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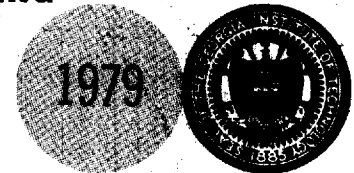
# MATHEMATICAL PROGRAMMING SOLUTION TO EXPANSION PLANNING PROBLEMS IN ELECTRIC UTILITIES

Leon F. McGinnis, Jr,  
Principal Investigator

Presented to  
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June, 1979

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June 25, 1979

Dr. William Brogan  
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Dear Dr. Brogan:

Enclosed is the final technical report for NSF Grant ENG77-06190, as well as a working paper describing other work which was supported by this grant. Never having worked on sponsored research, I have found this experience to be quite educational and very enjoyable.

The research is continuing, primarily through the dissertation research of Ms. J. C. Ammons. As further results and reports become available, they will be forwarded to you.

Sincerely,

Leon F. McGinnis, Jr. ✓  
Assistant Professor

LFM:vld

enclosures

Please read instructions on reverse carefully before completing this form.

1. INSTITUTION AND ADDRESS Georgia Institute of Tech. Atlanta, Georgia 30332		2. NSF PROGRAM System Theory and Applications	3. PRINCIPAL INVESTIGATOR(S) Leon F. McGinnis, Jr.
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9. SUMMARY (ATTACH LIST OF PUBLICATIONS TO FORM)

Optimization models and procedures should play an important role in the long range planning of generation capacity for large electric utilities. While optimization is widely used to solve problems of unit commitment, maintenance scheduling and the like, it is not currently widely accepted in generation capacity planning. The reasons for this lack of acceptance are that computationally tractable models do not provide adequate problem representation and more adequate models are at present not computationally feasible on realistic problems. Based on a review of available results, it is concluded that mixed integer programming models provide the best opportunity for adequately representing the complex features of the generation planning problem. In order to achieve greater realism in these models, a new approach to the production cost estimation problem is developed. Empirical results demonstrate the new technique to be comparable to a large scale production costing simulator in the accuracy of system production cost estimates. The new approach is based on aggregation of units via an optimization submodel and is shown to be compatible with mixed integer programming formulations of the generation planning problem. A new mixed integer programming model is formulated incorporating the new production costing technique and explicit capital budgeting constraints. Reliability requirements are enforced by a capacity reserve constraint and constraints on plant mix. Exact and approximate solution procedures are developed to exploit the special mathematical structure of the new model. It is anticipated that these algorithms will prove computationally attractive for use in long range expansion planning. The design of a comprehensive computerized planning system is also discussed. Design guidelines and a system architecture are proposed.

9. SIGNATURE OF PRINCIPAL INVESTIGATOR/ PROJECT DIRECTOR	TYPED OR PRINTED NAME F. McGinnis, Jr.	DATE 6/25/79
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Mathematical Programming Solution to  
Expansion Planning Problems in Electric Utilities

Presented to the  
National Science Foundation  
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## TABLE OF CONTENTS

	<u>page</u>
1. The Generation Capacity Planning Problem . . . . .	1
1.1 The Role of Models in Generation Planning . . . . .	3
1.2 Issues in Generation Planning . . . . .	6
1.2.1 Economic Issues . . . . .	6
1.2.2 Technological Issues . . . . .	10
1.3 Scope and Objectives . . . . .	13
2. A Survey of Optimization Models for Generation Capacity Planning .	16
2.1 Linear and Nonlinear Programming Models . . . . .	18
2.2 Dynamic Programming Models . . . . .	22
2.3 Mixed Integer Programming Models . . . . .	25
2.4 Summary . . . . .	27
References . . . . .	31
3. A Production Costing Model for Electric Utilities . . . . .	33
3.1 Production Costing for a Single Time Period . . . . .	36
3.1.1 Marginal Cost Approach for Continuous Units . . . . .	37
3.1.2 Continuous Units - Marginal Cost Curve Algorithm . . . . .	39
3.1.3 Cost Approach for Discrete Units . . . . .	44
3.1.4 Discrete Units - Dynamic Programming Algorithm . . . . .	46
3.1.5 Developing the Overall System Cost vs. Output Curve . . . . .	49
3.1.6 Computing Estimated Cost for One Period . . . . .	51
3.2 Empirical Evaluation . . . . .	51
3.3 Incorporation into Capacity Expansion Planning Model . . . . .	54
3.4 Summary . . . . .	55
References . . . . .	59
4. A Mixed Integer Programming Model for Generation Capacity Planning . . . . .	62
4.1 Problem Specification . . . . .	65
4.2 Model Formulation . . . . .	69
4.2.1 Production Costing Sector . . . . .	70
4.2.2 Budget Oriented Constraints . . . . .	72
4.2.3 Reliability Oriented Constraints . . . . .	73
4.2.4 Objective Function . . . . .	75
4.3 Model Use . . . . .	76
4.4 Parameter Specification . . . . .	78
4.5 Summary . . . . .	79
References . . . . .	83
5. Algorithms for Solving the Generation Planning Problem . . . . .	84
5.1 Benders' Partitioning Procedure . . . . .	86
5.2 An Exact Algorithm . . . . .	88
5.2.1 Solving the Subproblem . . . . .	88
5.2.2 Solving the Master Problem . . . . .	92
5.3 A Variant of the Exact Algorithm . . . . .	93
5.4 An Approximate Algorithm . . . . .	94
5.5 Summary . . . . .	95
References . . . . .	98

	<u>page</u>
6. Design of a User Oriented System for Generation Capacity	
Planning . . . . .	100
6.1 Overview . . . . .	101
6.2 Data Base Design . . . . .	103
6.3 System Functions . . . . .	104
References . . . . .	107
7. Summary and Recommendations . . . . .	108
References . . . . .	111

APPENDICES

A Notation and Terminology . . . . .	112
B The Loss of Load Probability (LOLP) Approach to Generation Reliability . . . . .	116

## Chapter 1

### The Generation Capacity Planning Problem

## The Generation Capacity Planning Problem

The electric utility industry faces some of the most challenging planning problems to be found. Implicit in its franchise is an obligation to meet the public's demand for electric energy, a demand whose growth has been quite rapid in the past four decades and shows little sign of declining. In addition the industry is extremely capital intensive and rapid expansion of capacity creates enormous requirements for investment funds.

Thus, electric utilities must not only correctly determine what the future demand for energy will look like, but also determine a sequence of expansion projects and feasible methods for their funding. Given the magnitude of investment and operating costs, even slight improvements in the expansion plan can impact significantly the consumers of electric energy and the industry's stockholders. It isn't surprising, then, that several different methods have been proposed for use by utilities in capacity expansion studies.

The research reported here focuses on the generation capacity planning problem and considers it independent from the problems of transmission and distribution. Although this approach could obviously lead to suboptimization, it is consistent with the philosophy of many major utility planning groups. Also, a fundamental precept guiding this research is that optimization models can and should play a vital role in the generation planning process. Therefore this report deals primarily with the role of optimization models in generation capacity planning and with the development and use of specific models in that process.



## 1.1 The Role of Models in Generation Planning

The long range generation planning problem involves not only technological and economic, but also social, environmental and purely political issues. Furthermore, all of these issues are clouded by uncertainty. No planning model or collection of planning models will ever be able to adequately cope with and resolve all these issues. Ultimately, the strategic decisions regarding expansion planning must be made by human decision makers. In this context, the proper role of optimization models for strategic planning is to allow the decision makers to explore various possible "futures" and various available strategies. This implies that good strategic planning models will have certain characteristics such as flexibility, ease of use, and economy of use.

Here, strategic or planning problems are contrasted with tactical or operating problems. In the electric utility industry two typical tactical problems are the day-to-day plant loading or dispatching problem and the scheduling of annual maintenance or planned outages. In these instances, mathematical models may very well be used to make the required decisions under a wide range of operating conditions. The primary distinctions between these examples of tactical and strategic problems is that tactical decisions are in effect over a shorter horizon, they may easily be revoked or changed and they involve much less uncertainty. In contrast, strategic decisions extend over a much longer horizon (e.g., twenty to forty years), they are not easily changed (at least those affecting the initial portion of the horizon) and they involve a great deal of uncertainty.

The recognition of the distinction between tactical and strategic problems is also a key element in developing optimization models for use in planning because it governs, to a large extent, the appropriate level

of modeling detail. For example, in maintenance scheduling, annual budget planning, or fuel stockpile positioning, it is important to have a good estimate of the energy produced by each generating unit during each week. In long range generation planning, however, such estimates are not really needed, as long as the total system production cost is correctly estimated.

Especially for the types of optimization models considered in this research, the magnitude of the problems that can be solved is inversely related to the level of detail in the model. Increasing the level of detail in the model implies that there will be a concomitant reduction in the size of problems that can be solved as measured by, say, the number of generating units and periods in the planning horizon. Primarily because of the state-of-the-art in computation, long range generation planning models should not incorporate greater detail than is actually needed.

There is a second, equally important, argument for limiting the level of detail in strategic planning models, which would be valid regardless of the capability for computing optimal solutions. The argument is based on the idea of spurious precision, i.e., a precise answer need not be accurate (better known is Garbage-In - Garbage-Out). In a long range generation planning model, regardless of the precision with which they are stated, detailed estimates of unit production beyond the first year or two cannot be regarded as accurate because of the enormous uncertainty. Thus, even if this level of detail could be included in such a model at no cost, it should not, because the resulting detailed estimates have little value. Obviously the model must incorporate production in some manner in order to estimate variable production cost; in this case the issue is the necessary level of detail.

The proper role and use of optimization models in generation planning,

as viewed in this research, can be summarized as follows. The role of optimization models is to facilitate the identification of the best strategies to follow under a variety of possible future events. Optimization models should be used to determine the appropriate strategies to employ in generation capacity expansion under a variety of possible future conditions. At this point, the results of the studies using optimization models should be used by the analyst to define a limited set of specific expansion plans. This limited set of expansion plans can then be analyzed in greater detail as a basis for actual decision making. This process is illustrated in Figure 1.1 which emphasizes that the models are used to provide better insight and information for the decision maker - they are not used to provide decisions.

For a variety of reasons, methods suggested in the literature (which is reviewed in Chapter 2) generally do not adopt this philosophy. In many cases, the methods do not employ optimization and are based on very detailed production costing simulators. Thus, they essentially omit the Phase I element of planning as illustrated in Figure 1.1. In other cases, the optimization is so expensive that the analyst is precluded from exploring more than one or two sets of conditions. Methods of this ilk typically do not employ a detailed Phase II evaluation.

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Figure 1.1 here  
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## 1.2 Issues in Generation Planning

From the previous discussion, it is clear that in designing a generation planning model a fundamental question is, "Which issues should the model address directly and which issues should be resolved outside the model?" The manner in which this question is answered naturally affects not only the quality of the resulting solutions but also the complexity of the model, its ease of solution, its data requirements and the cost of its use.

The following discussion addresses those issues which have been or could be reasonably incorporated in a generation planning model. The choice of issues is biased by an engineering point of view, i.e., political, social and environmental issues are not directly incorporated. It is assumed that if necessary, they may be incorporated by appropriately constraining the solution, or by using a different criterion.

### 1.2.1 Economic Issues

One of the basic economic trade-offs in generation planning is between the initial cost of a facility and its subsequent operating costs. Facilities, such as combustion turbines, with low initial costs per megawatt (MW) typically have large operating (primarily fuel) costs per megawatt-hour (MWH), while facilities such as nuclear or hydro-powered generators with low operating costs typically have large initial costs.

In a steady-state, no-growth system, depreciation of existing facilities would allow the utility to accumulate capital for replacement from current operating revenues. Such is certainly not the case at present because of inflation and the rapidly expanding demand for electric energy.

In this situation either current operating revenues must include part of the cost of expansion or investor funds must be attracted.

Suppose that all construction funds come from current operating revenues. There would be an obvious pressure to minimize initial construction costs resulting in higher future operating costs. Since these higher costs would be reflected in the cost to consumers there would be added pressure to reduce construction costs, etc. The problem is further aggravated by ever-increasing costs for fossil fuels, which are the major energy source for electric utilities.

On the other hand, suppose that all construction funds come from investors, either through equity or bonded indebtedness. If this source of construction funding were unconstrained, one could argue that there would be pressure to install facilities with low future operating costs, since doing so would minimize the impact of future increases in fuel costs. In the past, most construction funds were generated by investors.

In reality, construction funds may come from both sources. The problem then would seem to be simply one of choosing the appropriate mix of funding. However, because the electric utility industry is regulated, the economic issues tend to be complicated by political issues. In recent years, many utilities have faced not only rising fuel and construction costs, but a reluctance on the part of regulatory agencies to grant rate increases. This creates a situation in which not only are current revenues not sufficient to fund construction, but at the same time investors are discouraged.

Thus, at the present time and in the foreseeable future, generation planning will have to cope with scarce construction funds. It seems unreasonable to expect a long range capacity planning model to incorporate the

complex economic-political interaction, so it is assumed that this issue is dealt with outside the model. Resulting estimates of the availability of construction funds, the cost of investor generated funds, and interest rates become exogenous inputs to the generation planning process. In particular, this information impacts the formulation of the mathematical model to be used in the planning process.

One manner in which the information can be incorporated into the model is by specifying for each period in the planning horizon the amount of funds to be made available for construction, i.e., by constraining the solution. To a certain extent, this treatment of the problem of scarce construction funds reduces it to a capital budgeting framework. Thus, expansion projects could be modelled in considerable detail with regard to the duration of construction, rate at which construction costs are incurred, differential inflation rates, etc. Furthermore, short-term borrowing and lending could also be incorporated.

A second manner in which the information affects the planning model is in the choice of the criterion to be used. Typically, the criterion has been taken to be the sum over the periods in the planning horizon of the discounted capital and operating costs. The exogenously specified discount rate has a crucial effect on the outcome of the planning process.

The criterion of minimizing total discounted costs re-emphasizes the fundamental trade-off between capital and operating costs. The use of this criterion requires the estimation of operating costs in future time periods. Even if the facilities to be used in future periods were specified a priori, the estimation of these costs would be a very difficult problem.

In the first place, demand for energy and power in future periods

must be estimated, along with the prices of various fuels. Given that all this information is available (and reliable) the problem is to determine what the costs would be, based on the operating characteristics for each generating unit, their fixed operating costs, maintenance scheduling and operating policy (e.g., spinning reserve requirements). It seems unreasonable to expect a long range capacity planning model to also provide maintenance scheduling and operating policies, so these are assumed to be determined outside the model.

Appendix A summarizes a mathematical description of the parameters developed in Chapter 4 to incorporate the various economic issues into the capacity planning model. The notation will be used consistently throughout this report.

### 1.2.2 Technological Issues

A fundamental problem in long range generation capacity planning is to predict what types of generation facilities will be developed, and what their operating characteristics will be. This includes, of course, refinements to existing types of facilities. This technology assessment problem must be resolved outside the planning model. It is assumed that the potential expansion projects are completely defined with respect to cost, construction time, feasible commissioning times and size options.

A more central technological issue is that of reliability of service. Generation, transmission and distribution facilities are subject to randomly occurring failures resulting from a variety of causes. Since these failures can't be completely controlled, the service provided is not 100% reliable, i.e., there is some probability that some of the demand occurring at an instant in time will not be served.

Reliability has become a key issue because of the expense of having large capacity reserves on hand to compensate for possible unit failures. Although this is widely recognized as a problem, there does not seem to be a good understanding or specification of what it means for the system to be reliable, what level of reliability is satisfactory, or what are the economics of unreliability.

The current practice is to equate unreliability with the loss of load probability (LOLP). Appendix B presents a mathematical development of LOLP. Essentially LOLP is the fraction of time during a year that system power capacity falls below the demand for power. A commonly used definition of an acceptable LOLP is one which corresponds to "one day in ten years," i.e.,  $LOLP = 1/3650$ .

The LOLP approach to reliability assessment has several shortcomings.



In the first place a given LOLP could result from an infinite variety of circumstances. At one extreme would be a system which was 100% reliable except for one very brief period in the year, during which it almost certainly could not meet the presented load. At the other extreme would be a system which was somewhat less reliable over the entire year, but had no periods during which demand would almost certainly fail to be served. Although the LOLP approach does not distinguish between these two extreme cases, it is quite unlikely that the two situations would appear as equivalent alternatives to any decision makers.

A perplexing problem with LOLP is how to treat various load management tactics. For example, is a voltage reduction, strictly speaking, a loss of load? Similar questions arise regarding public appeals, selective curtailment of service, etc. Operating policies can impact LOLP, for example, reductions in spinning reserves. Likewise events such as fuel shortages can cause a loss of load even though there is no failure of generation equipment.

Finally, the LOLP approach does not explicitly recognize the cost of reliability or its economic benefits. There is very little evidence available for use in judging the merit of a given level of reliability. Of course, in the traditional approach, i.e., planning to meet all demand at all times, there was no need to consider the cost of reliability. The cost of capacity, however, is forcing explicit consideration of the economic costs of various types of load losses.

In terms of long range capacity planning models, the issue of reliability is perhaps the most difficult to deal with. It must be incorporated in some manner, or else the resulting plans may not be feasible. The usual approach avoids explicit consideration of economics by requiring either a

minimum reserve capacity (actually capacity in excess of anticipated peak demand) or by specifying a minimum acceptable LOLP. Although most optimization models use the reserve requirement approach, at least one recent model incorporates a yearly constraint on LOLP.

An alternative approach to reliability requirements is suggested and explored in the research reported here. The approach is based on the rationale that system reliability can be controlled by controlling plant mix, i.e., the types and sizes of units in the system. In essence, this approach requires the a priori specification of an appropriate set of plant mix constraints.

### 1.3 Scope and Objectives

As indicated in the introduction, this research considers the generation capacity expansion problem to be independent of the problems of transmission and distribution. Furthermore, no consideration is given to the problems of data gathering, including forecasting and estimation, or to problems of implementing expansion plans. Evidently, these problems are currently resolved in some manner, and the current methods are assumed to be adequate. The scope of this research is further restricted to long range planning models and methods which are based on or make some use of optimization.

The objective of the research reported here is the development of an optimization model and methodology for use in long range generation capacity planning. The form of the model and the type of methodology developed are directly related to the concept of the expansion planning process described by Figure 1.1.

The remainder of this report is organized as follows. A brief survey of the literature on generation expansion planning models is presented in Chapter 2. Particular attention is given to the issues of model adequacy, capacity for parametric analysis and "what if?" studies, and the cost and timeliness of results. Chapter 3 describes a new approach to production cost estimation and presents some empirical evidence for its effectiveness. In Chapter 4 this new approach to production costing is used to develop a mixed integer programming model of the generation capacity planning problem. Chapter 4 also contains a discussion of a new approach to reliability based on constraining generation plant mix. In Chapter 5, the methodology is developed for obtaining optimal and approximate solutions to the problem as modelled in Chapter 4. The design of an integrated system implementing

these results is addressed in Chapter 6. Finally Chapter 7 summarizes the research results and points toward further work.

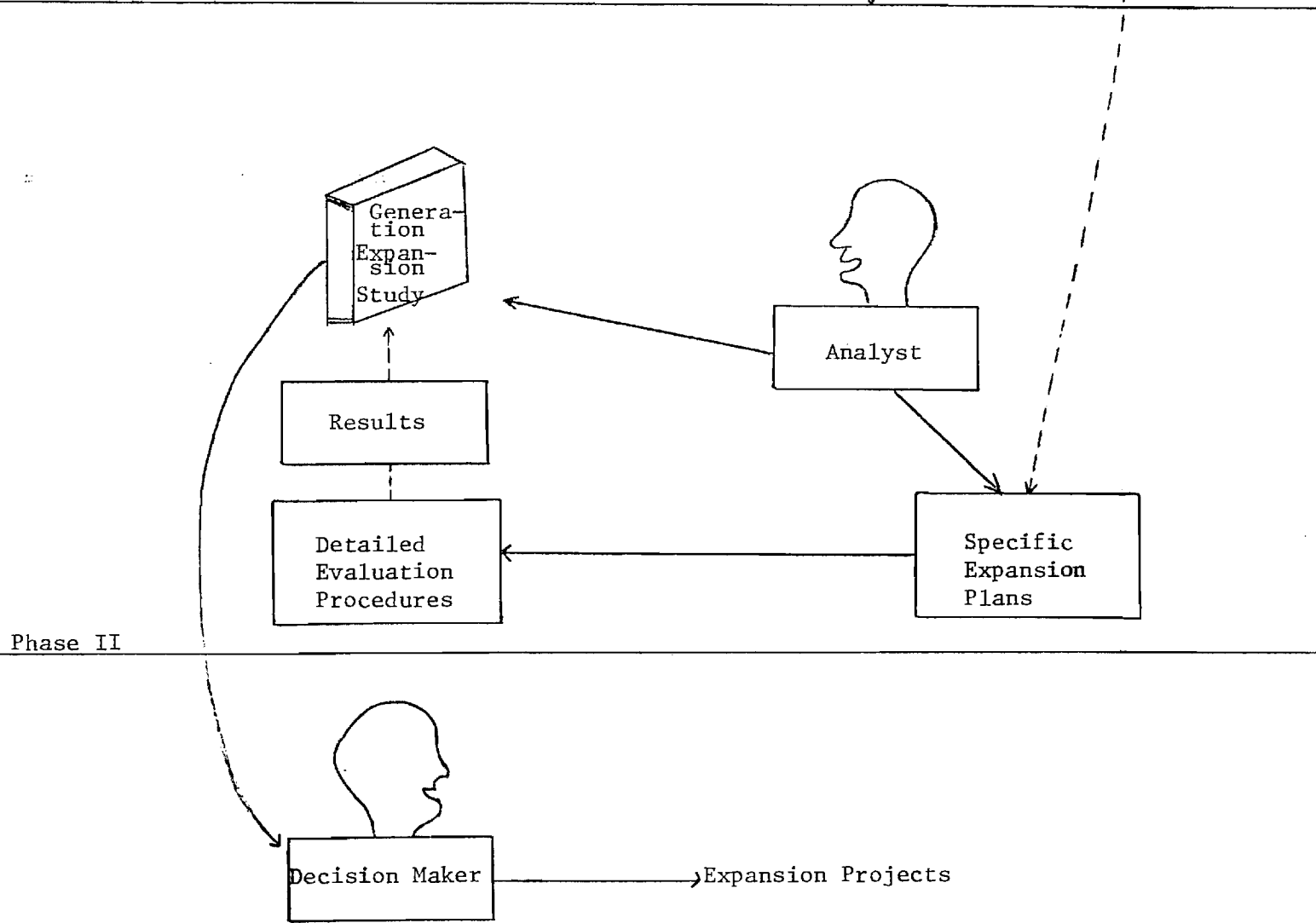
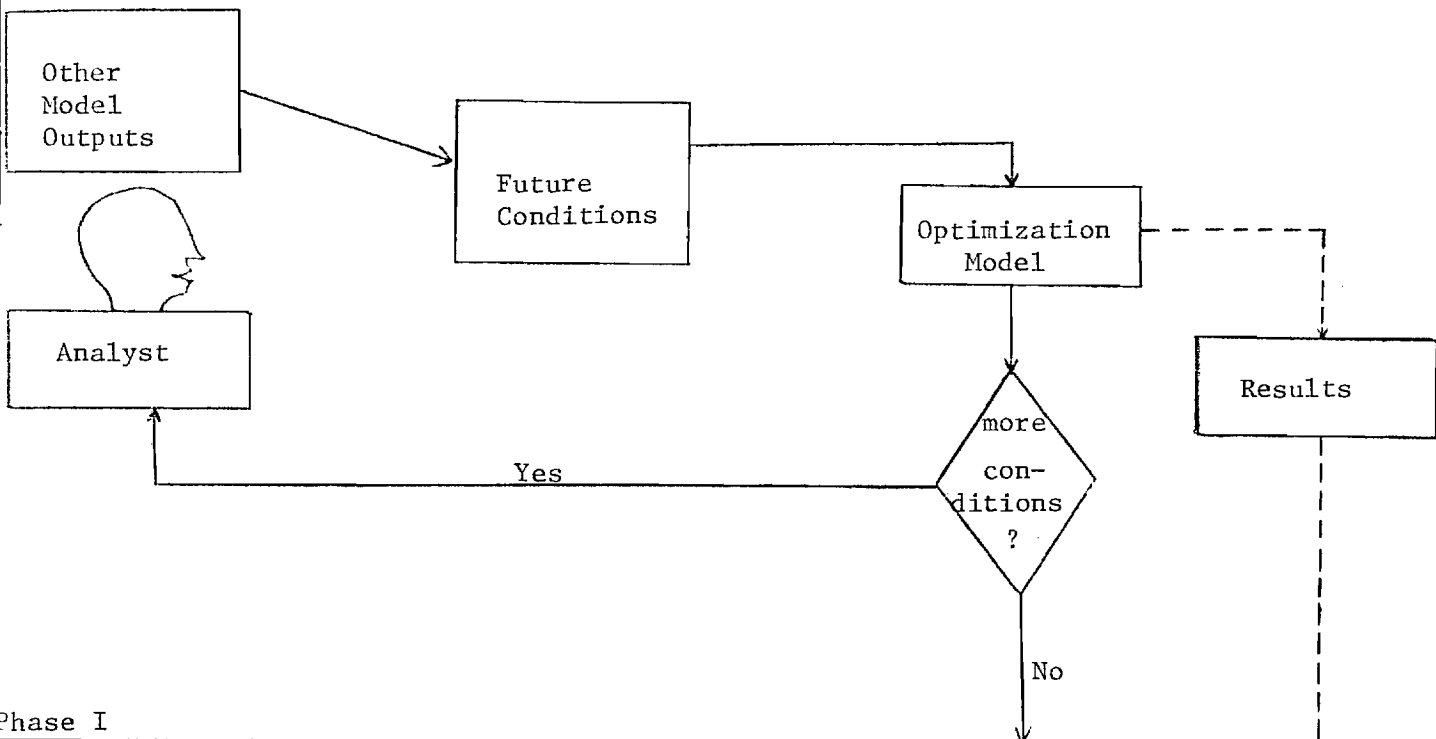


Figure 1.1. The Expansion Planning Process

## Chapter 2

### A Survey of Optimization Models for Generation Capacity Planning

## A Survey of Optimization Models for Generation Capacity Planning

The problem of planning generation capacity expansion can be stated in general very simply: Determine the types and sizes of units to be added within the planning horizon, say, thirty years, so that the system is adequately reliable, financial resources are not exceeded and an appropriate measure of total cost is minimized. This would appear to be a problem which is ideally suited to treatment via optimization models, and in fact, quite a number of optimization models have been proposed. None of these optimization models, however, has gained the level of acceptance of the simulation based heuristics such as OGP [ 1, 8, 13] or WASP [12].

One reason why there is no widely accepted optimization model is that at the detailed level, the statement of the generation planning problem becomes incredibly complex. For example, no practical technique is currently available for modelling system reliability in closed form. Modelling the system operating costs is also quite complicated, since it involves a constantly changing demand for energy and the operation of a large system of interconnected generating units, each having its own operating requirements and characteristics. Because of these and other complicating factors, a number of modelling assumptions are required. In addition, simplifying assumptions and aggregations are often required to make the model computationally tractable.

Each different set of modelling assumptions leads to a different optimization model, giving different types of results. In contrast, the simulation based approaches require fewer decisions in modelling because they essentially attempt to replicate the actual system. Thus, there is more unanimity of opinion regarding their validity and usefulness.

The purpose of this brief survey is to identify the different types of optimization models that have been proposed, the major modelling assumptions required and the relative advantages and disadvantages of each. This survey is intended to be indicative of the types of models in the literature rather than a comprehensive catalog of all published results. The survey is organized into three sections by solution methodology: linear and non-linear programming models, dynamic programming models, and mixed integer programming models. A final section compares and contrasts the various approaches.

### 2.1 Linear and Nonlinear Programming Models

The historical evolution and state of practice for linear and non-linear programming models is described in detail in Anderson's widely referenced paper [ 2 ]. The key assumptions embodied in the formulations discussed by Anderson are:

- (1) System demand can be adequately represented by a load duration curve (LDC).
- (2) Reliability requirements can be satisfied by a specified generation capacity margin.
- (3) All units of a given class (hydro, fossil, nuclear, etc.) can be lumped together for purposes of production costing, and each class has a constant marginal production cost.
- (4) Capacity additions in any positive amount are permitted and costs of capacity are linear (constant cost in \$/mw added).

Using the notation given in Appendix A, these assumptions lead to the following linear programming problem:



$$\text{minimize } z = \sum_{j=1}^J F_j z_j + \sum_{t=1}^T \sum_{j \in J_t} \sum_{i=1}^I \theta_j c_{jti} x_{jti} \quad (1)$$

subject to:

$$\sum_{j \in J_t} x_{jti} \geq P_{ti} \quad t = 1, \dots, T \quad (2)$$

$$i = 1, \dots, I$$

$$x_{jti} \leq a_{jt} z_j \quad j = 1, \dots, J \quad (3)$$

$$t = 1, \dots, T$$

$$i = 1, \dots, I$$

$$\sum_{j=1}^{J_t} z_j \geq P_{t1} (1 + m) \quad t = 1, \dots, T \quad (4)$$

$$i = 1, \dots, I$$

$$z_j, x_{jti} \geq 0$$

where

$z_j$  = installed capacity in category  $j$

$J$  = index set for categories; there is a category for every combination of generation class and vintage (year of commissioning)

$J_t$  = subset of category indices corresponding to a vintage less than or equal to  $t$

$x_{jti}$  = power output of category  $j$  in increment  $i$  of the LDC for period  $t$

$c_{jti}$  = discounted marginal cost associated with  $x_{jti}$

$F_j$  = discounted capital costs per unit of capacity in category  $j$ .

The major shortcomings of this model, as discussed by Anderson, have to do with its computational requirements. In particular, the constraints

(3) result in a large LP model. For example, if there are 20 categories, 10 segments in the LDC and 6 periods, there will be 1200 constraints of this type and about 1500 constraints in all. This growth in the size of the LP model was viewed as undesirable and lead to other formulations.

The "z-substitutes" method reported by Anderson in [ 2 ] replaced the variables  $x_{jti}$  by variables  $r_{jti}$  representing the reduction in output of unit  $j$  between increments  $i$  and  $i + 1$  of the LDC:

$$r_{jti} = x_{jti} - x_{jt,i+1} \geq 0 \quad i = 1, \dots, I - 1$$

Since the sum of the power reductions cannot exceed the power capacity, the JTI constraints (3) could be replaced by only JT constraints of the form

$$\sum_{i=1}^I r_{jti} \leq a_{jt} z_j \quad j = 1, \dots, J \quad (3')$$

$$t = 1, \dots, T$$

This change in the formulation eliminated the difficulty of model size.

The state-of-the-art in linear programming software is such that the modified LP model presents no serious computational problems. There are, however, other problems with this formulation. First of all, it requires aggregating the generating units in such a way that important differences in operating characteristics (because of size, design, etc.) and fuel costs (because of fuel type, environmental restrictions or unit location) cannot be considered. This is in addition to the assumption of constant marginal production costs. Thus, the production cost estimation sector of the model is open to question regarding the accuracy of its results.

Beglari and Laughton [ 3 ] present an interesting approach to dealing with this difficulty. In their combined costs approach, they first use a linear programming model similar to the one described by Anderson to

determine capacity additions. Then they simulate the power system with the proposed capacity additions to update the cost factor elements of the LP model. This process is repeated until "convergence is reached," which, as they report, usually takes between two and four iterations.

The second major shortcoming of the classical LP formulation is in the representation of construction financing. Although Anderson does not present it, a construct budget constraint could easily be formulated. In the early years of the planning horizon, however, capacity additions are very lumpy, especially for large high-cost units such as nuclear. Linear programming techniques cannot adequately cope with the lumpy nature of these additions (although Anderson does mention the use of integer variables and integer programming techniques).

On the other hand, linear programming models have several significant advantages. First of all, there are numerous high-quality state-of-the-art software systems for solving linear programming problems. These systems generally provide facilities for matrix generation, report writing, and most importantly, post-optimality analyses. Thus, linear programming models have a clear edge over other optimization approaches simply because of their ease of use and software support. It should be noted that the basic LP model described by Anderson is the point of departure for most of the mixed integer programming models described below.

The nonlinear programming (NLP) models, as described by Anderson, were formulated primarily in response to the size of the initial LP models. In the NLP models, preprocessing of the unit data allows defining a "merit order" for unit loading. Availability factors are ignored and the energy output of each unit is estimated by assigning it in merit order to a "slice" of the LDC, as illustrated in Figure 2.1. The resulting model has only peak

power requirements on generating capacity but has a convex, nonseparable objective function.

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 Figure 2.1 here  
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The nonlinear programming models do not provide a better (more adequate) representation of the generation capacity planning model. Furthermore, it is hard to argue that they provide any significant computational advantage, especially since the post-optimality capabilities of LP models are lost. For these reasons nonlinear programming models will not be considered further.

## 2.2 Dynamic Programming Models

The process of selecting generation units to add capacity to an existing system might be viewed as a sequential decision process. With this point of view it is natural to consider dynamic programming (DP) models of the capacity planning problem. Henault, et al. [ 9 ] propose a DP model in which the state variable is the "system configuration," thus, every possible state is specified a priori. They include in "system configuration" the location, size, and performance of not only generation but transmission equipment as well.

Because of the way in which the state space is defined, any configuration not meeting reliability requirements can be eliminated. The objective is to minimize the sum of (1) discounted costs of incremental state changes, (2) total discounted operating costs and (3) penalty costs associated with violating reliability requirements. The operating cost estimate is based on the (expected) peak demand and a load factor. The authors assume that capacity additions take one period or less to implement, although this does not appear to be a limiting assumption.

The model is limited, however, in that it cannot explicitly deal with constraints which link the planning periods. For example, if projects do require several periods to implement and there are construction budget constraints then the periods are linked together through these constraints. Requiring all possible states to be specified a priori has some clear advantages, in that it allows considerable generality in the specification of the cost functions, etc. At the same time, it is a severe limitation since, in essence, the analyst specifies the solution in setting up the model. Moreover, it means that the number of configurations is likely to be quite small.

The formulation used by Irisari [10] also employs a single state variable; however, in this case the state is the total installed capacity. Irisari assigns a value or "worth" to reliability as measured by LOLP thus treating reliability as part of the criterion rather than as a constraining factor. Irisari assumes a constant load factor for each generating unit and a constant marginal cost in determining the system operating costs, which seems to be a rather severe simplification. Irisari does incorporate a period by period budget constraint, so there is an implicit assumption that capacity additions do not impact the budget except in one period.

Irisari's model could be considered a simplified version of one developed by Petersen [16] which employs a four dimensional state vector whose components are the system's power capacities in four categories: hydro, thermal, nuclear, and peaking turbine. The decision variable is a four dimensional vector representing additions in each category. Petersen minimizes the sum of allocated capital costs and total discounted operating costs over the planning horizon. While this model is flexible with regard to the system configurations, it suffers from the same limitation as Henault

et al., i.e., it does not permit any constraint which links the periods together. Petersen gives a fairly detailed account of the techniques used to reduce the computational effort and reports that problems with 20 year planning horizons could be solved in 5 to 10 minutes (Univac 1108).

Petersen uses a "very efficient algorithm developed by Debanné," [ 7 ] to determine the production costs for a given system state. The procedures developed by Booth [ 5, 6 ] and Jenkins and Joy [12], on the other hand, use detailed production costing simulators in conjunction with dynamic programming. Because of the computational burden imposed, the WASP system developed by Jenkins and Joy [12] allows the state space at each stage (planning period) to be arbitrarily curtailed, possibly resulting in sub-optimization.

It is interesting to note that one-period-at-a-time optimization, or optimization with limited look ahead [ 8 ] is claimed to give results which are comparable to dynamic programming algorithms. There is not, however, any hard evidence in the published literature to support this conjecture. Irisari [10] also gives a similar static heuristic procedure.

In summary, the dynamic programming models in general require the units to be aggregated somehow, either by category as in Petersen [16] or into configurations as in Henault et al. [ 9 ]. In the one case, flexibility is gained at the cost of modelling detail, while in the other, much detail can be incorporated but flexibility is sacrificed. The limiting factor in DP models is the inability to cope with period linking constraints. The positive aspect of DP models is that within this limitation they allow considerable flexibility in the specification of the cost functions.

### 2.3 Mixed Integer Programming Models

Mixed integer programming (MIP) models are usually based on a linear programming formulation similar to the one given by Anderson [ 2 ], with the addition of zero-one variables to account for the lumpy (or fixed charge type) investments required for most generation units. The most significant differences between the MIP models discussed below are in scope (generation only or generation plus transmission), plant mix (whether or not hydro is included), and the treatment of reliability requirements.

The models of primary interest in this report all deal directly with the dynamic nature of the decision process and have planning horizons with four to nineteen periods representing from nineteen to thirty years. A somewhat different model has been suggested by Scherer and Joe [20] for dealing with a static (one decision period) problem of capacity expansion. Their goal was to choose an expansion plan to minimize expansion cost while satisfying an explicit LOLP constraint. The latter was accomplished by enumerating all  $2^n$  system states, where  $n$  is the number of existing plus potential generation units. The model holds some theoretical interest, but has little practical value for large systems.

The MIP models proposed by Noonan [14], Sawey [18], Rowse [17], and Iwayemi [11] exhibit a range of features corresponding to different assumptions and different planning situations. For example, Sawey and Iwayemi include expansion of the transmission network while Noonan and Rowse do not. Only Sawey ignores hydro expansion and only Noonan considers pumped hydro projects.

All four models assume a constant marginal production cost. Rowse, Sawey and Iwayemi allow each existing unit to be individually represented in the model, while Noonan lumps units by category (i.e., hydro, fossil,

peaking turbine, etc.). Rowse and Noonan represent demand as a LDC with 3 and 15 increments, respectively. Sawey and Iwayemi do not use LDC's because they deal with distributed demands, so their demand models are essentially the actual demand, approximate by discrete changes. They use two and four modes (or demand levels) respectively.

The description of the expansion alternatives is somewhat general in all four models. Expansions are identified either by site (Rowse, Sawey) or by category (Noonan, Iwayemi). The size of the expansion may be discrete or continuous within specified limits. Sawey's model is the only one with an explicit capital budget constraint, and his is a one-year budget, i.e., there is no carryover of funds and all project expenditures are assumed to occur in the year of commissioning.

The treatment of reliability varies dramatically between the four models. Iwayemi ignores reliability altogether. Rowse uses a simple reserve or capacity margin constraint. Sawey suggests three different approaches, including a capacity margin constraint, but does not indicate which method was used in his case study. Sawey's other two suggestions were to

- (1) limit the size of any expansion unit to some fraction of the existing system capacity;
- (2) simply inflate the demands to induce a spinning reserve (note that this has an effect on operating cost estimates).

The most sophisticated approach to reliability is taken by Noonan, who uses an explicit stochastic constraint on LOLP. This results in a non-linear constraint set which he subsequently linearized.

Sawey and Rowse used the OPHELIE MIXED system and Iwayemi used APEX III system to solve their respective mixed integer problems. Noonan,



however, developed a special purpose algorithm based on Benders' partitioning [4]. Each of the four models was implemented in a case study and Table 2.1 summarizes some interesting data about the case studies.

- - - - -  
Table 2.1 here  
- - - - -

About the only conclusion to be drawn from Table 2.1 is that the specialized solution procedure seems to permit greater detail in modelling (more time periods, load curve increments and expansion options) for a similar computational effort. This points out a major hindrance to widespread acceptance of MIP models, viz., the lack of computationally effective widely available algorithms for sufficiently general formulations.

MIP models of the generation capacity expansion problem have some of the desirable properties of LP models, particularly modelling flexibility in characterizing the expansion options and in constraining the solution. The most significant weakness of the MIP models per se is in the production costing sector, where the assumption of constant marginal production cost is overly restrictive. In addition, at the current state of practice, MIP models are not computationally attractive when compared to LP models. Their attractiveness improves considerably, however, when compared to simulation approaches to expansion planning.

#### 2.4 Summary

In comparing the three categories of optimization models, the following conclusions are reached:

- (1) LP models currently offer the greatest flexibility and have the most highly developed solution procedures. LP models are computationally feasible for in-depth parametric studies.

- (2) DP models admit considerable flexibility in their cost structures. Unfortunately, computational considerations usually require excessive aggregation. DP solution procedures must be "tailor-made" and do not offer the post-optimality capabilities of linear programming.
- (3) MIP models have the same modelling flexibility offered by LP models. Unfortunately the general purpose solution procedures are not currently adequate for detailed, large scale models. Special purpose procedures require computational effort comparable to that for DP models.

MIP models seem to offer the best chance for overcoming the inherent limitation of linear programming models, i.e., the inability to cope with large fixed costs. There is need for additional research along two lines:

- (1) improving the production costing sector of the model, especially relaxing the assumption of constant marginal production costs;
- (2) developing more computationally efficient solution procedures and techniques for post-optimality analysis.

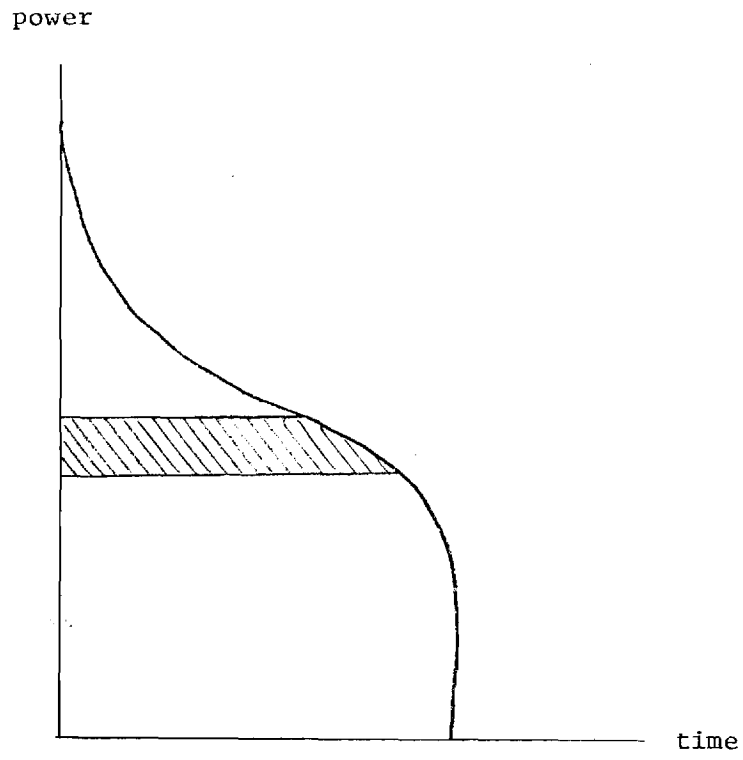


Figure 2.1. Merit Order Loading

Table 2.1. Statistics for Four Case Studies

STATISTIC	Iwayemi	Noonan	Rowse	Sawey
Solution procedure	APEX III	Special	OPHELIE	OPHELIE
Number of periods	6	19	4	15
Period length	5	1	3	1
Segments in load curve	4	15	3	2
Expansion projects per year	7	20	6	7
Binary variables	30	380,0*	?	86
Continuous variables	262	38,255*	?	1294
Constraints	198	19+,27*	?	973
Solution time (CPU sec.)	?	426	~ 450	600
Machine	CDC 6600	CDC 6600	CDC?	CDC?

\*Benders' master problem and each of 19 subproblem LP's

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## Chapter 3

### A Production Costing Model for Electric Utilities

## A Production Costing Model for Electric Utilities

The long range planning of generation capacity for electric power systems is a complex problem requiring the contributions of many technical fields in its solution. In particular, there has been considerable interest, as evidenced by [20, 27], in the use of management science techniques for studying these problems.

Models of the long range generation capacity planning problem (hereafter, the generation planning problem, or GPP) have been proposed, based on linear programming [1, 3, 6, 18, 19], nonlinear programming [7, 26], dynamic programming [15, 22], and mixed integer programming [12, 20, 24] formulations, as well as simulation procedures [8, 9, 16] and procedures which attempt to combine simulation with optimization [2, 4, 11, 21]. A fundamental issue dealt with in all these models is the economic trade-off between capital costs and operating costs. This paper addresses the operating cost sector in models for the long range GPP.

The approach taken here represents a significant departure from the traditional approach in the following sense. Traditionally, especially in simulation models, the production costing sector has been quite detailed, usually representing each generating unit in the system as a distinct entity. As a result, the models have been required to incorporate considerable detail regarding unit operations, and quite often are capable of producing detailed accounting type reports for each unit. In short, the models have been very precise in the production costing sector.

The price extracted for having a precise representation is two-fold. In the first place, the greater the modelling detail, the greater the burden for parameter estimation and data base management. Secondly, the solution techniques applied to these models become ever more costly as the



level of detail increases. Especially in long range planning (twenty years or more) one must question whether or not the price paid for the precise model can be justified, simply because of the uncertainty associated with the estimates of parameter values.

On the other hand, the models are of no value if they are not accurate, i.e., when the model indicates economic superiority for one plan over another, it should be correct. Thus, for long range planning, one may desire a computationally tractable model that is accurate even though it may not be precise in the sense discussed above. In the face of massive uncertainty regarding fuel availability and costs, equipment costs, technological developments, etc., such a model could be used to develop strategies, with detailed evaluation and planning relying on more detailed, or precise, models.

With this in mind, the research reported here is directed toward developing an approach to the production costing sector which is computationally tractable while at the same time meets the requirement for accuracy. The basic idea is one which many experienced system planners will initially scoff at, but the empirical results indicate success in both computational efficiency and accuracy. The approach is described below assuming that production costs are required for only one time period. The basic approach is then evaluated by comparing the results obtained to those obtained using a large scale simulation for a major southeastern utility system. Finally, the adaptation to a mixed integer programming formulation of the generation capacity planning problem is discussed.

### 3.1 Production Costing for a Single Time Period

The goal of this production costing model is to efficiently provide a reasonable estimate of the overall system operating cost for each time period in the horizon. The behavior of individual units and any other information is considered extraneous to the overall expansion planning problem. For the one period production costing model, three basic assumptions are made. First, it is assumed that a reliable demand forecast is available in the form of a load duration curve. Next, units can be classified as being one of two operating types: (1) continuous levels of output between a minimum and maximum capacity, and (2) units which are either "on" or "off," i.e., they have one discrete production level. Finally it is assumed that the given system configuration meets reliability criteria and maintenance scheduling feasibility.

The approach described here for obtaining an estimate of production cost for the system consists of two general steps, determining a system production cost function and computing expected production cost. For the initial step, production cost as a function of production level is developed first for two aggregate units, an aggregate continuous production unit and an aggregate discrete production unit. To obtain the system cost for the period, the two aggregate cost curves are combined into a system production cost curve. The product of the system cost function and the load duration function is integrated with respect to time to compute the desired cost estimate. Since the second step is simply computational, interest is centered in developing the system production cost function.

Establishing the cost-output relationship for continuous units is performed using a marginal costing approach illustrated below. For the second type, or "discrete" units, the production cost relationship is established

using dynamic programming principles described later. The overall production cost curve is then derived by solving a simple optimization problem where demand is varied parametrically over the relevant range.

### 3.1.1 Marginal Cost Approach for Continuous Units

All generation units in a power system are assumed to have individual production (or fuel) costs which assume the quadratic form:

$$PC = Ax^2 + Bx + C$$

where PC is the production cost per hour and x is the production level. For this analysis, a continuous unit is one which can produce power x at a continuous level between prescribed upper and lower bounds.

For a system which has only continuous units, all of which are considered to be operating within their production limits, the production costing problem at a given demand level, D, may be formulated as a convex optimization problem (assuming  $A > 0$ ,  $B > 0$ ):

$$(P1) \quad \text{minimize} \quad TC(D) = \sum_i (A_i x_i^2 + B_i x_i + C_i) \quad (1)$$

$$\text{subject to:} \quad \sum_i x_i = D \quad (2)$$

$$L_i \leq x_i \leq U_i \quad i = 1, 2, \dots, N \quad (3)$$

where

$TC(D)$  = hourly production cost at production level D

$A_i, B_i, C_i$  = coefficients of quadratic production cost curve for unit i

$x_i$  = production or output level of unit i

$L_i$  = lower bound on production for unit i

$U_i$  = upper bound on production for unit i

Solving (P1) for every possible  $D$  between  $\sum_i L_i$  and  $\sum_i U_i$  will yield the desired relationship between production cost and output.

Because the objective in (P1) is convex, a Kuhn-Tucker point will provide an optimal solution. In order to characterize the Kuhn-Tucker points in a helpful way, it is convenient to restate (P1) with the associated dual variables shown in brackets:

$$(P2) \quad \text{minimize} \quad \sum_i A_i x_i^2 + B_i x_i + C_i$$

$$\text{subject to: } D - \sum_i x_i = 0 \quad [u] \quad (4)$$

$$x_i - L_i \leq 0 \quad i = 1, 2, \dots, N \quad [v_i] \quad (5)$$

$$U_i - x_i \leq 0 \quad i = 1, 2, \dots, N \quad [w_i] \quad (6)$$

Letting  $\mu_i(x_i)$  denote the marginal production cost ( $2A_i x_i + B_i$ ) for unit  $i$ ,  $i = 1, 2, \dots, N$ , the Kuhn-Tucker conditions for (P2) can be written as:

$$D - \sum_i x_i = 0; \quad x_i \geq L_i; \quad x_i \leq U_i \quad i = 1, 2, \dots, N \quad (7)$$

$$v_i, w_i \geq 0 \quad i = 1, 2, \dots, N \quad (8)$$

$$v_i(x_i - L_i) = 0 \quad i = 1, 2, \dots, N \quad (9)$$

$$w_i(U_i - x_i) = 0 \quad i = 1, 2, \dots, N \quad (10)$$

$$\mu_i(x_i) - u + v_i - w_i = 0 \quad i = 1, 2, \dots, N \quad (11)$$

From the conditions (7) - (11) it is clear that:

$$v_i > 0 \quad \text{only if} \quad x_i = L_i \quad (12)$$

$$w_i > 0 \text{ only if } x_i = U_i \quad (13)$$

$$\text{if } L_i < x_i < U_i \text{ then } \mu_i(x_i) = u \quad (14)$$

Now a simple characterization of an optimal solution to (P2) is at hand. Corresponding to each demand level,  $D$ , there is a "system marginal cost,"  $u$ . In satisfying the demand  $D$ , each individual unit operates in one of three states: either  $x_i = L_i$ ,  $L_i < x_i < U_i$ , or  $x_i = U_i$ . If  $x_i = L_i$  then from (11) and (12) we have  $\mu_i(x_i) \geq u$ . If  $L_i < x_i < U_i$ , then from (11) and (14) we have  $\mu_i(x_i) = u$ . If  $x_i = U_i$ , then from (11) and (13) we have  $\mu_i(x_i) \leq u$ .

Thus, (P2) can be solved for a given  $D$  by searching for the optimal value for  $u$ . For each value of  $u$ , the corresponding "optimal"  $x_i$  can be readily determined from the above conditions on  $\mu_i(x_i)$ , i.e., choose  $x_i$  so that  $\mu_i(x_i) = u$  unless this violates the bound constraints on  $x_i$ , in which case it is pegged at the appropriate bound.

Note that it is a simple matter to solve (P2) for a range of values of  $D$ . For example, start with  $D = \sum_i L_i$ , and determine the optimal solution. Increasing  $D$  above this level corresponds to increasing  $u$ , the "system" marginal cost. As  $u$  increases, individual units change their production status, i.e., they begin to produce above  $L_i$  or they get pegged at  $U_i$ . Since these changes are easy to keep track of, it is easy to generate the entire aggregate marginal production cost or total production cost function. Note that if there are  $n$  units in the system there will be at most  $2n$  breakpoints in the piecewise linear aggregate marginal cost curve, one for each status change for each unit.

### 3.1.2 Continuous Units - Marginal Cost Curve Algorithm

The desired result of this procedure is the specification of the relationship between marginal cost and demand level for all continuous

units in the system so that total cost may then be determined. Due to the piecewise linearity of the aggregate marginal cost curve, it may be recorded as a series of breakpoints. Breakpoints exist where the marginal cost corresponds (1) to a level at which a unit produces above its lower bound production level, and (2) to a level at which a unit's output must become set at its upper bound production level. The algorithm proceeds by incrementing marginal cost to the next breakpoint quantity and then calculating output at that point. Steps in the routine are as follows:

0. [Initialize]

For each unit  $i \in I$  determine

$$\mu_i^U \leftarrow 2A_i U_i + B_i$$

$$\mu_i^L \leftarrow 2A_i L_i + B_i$$

1. [Find smallest marginal cost]

$$\mu \leftarrow \min_i (\mu_i^L)$$

2. [Partition units]

$$\text{For all } i \in J = \{i: \mu_i^L = \mu\} \quad x_i \leftarrow L_i$$

$$\text{For all } i \in K = \{i: \mu_i^L > \mu\} \quad x_i \leftarrow L_i$$

$$\text{For all } i \in L = \{i: \mu_i^U \leq \mu\} \quad x_i \leftarrow U_i$$

(Note: initially,  $L = \emptyset$ )

3. [Initial breakpoint on curve]

$$t \leftarrow 1$$

$$\mu_t \leftarrow \mu$$

$$D_t \leftarrow \sum_i L_i$$

4. [Find next breakpoint]

a. For each  $i \in J$  determine  $\delta_i \leftarrow 2A_i[U_i - x_i]$

For each  $i \in K$  determine  $\delta_i \leftarrow 2A_i L_i + B_i - \mu_t$

(the  $\delta$ 's are the minimum change in  $\mu$  that would result  
in a breakpoint from unit  $i$ )

$$t \leftarrow t + 1$$

b. Set  $\delta_J \leftarrow \delta_p \leftarrow \min_{i \in J} (\delta_i, \infty)$

$$\delta_K \leftarrow \delta_q \leftarrow \min_{i \in K} (\delta_i, \infty)$$

c. If  $\delta_J < \delta_K$  then:  $\mu_t = \mu_{t-1} + \delta_J$

(unit  $p$  reaches  
its upper  
limit next)

$$J \leftarrow J - \{p\}$$

$$L \leftarrow L + \{p\}$$

$$x_i \leftarrow (\mu_t - B_i)/2A_i \quad i \in J$$

$$x_p \leftarrow U_p$$

$$D_t \leftarrow \sum_i x_i$$

else:  $\mu_t = \mu_{t-1} + \delta_K$

(unit  $q$  starts  
to produce  
above its lower  
limit next)

$$K \leftarrow K - \{q\}$$

$$J \leftarrow J + \{q\}$$

$$x_i \leftarrow (\mu_t - B_i)/2A_i \quad i \in J$$

$$D_t \leftarrow \sum_i x_i$$

d. If  $J = \emptyset$  stop; else repeat 4.

### EXAMPLE

<u>UNIT NO.</u>	<u>L<sub>i</sub></u>	<u>U<sub>i</sub></u>	<u>A<sub>i</sub></u>	<u>B<sub>i</sub></u>	<u>C<sub>i</sub></u>
1	10	20	1	1	1
2	15	40	1	3	1
3	10	30	1	2	1

0. <u>UNIT NO.</u>	<u>L<sub>i</sub></u>	<u>U<sub>i</sub></u>
1	21	41
2	33	83
3	22	62

1.  $\mu \leftarrow \min (21, 33, 22) = 21$
2.  $J = \{1\} \quad x_1 = 10$   
 $K = \{2, 3\} \quad x_2 = 15, x_3 = 10$   
 $L = \emptyset$
3.  $t \leftarrow 1 \quad \mu_1 \leftarrow 21 \quad D_1 \leftarrow 10 + 15 + 10 = 35$
4. a.  $\delta_1 \leftarrow 2(1)(20 - 10) = 20$   
 $\delta_2 \leftarrow 2(1)(15) + 3 - 21 = 12$   
 $\delta_3 \leftarrow 2(1)(10) + 2 - 21 = 1$   
 $t \leftarrow 1 + 1 = 2$ 
  - b.  $\delta_J \leftarrow \delta_p \leftarrow \min (20)$   
 $\delta_K \leftarrow \delta_q \leftarrow \min (12, 1)$
  - c.  $\delta_J > \delta_K \quad \mu_2 \leftarrow 21 + 1 = 22$   
 $K = \{2\}$   
 $J = \{1, 3\}$   
 $x_1 \leftarrow (22 - 1)/2 = 10.5$   
 $x_2 \leftarrow (22 - 2)/2 = 10$   
 $D_2 \leftarrow 10.5 + 10 + 15 = 35.5$
  - d.  $J \neq \emptyset$  and  $K \neq \emptyset$ . Go to step 4.
4. a.  $\delta_1 \leftarrow 2(1)[20 - 10.5] = 19$   
 $\delta_3 \leftarrow 2(1)[30 - 10] = 40$   
 $\delta_2 \leftarrow 2(1)(15) + 3 - 22 = 11$   
 $t \leftarrow 3$ 
  - b.  $\delta_J \leftarrow 19 \leftarrow \min (19, 40)$   
 $\delta_K \leftarrow 11 \leftarrow \min (11)$
  - c.  $\delta_J > \delta_K \quad \mu_3 = 22 + 11 = 33$   
 $K \leftarrow \emptyset$   
 $J \leftarrow \{1, 2, 3\}$   
 $x_1 \leftarrow (33 - 1)/2 = 16$   
 $x_2 \leftarrow (33 - 3)/2 = 15$   
 $x_3 \leftarrow (33 - 2)/2 = 15.5$   
 $D_3 \leftarrow 16 + 15 + 15.5 = 46.5$
  - d.  $J \neq \emptyset$  and  $K \neq \emptyset$ . Go to step 4.



4. a.  $\delta_1 \leftarrow 2(20 - 16) = 8$   
 $\delta_2 \leftarrow 2(40 - 15) = 50$   
 $\delta_3 \leftarrow 2(30 - 15.5) = 14.5$   
 $t \leftarrow 4$
- b.  $\delta_J \leftarrow \delta_1 \leftarrow \min(8, 50, 14.5)$   
 $\delta_K \leftarrow \infty$
- c.  $\delta_J < \delta_K$      $\mu_4 = 33 + 8 = 41$   
 $J \leftarrow \{2, 3\}$   
 $L \leftarrow \{1\}$   
 $x_2 \leftarrow (41 - 3)/2 = 19$   
 $x_3 \leftarrow (41 - 2)/2 = 19.5$   
 $\mu_1 \leftarrow 20$   
 $D_4 \leftarrow 20 + 19 + 19.5 = 58.5$
- d.  $J \neq \emptyset$ . Go to step 4.
4. a.  $\delta_2 \leftarrow 2(40 - 19) = 22$   
 $\delta_3 \leftarrow 2(30 - 19.5) = 21$   
 $t \leftarrow 5$
- b.  $\delta_J \leftarrow \delta_3 \leftarrow \min(22, 21)$   
 $\delta_K \leftarrow \infty$
- c.  $\delta_J < \delta_K$      $\mu_5 = 41 + 21 = 62$   
 $J \leftarrow \{2\}$   
 $L \leftarrow \{1, 3\}$   
 $x_2 \leftarrow (62 - 3)/2 = 29.5$   
 $x_3 \leftarrow 30$   
 $D_5 \leftarrow 20 + 29.5 + 30 = 79.5$
- d.  $J \neq \emptyset$ . Go to step 4.
4. a.  $\delta_2 \leftarrow 2(40 - 29.5) = 21$   
 $t \leftarrow 6$
- b.  $\delta_J \leftarrow \delta_2 = 21$   
 $\delta_K \leftarrow \infty$
- c.  $\delta_J < \delta_K$      $\mu_6 = 62 + 21 = 83$   
 $J \leftarrow \emptyset$

$$L \leftarrow \{1, 2, 3\}$$

$$x_2 \leftarrow 40$$

$$D_6 \leftarrow 20 + 40 + 30 = 90$$

d.  $J = \emptyset$ . Stop.

Breakpoints on curve are:

(35, 21) (35.5, 22) (46.5, 33) (58.5, 41) (79.5, 62) (90, 83)

Using this algorithm, the marginal cost-output relationship is established for all continuous units. The total production cost curve can be developed by simultaneously recording total cost at each breakpoint on the marginal cost curve. In order to complete the costing for the entire system, the next step is to establish the total cost-demand relationship for the category of units which operate in a discrete fashion. This will be done with the following technique.

### 3.1.3 Cost Approach for Discrete Units

To be considered in this category are those units which are assumed to be operated at discrete production levels. Consequently their production costs are discontinuous; i.e. there are only certain cost-output values to consider. Only two such values are considered, one associated with lower bound operation ( $AL_i^2 + BL_i + C_i$ ) and the other with the upper bound operation ( $AU_i^2 + BU_i + C_i$ ). To simplify, this discussion will further assume a zero value for all lower bound outputs, although it is easy to modify the analysis to include nonzero lower bounds. This category is intended to include gas-fired or peaking turbines.

For a given demand level,  $D$ , the production costing problem for a system with only discrete units may be formulated as:

$$(P3) \quad \text{minimize} \quad TC(D) = \sum_i c_i y_i \quad (15)$$

$$\begin{aligned} \text{subject to:} \quad & \sum_i U_i y_i = D & (16) \\ & y_i = 0, 1 \quad \forall i \end{aligned}$$

where

TC(D) = total production cost per hour for the system at output level D

$$c_i = A_i U_i^2 + B_i U_i + C_i = \text{cost per hour of producing for unit } i$$

$U_i$  = upper bound production level for unit  $i$

$$y_i = \begin{cases} 1 & \text{if unit } i \text{ produces at upper bound level} \\ 0 & \text{if unit } i \text{ produces at lower bound level} \end{cases}$$

However, (P3) is a binary knapsack problem for which the following recursive relationship may be applied:

$$f_k(b) = \min_{y_k=0,1} c_k y_k + \min \sum_{j=1}^{k-1} c_j y_j \quad (17)$$

$$\text{subject to:} \quad \sum_{j=1}^{k-1} U_j y_j = b - U_k y_k \quad (18)$$

$$y_j = 0, 1 \quad j = 1, \dots, k-1$$

In a straightforward manner, the recursion can be rewritten as

$$f_k(b) = \min_{y_k=0,1} (c_k y_k + f_{k-1}(b - U_k y_k)) \quad \begin{array}{l} k = 1, 2, \dots, N, \\ b = 0, 1, \dots, D \end{array} \quad (19)$$

Since the output constraint (16) has been written as an equality,  $f_k(b_n)$  will only be defined for certain discrete values of  $b$ ; in particular those which satisfy

$$\sum_{i=1}^k U_i y_i = b$$

for some specification of the binary variables. Thus, the state space may be reduced appropriately. Also, at any stage of the solution, the particular solution  $\{y_i: i = 1, \dots, k\}$  which gives the minimum cost is not important. Rather, determining this minimum cost is the issue of concern. Thus, solution tables need not be maintained at each stage of the recursive solution. These two observations lead to a computationally efficient procedure for solving (P3).

### 3.1.4 Discrete Units - Dynamic Programming Algorithm

The algorithm for generating the discrete aggregate production cost is presented below. The algorithm considers each discrete unit in turn and examines all current output levels to see if a new output level can be created or if a lower cost has been found for an existing output level.

0. [Initialize]

$$\beta_0 \leftarrow \{0\}, t_0 \leftarrow 0, d_0 \leftarrow 0$$

$$k \leftarrow 0, I \leftarrow 1$$

1. [Select the next unit]

$$k \leftarrow k + 1$$

$$\beta_k \leftarrow \beta_{k-1}$$

2. [Attempt to modify existing points or create new ones]

For each  $b_p \in \beta_{k-1}$  in descending order

DO

Compute  $\delta \leftarrow b_p + U_k$

IF  $\delta = d_q \in \beta_{k-1}$  THEN  $t_q \leftarrow \min \{t_q, t_q + c_k\}$  (Modify existing point)

ELSE DO

$$\beta_k \leftarrow \beta_k + \{\delta\}$$

(Create a new point)

$$I \leftarrow I + 1$$

$$t_I \leftarrow t_p + c_k$$

$$d_I \leftarrow \delta$$

END

END

3. If  $t < n_d$  go to 1, else STOP.

EXAMPLE

<u>UNIT NO.</u>	<u><math>c_i</math></u>	<u><math>a_i</math></u>
1	3	2
2	4	2
3	2	3

0. [Initialize]

$$\beta_0 \leftarrow \{0\}; t_0 \leftarrow 0, d_0 \leftarrow 0; k \leftarrow 0; I \leftarrow 1$$

1. [Select Unit 1]

1.  $\beta_1 \leftarrow \{1\}; k \leftarrow 1$

2.  $\delta \leftarrow 0 + 2 = 2$

$$\delta \notin \beta_0 \Rightarrow \beta \leftarrow \{0, 2\} \quad (\text{Create a new point})$$

$$I \leftarrow 1 + 1 = 2$$

$$t_1 \leftarrow 0 + 3 = 3$$

$$d_1 \leftarrow 2$$

3.  $1 < 3$  so go to step 1.

$$\text{Current } (d_i, t_i) \text{ set} \longrightarrow (0, 0) (2, 3)$$

2. [Select Unit 2]

1.  $\beta_2 \leftarrow \{0, 2\}, k \leftarrow 2$

2.  $\delta = 2 + 2 = 4 \notin \beta_1 \Rightarrow \beta_2 = \{0, 2, 4\} \quad (\text{Create a new point})$

$$I \leftarrow 2 + 1 = 3$$

$$t_2 \leftarrow 3 + 4 = 7$$

$$d_2 \leftarrow 4$$

$$\delta = 0 + 2 = d_1 \in \beta_1 \Rightarrow t_1 = \min \{3, 4\} = 3 \quad (\text{Check existing point})$$

$$\text{Current } (d_i, t_i) \text{ set} \longrightarrow (0, 0) (2, 3) (4, 7)$$

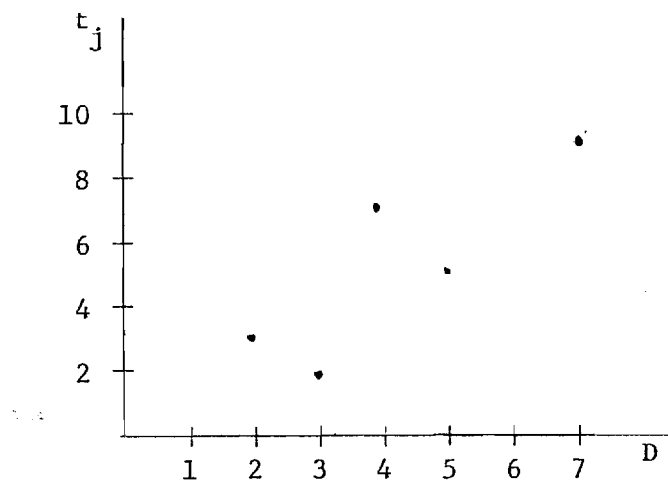


Figure 3.1

## 3. [Select Unit 3]

1.  $\beta_3 \leftarrow \{0, 2, 4\}$ ,  $k \leftarrow 3$

2.  $\delta \leftarrow 4 + 3 = 7 \notin \beta_2 \Rightarrow \beta_3 \leftarrow \{0, 2, 4, 7\}$  (Create a new point)

$I \leftarrow 4$

$t_3 \leftarrow 7 + 2 = 9$

$d_3 \leftarrow 7$

$\delta \leftarrow 2 + 3 = 5 \notin \beta_2 \Rightarrow \beta_3 \leftarrow \{0, 2, 4, 5, 7\}$  (Create a new point)

$I \leftarrow 5$

$t_4 \leftarrow 3 + 2 = 5$

$d_4 \leftarrow 5$

$\delta \leftarrow 0 + 3 = 3 \notin \beta_2 \Rightarrow \beta_3 \leftarrow \{0, 2, 3, 4, 5, 7\}$  (Create a new point)

$I \leftarrow 6$

$t_5 \leftarrow 0 + 2 = 2$

$d_5 \leftarrow 3$

3.  $3 = 3$  STOP.

The  $(d_i, t_i)$  points on the curve are (0, 0) (2, 3) (3, 2) (4, 7) (5, 5) (7, 9) and are shown graphically in Figure 3.1.

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Figure 3.1 here  
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Note that the costs determined for the aggregate discrete unit are not necessarily monotone with output. This is primarily an end point difficulty, occurring for small values of output and possibly for values near the maximum aggregate output.

3.1.5 Developing the Overall System Cost vs. Output Curve

The two previous sections have demonstrated methodologies which yield one production cost-output curve for all continuous units and one set of points for production cost versus output for all discrete units. Combining these two curves into one overall total production cost versus output relationship is the next step. For any output level  $D$ , this problem may be

formulated as:

$$(P4) \quad \text{minimize} \quad K(D) = TC_1(x) + \sum_j t_j y_j \quad (20)$$

$$\text{subject to:} \quad x + \sum_j d_j y_j = D \quad (21)$$

$$\sum_j y_j = 1 \quad (22)$$

$$L \leq x \leq U \quad (23)$$

$$y_j = 0, 1$$

where

$x$  = level of production for aggregate continuous unit

$TC_1(x)$  = cost of production at level  $x$  for aggregate continuous unit

$t_j$  = production cost for the  $j^{\text{th}}$  discrete point

$$y_j = \begin{cases} 1 & \text{if the } j^{\text{th}} \text{ discrete point is used} \\ 0 & \text{otherwise} \end{cases}$$

$d_j$  = production level for the  $j^{\text{th}}$  discrete point

$U$  = upper bound on continuous units' production

$L$  = lower bound on continuous units' production

However, (P4) may be written as

$$(P5) \quad K(D) = \min_{j \in J} [t_j y_j + TC(D - d_j)] \quad (24)$$

$$j \in J \forall j: L \leq (D - d_j) \leq U \quad (25)$$

The optimal solution to (P5) will be piecewise convex as illustrated in Figure 3.2.

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 Figure 3.2 here  
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For any given demand level,  $D$ , (P5) is easily solved by simply trying each of the discrete output levels (including zero) and choosing the one which leads to the best total cost. Again, some computational efficiencies can be realized by solving (P5) parametrically, and this is greatly simplified if the discrete points have been "smoothed" so that they are monotone.

### 3.1.6 Computing Estimated Cost for One Period

The final step in the cost estimating procedure is to combine the aggregated system production cost curve with demand information. In standard fashion, suppose that the load duration curve is given with intervals of width  $\theta$ , and load values  $P_i$ ,  $i = 1, 2, \dots, 8760/\theta$ . Then the estimated production cost is given by

$$EPC = \theta \sum_i K(P_i) \quad (26)$$

### 3.2 Empirical Evaluation

The production costing procedure was implemented in FORTRAN IV on a CDC Cyber 74 and tested using data from a large southeastern utility. As shown in Table 1, computational results from this method compare very favorably with those obtained by the utility using a conventional, large scale simulation procedure. For purposes of long range (20-40 years) studies, the accuracy of the method along with its relatively small computational requirements make it quite attractive for implementation within a capacity planning model.

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Table 1 here  
- - - - -

Although the small sample size precludes statistical analysis, the

computational requirements of the method can be described as follows:

$$\text{cpu} = f \left[ T \left[ 1 + 5n_c + 2^{n_d} + \left( \frac{7680}{\theta} \right) (\delta) (2n_c) \right] \right]$$

$$\text{words} \approx 9(n_c + n_d) + 11n_c + n_d + 3\delta$$

where

cpu = execution time

T = number of years in study

$n_c$  = number of continuous units

$n_d$  = number of discrete units

$\delta$  = number of points in discrete units cost function

The storage words include only the major arrays. Other variables and the object code require a constant, approximately 1200 words.

Obviously, both storage and execution time are significantly affected by the number of points in the discrete units cost function. For the sample data there were 37 discrete units which resulted in 1280 points in the associated aggregate production cost function. The number of points,  $\delta$ , can increase quite rapidly with the number of units, in fact the maximum is  $2^{n_d}$ . If  $n_d$  is large, then some method for reducing  $\delta$  is needed, and fortunately, several methods are readily available. The most effective of these appears to be interval partitioning [22].

Certain issues of interest were faced in implementing the aggregation method. One of these issues involved choosing the magnitude of  $\theta$ , the increment size for the load duration curve. For the computational results reported in Table 1, the load duration curves were used as received from the utility, and consisted of 68 segments of unequal duration.

Another implementation issue was how to derate individual unit

capacity to reflect the expected outage rate. Derating was accomplished for the continuous units by assuming more outage on the continuous units when their output is low, and less outage as output increases to maximum.

This approach is based on the observation that scheduled outage for maintenance, refueling, etc. typically occurs during seasons of relatively low demand, e.g. Spring and Fall. As system output approaches its peak, it is desirable to have all units operational. It may be observed, however, that during peak demand unanticipated outages have undesirable effects on system reliability and corresponding cost estimates. For the purposes of this production costing, the impact of unanticipated outages on computational results are shown to have negligible influence in determining a credible production estimate and thus may be reasonably ignored.

Discrete units were not derated under the assumption that typically they are not base load suppliers since their operating costs are relatively higher so their downtime will be scheduled during periods of low demand. Incorporating this technique for handling capacity derating into the overall procedure was not only intuitive and computationally appealing, but provided reasonable results.

A final implementation issue derived from the large number of production points (levels of  $b$  in Eq. 19) obtained for the discrete units cost curve. When solving for total annual production cost ( $P_5$ ), certain low cost points tended to dominate every solution value. Searching the entire list became computationally undesirable. Therefore, the total cost curve for discrete units was represented in the overall aggregate costing by a reduced list. This smaller set was obtained by dividing the discrete total cost curve into segments containing ten adjacent demand-level points and selecting from the ten the point with minimum cost for inclusion in the aggregate costing step.

All of these implementation issues were resolved in such a way that the desired result of relatively accurate and computationally efficient results were obtained. Significantly, this procedure provides a reasonable approach to the production costing requirements for long range capacity expansion planning. Integration of the production costing results into an expansion planning model is considered in the following section.

### 3.3 Incorporation into Capacity Expansion Planning Model

The generation expansion planning problem may be formulated as a constrained optimization problem. The objective is to minimize total costs including production costs and capital outlays. Constraints for the problem include reliability guarantees, budget restrictions, system configuration requirements, and demand satisfaction. Decision variables fall into two broad categories: (1) system configuration specification (a "yes-no" or integer variable for each unit for every time period), and (2) production or output levels for each unit for every time period (a continuous variable).

It has been observed that this type of formulation can be decomposed into two related subproblems: an integer subproblem for project selection and scheduling and a continuous subproblem for production costing [20]. Each distinct integer subproblem solution defines an associated distinct continuous subproblem or production costing problem. The production costing procedure presented here can be incorporated in the continuous subproblem in the following way.

The continuous subproblem for one time period may be modelled as a network flow problem as illustrated in Figure 3.3. In this formulation it is assumed that the load duration segments all have the same width, which

may lead to more segments than are required by some other models [ 3 ].

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Figure 3.3 here  
- - - - -

The annual energy capacity of the existing system and proposed expansion projects are given by  $E_i$ ,  $i = 0, \dots, n$ . For each production unit (existing system or proposed addition) and for energy import ( $n + 1$ st unit) the multiple arcs represent piecewise linear convex production costs, in power terms. Within each segment of the load duration curve, the power requirement is  $P_t$ ,  $t = 1, \dots, 8760/\theta$ .

The derating scheme discussed earlier may also be applied to the network model. All that is required is that for each unit there must be different capacities and costs for each interval of the load duration curve. Note that the complete model is simply a convex cost transportation problem, which can be readily solved using any one of a number of modern network flow codes, e.g., GNET [10], NETFLO [14] or PNET [13].

#### 3.4 Summary

Production cost estimation is a crucial component of any generation capacity expansion model. The accuracy of the estimates is of obvious concern to system planners. In addition, the computational effort required for the estimation is a key factor since it determines to a large degree how frequently or extensively the model can be used.

This paper has presented a new approach to production cost estimation which has considerable promise for enhancing GPP models. While the model does not provide detailed estimates of unit loads, it does provide accurate estimates of total production cost. Furthermore, the computational requirements are quite modest and the approach is flexible enough to be incorporated in a variety of different models.

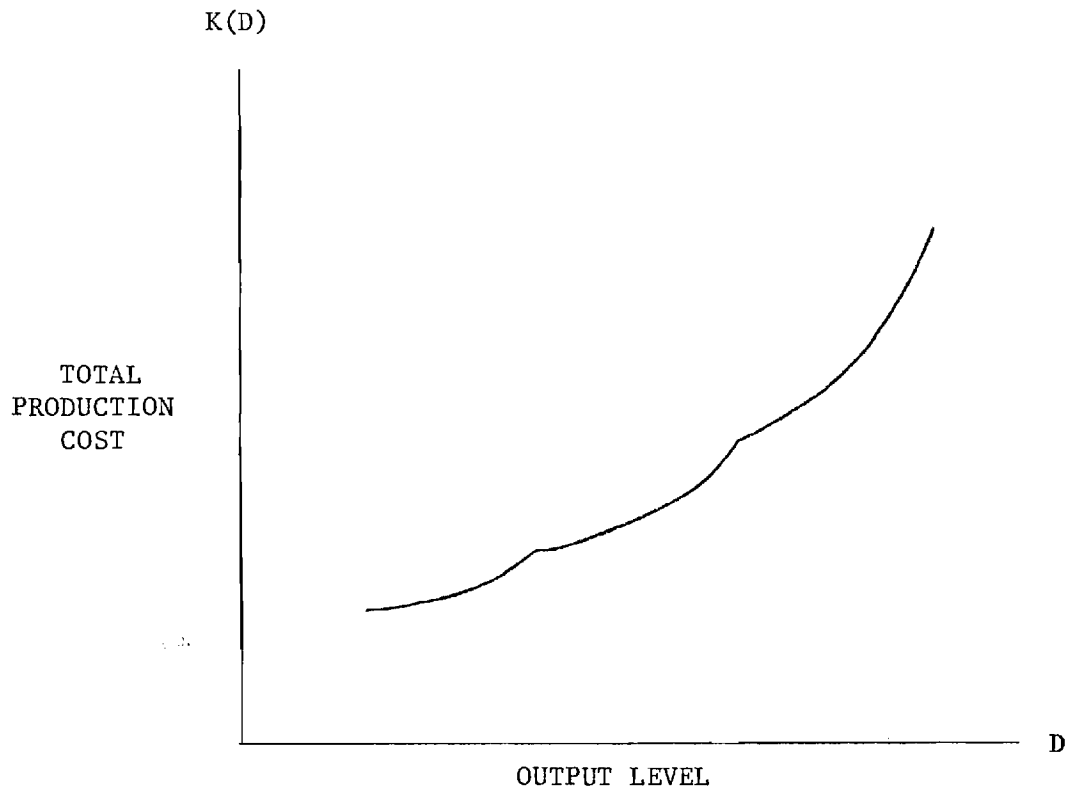
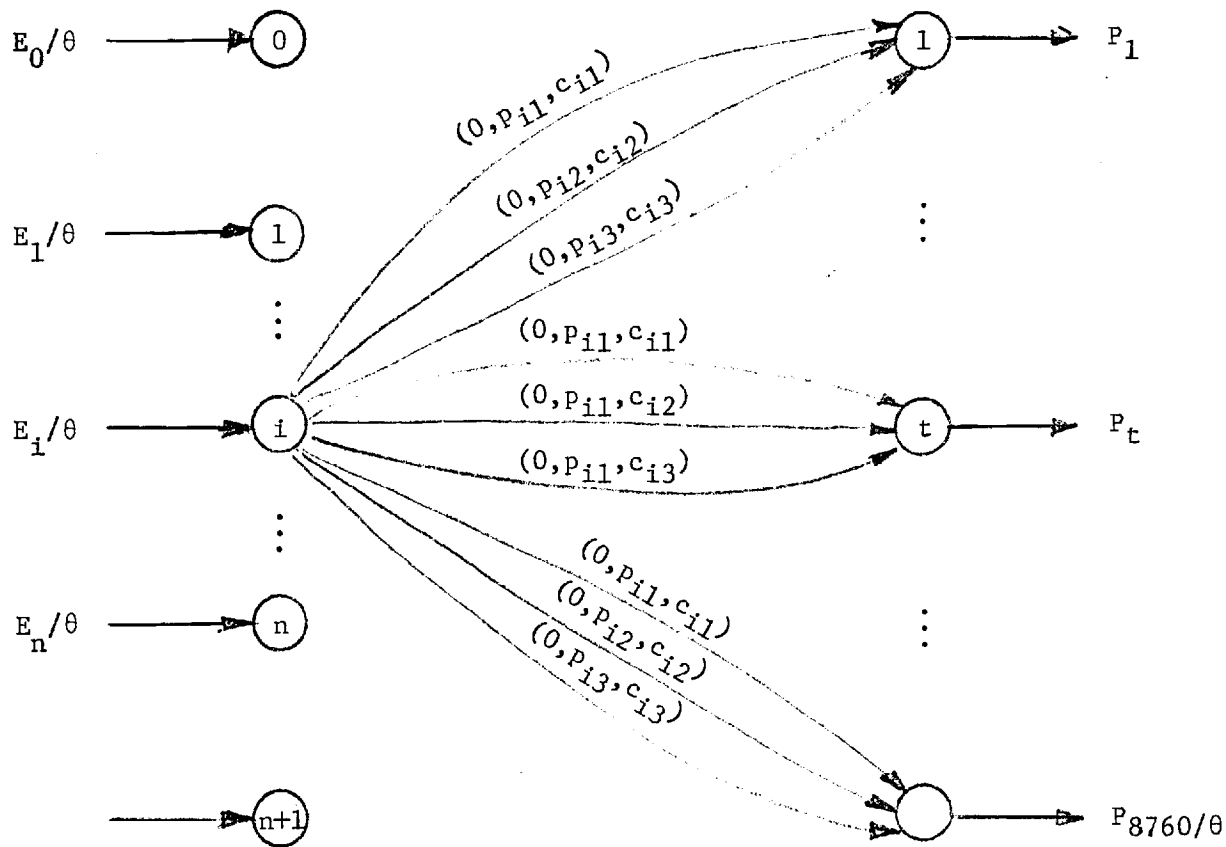


Figure 3.2. Typical Total Cost vs. Output Curve



(a, b, c)

a = lower bound on flow

b = upper bound on flow

c = per unit flow cost

Figure 3.3. Network Flow Model for Production Costing

Table 1. Computational Results Using Two Production Costing Methods for 4 Year Study of Large Southeastern Utility

	<u>Year</u>	<u>Utility's Simulation Method</u>	<u>Aggregation Procedure</u>	<u>Difference</u>	<u>Per Cent Difference</u>
Estimated Total Operating Cost (\$1000)	19 x 1	1,108,095	1,080,042	-28,053	-2.532
	19 x 2	1,334,934	1,340,301	5,367	.402
	19 x 3	1,535,235	1,548,168	12,932	.842
	19 x 4	1,770,274	1,746,355	-23,919	-1.351
	TOTAL	5,748,538	5,714,865	-33,673	-.586
Execution Time		45 minutes (AM DOL V6)	11.4 seconds (CDC Cyber 74) (FTN, OPT=2)		
Core Requirements		?	Program 1146 words Arrays 9668 words		



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## Chapter 4

### A Mixed Integer Programming Model for Generation Capacity Planning

## A Mixed Integer Programming Model for Generation Capacity Planning

Long range planning for generation capacity in electric utilities is a very complicated and complex problem. Solutions to the planning problem specify for the near term (one to ten years) the expansion projects to be funded, and indicate for the longer term (ten to forty years) the sizes and types of units to add. To obtain the data for the problem, many different time series must be estimated (demand, fuel availability and cost, construction cost, etc.), the available technologies must be predicted, and the future availability of construction funds must be determined. The problem of looking ahead twenty to forty years to predict economic and technical factors alone is an enormous one.

Given the necessary data regarding future events, a solution to the generation planning problem requires a correct assessment of the impacts associated with the very "lumpy" investments being considered, and must satisfy several very complex constraints. The solution must allow the presented loads to be met, must provide a reasonable level of reliability and must not exceed the budgets predicted over the planning horizon.

Because the generation planning problem is a problem of resource allocation and scheduling in the face of resource and technical constraints, it is natural to try to approach the problem via optimization models. Such models have been suggested by several authors, based on linear programming [ 1, 2, 6], nonlinear programming [ 3, 10], dynamic programming [ 5, 8 ] and mixed integer programming [ 4, 7, 9 ] methods. Each of these models, however, has one or more shortcomings which have resulted in diminished usefulness.

First of all, model adequacy is a problem in many cases. In particular,

some current technologies favor very large generating units. The resulting investments are discrete in nature and simply cannot be considered adequately by models based on continuity assumptions. Thus, linear and non-linear programming models are typically inadequate when these very lumpy resource allocations are to be made.

Another effect of large generating units is relatively long construction periods. In the presence of construction budgets, this means that several periods are "tied together" through the construction budget constraints. Although this does not prevent the use of dynamic programming based models, it does result in state space dimensionality problems. Thus, models based on dynamic programming which incorporate budget restrictions are likely to be severely penalized in computational effort required.

The second major shortcoming of previous models is that when model adequacy is satisfied, the computational effort required when using the model is generally excessive. Because there is so much uncertainty regarding the future, it is desirable to employ post-optimality studies (such as parametric analysis) to determine the robustness or stability of the solutions. Especially for mixed integer programming models, this has been computationally infeasible.

Thus, although mixed integer programming models seem best suited in terms of model adequacy, their computational requirements have limited their utility. This paper introduces a new mixed integer programming model designed in an attempt to overcome this problem. The new model differs from existing models in several respects. First, it incorporates annual limits on new construction funds while allowing carryover of unspent funds from the previous year. Second, it incorporates a new simplified yet accurate approach to production costing. The new approach is computationally

compatible with a large scale mixed integer programming model. Third, the new model treats reliability requirements in a novel fashion and one that is easily understood by system planners.

#### 4.1 Problem Specification

There are many different ways to state the generation planning problem, depending on how various aspects of the problem are treated. This section of the report presents a detailed account of the various problem characteristics which will be incorporated in the mathematical model of the following section.

##### Planning Horizon

It is assumed that the planning horizon extends over  $T$  periods, which need not have the same duration. Let  $\delta_t$ ,  $t = 1, \dots, T$  denote the duration of period  $t$  in years. It may prove desirable to use longer durations for periods further in the future.

In the following discussion, the parameters will generally not be indexed by period, for simplicity. It should be understood that whenever appropriate, a parameter may take on different values in each period. Also, costs and commissionings are assumed to occur at the end of the associated period.

##### Demand

The generation planning problem is driven by the demand for energy and is significantly affected by the distribution of the demand. For the purposes of a long range planning model the demand for energy can be described by a load duration curve (LDC). The LDC for any period is represented by a set of discrete values as illustrated in Figure 4.1 where:

$$\theta_i = \text{width (in hours) of the } i^{\text{th}} \text{ interval of the LDC}$$

$P_i$  = power demand in interval  $i$  (MW)

$P_i \theta_i$  = energy demanded in interval  $i$  (MWH)

The LDC may represent a period of any desired length, e.g., a week, month, year, etc. The period represented by the LDC need not be the same as the period in the planning model. For example, if a two year planning period is used, a "typical" one-year LDC could be employed, provided the cost estimates were multiplied by a factor of two. If  $\lambda$  is the time covered by the representative LDC, then  $\delta_t/\lambda$  LDC's are required in period  $t$ .

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 Figure 4.1 here  
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### Generating Units

The demand for energy is satisfied by operating a system of generation units. It is assumed that the variable operating costs for each unit in \$/mw can be described as a quadratic function:

$$V_j(x) = A_j x^2 + B_j x + C_j$$

where

$V_j(\cdot)$  = variable production cost function for unit  $j$  (\$/MW)

$x$  = production level (MW)

$A_j, B_j, C_j$  = unit specific parameters.

The coefficients  $A_j, B_j,$  and  $C_j$  can take on different values in different periods to reflect unit aging or anticipated changes in fuel costs. It is further assumed that when operating, a unit must operate between two limiting power levels, i.e.,

$$L_j \leq x \leq U_j$$

where

$L_j$  = lower limit on production for unit  $j$



$U_j$  = upper limit on production for unit  $j$

These limits are not absolute, and should be set in light of the quadratic approximation to variable costs.

There are fixed costs associated with the generation units. The definition of these fixed costs in a long range planning model is a subject of some controversy, especially in a model which aggregates to a considerable extent. It is assumed here that the planning period fixed costs include the usual taxes, interest, etc., as well as the fixed operating costs. That is, the costs of start up, shut down, and maintenance that cannot be incorporated into the variable production cost are assumed to be represented in the single planning period fixed cost:

$F_j$  = planning period fixed costs associated with operating  
unit  $j$ .

To some extent, the fixed costs depend on how the unit is operated. In this situation it would be prudent to determine whether or not the solution were sensitive to the precise values of the  $F_j$ .

Generating units are subject to both planned and forced outages. It is assumed that the resulting availability rate,  $a_j$ , is known for each unit (possibly in each planning period). The availability rate is simply the expected number of hours the unit will be available divided by the number of hours in the period.

### Expansion Units

Let  $J$  be the index set for all generating units and let  $J_e$  be the subset of indices for expansion units which may be selected or have already been selected but not yet commissioned. For any unit in  $J_e$ , there is a commissioning "window," i.e., a time period during which the unit must be

commissioned, assuming it is selected for construction (or has already begun construction). Let  $w_j$  denote the index set of time periods in the commissioning window for unit  $j$ ,  $j \in J_e$ .

#### Construction Costs

Regardless of the period  $t \in w_j$  when expansion unit  $j$  is completed, the construction period has the same length,  $d_j$  years. Construction costs are estimated in then-current dollars, and their magnitude depends on the commissioning date. If expansion unit  $j$  is selected for commissioning in period  $t \in w_j$  then the construction costs in each period are denoted  $c_{jkt}$  where  $k = t - d_j + 1, t - d_j + 2, \dots, t$ , assuming planning periods of one year. If the planning periods have other durations, the annual construction costs must be converted as appropriate.

#### Construction Funds

The availability in period  $t$  of new funds for construction is given by  $B_t$ , specified in then-current dollars. Construction funds not fully utilized in period  $t$  may be carried forward into period  $t + 1$ . Such funds are assumed to grow according to a short term investment rate,  $r_{t+1}$ .

#### Reliability

System reliability is commonly measured by the loss of load probability, or LOLP. When the types and sizes of units are given, along with their individual forced outage rates, controlling LOLP amounts to controlling the generation plant mix. Thus, it is assumed that suitable constraints on unit size, proportions of capacity in various classes, etc., can be specified. (The derivation and specification of these constraints is discussed in more detail in following sections.)

Another common practice in capacity planning is to require a certain

capacity margin, i.e., generation capacity in excess of the predicted peak demand. If a margin is required it is denoted by  $m_t$ , such that  $(1 + m_t)$  times the peak power demanded is the required generation capacity.

#### 4.2 Model Formulation

A guiding principle in the model formulation is that the purpose of the model is to reveal the "optimal" (for given parameters) decisions for selecting and commissioning generation units. Thus, the specific details of unit operations are not of interest per se. Rather, they are only of interest as they impact the estimated system production costs.

Following this principle, the units in the existing system are treated in a new and novel way. All units in the existing system are aggregated as discussed in Chapter 3, with the resulting production cost being given by:

$$\begin{aligned} F_{at} &= \text{annual fixed operating costs in period } t \\ &= \sum_{j \in J \setminus J_e} F_{jt} \end{aligned}$$

$$\begin{aligned} V_{at}(x) &= \text{aggregate variable production cost (\$/mw) in period } t \\ &\text{for } j \in J \setminus J_e \end{aligned}$$

As demonstrated in Chapter 3, this approach still yields accurate estimates of the associated production costs, even though it obscures the details of individual units' operations.

Similarly, the production costs for the potential expansion units are:

$$\begin{aligned} F_{jt} &= \text{annual fixed operating costs in period } t, j \in J_e \\ V_{jt}(x) &= \text{variable production cost in period } t (\$/mw), j \in J_e \end{aligned}$$

Also, there are production limits on the aggregate units:

$$L_{at} \leq x \leq U_{at}$$

and the potential expansion units (if they are chosen):

$$L_{jt} \leq x \leq U_{jt} \quad j \in J_e$$

The detailed development of the model is presented in the following sections. Section 4.2.1 deals with the constraints in the production costing sector of the model. Other constraints are discussed in the sections 4.2.2 and 4.2.3. The important issue of criterion specification is addressed in section 4.2.4.

#### 4.2.1 Production Costing Sector

The production costing sector of the model provides the necessary estimates of variable production costs. This sector actually "drives" the model since it is where the demand for electricity is incorporated.

The requirement to satisfy demand as represented by the series of load duration curves can be stated as:

$$x_{ait} + \sum_{j \in J_e} x_{jit} = P_{it} \quad \begin{array}{l} i = 1, \dots, \lambda/\theta \equiv I \\ t = 1, \dots, T \end{array} \quad (1)$$

where

$x_{ait}$  = power output of the aggregate unit during interval  $i$  of the LDC for period  $t$

$x_{jit}$  = power output of expansion unit  $j$  during interval  $i$  of the LDC for period  $t$ .

Note that a uniform increment length,  $\theta$ , a uniform LDC duration,  $\lambda$ , and a uniform number of increments,  $I$ , for each LDC has been assumed. In addition, no expansion unit may be operated prior to its commissioning, and if selected, is available in every period following its commissioning. This

requirement generates the following constraints:

$$L_{jt} \sum_{\substack{\tau \in w_j \\ \tau \leq t}} y_{j\tau} \leq x_{jit} \leq U_{jt} \sum_{\substack{\tau \in w_j \\ \tau \leq t}} y_{j\tau} \quad j \in J_e \quad (2) \\ i = 1, \dots, I \\ t = 1, \dots, T$$

where

$$y_{j\tau} = \begin{cases} 1 & \text{if unit } j \text{ is commissioned in period } \tau \\ 0 & \text{otherwise} \end{cases}$$

For simplicity, it is assumed that  $x_{jit}$  is defined to be zero for periods  $t$  prior to the commissioning window,  $w_j$ .

In order to represent the availability factor, the expansion units are constrained with respect to total energy produced in any period:

$$\sum_{i=1}^I x_{jit} \leq \frac{1}{\theta} U_{jt} a_{jt} \quad j \in J_e \quad (3) \\ t = 1, \dots, T$$

If desirable or necessary, the aggregate unit representing the existing system may be similarly constrained.

Constraints (1)-(3) define the production costing sector of the generation capacity planning model. Because of the assumptions regarding the LDC representation, the production costing sector of the model exhibits a special structure. In later sections of this report, the special structure will be examined and methods will be presented for exploiting the special structure in a solution procedure.

The decisions represented by the variables  $y_{j\tau}$ , i.e., the selection and commissioning decisions are subject to constraints other than those in the production costing sector of the model. There are two types of additional constraints of primary interest here: budget oriented constraints and reliability oriented constraints.

#### 4.2.2 Budget Oriented Constraints

The simplest formulation of the construction budget constraint is:

$$\sum_{j \in J_e} \sum_{p \geq t} c_{jtp} y_{jp} + S_t - (1 + r_t) S_{t-1} = B_t \quad t = 1, \dots, T \quad (4)$$

Constraint (4) allows the carryover of excess construction funds with a short term growth (or investment return) rate of  $r_t$ . This formulation is somewhat restrictive since it requires the annual construction budgets to be specified a priori.

A more complex formulation would treat the budget amount as a decision variable, at least partially. To see how this could be accomplished define the following variables:

$$z_t = \begin{cases} 1 & \text{if additional construction funds are obtained for period } t \\ 0 & \text{otherwise} \end{cases}$$

$\beta_t$  = amount of additional budget funds obtained in period  $t$

Now, constrain  $\beta_t$  between appropriate limits, e.g.:

$$\beta_t^{\min} z_t \leq \beta_t \leq \beta_t^{\max} z_t$$

The extended budget constraint formulation is then

$$\sum_{j \in J_e} \sum_{\substack{p \geq t \\ t \in w_j}} c_{jtp} y_{jp} + S_t - (1 + r_t) S_{t-1} - \beta_t = B_t \quad t = 1, \dots, T$$

$$\beta_t^{\min} z_t \leq \beta_t \leq \beta_t^{\max} z_t \quad t = 1, \dots, T$$

$$z_t = 0 \text{ or } 1 \quad t = 1, \dots, T$$

The reason for defining the zero-one variables,  $z_t$ , is to allow for non-zero lower limits on budget expansion and to allow for fixed costs of

obtaining additional funds.

Although the extended formulation has the obviously desirable property of greater generality, it also has some drawbacks. In particular, it may prove to be computationally undesirable, since it adds another dimension in the decision space. Therefore, the formulation in constraint (4) will be adopted in subsequent developments.

#### 4.2.3 Reliability Oriented Constraints

The reserve margin constraint is the simplest of the reliability oriented constraints and has the following form:

$$U_{at} + \sum_{j \in J_e} U_{jt} \left( \sum_{\tau \leq t} y_{j\tau} \right) \geq L_{1t} (1 + m_t) \quad t = 1, \dots, T \quad (5)$$

Constraint (5) requires that if all units are operated at their maximum rated power, their output exceeds the period's peak power requirement by at least  $m_t \cdot 100\%$ .

As discussed earlier, the other reliability oriented constraints are the plant mix constraints. The idea behind these constraints is that because generation units are discrete entities, constraints on composition can serve the purpose of constraining LOLP.

For example, it is widely recognized that it would probably be undesirable to have a system made up of only a few very large generating units even if they were individually quite reliable. The reason is that if any one unit failed, it would represent a large fraction of total capacity, and thus very large reserve margins would be required. In contrast, relatively small reserve margins might be feasible with much less reliable units, provided that each unit is small enough.

Instead of giving a completely general formulation of the unit mix

constraints, several specific examples will be given. One type of unit mix constraint would restrict the total capacity in some category to be less than a specified fraction of the total system capacity. For example, suppose nuclear fueled generation is to be restricted to less than 40 percent of total capacity. Letting  $J_N$  denote nuclear units, the appropriate constraints are:

$$\sum_{\substack{j \in J_N \\ j \notin J_e}} U_{jt} + \sum_{j \in J_N \cap J_e} U_{jt} \left( \sum_{\tau \leq t} y_{j\tau} \right) \leq .40 \left[ U_a + \sum_{j \in J_e} U_{jt} \sum_{\tau \leq t} y_{j\tau} \right]$$

$$t = 1, 2, \dots, T$$

This formulation yields a mix constraint for each period. Because of the nature of the problem, it may be just as effective to specify the constraint only for four or five periods in the planning horizon, rather than for every period.

A second type of unit mix constraint enforces a proportionality relationship between two classes of generating units. For example, suppose that fossil fuel units are required to be more than 60 percent of the capacity of nuclear units. Letting  $J_F$  denote fossil units, this constraint would be written:

$$\sum_{\substack{j \in J_F \\ j \notin J_e}} U_{jt} + \sum_{j \in J_F \cap J_e} U_{jt} \left( \sum_{\tau \leq t} y_{j\tau} \right) \geq .60 \left[ \sum_{\substack{j \in J_N \\ j \notin J_e}} U_{jt} + \sum_{j \in J_N \cap J_e} U_{jt} \left( \sum_{\tau \leq t} y_{j\tau} \right) \right]$$

$$t = 1, 2, \dots, T$$

Again, it may be desirable to drop all but four or five of the constraints.

Note that these constraints are stated with regard to categories of units. In the examples given above, the categories were based on energy source. It would be just as easy to specify other categories, for example,



based on the magnitude of the forced outage rates.

A third type of constraint would force the solution to "compensate" for selecting a very large or very unreliable unit. Letting  $\alpha_{jt}$  be the unit forced outage rate, this constraint is:

$$\sum_{j \in J_e} (1 - \alpha_{jt}) U_{jt} \sum_{\tau \leq t} y_{j\tau} \geq (1 - \alpha) \left[ \sum_{j \in J_e} U_{jt} \sum_{\tau \leq t} y_{j\tau} + U_{at} \right]$$

$t = 1, 2, \dots, T$

or

$$\sum_{j \in J_e} (\alpha - \alpha_{jt}) U_{jt} \sum_{\tau \leq t} y_{j\tau} \geq (1 - \alpha) U_{at}$$

In this constraint  $\alpha$  is a parameter representing a desired "average" or capacity weighted forced outage rate. Choosing a relatively smaller value for  $\alpha$  would result in relatively more compensation in the solution. As with the other two reliability oriented constraints, this one could be specified for only a few time periods in the planning horizon.

One last set of constraints must be added. Observe that thus far, nothing prohibits more than one  $y_{jt} = 1$  for a given expansion unit, i.e., it might be commissioned more than once. While this is a practical impossibility, the model might call for such a solution in order to satisfy the reliability oriented constraints. Thus, the following constraints are required:

$$\sum_{t \in W_j} y_{jt} \leq 1 \quad j \in J_e \quad (6)$$

#### 4.2.4 Objective Function

A widely accepted criterion for the generation planning problem is the minimization of the total discounted costs, including operating costs and expansion costs. The present formulation, however, treats the construction

budget as fixed, so that expansion costs are not properly included in the objective function, with two minor exceptions.

If there are unspent construction funds at the end of period T, they can be treated as a savings and netted out against the operating costs. Thus, all costs (operating as well as construction) should be estimated in then-current dollars, and discounted by an appropriate factor, say  $\gamma$ . The resulting objective function to be minimized is the discounted total cost:

$$DTC = \sum_{t=1}^T \left\{ \gamma^t \left[ F_{at} + \theta \sum_{i=1}^I v_{at}(x_{ait}) + \sum_{j \in J_e} \left( F_{jt} + \sum_{\tau \leq t} y_{j\tau} + \theta \sum_{i=1}^I v_{jt}(x_{jit}) \right) \right] \right\} - \gamma^T S_T \quad (7)$$

If the extended formulation of the budget constraints is to be used, then the costs associated with obtaining additional funds must be included in the objective function. This formulation can be viewed as equivalent to the minimization of total discounted costs by letting the  $B_t$  be zero and appropriately specifying the costs associated with  $z_t$  and  $\beta_t$ .

The objective function as stated in (7) is highly nonlinear. In addition to the fixed costs, it has convex variable costs. The solution procedures discussed in Chapters 5 and 6 exploit the problem's special structure to overcome the nonlinearities. It is assumed, however that the convex cost functions,  $V_{at}(\cdot)$  and  $V_{jt}(\cdot)$ ,  $j \in J_e$  are piecewise linearized as illustrated in Figure 4.2.

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Figure 4.2 here  
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### 4.3 Model Use

The complete mixed integer programming model for the generation planning problem is

$$\text{minimize DTC} \quad (8)$$

subject to:

$$x_{ait} + \sum_{j \in J_e} x_{jit} = P_{it} \quad i = 1, \dots, I \quad (9)$$

$$t = 1, \dots, T$$

$$L_{jt} \sum_{\substack{\tau \in W_j \\ \tau \leq t}} y_{j\tau} \leq x_{jit} \leq U_{jt} \sum_{\substack{\tau \in W_j \\ \tau \leq t}} y_{j\tau} \quad j \in J_e \quad (10)$$

$$i = 1, \dots, I$$

$$t = 1, \dots, T$$

$$\sum_{i=1}^I x_{jit} \leq \frac{1}{\theta} U_{jt} a_{jt} \quad j \in J_e \quad (11)$$

$$t = 1, \dots, T$$

$$\sum_{j \in J_e} \sum_{p \geq t} c_{jtp} y_{jp} + S_t - (1 + r_t) S_{t-1} = B_t \quad t = 1, \dots, T \quad (12)$$

$$U_{at} + \sum_{j \in J_e} U_{jt} \sum_{\tau \leq t} y_{jt} \geq L_{lt} (1 + m_t) \quad t = 1, \dots, T \quad (13)$$

$$\bar{R}y \geq \bar{r} \quad (14)$$

$$\sum_{\tau \in W_j} y_{j\tau} \leq 1 \quad j \in J_e \quad (15)$$

$$y_{jt} = 0 \text{ or } 1, \quad S_t \geq 0 \quad (16)$$

where constraints (14) are simply a generalized representation of the reliability oriented constraints.

This model represents a departure from existing models in several important respects. First, it incorporates a detailed capital budgeting type constraint on construction expenditures. Thus the analyst is required to specify the construction budgets for each year in the planning horizon.

The second major departure is that reliability requirements are enforced implicitly. The analyst's input, therefore is an important and

essential component in the model. As illustrated in Figure 4.3 the evolution of the model involves interaction by the analyst in an iterative fashion in order to generate an appropriate set of reliability constraints. This type of interaction is viewed here as the proper use for optimization models for long range planning.

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 Figure 4.3 here  
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A third departure is that even though sufficient expansion projects are defined, there may be no feasible solution because of either the budget constraints or the reliability constraints. This implies that an iterative use of the model is mandatory, at least in those situations where no feasible solution is discovered on the first try.

Finally, the model departs from existing mixed integer programming models in its production costing sector. The LDC is specified in such a way that a special, readily exploitable structure results. Within this framework, aggregation has been used to give a computationally superior model without sacrificing accuracy. In addition, the nonlinear production costs have been treated as such instead of linearized as in most other mixed integer programming models.

#### 4.4 Parameter Specification

Two points need to be made regarding parameter specification. The first is that there are a number of critical solution controlling parameters, namely:

$\gamma$ : the discount factor

$r_t$ : short term investment return rate

$B_t$ : construction budget

and the factors used in the reliability constraints (e.g.,  $\alpha$ , the target

system average forced outage rate). It is anticipated that these types of parameters are studied to determine the impact on the solution of changes in their values.

In addition, the model requires specifying a large number of cost parameters in then-current dollars. The practical approach to this parameter specification problem is as follows. First, estimate all costs in current dollars. Next, separate costs into different categories based on the assumption that costs in a given category will inflate (or deflate) at the same rate. Finally, specify the appropriate inflation rates for each cost category. With this data base it is a simple matter to generate the required cost parameters for the model. Moreover, it becomes a simple matter to study alternative assumptions about differential inflation rates.

#### 4.5 Summary

This chapter contains a detailed discussion and specification of a mixed integer programming model for the long range capacity planning problem. The model differs significantly from other mixed integer programming models for GPP, both in the structure available to be exploited in a solution procedure and in the required manner of use. The ultimate evaluation of this new model therefore depends on its computational requirements and on its accessibility to utility system planners.

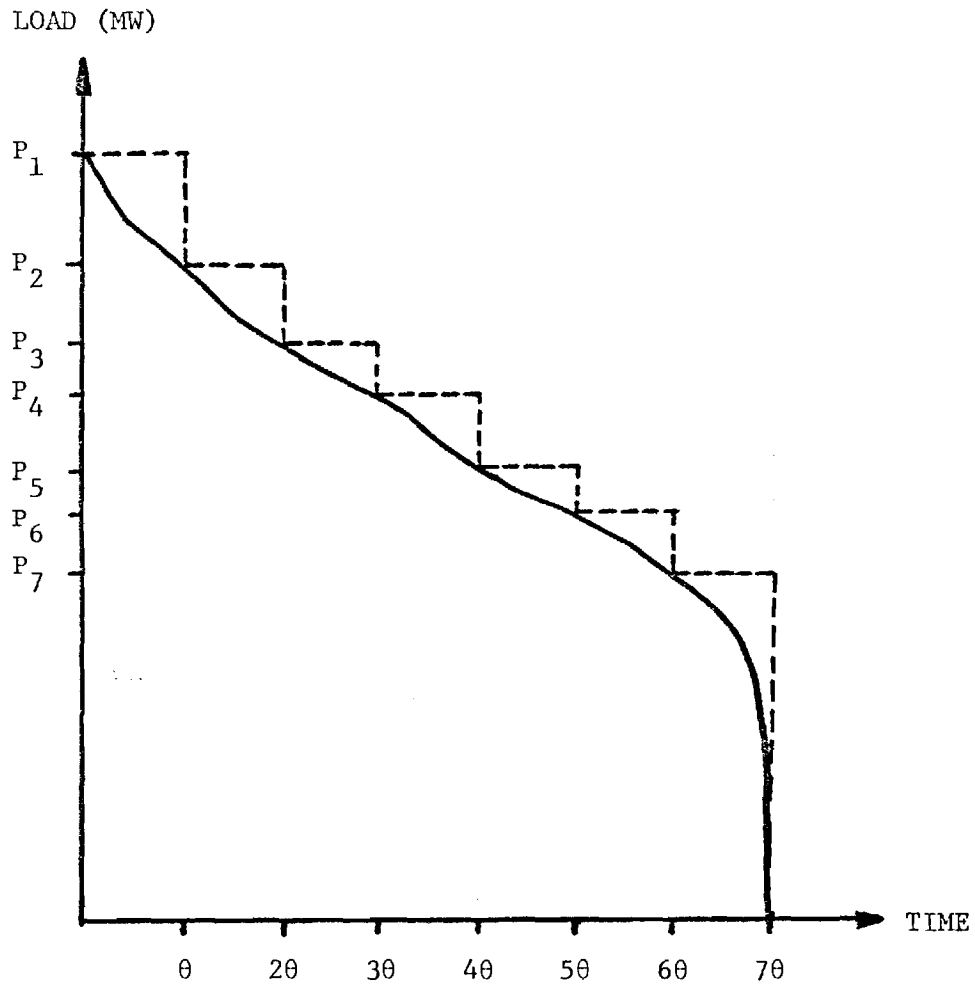


Figure 4.1. Load Duration Curve Approximation for Period p

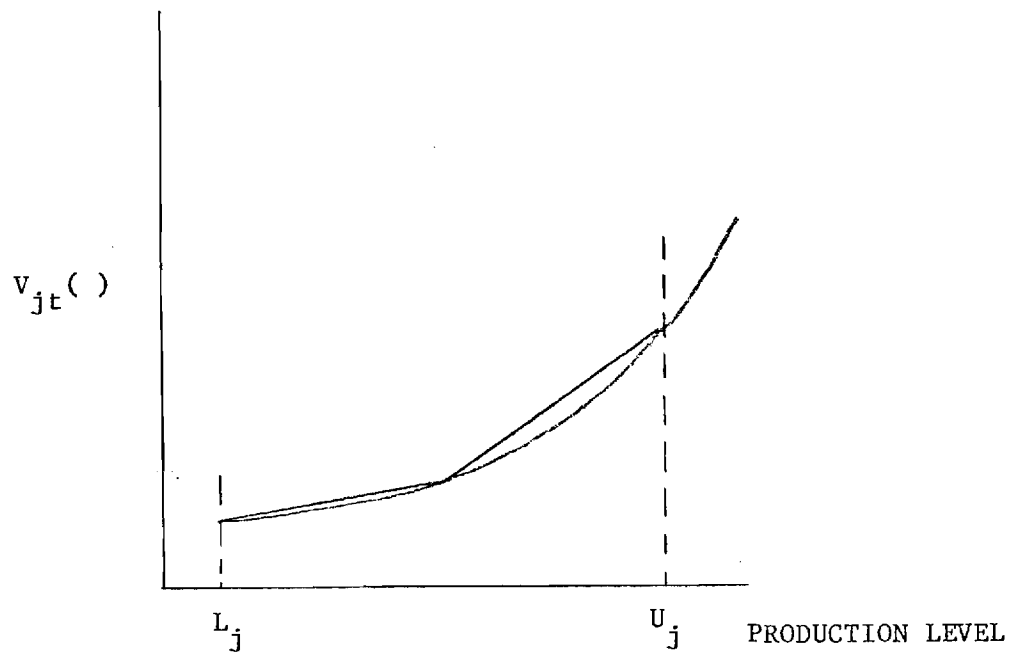


Figure 4.2. Illustration of Piecewise Linearization for Convex Cost Function  $V_{jt}(\cdot)$

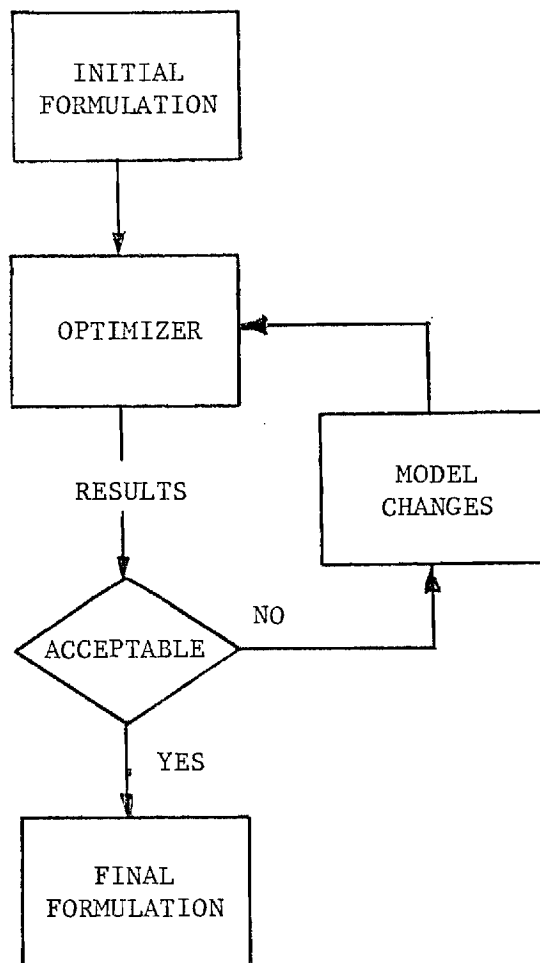


Figure 4.3. Iterative Model Development



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## Chapter 5

### Algorithms for Solving the Generation Planning Problem

Algorithms for Solving the  
Generation Planning Problem

In Chapter 4, a new mixed integer programming model was formulated for the long range generation capacity planning problem. The purpose of this chapter is to present the development and analysis of solution algorithms for this new formulation. The algorithms are based on Benders' partitioning procedure [3] and exploit the special structure of the GPP model. To simplify the presentation of the algorithm, it will be helpful to restate the generation capacity planning model in vector form as:

$$P: \quad \text{minimize} \quad z = \underline{F}\underline{y} + \underline{1}'\underline{V}(\underline{x}) - \underline{\gamma}'\underline{S}_T + K \quad (1)$$

$$\text{subject to:} \quad \underline{C}\underline{y} + \underline{D}\underline{S} = \underline{B} \quad (2)$$

$$\underline{R}\underline{y} \quad \geq \underline{r} \quad (3)$$

$$\underline{G}\underline{y} + \underline{A}\underline{x} \geq \underline{b} \quad (4)$$

$$\underline{y} \in \bar{\underline{Y}}, \quad \underline{S} \geq \underline{0} \quad (5)$$

In the objective function,  $\underline{1}'$  is a row vector of ones,  $\underline{V}(\underline{x})$  is a column vector and  $K$  represents the sum of all constant terms. Note that  $\underline{v}$  is a vector of convex functions. Constraints (2) and (3) are, respectively, the budget and reliability constraints, while constraints (4) are the production costing sector constraints. The set  $\bar{\underline{Y}}$  is defined as:

$$\bar{\underline{Y}} = \left\{ \underline{y}: \sum_{t \in \omega_j} y_{jt} \leq 1; y_{jt} = 0 \text{ or } 1 \right\} \quad (6)$$

It should be noted that Benders' partitioning has been previously considered as an approach to solving the generation planning problem. Noonan and Giglio [16] applied it to a model with an explicit, nonlinear LOLP

constraint and linear production costs. Bloom [4] discusses the application of Benders' partitioning to simplified versions of the models presented by Noonan and Giglio, Phillips, et al. [18], and Schweppe, et al. [19].

### 5.1 Benders' Partitioning Procedure

Problem P can be rearranged in such a manner that the selection and commissioning decisions, the  $\underline{y}$ 's, and the production decisions, the  $\underline{x}$ 's, are partitioned into two distinct yet related problems. The result of the partitioning is:

$$\begin{array}{l}
 \text{Pl: minimize} \\
 \underline{y} \in \bar{\underline{Y}} \\
 \\
 \underline{C}\underline{y} + \underline{D}\underline{S} = \underline{B} \\
 \\
 \underline{R}\underline{y} \geq \underline{r} \\
 \\
 \underline{S} \geq \underline{0}
 \end{array}
 \left\{ \begin{array}{l}
 \underline{F}\underline{y} - \gamma^T \underline{S}_T + \text{minimum } \underline{l}'\underline{v}(\underline{x}) + K \\
 \\
 \underline{A}\underline{x} \geq \underline{b} - \underline{G}\underline{y}
 \end{array} \right. \quad (7)$$

This arrangement of the problem isolates the integer variables, or complicating variables, to use Geoffrion's terminology [7]. If the complicating variables,  $\underline{y}$ , are fixed, the remaining problem in  $\underline{x}$  is a concave program and may be readily solved. This is precisely the structure for which Benders' partitioning algorithm [3], and Geoffrion's generalization [7] of it were developed.

Using Geoffrion's results [ ], problem P can be restated in the following equivalent form:

$$P2: \text{ minimize } y_0 \quad (8)$$

$$y_0 \geq (\underline{F} + \underline{uG})\underline{y} + \underline{u}(\underline{Ax} - \underline{b}) + \underline{1}'\underline{v}(\underline{x}) - \underline{\gamma}'\underline{S}_T + K \quad (9)$$

$$\underline{Cy} + \underline{DS} = \underline{B} \quad (10)$$

$$\underline{Ry} \geq \underline{r} \quad (11)$$

$$\underline{y} \in \bar{\underline{Y}}, \quad \underline{u} \geq 0 \quad (12)$$

Geoffrion's generalized Benders' partitioning algorithm does not solve P2 directly. Instead, it adds the constraints (9) in a sequential fashion, much like a cutting plane procedure. Thus, the constraint set represented by (9) can be indexed, say on  $q = 1, \dots, Q$  and written as:

$$y_0 \geq (\underline{F} + \underline{u}_q \underline{G})\underline{y} + \underline{u}_q (\underline{Ax}_q - \underline{b}) + \underline{1}'\underline{v}(\underline{x}_q) - \underline{\gamma}'\underline{S}_{Tq} + K_q \quad (9')$$

or simplifying:

$$y_0 \geq \underline{\beta}_q \underline{y} + \delta_q \quad q = 1, \dots, Q \quad (9'')$$

The cuts or Benders constraints, (9''), are generated by solving the production costing subproblem for a specified  $\hat{\underline{y}}$ :

$$SP(\hat{\underline{y}}): \quad v(\hat{\underline{y}}) = \text{minimum} \quad \underline{1}'\underline{v}(\underline{x}) \quad (13)$$

$$\text{subject to: } \underline{Ax} \geq \underline{b} - \underline{G}\hat{\underline{y}} \quad (14)$$

Because of the reserve margin constraints, if  $\hat{\underline{y}}$  is feasible in (2), (3) and (5), then  $SP(\hat{\underline{y}})$  will have a feasible solution, which simplifies the generalized Benders' procedure.

The generalized Benders' procedure for solving problem P can now be stated [7]:

STEP 1: For a  $\hat{y}$  satisfying (2), (3), and (5), solve  $SP(\hat{y})$ , obtaining the optimal dual vector  $\underline{u}$ . Set  $q \leftarrow 1$ ,  $\underline{u}_q \leftarrow \underline{u}$ ,  $LBD \leftarrow v(\hat{y})$ .

Select a convergence tolerance parameter,  $\epsilon \geq 0$ .

STEP 2: Solve the current master problem, P2, with (9) replaced by (9''), obtaining the solution  $(\hat{y}, \hat{y}_0)$ . If  $LBD \geq \hat{y}_0 - \epsilon$ , then STOP.

STEP 3: Solve the revised subproblem  $SP(\hat{y})$  obtaining the optimal dual vector,  $\underline{u}$ . If  $v(\hat{y}) \geq \hat{y}_0 - \epsilon$ , then STOP. Otherwise, set  $q \leftarrow q + 1$ ,  $\underline{u}_q \leftarrow \underline{u}$ ,  $LBD \leftarrow \max(LBD, v(\hat{y}))$ . Return to STEP 2.

Geoffrion shows that this procedure converges finitely for problems in the class considered here, i.e., where  $y$  is required to be a zero-one vector.

## 5.2 An Exact Algorithm

An exact algorithm for implementing the generalized Benders' procedure for solving problem P requires: (1) a method for solving  $SP(\hat{y})$ , and (2) a method for solving the revised master problem for a given set of Benders' constraints. As indicated in Chapter 4, the production costing sector of the model was formulated in such a way that solving the subproblems would be a simple task. Unfortunately the revised master problem does not appear to have an easily exploitable structure.

### 5.2.1 Solving the Subproblem

For a given  $\hat{y}$ , the subproblem to be solved is:

$$SP(\hat{y}): \quad v(\hat{y}) = \text{minimum} \sum_{t=1}^T \gamma^t \theta \sum_{i=1}^I \left[ v_{at}(x_{ait}) + \sum_{j \in J_e} v_{jt}(x_{jit}) \right] \quad (15)$$

subject to:

$$x_{ait} + \sum_{j \in J_e} x_{jit} = P_{it} \quad \begin{array}{l} i = 1, \dots, I \\ t = 1, \dots, T \end{array} \quad (16)$$

$$\sum_{i=1}^I x_{jit} \leq \frac{1}{\theta} U_{jt} a_{jt} \quad \begin{array}{l} j \in J_e \cup \{a\} \\ t = 1, \dots, T \end{array} \quad (17)$$

$$x_{jit} \leq U_{jt} \sum_{\substack{\tau \in w_j \\ \tau \leq t}} \hat{y}_{j\tau} \quad \begin{array}{l} j \in J_e \\ i = 1, \dots, I \\ t = 1, \dots, T \end{array} \quad (18)$$

$$x_{jit} \geq L_{jt} \sum_{\substack{\tau \in w_j \\ \tau \leq t}} \hat{y}_{j\tau} \quad \begin{array}{l} j \in J_e \\ i = 1, \dots, I \\ t = 1, \dots, T \end{array} \quad (19)$$

$$L_{at} \leq x_{ait} \leq U_{at} \quad t = 1, \dots, T \quad (20)$$

$SP(\hat{y})$  has several useful properties. First of all, the problem naturally decouples into independent subproblems for each period,  $t = 1, \dots, T$ . Thus, the problem may be solved by solving  $T$  smaller, more manageable problems.

A second, less obvious, property is that the problem can be easily transformed into a network flow problem. The advantage of doing so is that there are quite efficient algorithms available for solving network flow problems, so the computational burden is eased. In order to demonstrate the network structure, drop the  $t$  subscript and let

$$\delta_j = \sum_{\substack{\tau \in w_j \\ \tau \leq t}} \hat{y}_{j\tau} .$$

Figure 5.1 illustrates a network flow formulation of the production costing subproblem for one period, where the LDC has three segments and there is one potential expansion unit in addition to the existing system. Each source of energy, i.e., the existing system or the expansion unit, is represented by two nodes. The arc between them has a maximum capacity, corresponding to constraint (17). The cost on this arc is always zero. Each increment of the LDC is also represented by a node, and the required flow out of the node represents the power demand constraints, (16). The arcs joining the unit nodes to the increment nodes represent the actual production of energy. The costs on the arc flows are taken directly from (15) except that the term  $\gamma\theta$  is omitted, since it can be factored out.

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 Figure 5.1 here  
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The bound constraints, (18) and (19), are the only aspect of  $SP(\hat{y})$  that is not directly translated into the network formulation. For computational reasons, it is desirable to enforce the bound constraints in a different way. As shown in the example, there are two arcs between expansion unit nodes and LDC increment nodes. The arc going from the generation unit node represents the production of energy by that unit. However, if in the given  $\hat{y}$ , the unit is not available, i.e.,  $\delta_j = 0$ , the flow will be forced into the associated arc going back into the generation unit node, thus will not be available to satisfy the power demand. With this modeling "trick" it will be necessary to adjust the optimal solution value for all those units not available in the current  $\hat{y}$ .

The purpose behind this somewhat cumbersome method of enforcing the bounds is the reduction of computational effort. With the network formulated as in Figure 5.1, a change in  $\hat{y}$  does not require a completely new solution to the network flow problem. A significant overhead is avoided



by simply changing the arc costs (by replacing  $\delta_j$ 's with their new values) and restarting the optimization from the most recent optimal solution. The efficacy of this technique has been demonstrated in many other mixed integer programming applications [10, 13, 14, 16].

Although the subproblem can be converted to a network flow problem, there is still the matter of the convex costs,  $V_j(x)$ . In order to fully exploit the state of the art technology for solving network flow problems [5, 11, 12] it will be necessary to use a piecewise linear approximation as shown in Figure 5.2. Note that the approximation can be made as tight as desired by choosing more linear segments. The effect in the network is to replace each production arc (those having costs  $V_j(x)$ ) by a set of arcs, one for each linear segment in the approximation.

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Figure 5.2 here  
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At this point, the production costing subproblem has been transformed into an equivalent linear programming problem having a computationally advantageous special structure. Noonan and Giglio [17] also employed a linear programming subproblem but their model did not incorporate convex production costs nor did it exploit the underlying network structure.

All that remains is to show how to recover the optimal dual vector,  $\underline{u}$ , from the solution to the piecewise linearized network flow problem. The optimal dual variables associated with constraints (16) and (17) are just the corresponding node potentials in the solution to the network flow problem.

The dual variable values associated with the bound constraints (18), (19) and (20) are easily determined by considering the Kuhn-Tucker conditions for optimality of  $SP(\hat{y})$ . Let  $\alpha_i$ , unrestricted;  $\beta_j \leq 0$ ,  $\gamma_{ji} \leq 0$ , and  $\lambda_{ji} \geq 0$  be, respectively, the dual variables for constraints (16), (17),

(18), and (19). Note that  $\alpha_i$  and  $\beta_j$  are available from the network solution.

The Kuhn-Tucker conditions give:

$$\frac{\partial}{\partial x_{ji}} V_j + \alpha_i + \beta_j + \gamma_{ji} + \lambda_{ji} = 0 \quad \begin{array}{l} j \in J_e \cup \{a\} \\ i = 1, \dots, I \end{array}$$

$$[x_{ji} - U_j \delta_j] \gamma_{ji} = 0 \quad \begin{array}{l} j \in J_e \cup \{a\} \\ i = 1, \dots, I \end{array}$$

$$[x_{ji} - L_j \delta_j] \lambda_{ji} = 0 \quad \begin{array}{l} j \in J_e \cup \{a\} \\ i = 1, \dots, I \end{array}$$

Approximating  $\frac{\partial}{\partial x_{ji}} V_j$  by the optimal reduced cost for  $x_{ji}$ , and noting that not both  $\gamma_{ji}$  and  $\lambda_{ji}$  may be non-zero gives directly the computational scheme for determining the rest of the vector  $\underline{u}$ .

### 5.2.2 Solving the Master Problem

At iteration  $Q$  of the generalized Benders' procedure, the master problem to be solved is:

$$\text{MP:} \quad \text{minimize } y_0 \quad (21)$$

$$y_0 \geq \underline{\beta}_q y + \delta_q \quad q = 1, \dots, Q \quad (22)$$

$$\underline{C}y + \underline{D}S = \underline{B} \quad (23)$$

$$\underline{R}y \geq \underline{r} \quad (24)$$

$$y \in \bar{Y}, \quad \underline{S} \geq \underline{0} \quad (25)$$

which is a mixed integer programming problem. Although MP is not formidably large (e.g. if  $T = 30$ ,  $|J_e| = 30$  and the average  $|w_j| = 5$ , MP has 150

0-1 variables, 31 continuous variables,  $60 + Q$  constraints and 30 generalized upper bounds) it also does not have any obviously exploitable structure.

Thus, it appears that a standard linear programming based branch and bound approach [9] would be the wisest choice for solving MP in the straightforward generalized Benders' partitioning procedure. Such a procedure is likely to be most successful when MP is tightly constrained by budget and reliability requirements.

### 5.3 A Variant of the Exact Algorithm

Many variations of Benders' partitioning have been suggested in the literature [2, 6, 10, 15], at least partially motivated by poor performance of the original procedure. In this section, the one proposed by Geoffrion and Graves [10] is considered for solving the generation planning problem.

Geoffrion and Graves (GG) propose using the master problem, not to generate lower bounds per se, but to generate new feasible  $\underline{y}$ 's for the subproblem. Feasibility for the Benders' constraints means that  $y_0$  must not exceed the best known feasible solution value, call it UBD. Thus the Benders' constraints are rewritten as:

$$\text{UBD} - \epsilon \geq \underline{\beta}_q \underline{y} + \delta_q \quad q = 1, \dots, Q \quad (26)$$

Obviously, if no such feasible solution can be found, the procedure terminates.

Since the master problem was only used to find feasible  $\underline{y}$ 's, GG observed that any objective function could be used. In their empirical evaluation, they found that using the most recently generated Benders' constraint as the objective function gave good results.

With these modifications, the master problem at iteration  $Q$  of the

procedure is:

$$\begin{aligned}
 \text{MP':} \quad & \text{minimize} \quad y_0 = \underline{\beta}_Q y \\
 & \text{subject to:} \quad \text{UBD} - \epsilon \geq \underline{\beta}_q y + \delta_q \quad q = 1, \dots, Q \\
 & \quad \underline{C}y + \underline{D}S = \underline{B} \\
 & \quad \underline{R}y \geq \underline{r} \\
 & \quad y \in \underline{\bar{Y}}, \quad \underline{S} \geq \underline{0}
 \end{aligned}$$

Note that the problem need not be solved to optimality; it may be terminated as soon as a feasible  $y$  is encountered.

#### 5.4 An Approximate Algorithm

The reason for considering an approximate algorithm for the generation planning problem is that there may be situations in which quickly finding good solutions is the object rather than spending more (perhaps much more) time to guarantee optimality. For example, in studying the effects of differing assumptions about fuel costs, it might be desirable to quickly examine many different possibilities. In situations such as this, having a good solution and some evidence indicating how good it is may be all that is needed.

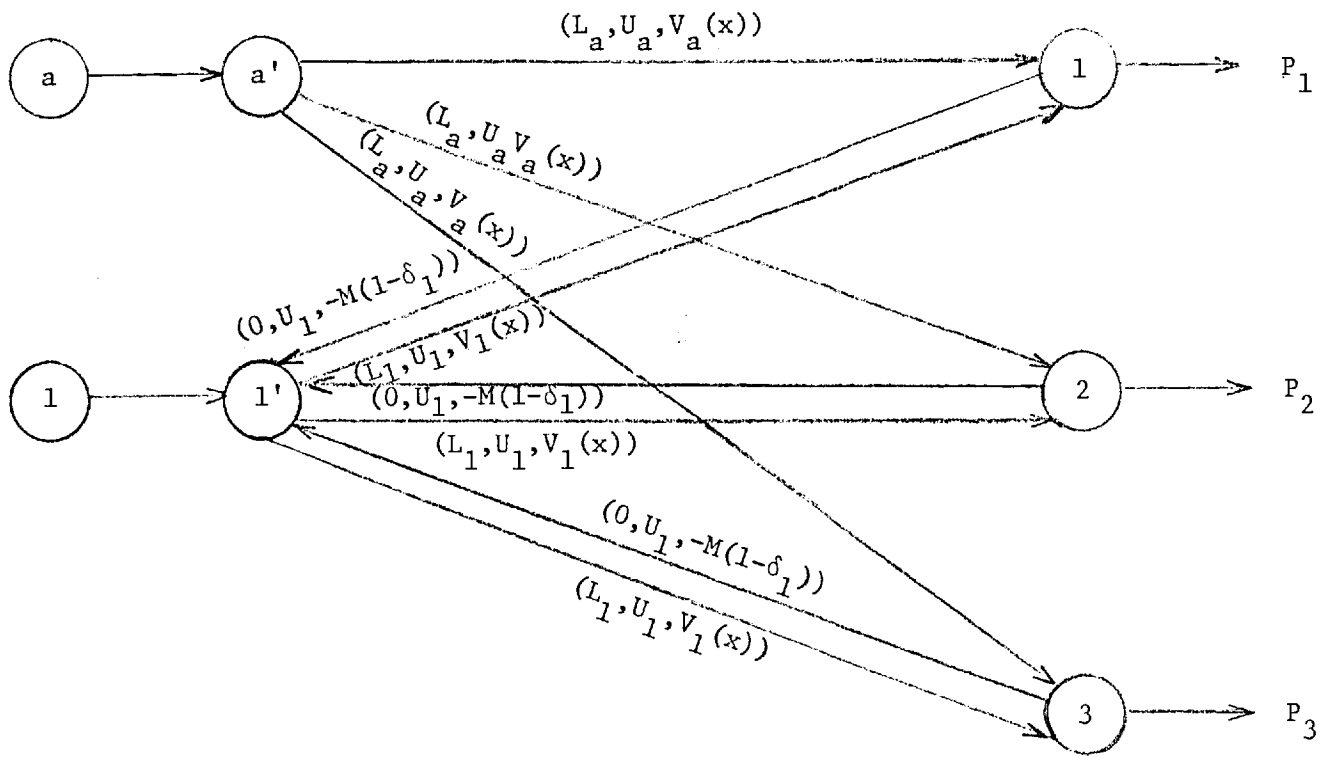
Several approaches to developing approximate solutions are available, based on the exact procedures described in the previous section. Note, however, that the goal of the approximate algorithm is to avoid large computational requirements. Thus, for example, simply using a large  $\epsilon$  value in the termination criterion of the exact algorithms does not constitute a desirable procedure, because it still requires solving a mixed integer programming problem.

An approach that has been used with some success in solving similar problems [1] is the following. Implement the generalized Benders' procedure except that instead of solving the master problem, MP, as a mixed-integer programming problem, solve it as an LP and then use some ad hoc rules to integerize the continuous solution. Additional rules are required to handle situations in which the same  $y$  is generated more than once.

In the work reported in [1], it was found that this approach usually led to very good or optimal solutions. Unfortunately, the lower bound obtained from the linear relaxation of MP was usually very weak so that the quality of the solutions could not be judged. Attempts to strengthen the bounds through Lagrangian relaxation [8] were generally unsuccessful.

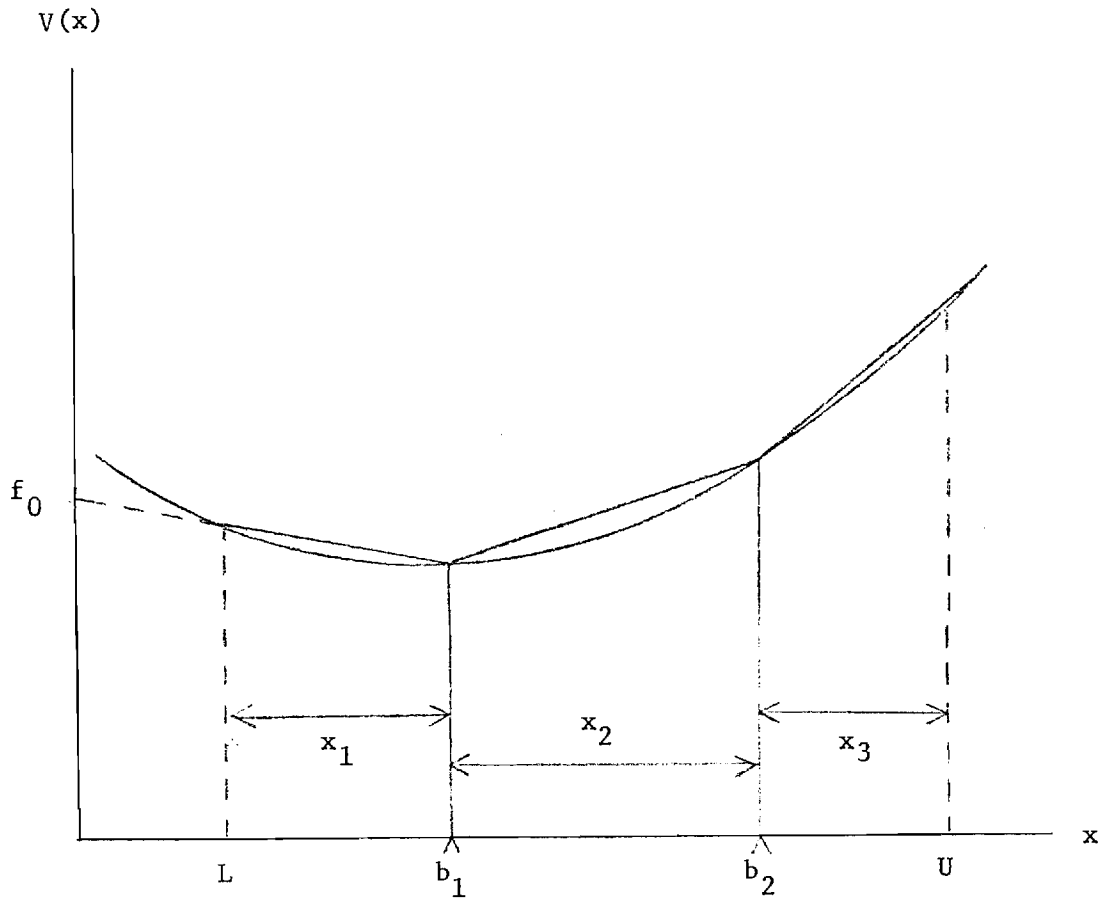
### 5.5 Summary

This chapter has described both exact and approximate algorithms for solving the generation capacity planning problem as formulated in Chapter 4. While these algorithms have not been empirically tested yet, there is reason to hope for good computational results. Similar applications of Benders' partitioning have been successful, and the model used here has been formulated to exploit the problem's special structure.



- (a, b, c)
- a: lower bound on flow
  - b: upper bound on flow
  - c: cost of flow

Figure 5.1. Subproblem Network Formulation Example



$$V(x) \approx f_0 + v_1 x_1 + v_2 x_2 + v_3 x_3$$

$$x_1 + x_2 + x_3 = x$$

$$L \leq x_1 \leq b_1$$

$$b_1 \leq x_2 \leq b_2$$

$$b_2 \leq x_3 \leq U$$

Figure 5.2. Piecewise Linear Approximation to  $V(x)$

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## Chapter 6

### Design of A User Oriented System for Generation Capacity Planning

## A User Oriented System for Generation Capacity Planning

The development of an optimization model and associated algorithms for generation capacity planning does not guarantee their use. In order for such developments to gain acceptance, there must be at least a design for the supporting data base and its management and more importantly, a design for the interface between the user and the algorithms. The purpose of this chapter is to propose some design guidelines for these aspects of an integrated generation capacity planning system.

### 6.1 Overview

One of the most important factors to consider in designing the system is that its users will probably have very little knowledge of the methodologies employed in the solution procedures. Their expertise will be in other areas, and it is crucial that their use of the system should magnify their capabilities in modelling, optimization, or computer programming. The user should not be required to undergo extensive training in order to learn how to use the system. This consideration leads to the first of the general guidelines:

- (1) The system should be terminal oriented, incorporating a conversational style prompting monitor.

Special purpose terminal monitors have been developed for a number of general purpose optimization systems, e.g., MPOS [2] and EZLP [3]. These monitors are actually nothing more than master programs with extensive I/O and capabilities for processing character string data. A desirable feature of the terminal monitor is the capability for the user to obtain from the

terminal monitor some basic instruction in how to use the system along with simple examples.

A second important factor to consider is that the system may be used to solve many different problems or variation of a given problem. Thus it is necessary to separate the data base and data base management functions from the solution procedures. The system must allow the user to specify the source (filename) for the data to be operated on:

- (2) The system should permit the user to specify (from the terminal) the filenames of the data to be used.

A third requirement is that the system be flexible in the reporting of results. For large scale generation capacity planning, it would be impractical to receive detailed solution reports at the terminal. It might be desirable, however, to have detailed reports in a hard form for further analysis. Thus the system should provide for appropriate aggregated and/or abbreviated reporting to the terminal and for flexible reporting to a high speed hard copy printer:

- (3) The system should provide levels of reporting for results both to the terminal and to hard copy printers.

A fourth and final design consideration is to make the system modular. There are two reasons for doing so. One is that a modular system is easier to code and debug. The other is that if the system is designed in a modular fashion, the more important functions can be implemented and used while ancillary functions are being developed:

- (4) The systems should be modularized.

## 6.2 Data Base Design

The existing widely-used generation planning systems, such as OGP [1], WASP [4], and others [6,7,8], all require much of the same basic data as is required by the optimization model proposed here. In a system based on the optimization model, it would be desirable, then, to have the same basic file organization. It is inevitably true, however, that no two utilities (or other potential users) will have exactly the same detailed file organization or structures. Thus, the system must be flexible as well as easily modified in the input of raw data.

This leads to the definition of a system function called "CREATE DATA." This function takes raw data in some specified format, modifies it according to user instructions, and creates a user data file in a standard format. For example, suppose the system is designed to operate on the following four files:

LOAD file:	contains load forecasts
UNIT file:	contains heat rate and output limit coefficients
PROJECT file:	contains base year construction costs and commissioning window data
PARAMETER file:	contains discount rates, inflation rates, budget limits, and plant mix constraints

In order to set up the PROJECT file, the CREATE DATA function may be required to access several of the user's existing data files.

It will generally be true that the best internal organization of the data in their four files will not necessarily be the most convenient for the solution procedure. This consideration leads to the definition of a new file and another system function, "CREATE PROBLEM FILE". This function takes as input the four data files described above and produces a single file containing all the data required by the solution procedure in a format

convenient for input to the procedure. The resulting PROBLEM file will then be the input source for the solution procedure.

The third system function is "SOLVE PROBLEM", which takes the PROBLEM file as input and generates a RESULTS file. This system function may require some direct input from the user, for example, to choose the solution technique (exact or approximate) or to determine when to stop the procedure.

The RESULTS file can be examined by the user through a fourth system function called "QUERY RESULTS". The primary purpose of this function is to reformat and summarize the results for easy assimilation by the user. This function is required because of the volume of results produced by the problem solver.

The final system function is "CREATE REPORTS", which uses the RESULTS file along with user instructions to create a set of reports which may be dumped to a hard copy printer. Again, the purpose of the function is to allow flexibility in reporting results so that only those results actually needed are printed.

### 6.3 System Functions

Figure 6.1 illustrates the fundamental structure of the system, including the five primary system functions and the six system specific files. Note that this system architecture allows for a great deal of flexibility in implementation. For example, the SOLVE PROBLEM function might allow the user limited abilities to override or specify alternative values for certain parameters in the problem file, e.g., a fuel cost inflation factor.

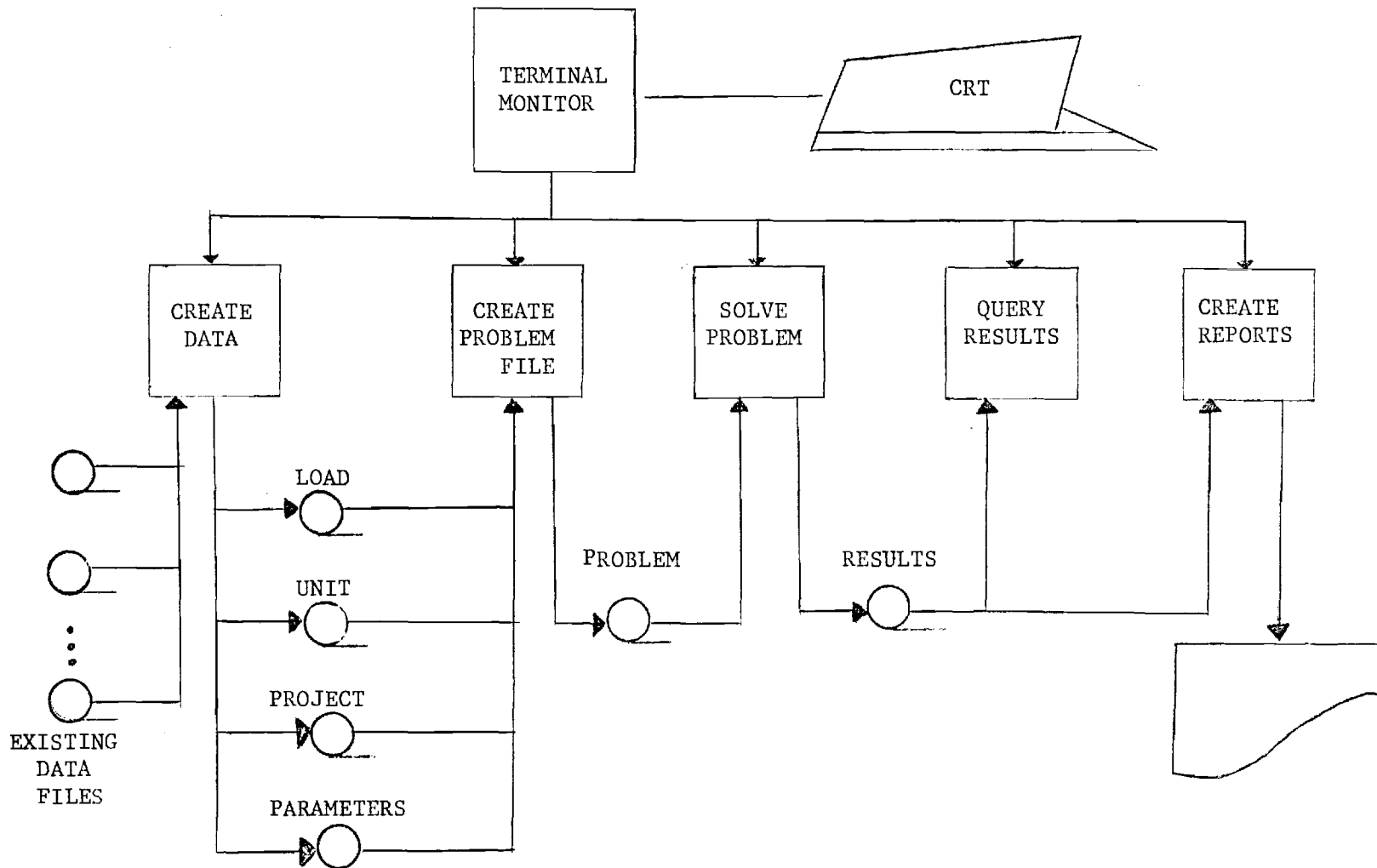


Figure 6.1 System Functions and Files

Because certain functions are likely to require considerable elapsed time, e.g., CREATE DATA or SOLVE PROBLEM when optimizing, it will be important to have a "batch" mode for the system functions. In other words, the user will specify all the user-supplied parameters but instead of the processing being done in a real-time or time-share mode, the work will be sent to the batch queue. In this way, the terminal can be freed for other uses.

Finally, as noted by Jordon and Schlaepfer [5], there are potential uses of interactive graphics in power system planning. This type of interactive processing also falls within the scope of the architecture as described.



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## Chapter 7

### Summary and Recommendations

### Summary and Recommendations

The research reported on here is directed toward developing optimization models and solution techniques for use in long range planning of generation capacity for electric utilities. The key accomplishments to date in this research effort are:

- (1) the development of a new technique for production cost estimation, its implementation and experimental validation;
- (2) the detailed development of a new mixed integer programming model of the generation capacity planning problem which explicitly incorporates construction budget considerations and treats reliability requirements in a novel way;
- (3) the general development of exact solution procedures and heuristic solution procedures which exploit the proposed model's special structure to achieve computational efficiency; and
- (4) the development of guidelines and an architecture for an integrated planning system based on the proposed model and solution techniques.

Item (1) in particular represents a significant departure from the traditional and currently accepted practice. Although the results of that work have been used here in a mixed integer programming model, the technique is equally valid in the context of dynamic programming approaches (such as WASP [1]) or system myopic heuristics (such as PEP [2]). In addition, the technique is potentially valuable in more comprehensive models which include a revenue generation sector [3]. Although there are some computational issues remaining to be settled, the research effort relative to item (1) is essentially complete.

Research on item (2) is continuing, specifically with regard to mechanisms for defining the appropriate plant mix constraints. Several questions remain to be answered. First of all, what types of feedback are needed and how should the feedback be used by the analyst in the iterative process of developing these constraints? In addition, are there any techniques that can allow a priori specification of a "good" set of plant mix constraints? The issue of reliability measurement for generation is itself an unresolved one, and it is recommended that further research on the approach proposed here represents a valuable contribution to understanding and solving the problem.

The value of the general developments of item (3) may only be determined through implementation and computational testing. In the implementation process, a number of interesting methodological issues will need to be resolved, especially in the solution of the Benders' master problem. This research is also continuing and computational results should be forthcoming in the near future.

In terms of practical value, the development of an integrated system based on the results of item (4) may prove to be most significant. One reason why mixed integer programming models are not more widely used in general is that the solution methodologies are not very accessible to the potential users. However, development of such a system holds more interest as a commercial venture than as a research endeavor, so no further research seems warranted.

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## Appendix A

### Notation and Terminology

$T$  = number of time periods in the study

$D$  = demand level faced by system of generation units

$TC(D)$  = hourly production cost at output level  $D$  for system

$A_i, B_i, C_i$  = coefficients of quadratic cost curve for unit  $i$

$x_i$  = production or output level of generation unit  $i$

$L_i$  = lower bound on production for unit  $i$

$U_i$  = upper bound on production for unit  $i$

$\delta_t$  = duration of period  $t$  in years for the generation capacity planning model

$\lambda$  = amount of time covered by the representative load duration curve

$\delta_t/\lambda$  = number of load duration curves required in period  $t$

EPC = estimated production cost for one time period in production costing routine

$P_i$  = power demand value for segment  $i$  of the load duration curve

$\theta_i$  = width of interval partitions on the load duration curve

$E_i$  = annual energy capacity of unit  $i$

$v_j(\cdot)$  = variable production cost function for unit  $j$  (\$/mw)

$F_j$  = planning period fixed costs associated with operating unit  $j$

$a_j$  = availability rate for unit  $j$  = (expected number of hours available)/(number of hours in period)

$J$  = index set of all units for the generation planning model

$J_e$  = subset of indices for expansion units

$w_j$  = index set of time periods in the commissioning window for unit  $j$ ,  $j \in J_e$

$d_j$  = length of time for construction of expansion unit  $j$

$B_t$  = new funds available in period  $t$  for construction

$S_t$  = excess construction funds accumulated in period  $t$

$c_{jkt}$  = construction cost for project  $j$  in period  $k$  if commissioning occurs in year  $t$

$r_t$  = short term investment rate of return at period  $t$

- $\gamma$  = discounting factor for costs
- $m_t$  = generation capacity required in excess of the predicted peak demand
- $\alpha_j$  = unit  $j$  forced outage rate
- $\alpha$  = desired "average" or capacity weighted forced outage rate
- $c_i$  = cost per hour of producing for discrete unit  $i$   
 $= A_i U_i^2 + B_i U_i + C_i$
- $b$  = output level prescribed in recursive relationship developed in algorithm for discrete units
- $f_k(b)$  = optimal solution value (minimum cost) at the  $k^{\text{th}}$  solution stage for output level  $b$
- $\beta_t$  = set of system output levels for which associated costs have been determined, given a system of discrete units 1 through  $t$
- $b_p$  = individual member of set  $\beta_t$
- $(d_i, t_i)$  = (demand, cost) point derived for the system of discrete units
- $n_d$  = number of discrete units
- $n_c$  = number of continuous units
- $n$  = number of units in the system
- $u$  = dual variable associated with the demand satisfaction requirement in the marginal cost approach for continuous units
- $v_i$  = dual variable associated with lower bound restriction on unit  $i$  in the marginal cost approach for continuous units
- $w_i$  = dual variable associated with upper bound restriction on unit  $i$  in the marginal cost approach for continuous units
- $\mu_i(x_i)$  = marginal production cost ( $2A_i x_i + B_i$ ) associated with unit  $i$  operating at level  $x_i$
- $\mu_i^U$  = marginal production cost associated with unit  $i$  when it is operating at its upper bound on production
- $\mu_i^L$  = marginal production cost associated with unit  $i$  when it is operating at its lower bound on production
- $\mu$  = variable representing marginal cost for entire system of continuous units at a specific output level
- $\delta_i$  = smallest change in continuous system marginal cost,  $\mu$ , which would force unit  $i$  either to assume an operation level above its lower bound or to become fixed at a production level corresponding to its upper bound in the production



$K(D)$  = hourly production cost at output level  $D$  for total system containing all continuous and discrete units

$x$  = production level for aggregate continuous unit

$TC_1(x)$  = cost of production level  $x$  for aggregate continuous unit

$t_j$  = production cost for the  $j^{\text{th}}$  discrete point

$d_j$  = production level for the  $j^{\text{th}}$  discrete point

$U$  = upper bound on production for aggregate continuous unit

$L$  = lower bound on production for aggregate continuous unit

Appendix B

The Loss of Load Probability (LOLP)  
Approach to Generation Reliability

The LOLP is essentially an estimate of the fraction of time during the year when the system will not be able to meet all demand because of a forced or unplanned outage. The development given below is based on descriptions found in [1, 2].

Suppose the vector  $\underline{x}$  represents the state of the system. If there are  $n$  units in the system then  $\underline{x}$  is an  $n$ -vector whose elements are one or zero, depending on whether or not the corresponding unit is available or in service. When the system is in state  $\underline{x}$ , there is a random variable,  $G(\underline{x})$ , which describes the instantaneous generation capacity resulting from the randomly occurring failures of the units in service. In principle, there is a different  $G(\underline{x})$  for each possible state,  $\underline{x}$ , which may occur.

For a given  $\underline{x}$ , the distribution function for  $G(\underline{x})$  is  $F_G(g|\underline{x})$  and is determined as follows. Let  $J_{\underline{x}}$  be the index set for units that are available in state  $\underline{x}$ . For each of the  $n$  units in the system, let  $U_j$  be the random variable describing unit availability, with distribution function  $F_j(u)$ . In practice,  $U$  is generally assumed to be discrete with only two values, full capacity and completely failed, with corresponding probabilities  $(1 - \alpha)$  and  $\alpha$ , where  $\alpha$  is the forced outage rate.  $F_G(g|\underline{x})$  is the convolution of all the  $F_j(u)$  for units  $j \in J_{\underline{x}}$ .

As an illustration, consider a system with four generating units having the capacities and forced outage rates given in Table B.1. Let  $\underline{x} = (0, 1, 1, 1)$ , i.e., only units 2, 3, and 4 are available.  $F_G(g|\underline{x})$  is determined by convoluting  $F_2(u)$  with  $F_3(u)$  to obtain an intermediate result,  $F_*(u)$ , then convoluting  $F_*(u)$  with  $F_4(u)$ . The calculations are illustrated in Table B.2.

Table B.1. Sample Problem

<u>Unit</u>	<u>Capacity</u>	<u>Forced Outage Rate</u>
1	40	0.010
2	45	0.015
3	50	0.005
4	55	0.010

Now let  $L(t)$  be a random variable describing the demand for power at time  $t$ . The expected value of this random variable is simply the forecasted demand. The distribution function for  $L(t)$  is  $F_L(\ell, t)$ . The margin, or excess of capacity over demand at time  $t$  is now a random variable,  $M(t|\underline{x})$ :

$$M(t|\underline{x}) = G(\underline{x}) - L(t)$$

whose distribution function is found by convolution ("\*" denotes the convolution operation):

$$F_M(m, t|\underline{x}) = F_G(g|\underline{x}) * F_L(\ell, t)$$

If the margin is less than zero,  $M(t|\underline{x}) < 0$ , then there is a loss of load, and the instantaneous probability of this event is simply  $F_M(0, t|\underline{x})$ .

Thus, it is a simple matter to determine LOLP, provided the system state  $\underline{x}$  doesn't vary:

$$LOLP_{\underline{x}} = \int_0^1 F_M(0, t|\underline{x}) dt$$

In reality  $\underline{x}$  does vary in response to anticipated changes in the load,  $L(t)$ , as well as to allow for planned maintenance. Further, because the changes in  $\underline{x}$  are not random, they can't be incorporated in the above model

Table B.2. Calculation of  $F_t(g|\underline{x})$ 

<u>u</u>	<u><math>P_2(u)</math></u>	<u>u</u>	<u><math>P_3(u)</math></u>	<u>u</u>	<u><math>P_*(u)</math></u>
0	.015	0	.005	0	.000075
45	.985	50	.995	45	.004925
				50	.014925
				95	.980075

<u>u</u>	<u><math>P_*(u)</math></u>	<u>u</u>	<u><math>P_4(u)</math></u>	<u>u</u>	<u><math>P(u)</math></u>	<u><math>F_G(g \underline{x})</math></u>
0	.000075	0	.01	0	.00000075	.00000075
45	.004925	55	.99	45	.00004925	.00005000
50	.014925			50	.00014925	.00019925
95	.980075			55	.00007425	.00027350
				95	.00980075	.01007425
				100	.00487575	.01495000
				105	.01477575	.02972575
				150	.97027425	1.0

by making  $\underline{x}$  a random variable with a known distribution function.

In order to allow for planned variations in  $\underline{x}$ , the time interval must be broken down into contiguous increments, each of which is as long as possible but still corresponds to only one state. Suppose there are  $m$  such increments. Let  $I$  be the index set for these increments with  $\underline{x}_i$  the state during increment  $i$ , which is defined as  $(\tau_{i-1}, \tau_i]$  where  $\tau_0 = 0$  and  $\tau_m = 1$ . Also, let  $d_i = \tau_i - \tau_{i-1}$ . Now to calculate the annual LOLP, simply calculate the LOLP for each increment, then combine them:

$$M(t|\underline{x}_i) = G(\underline{x}_i) - L(t)$$

$$F_M(m, t|\underline{x}_i) = F_G(g|\underline{x}_i) * F_L(\ell, t)$$

$$LOLP_i = \frac{1}{d_i} \int_{\tau_{i-1}}^{\tau_i} F_M(0, t|\underline{x}_i) dt$$

$$LOLP = \sum_{i \in I} LOLP_i(d_i)$$

REFERENCES

1. Albrecht, P. F., "Overview of power system reliability," EPRI/Engineering Foundation Conference on Power System Reliability - Research Needs and Priorities, Pacific Grove, California, 1978.
2. Irisari, G., "Power system expansion planning," unpublished Ph.D. dissertation, Purdue University, 1975.