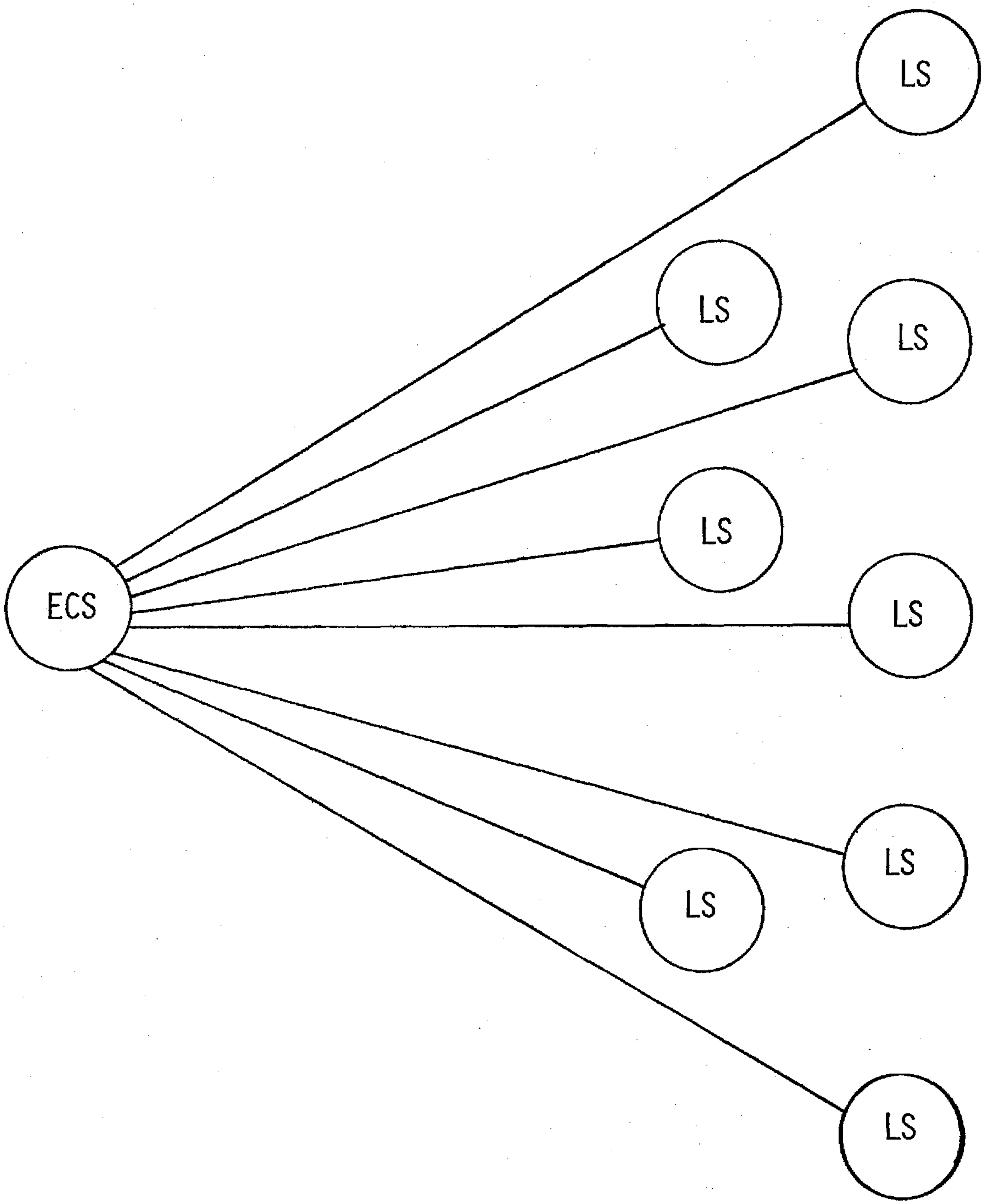
The answer to this question is that there is now a whole new area of interest available for investigation. Now that a mathematical model which can provide usable measurement data is available, the obvious next step is to use this model to perform sensitivity analysis upon the system. An example of this was given when the system was permitted the probability of repair in one transition. Through sensitivity analysis, it is possible to study the effects upon the system of changing any of the myriad of parameters which affect system reliability, maintainability, etc. This provides the opportunity to obtain a definitive measure of the value of various attributes of the system. These measurements are of value in making cost effectiveness studies for new systems procurement or for product improvement programs. By insuring the maximum benefit is realized for the resources committed, time and money can be allocated where the greatest benefits will be realized.

Another use for the measurement system developed as part of the model is in determining priorities for resources when reconstituting a unit after damage or destruction of assets due to enemy attack or other causes. By defining the states of the system to enable the modeler to obtain a measurement of the attributes of interest, a quick sensitivity analysis can be performed providing to the commander data as to how best to utilize available assets to insure this gracefully degrading system operates at as high a state as possible.

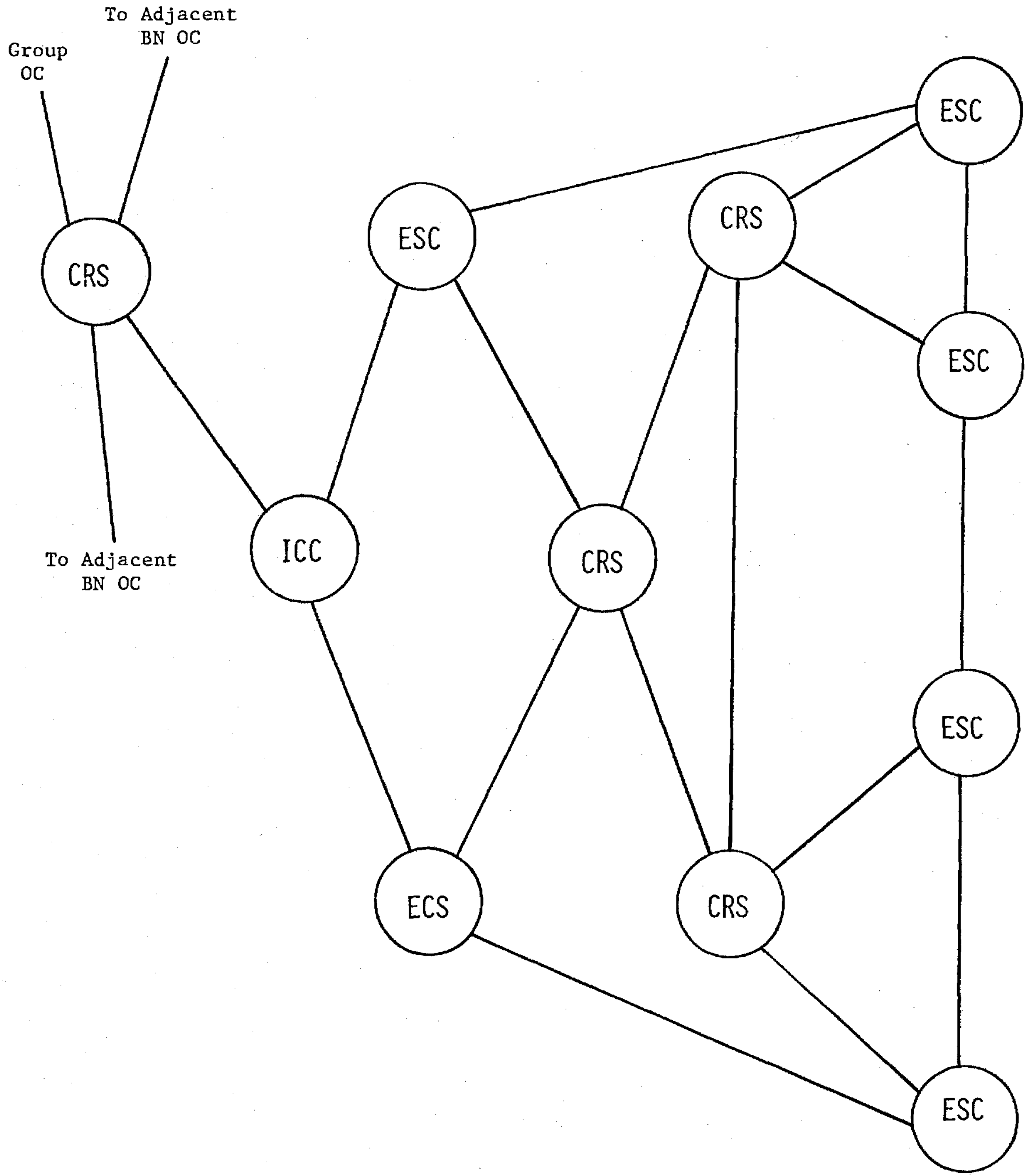
This gives rise to a very powerful feature of the Markov model. The results obtained from the model are dependent upon the definitions of the states of the model as determined by the modeler. The model is flexible in that the modeler is able to supply his own definitions for the states to obtain information concerning the parameters of interest to him. This permits the model to be used in making an exhaustive study of all aspects of the system while retaining the simplicity of being easily understood and the results readily available for application.

In conclusion, the Markov model provides another tool to the analyst, and potentially to the user, to enable him to make a better decision based on more available, accurate data. And that, after all, is the purpose of a model.

Appendix 1: Battery Communications Network



Appendix 2: Battery-Battalion Communications Network



Appendix 3: Battery Model, No Repair Authorized

A - Initial State
B - Probability of Having a Number of Nodes Operational Given the Initial State
C - Number of Nodes Operational
D - Probabilities of One-Step Transition from Initial Number of Operational Nodes
E - Number of Nodes Operational After One-Step Transition
F - State After One-Step Transition
G - Probability of Being in State After One-Step Transition

A	B	C	D	E	F	G
4	.3	0	1	0	4	.3
	.7	1	.3	0	4	.21
			.7	1	4	.49
3	.09	2	.09	0	4	.0081
			.42	1	4	.0378
			.49	2	3	.0441
	.42	3	.027	0	4	.01134
			.189	1	4	.07938
			.441	2	3	.18522
			.343	3	3	.14406
	.49	4	.0081	0	4	.003969
			.0756	1	4	.037044
			.2646	2	3	.129654
			.4116	3	3	.201684
			.2401	4	3	.117649

Appendix 3 (continued)

A	B	C	D	E	F	G
2	.3	5	.0024	0	4	.00072
			.0284	1	4	.00852
			.1323	2	3	.03969
			.3087	3	3	.09261
			.3602	4	3	.10806
			.1681	5	2	.05043
	.7	6	.0007	0	4	.00049
			.0102	1	4	.00714
			.0595	2	3	.04165
			.1852	3	3	.12964
			.3241	4	3	.22687
			.3025	5	2	.21175
			.1176	6	2	.08232
1	.3	7	.0002	0	4	.00006
			.0036	1	4	.00108
			.0250	2	3	.00750
			.0972	3	3	.02916
			.2269	4	3	.06807
			.3177	5	2	.09531
			.2471	6	2	.07413
			.0824	7	1	.02472
	.7	8	.0001	0	4	.00007
			.0012	0	4	.00084
			.0100	2	3	.007
			.0467	3	3	.02869
			.1361	4	3	.09527
			.2541	5	2	.15787
			.2965	6	2	.20755
			.1977	7	1	.13839
			.0576	8	1	.04032

Appendix 3 (continued)

One Step Transition Matrix

	1	2	3	4
1	.20343	.53486	.25966	.00205
2	0	.3445	.63863	.01687
3	0	0	.822367	.177633
4	0	0	0	1

Steady State Equations

$$\pi_1 = .20343\pi_1$$

$$\pi_2 = .53486\pi_1 + .3445\pi_2$$

$$\pi_3 = .25966\pi_1 + .63863\pi_2 + .822367\pi_3$$

$$\pi_4 = .00205\pi_1 + .01687\pi_2 + .177633\pi_3 + \pi_4$$

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$

Steady State Probabilities

$$\pi_1 = 0$$

$$\pi_2 = 0$$

$$\pi_3 = 0$$

$$\pi_4 = 1$$

Appendix 4: Battery Model, Repair Authorized

A - Initial State
B - Probability of having a Number of Nodes Operational Given the Initial State
C - Number of Nodes Operational
D - Probability of One-Step Transition from Initial Number of Operational Nodes
E - Number of Nodes Operational After Considering Repair
F - Probability of Repair
G - Final Number of Nodes Operational with Repair Considered
H - State after One-Step Transition with Repair Considered
I - Probability of Being in "State" after One-Step Transition

A	B	C	D	E	F	G	H	I
4	.3	0	1	0	.4	0	4	.12
			1	0	.6	1	4	.18
	.7	1	.3	0	.4	0	4	.084
			.3	0	.6	1	4	.126
			.7	1	.4	1	4	.196
			.7	1	.6	2	3	.294

Appendix 4 (continued)

A	B	C	D	E	F	G	H	I
			.09	0	.4	0	4	.00324
			.09	0	.6	1	4	.00486
			.42	1	.4	1	4	.01512
	.09	2						
			.42	1	.6	2	3	.02268
			.49	2	.4	2	3	.01764
			.49	2	.6	3	3	.02646
			.027	0	.4	0	4	.004536
			.027	0	.6	1	4	.006804
			.1890	1	.4	1	4	.031752
			.1890	1	.6	2	3	.047628
3	.42	3						
			.4410	2	.4	2	3	.074088
			.4410	2	.6	3	3	.111132
			.3430	3	.4	3	3	.057624
			.3430	3	.6	4	3	.086436
			.0081	0	.4	0	4	.0015876
			.0081	0	.6	1	4	.0023814
			.0756	1	.4	1	4	.0148176
			.0756	1	.6	2	3	.00222264
			.2646	2	.4	2	3	.0518616
	.49	4						
			.2646	2	.6	3	3	.0777924
			.4116	3	.4	3	3	.0806736
			.4116	3	.6	4	3	.1210104
			.2401	4	.4	4	3	.0470596
			.2401	4	.6	5	2	.0705894

Appendix 4 (continued)

A	B	C	D	E	F	G	H	I
2	.3	5	.0024	0	.4	0	4	.000288
			.0024	0	.6	1	4	.000432
			.0284	1	.4	1	4	.003408
			.0284	1	.6	2	3	.005112
			.1323	2	.4	2	3	.015876
			.1323	2	.6	3	3	.023814
			.3087	3	.4	3	3	.037044
			.3087	3	.6	4	3	.055566
			.3602	4	.4	4	3	.043224
			.3602	4	.6	5	2	.064836
			.1681	5	.4	5	2	.020172
			.1681	5	.6	6	2	.030258
	.7	6	.0007	0	.4	0	4	.000196
			.0007	0	.6	1	4	.000294
			.0102	1	.4	1	4	.002856
			.0102	1	.6	2	3	.004284
			.0595	2	.4	2	3	.01666
			.0595	2	.6	3	3	.02499
			.1852	3	.4	3	3	.051856
			.1852	3	.5	4	3	.077784
			.3241	4	.4	4	3	.090748
			.3241	4	.6	5	2	.136122
			.3025	5	.4	5	2	.0847
			.3025	5	.6	6	2	.12705
			.1176	6	.4	6	2	.032928
			.1176	6	.6	7	1	.049392

Appendix 4 (continued)

A	B	C	D	E	F	G	H	I
1	.3	7	.0002	0	.4	0	4	.000024
			.0002	0	.6	1	4	.000036
			.0036	1	.4	1	4	.000432
			.0036	1	.6	2	3	.000648
			.0250	2	.4	2	3	.003
			.0250	2	.6	3	3	.0045
			.0972	3	.4	3	3	.011664
			.0972	3	.6	4	3	.017496
			.2269	4	.4	4	3	.027228
			.2269	4	.6	5	2	.040842
			.3177	5	.4	5	2	.038124
			.3177	5	.6	6	2	.057186
			.2471	6	.4	6	2	.029652
			.2471	6	.6	7	1	.044478
			.0824	7	.4	7	1	.009888
			.0824	7	.6	8	1	.014832
	.7	8	.0001	0	.4	0	4	.000028
			.0001	0	.6	1	4	.000042
			.0012	1	.4	1	4	.000336
			.0012	1	.6	2	3	.000504
			.0100	2	.4	2	3	.0028
			.0100	2	.6	3	3	.0042
			.0467	3	.4	3	3	.013076
			.0467	3	.6	4	3	.019614
			.1361	4	.4	4	3	.038108
			.1361	4	.6	3	2	.057162
			.2541	5	.4	5	2	.071148
			.2541	5	.6	6	2	.106722
			.2965	6	.4	6	2	.08302
			.2965	6	.6	7	1	.12453
			.1977	7	.4	7	1	.055356
			.1977	7	.6	8	1	.083034
			.0576	8	1	8	1	.04032

Appendix 4 (continued)

One-Step Transition Matrix

	1	2	3	4
1	.372438	.483856	.142808	.000898
2	.049392	.489472	.453662	.007474
3	0	.0705894	.837008	.093402
4	0	0	.294	.706

Steady State Equations

$$\pi_1 = .372438\pi_1 + .049392\pi_2$$

$$\pi_2 = .483856\pi_1 + .489472\pi_2 + .0705894\pi_3$$

$$\pi_3 = .142808\pi_1 + .453662\pi_2 + .837008\pi_3 + .294\pi_4$$

$$\pi_4 = .000898\pi_1 + .007474\pi_2 + .0934026\pi_3 + .706\pi_4$$

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$

Steady State Probabilities

$$\pi_1 = .0079493296$$

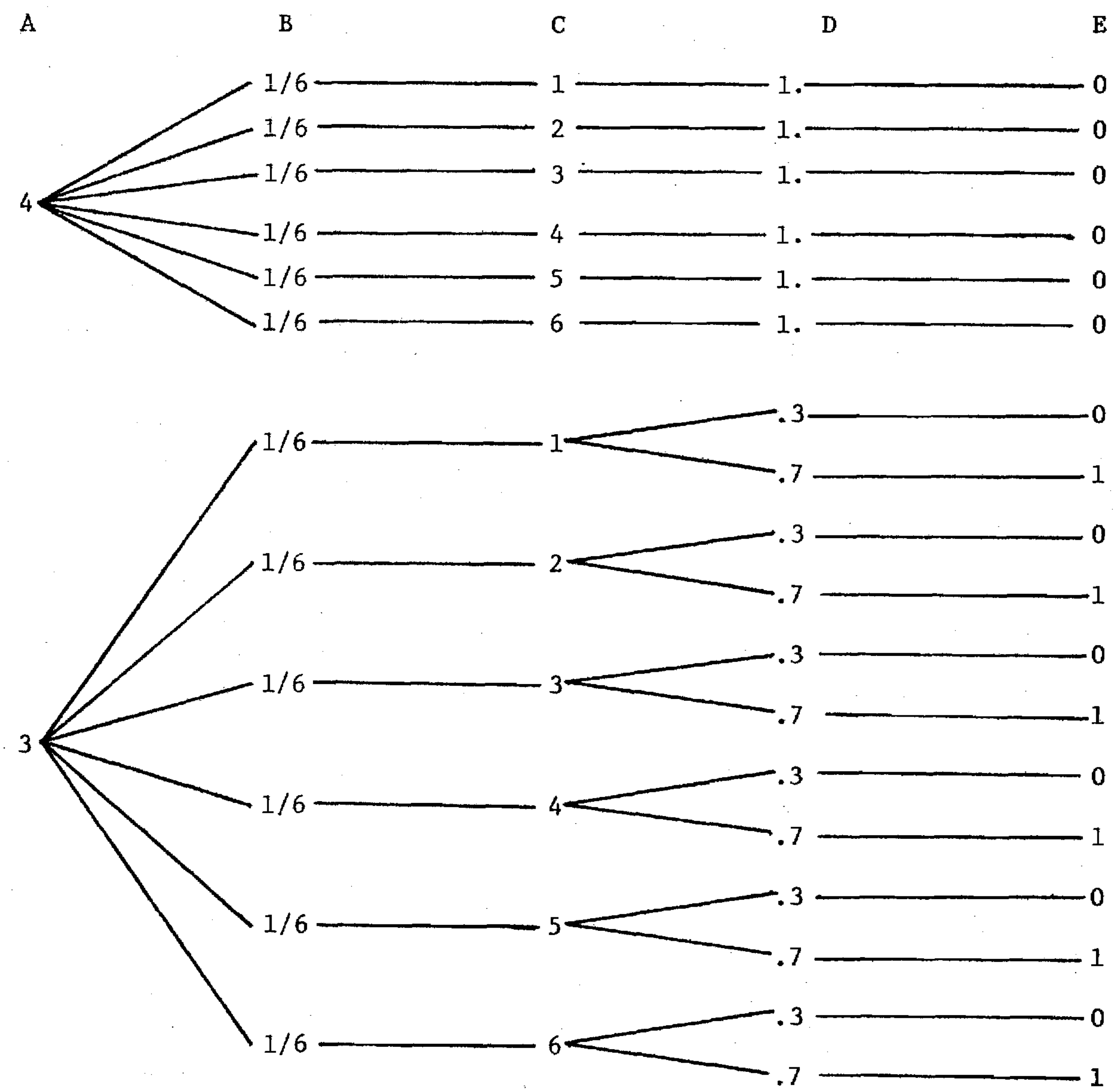
$$\pi_2 = .1010021299$$

$$\pi_3 = .6759950437$$

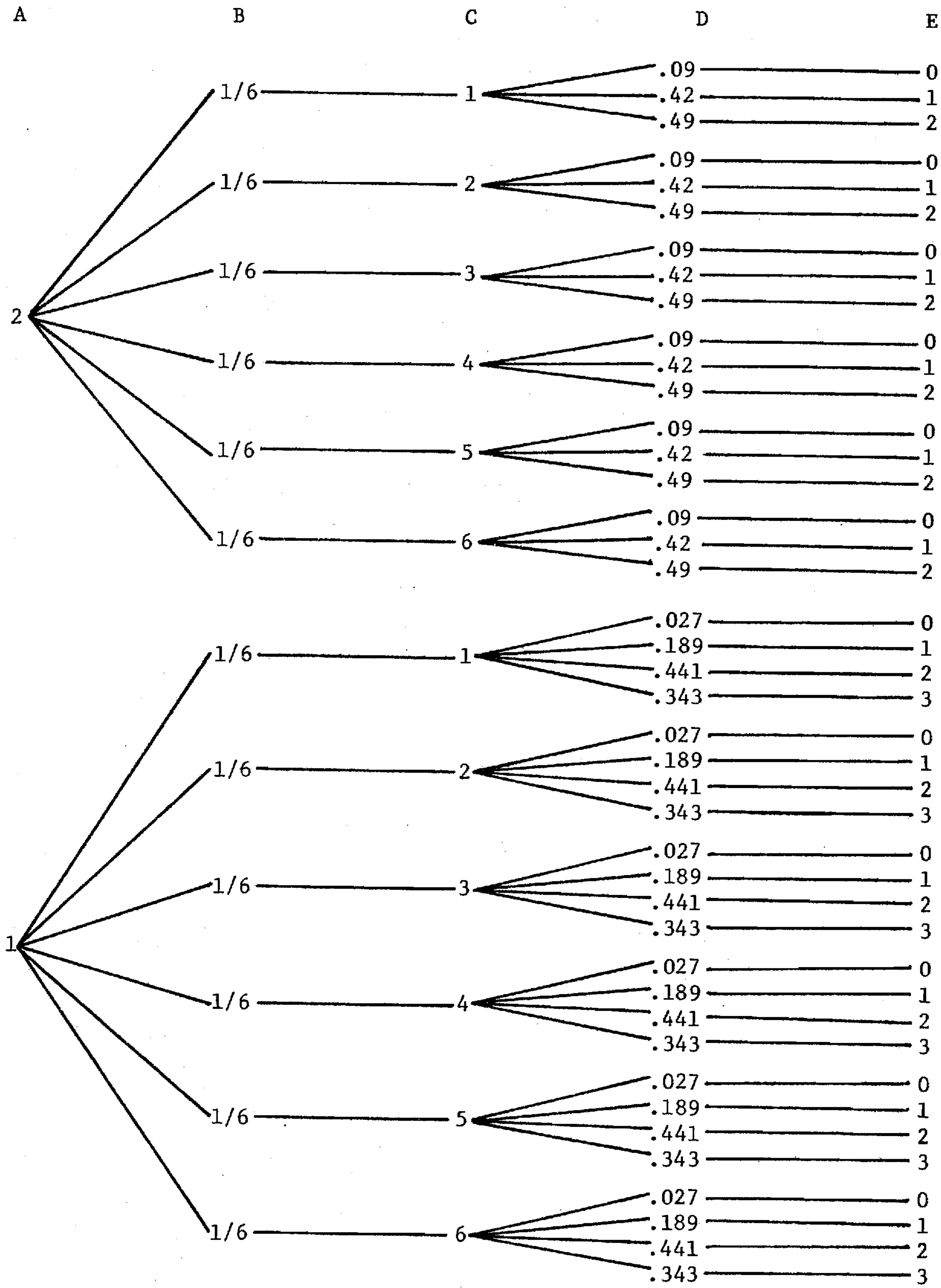
$$\pi_4 = .2150534967$$

Appendix 5: Battery-Battalion Model, No Repair Authorized

A - Initial State of the System
B - Probability that this Node will Affect State of System, Uniform Distribution Equal to 1/n where Equal the Number of Nodes in the System
C - The Node Number, Each Node Corresponds to a Battery in the System.
D - The Probability that the Node will have a Certain Number of Communications Links Operational.
E - The Number of Communications Links Operational for Each Node, This Determines the State of the System.



Appendix 5 (continued)



Appendix 5 (continued)

One-Step Transition Matrix

	1	2	3	4
1	.343	.441	.189	.027
2	0	.49	.42	.09
3	0	0	.7	.3
4	0	0	0	1

Steady State Equations

$$\pi_1 = .343\pi_1$$

$$\pi_2 = .441\pi_1 + .49\pi_2$$

$$\pi_3 = .189\pi_1 + .42\pi_2 + .7\pi_3$$

$$\pi_4 = .027\pi_1 + .09\pi_2 + .3\pi_3 + \pi_4$$

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$

Steady State Probabilities

$$\pi_1 = 0$$

$$\pi_2 = 0$$

$$\pi_3 = 0$$

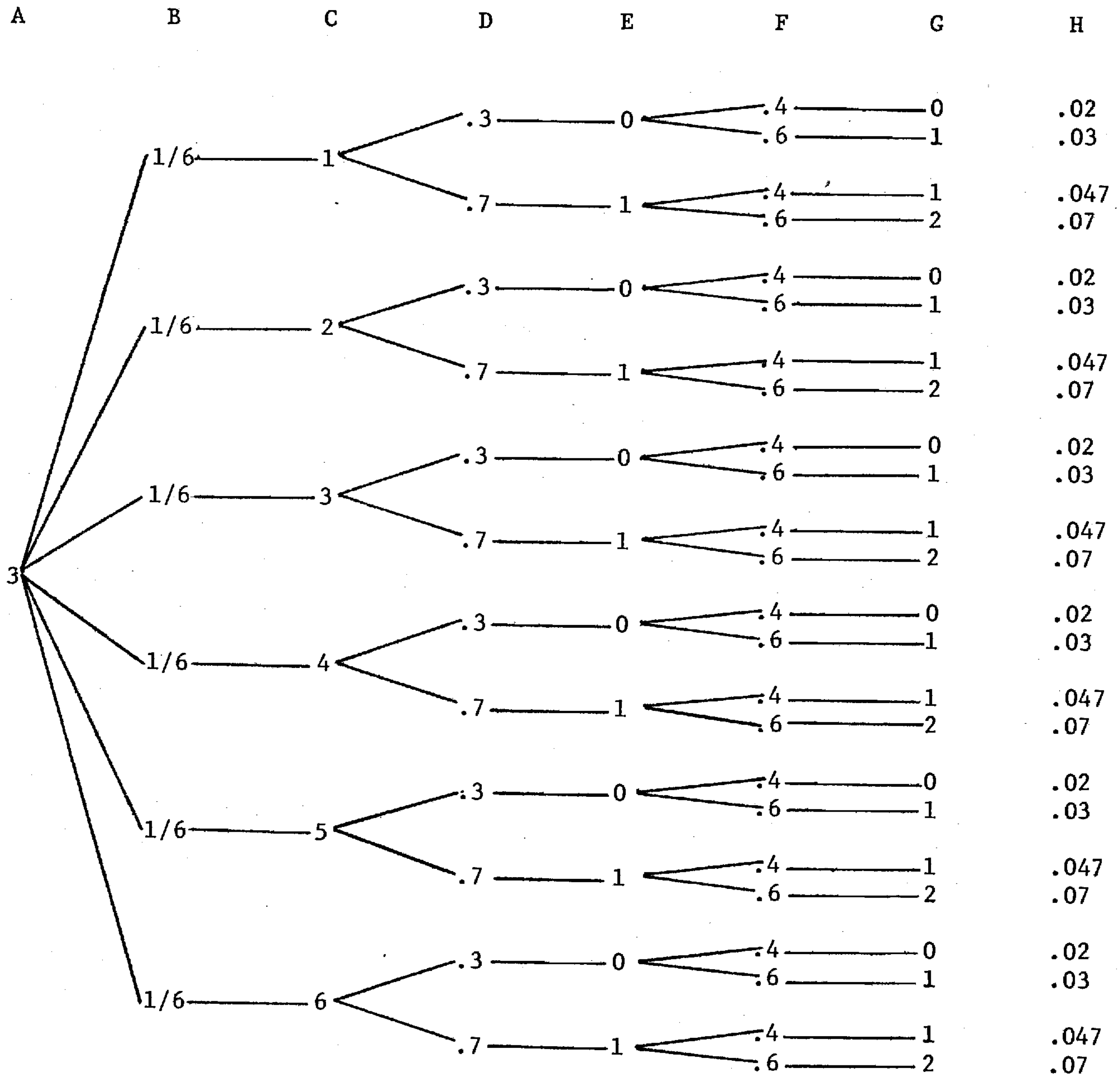
$$\pi_4 = 1$$

Appendix 6: Battery-Battalion Model, Repair Authorized

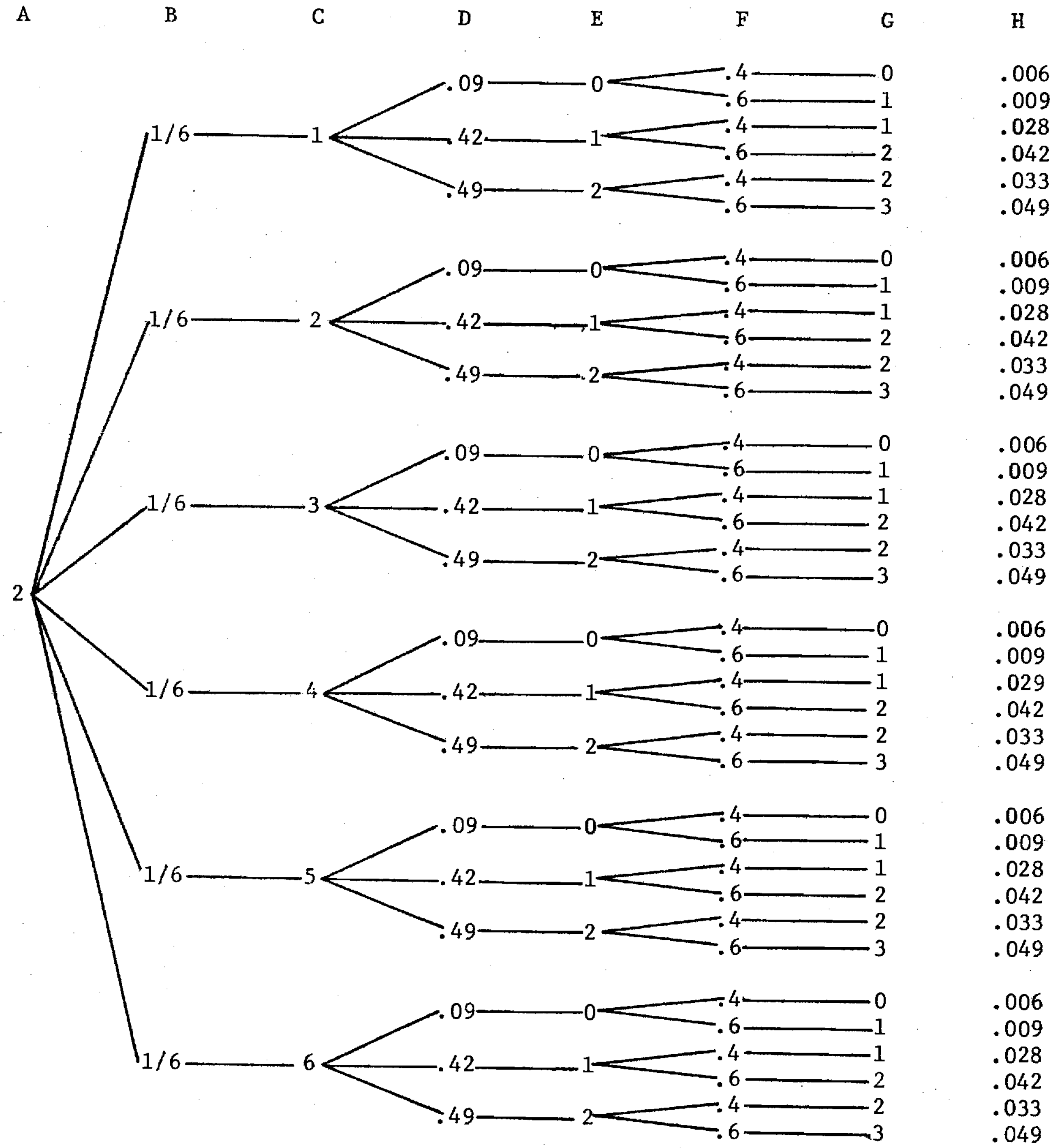
A - Initial State of the System
B - Probability that this Node will Affect State of System, Uniform Distribution Equal to 1/n where n Equal the Number of Nodes in the System
C - The Node Number, Each Node Corresponds to a Battery in the System
D - The Probability that a Node Will Have a Certain Number of Links Operational in One Transition
E - The Number of Communications Links Operational for each Node
F - Probability of Transition from E to G
G - Number of Communications Links Operational for Each Node when Repair is Considered. This defines the State of the Node and System after One Transition.
H - Probability of Node Transition from A to G in One Step

A	B	C	D	E	F	G	H
4	1/6	1	1.	0	.4	0	.067
					.6	1	.1
	1/6	2	1.	0	.4	0	.067
					.6	1	.1
	1/6	3	1.	0	.4	0	.067
					.6	1	.1
	1/6	4	1.	0	.4	0	.067
					.6	1	.1
	1/6	5	1.	0	.4	0	.067
					.6	1	.1
	1/6	6	1.	0	.4	0	.067
					.6	1	.1

Appendix 6 (continued)



Appendix 6 (continued)



Appendix 6 (continued)

A	B	C	D	E	F	G	H
1	1/6	1	.027	0	.4	0	.0018
					.6	1	.0027
			.189	1	.4	1	.0126
					.6	2	.0189
			.441	2	.4	2	.0294
					.6	3	.0441
			.343	3	1	3	.0572
	1/6	2	.027	0	.4	0	.0018
					.6	1	.0027
			.189	1	.4	1	.0126
					.6	2	.0189
			.441	2	.4	2	.0294
					.6	3	.0441
			.343	3	1	3	.0572
	1/6	3	.027	0	.4	0	.0018
					.6	1	.0027
			.189	1	.4	1	.0126
					.6	2	.0189
			.441	2	.4	2	.0294
					.6	3	.0441
			.343	3	1	3	.0572
	1/6	4	.027	0	.4	0	.0018
					.6	1	.0027
			.189	1	.4	1	.0126
					.6	2	.0189
			.441	2	.4	2	.0294
					.6	3	.0441
			.343	3	1	3	.0572
	1/6	5	.027	0	.4	0	.0018
					.6	1	.0027
			.189	1	.4	1	.0126
					.6	2	.0189
			.441	2	.4	2	.0294
					.6	3	.0441
			.343	3	1	3	.0572
	1/6	6	.027	0	.4	0	.0018
					.6	1	.0027
			.189	1	.4	1	.0126
					.6	2	.0189
			.441	2	.4	2	.0294
					.6	3	.0441
			.343	3	1	3	.0572

Appendix 6 (continued)

One-Step Transition Matrix

	1	2	3	4
1	.6078	.2898	.0918	.0108
2	.294	.4518	.222	.036
3	0	.42	.462	.12
4	0	0	.6	.4

Steady State Equations

$$\pi_1 = .6078\pi_1 + .294\pi_2$$

$$\pi_2 = .2898\pi_1 + .4518\pi_2 + .42\pi_3$$

$$\pi_3 = .0918\pi_1 + .222\pi_2 + .462\pi_3 + .6\pi_4$$

$$\pi_4 = .0108\pi_1 + .036\pi_2 + .12\pi_3 + .4\pi_4$$

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$$

Steady State Probabilities

$$\pi_1 = .2395324439$$

$$\pi_2 = .3195395391$$

$$\pi_3 = .2517977931$$

$$\pi_4 = .1891302239$$

Bibliography

1. Bazaraa, M.S. and Jarvis, J.J., "Linear Programming and Network Flows," John Wiley and Sons, Inc., 1977.

2. Clement, George H., "Weapon System Philosophy," Paper presented at AGARD Seminar on Guidance and Control of Guided Missiles; Venice, Italy, September 1956.

3. Fishlow, Albert; Goodman, Nelson, and Perkins, C. C., "The Flow and Functions of Communications in Weapon Systems," Project WESCOM, 21 January 1957.

4. "FM 44-1, U.S. Army Air Defense Artillery Employment," with Change 1, Headquarters, Department of the Army, 25 March 1976.

5. Frank, H. and Frisch, I.T., "Analysis and Design of Surviveable Networks," IEEE Trans. Comm. Tech., Vol. Com-18, pp. 501-519, October 1970.

6. Fran, H., "Vulnerability of Communications Networks," IEEE Trans. Comm. Tech., Vol. Com-15, pp. 778-789, December 1967.

7. Gay, F. A., "Performance Modeling for Gracefully Degrading Systems," Ph.D. Dissertation, Computer Science Department, Northwestern University, June 1979.

8. Kennington, Jeff L. and Helgason, Richard A., "Algorithms for Network Programming," John Wiley and Sons, Inc., 1980.

9. Martin, Edith W., "Operational Surviveability in Gracefully Degrading Distributed Processing Systems," Ph.D. Dissertation, School of Information and Computer Science, Georgia Institute of Technology, December 1980.

10. "Micro Force Structure Planning," OAVC of SA-FPA, 14 March 1969.

11. Montgomery, D. C. and Peck, E. A., "Introduction to Regression Analysis," John Wiley and Sons, 1981.

12. "Patriot Air Defense Artillery Battalion Communications," Draft, U.S. Army Patriot Progress Office, 1980.

13. "Patriot Air Defense System, System Description and Equipment Summary," Raytheon Company, Missile System Division, April 1980.

14. "Project WESCOM, Communications Systems for Weapons Systems," The Institute for Cooperative Research, University of Pennsylvania, Philadelphia, Pa., August 22, 1955.

15. Ross, Sheldon M., "Introduction to Probability Models," 2d ed, Academic Press, 1980.