GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT INITIATION

Gr Cd
NSF

Date: 7/6/77

Project Title: *Mathematical Models for Complex Granular Media*

Project No: E-23-628 (Co-project to E-20-615/Sowers/CE)

Project Director: Dr. Stephen L. Passman

Sponsor: National Science Foundation

Agreement Period: From 7/1/77 Until 12/31/79
(24 months budget period plus 6 months flexibility period)

Type Agreement: Grant No. ENG76-84490

			CE		ES&M	Total
Amount:	NSF	E-20-615	$11,157	E-23-628	$45,443	$56,600
	GIT		--	E-23-328	3,165	3,156
	TOTAL		$11,157		$48,608	$59,765

Reports Required: Annual Technical Letter Report; Final Technical Letter Report; Summary of Completed Project

Sponsor Contact Person (s):

Technical Matters	Contractual Matters (thru OCA)
Dr. Clifford J. Astill, Director Solid Mechanics Program Engineering Mechanics Section Division of Engineering National Science Foundation Washington, D. C. 20550	Ms. Mary Frances O'Connell Grants Administrator National Science Foundation Washington, D. C. 20550 (202) 632-2858

Defense Priority Rating: none

Assigned to: Engineering Science and Mechanics (School/Laboratory)

COPIES TO:

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CA 3 (3/76)

GEORGIA INSTITUTE OF TECHNOLOGY
OFFICE OF CONTRACT ADMINISTRATION

SPONSORED PROJECT TERMINATION

Date: December 16, 1980

Project Title: Mathematical Models for Complex Granular Media

Co-Project Nos: E-23-628 and E-20-615

Co-Project Directors: Dr. Stephen L. Passman and Prof. George F. Sowers

Sponsor: National Science Foundation

Effective Termination Date: 12/31/79

Clearance of Accounting Charges: - - - -

Grant/Contract Closeout Actions Remaining:

- _ Final Invoice and Closing Documents
- X Final Fiscal Report Accounting (FCTR)
- X Final Report of Inventions (If positive)
- _ Govt. Property Inventory & Related Certificate
- _ Classified Material Certificate
- _ Other ______

Assigned to: Engineering Science and Mechanics and Civil Engineering (School/Laboratory)

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CA-4 (1/79)

E 23-6[illegible]

NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

FINAL PROJECT REPORT
NSF FORM 98A

PLEASE READ INSTRUCTIONS ON REVERSE BEFORE COMPLETING

PART I–PROJECT IDENTIFICATION INFORMATION

1. Institution and Address eorgia Institute of Technology tlanta, GA 30332	2. NSF Program Solid Mechanics	3. NSF Award Number ENG76-84490
	4. Award Period From 7/1/77 To 12/31/79	5. Cumulative Award Amount $56,600

6. Project Title
athematical Models for Complex Granular Media

PART II–SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

Errors were found in the previous model of Goodman and Cowin for dry granular materials which required that a new model be constructed. This was done, and a number of boundary-value problems were solved. The solutions agree well with experiment. A mixture model based on the work of Passman was constructed and the dynamic boundary-value problem associated with the simple shearing experiment was solved. Experimental work for the material considered a model appropriate to, e.g., a saturated sand-water mixture, is meager, but the results agree qualitatively with the existing experiments. The model shows promise in explaining theoretically, for the first time, various geological phenomena, including mudslides, both on the earth's surface and undersea. Work is proceeding on modelling the vane shear experiment.

PART III–TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM	
				Check (✓)	Approx. Date
a. Abstracts of Theses					
b. Publication Citations		✓			
c. Data on Scientific Collaborators					
d. Information on Inventions					
e. Technical Description of Project and Results	▨				
f. Other *(specify)*	▨				

2. Principal Investigator/Project Director Name *(Typed)* Stephen L. Passman	3. Principal Investigator/Project ~~Director~~ Signature	4. Date

NSF Form 98A (5-78) Supersedes All Previous Editions

Form Approved OMB No. 99R0013

Publications, ENG76-84490

1. S. L. Passman and J. P. Thomas, Jr., On the linear theory of flow of granular media, Developments in Theoretical and Applied Mechanics 9, 193-202, 1978. Abstract also appears in Journal of Rheology 23, 103, 1979.

2. S. L. Passman, J. T. Jenkins, and J. P. Thomas, Jr., Flow of a granular material in a vertical channel, Proceedings of the U. S.-Japan Seminar on Continuum-Mechanical and Statistical Approaches in the Mechanics of Granular Materials, Gakujutsu Bunken Fukya-Kai, Tokyo, 171-180, 1978.

3. A. W. Marris and S. L. Passman, Some velocity-universal motions of granular materials, Proceedings of the U. S.-Japan Seminar on Continuum-Mechanical and Statistical Approaches in the Mechanics of Granular Materials, Gakujutsu Bunken Fukya-Kai-, Tokyo, 218-221, 1978.

* 4. S. L. Passman, J. W. Nunziato, P. B. Bailey, and J. P. Thomas, Jr., Shearing flows of granular materials, J. Engr. Mech. Division, ASCE, 778-783, 1980. Similar work appears as ASCE Preprint 3708, 1979.

* 5. J. W. Nunziato and S. L. Passman, Gravitational flows of granular materials with incompressible grains, J. Rheology 24, 395-420, 1980.

* 6. S. L. Passman, D. E. Grady, and J. B. Rundle, The role of inertia in the fracture of rock, J. Appl. Physics 51, 4070-4075, 1980.

* 7. S. L. Passman, J. W. Nunziato, P. B. Bailey, and E. K. Walsh, A mixture theory for suspensions, Rheology, Vol. 2, edited by G. Astarita, G. Marrucci, and L. Nicolais, Plenum Press, New York, 583-590, 1980.

* 8. J. W. Nunziato and S. L. Passman, A multiphase mixture theory for fluid-saturated granular materials, Proceedings of the Second International Symposium on the Mechanical Behavior of Structured Media, Ottawa, 1981.

* 9. S. L. Passman and J. W. Nunziato, Shearing flow of a fluid-saturated granular material, Proceedings of the Second International Symposium on the Mechanical Behavior of Structured Media, Ottawa, 1981.

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* Those papers marked with an asterisk are part of the research program described in the proposal, but were supported by the U. S. Department of Energy under Contract DE-AC04-76-DP00789 to Sandia National Laboratories.

E-23-628

ENGINEERING COLLEGE

SCHOOL OF ENGINEERING SCIENCE
AND MECHANICS
225 NORTH AVENUE, N.W.
ATLANTA, GEORGIA 30332

May 12, 1978

Dr. Clifford J. Astill
Solid Mechanics Program
National Science Foundation
Washington, D.C. 20550

Subject: Letter report, Grant ENG76-84490, July 1, 1977 to June 30, 1978

Dear Dr. Astill:

By the time the subject Grant had been approved, I had considered in detail the method of solution of boundary-value problems for the theory I was to generalize and had decided that it left one of the equations of the theory unsolved, thus casting some doubt upon the validity of the whole theory. I have worked on that question for much of the last year, and it is now resolved to my satisfaction.

In a talk given to the Society of Rheology in October, 1977, I set forth my ideas about the possible weakness of the theory for the first time. I had concluded that, in a channel or pipe, the only possible motion of a dry granular material according to Goodman and Cowin's theory is a rigid motion. I found this to be an unsatisfactory conclusion. Several experimentists in the room, to the contrary, did not. To exemplify this, I enclose material from Dr. E. B. Bagley, Chief, Engineering and Development Laboratory, Northern Regional Research Center, U.S.D.A.

In my further research, I have had valuable advice on the physical phenomena from Professor G. Sowers. This advice encouraged me to reformulate the equations to allow for incompressible grains. The results from this model, which differs very little from that of Goodman and Cowin except in some small details which do have large effects on the nature of the solutions, are just now becoming available. In particular, I have again worked the problems of flow in a channel and in a tube, this time under the influence of gravity. The results indicate plug flow with a small but finite shear layer near the boundary, a result which both Sowers and I consider physically appropriate. I have assigned the problem of flow between concentric cylinders to Lt. John P. Thomas, Jr., and results are expected soon. These should have application to dry fluid drives and couplings, such as the "Flexidyne" drives manufactured by the Reliance Electric Company, Mishawaka, Indiana.

During the period July 1978-August 1979, I will be on leave from the Georgia Institute of Technology, and will be with the Geomechanics Division, Sandia Laboratories, Albuquerque, New Mexico. Although I will draw no funds from the Grant, I anticipate that I will spend a substantial portion of my time constructing mixture theories appropriate to granular and other geological materials, in accordance with the original proposal.

Mr. John P. Thomas, Jr., listed as a prospective graduate assistant in the proposal, has been called to active duty with the Air Force. He is continuing research on the subject of the Grant on his own time, and has served as co-author of some of the work published. He anticipates continuing to do so, and now that it is clear how to go about solving boundary-value problems in the linear theory, will examine as many as time permits.

In order to get some reasonable experimental data for the constants in the theory we have studied thus far, it is useful to know about controllable deformations. I have looked into this idea with my colleague, Regents' Professor A. W. Marris, and anticipate publishing a note on the concept. The heavy mathematical work required to find large classes of such solutions is Marris' specialty. A separate proposal including such work has been prepared and submitted to N.S.F.

I wish to acknowledge, in addition to my colleagues Marris, Sowers and Thomas, Professor S. C. Cowin of Tulane University and Professor J. T. Jenkins of Cornell University, with whom I have on several occasions debated the mathematical and physical aspects of the theory. Such interaction is possible largely because of the travel support supplied by N.S.F. and the Georgia Institute of Technology. In the next month, travel will be undertaken to several meetings outside of the U.S. to present results. These presentations are listed in an enclosure. Particular note should be taken of travel to Japan (supported by a separate N.S.F. grant), where the most active workers, principally continuum theoreticians, from the U.S., will meet with the most active workers, principally statistical theoreticians and experimentists, from Japan.

A list of papers published and talks given under the auspices of this Grant is enclosed.

Sincerely,

S. L. Passman
Associate Professor

pm

Enclosures

N.S.F. Grant ENG76-84490, July 1, 1977 to June 30, 1978

Papers

1. S. L. Passman and J. P. Thomas, Jr., On the Linear Theory of Flow of Granular Media. Developments in Theoretical and Applied Mechanics 9, 325-332, 1978.

2. S. L. Passman, J. T. Jenkins, and J. P. Thomas, Jr., Flow of a Granular Material in a Vertical Channel. Preprint, U.S.-Japan Seminar, Continuum Mechanical and Statistical Approaches in the Mechanics of Granular Materials, 193-202, 1978. (An enlarged version will appear in the Proceedings of this meeting.)

3. A. W. Marris and S. L. Passman, Some Velocity Universal Motions of Granular Materials. (To appear in the Proceedings cited above.)

Lectures

L1. Exact Solutions in the Flows of Granular Materials. Invited lecture at Engineering Science Department, University of Florida, Gainesville, May, 1977.

L2. Exact Solutions in the Flows of Granular Materials. Invited lecture at Sandia Laboratories, Albuquerque, N.M., May, 1977.

L3. Exact Solutions for Helical Flows of Granular Materials. National Meeting, Society for Industrial and Applied Mathematics, Philadelphia, June, 1977, (Abstract appears in Abstract Booklet.) Presented by J. P. Thomas, Jr.

L4. An Anomaly in the Theory of Granular Media. Annual Meeting, Society of Rheology, Madison, October, 1977. (Abstract appears in Abstract Booklet.)

L5. Exact d'Alembert Solutions of the Navier-Stokes Equations. National Meeting, Society for Industrial and Applied Mathematics, Albuquerque, October, 1977. (Abstract appears in Abstract Booklet) Presented by J. P. Thomas, Jr.

L6. Universal Motions of Granular Materials. Invited lecture at Sandia Laboratories, March, 1978.

L7. On the Linear Theory of Flow of Granular Media. Ninth Southeastern Conference on Theoretical and Applied Mechanics, Nashville, May, 1978. Presented by J. P. Thomas, Jr.

L8. Universal Motions of Granular Materials. Meeting of Society for Natural Philosophy, Pisa, Italy, May, 1978.

L9. Recent Developments in the Theory of Flowing Granular Materials. Invited lecture at the Marmara Research Institute, Istanbul, Turkey, May, 1978.

L10. Recent Developments in the Theory of Flowing Granular Materials. Invited lecture at the Weizmann Institute of Science, Rehovot, Israel, May, 1978.

L11. Flow of a Granular Material in a Vertical Channel. Invited lecture at the U.S. Japan Seminar (Supported by N.S.F. and the Japan Society for the Promotion of Science) Continuum Mechanical and Statistical Approaches in the Mechanics of Granular Materials, Sendai, Japan, June, 1978.

L12. Some Velocity Universal Motions of Granular Materials. Presented at the same Seminar cited in L11.

AGRICULTURAL RESEARCH SERVICE
NORTH CENTRAL REGION
OF UNITED STATES DEPARTMENT OF AGRICULTURE

NORTHERN REGIONAL RESEARCH CENTER
1815 NORTH UNIVERSITY STREET
PEORIA, ILLINOIS 61604

November 18, 1977

Dr. S. L. Passman
School of Engineering Science
and Mechanics
Georgia Institute of Technology
Atlanta, GA 30332

Dear Dr. Passman:

As I promised, I am sending you a copy of our Director's Notes Issue 1311, November 14, 1977. I hope that my description of events is reasonably accurate and meets with your approval. The material is circulated internally in ARS.

I am still getting together some of the references given in your paper in order to address myself to some of the extrusion problems pertinent to our work. In particular, the question of the pressure which can be generated at the end of an extruder (just prior to the die entry) is one that intrigues me. In this connection, however, I do not have a good physical "feel" for p and $\hat{p}$; nor do I see clearly what bulk measurements should be done to predict these pressure values. If you have any comments or suggestions I would be pleased to hear them.

Sincerely,

E. B. Bagley, Chief
Engineering & Development
Laboratory

Enclosure
Newsletter 1311

NORTHERN REGIONAL RESEARCH CENTER
Notes from the Director
November 14, 1977 Issue 1311

BGY Reference Standard Supplied for Black Light Test. Since aflatoxin was reported earlier this year in southeastern corn, almost 1,000 requests from 27 states and 3 foreign countries for BGY reference standards have been filled by Fermentation Laboratory personnel. The standards are vials containing an optical brightener with the same bright greenish-yellow fluorescence (BGY) under ultraviolet light (365 nm) as that associated with *Aspergillus flavus* and possible aflatoxin in corn. Corn is inspected for BGY to identify lots that should be tested further for aflatoxin. BGY standards were supplied to country and terminal elevators, farmers' organizations, grain and feed associations, universities, and action and regulatory agencies such as Federal Grain Inspection Service, Extension Service, and Food and Drug Administration.

Theory & Experiment--The Twain Meet. In the study of complex materials, for example in food rheology and processing, the theoretician and experimentalist are often at odds. An exception to this unhappy state occurred last month at the Society of Rheology meeting in Madison, WI. Professor S. L. Passman, Georgia Institute of Technology, presented a theoretical paper entitled "On the Linear Theory of Granular Material." After a rather complex theoretical treatment, Dr. Passman came to the conclusion that granular material does not flow through a pipe in what is termed the "Poiseuille Mode." Dr. Passman was very disturbed at having arrived at what seemed to him to be a physically incorrect result. By an odd coincidence, however, the paper on soybean extrusion presented at this same meeting (B. Jasberg, E. B. Bagley, and G. C. Mustakas, ED) showed that Dr. Passman's conclusion was correct. When soybean flakes or flour (granular at the temperatures and moisture levels involved) are extruded through a pipe, Poiseuille flow is not observed and the material passes through the tube in plug flow.

In a letter to Ed Bagley later, Dr. Passman notes that although the work he reported at Madison was supported by NSF and USAF, his interest in granular material was initially motivated by USDA, when Dr. P. A. C. Raats (ARS, Salinity Laboratory, Riverside, CA) arranged for him to serve in 1973 as Visiting Assistant Professor in Soil Science and member of the Mathematics Research Center, University of Wisconsin. Thus, Dr. Passman's down-to-earth research in soil science rather unexpectedly surfaces in food extrusion studies.

62nd AACC Meeting. Dietary fiber is very much in the thoughts of the cereal and food industry, as evidenced by the 200 people at a symposium on Developments in Food Fiber at the 62nd meeting of the American Association of Cereal Chemists (AACC) held October 23-27 in San Francisco. A new AACC attendance record of more than 1,600 was established. F. R. Dintzis (CSF) presented an invited paper on "Human Gastrointestinal Action upon Wheat, Corn, and Soy Hull Bran." The research represented a cooperative effort involving three ARS facilities: NRRC; the Human Nutrition Laboratory (HNL) at Grand Forks, North Dakota; and the Spring and Durum Wheat Quality Laboratory at Fargo. The audience was interested not only in what fiber does to them, but also in what they do to fiber. At the annual business meeting, First Approval was given to the new AACC method for insoluble dietary fiber. The Dietary Fiber Committee is now working on a method for determining soluble fiber. On his way to the AACC meeting, Dr. Dintzis visited HNL. He gave a seminar on NRRC's dietary fiber work and met with HNL Director Dr. Harold Sandstead and his colleagues to discuss research and the continuing smooth operation of our cooperative efforts.

Jonas M. Sykes, The Johns Hopkins University, Baltimore, MD 21205

SYM. ON APPLIED MATHEMATICS OF THE
BIOLOGICAL AND SOCIAL SCIENCES II -
WED. 2:15 PM

Dynamics of Populations with Changing Vital Rates

Instead of pursuing models for the actual or imagined dynamics of populations, jectories resulting from more or less arbitrary sequences of vital rates may be sidered. Results so obtained are equally applicable to deterministic models and sample paths of stochastic processes chosen to model, say, population dynamics in dom environments. Attention here is concentrated on the demographic concepts of odicity, stationarity, and the net maternity function as they arise in the usual crete model for the dynamics of a population with age structure. In particular, behavior of products of 2x2 population projection (or "Leslie") matrices and of vector-valued processes generated by those products is considered. Starting from ducts of 2x2 stochastic matrices, explicit results are obtained for products rep- enting various constraints on reproductive behavior which intuitively imply "re- cement" of a population or "zero population growth." The main result is that re- cement fertility for all individuals says little about stationarity of the popu- tion they comprise; a further implication is that measures such as the intrinsic e of natural increase (or "Malthusian parameter") appear to be valid only when al rates are constant over time, and have no obvious generalization to the dyna- s of populations with changing vital rates.

N. Swarztrauber, National Center for Atmospheric Research, Boulder, CO 80303

SESSION 6C - 3:30 PM

A Spectral Approximation for Discrete Scalar and Vector Functions on the Surface of the Sphere

A spectral approximation using spherical harmonics is determined for a discrete tion defined on a uniform spherical grid. The discrete basis is selected as ose harmonics which do not alias on the grid. It is shown that the number of in function is about half the number of grid points with the result that the ximation may not fit all of the discrete function values. A high order ase least squares approximation is obtained, which has certain desirable ing characteristics near the poles.

Although a vector field and its derivatives are continuous in Cartesian ordinates, in spherical coordinates the field or one of its derivatives may be continuous. Therefore, a straight forward spectral analysis may result in a ly convergent series. A method is presented for the approximation of vector lds which is independent of the discontinuities induced by the spherical coordinate tem. Model applications are discussed.

J. P. Thomas, Wright-Patterson Air Force Base, OH 45433, and S. L. Passman, Georgia Institute of Technology, Atlanta, GA 30332. (Introduced by W. F. Ames).

SESSION 3A - 5:00 PM

Exact Solutions for Helical Flows of Granular Materials

A theory for the flow of granular materials in three dimensions has recently been proposed. This theory is complete and rational in that it satisfies invariance principles required of theories of materials, and it allows for solution of boundary-value problems in the usual sense. We consider the linear constitutive theory, but retain the non-linear convective term in the acceleration, thus obtaining a set of non-linear partial differential equations. We consider helical flows, which include as special cases many of those commonly used to measure viscosity of fluids. For such flows we give exact solutions to the equations for granular materials for a variety of boundary conditions. In some cases this theory allows for exact solutions with the non-linear convective term in the acceleration considered, whereas in the theory of fluids the only known exact solutions are for the case of linearized acceleration. In other cases the fact that the granular material has a finite yield point leads to the non-existence of solutions of the type considered, whereas exact solutions do exist for fluids. Physical implications of these facts are discussed.

David McJanaugh, Dept. of Mathematics, University of Missouri, Columbia, MO, 65211 and Bill Thompson, Babcock & Wilcox Company, Nuclear Power Generation Division, Lynchburg, VA 24505

SESSION 5B - 10:35 AM

On the Stability of Adams Methods

Nonlinear optimization and root-finding procedures were used to locate Adams-type methods with increased ranges of stability. $PE(CE)^m$ and $P(EC)^m$ methods, where P is an Adams Bashforth predictor and C is an Adams-type corrector, with order up to 10 are given and discussed for m = 1,2,3. Results for both absolute and relative stability are given and compared. The methods were obtained by maximizing the size of the largest interval of stability with the origin as right endpoint for both types of stability. The complex regions of stability are also given and compared with those of the usual Adams methods.

Henry C. Tuckwell (Introduced by R.M. Miura) and Robert M. Miura, Department of Mathematics, University of British Columbia, Vancouver, B.C., Canada V6T 1W5

SESSION 3C - 4:45 PM

Nonlinear Slow Potential Waves in Cortical Neuronal Structures

Slow potential waves in brain structures can be elicited by mechanical, chemical, or electrical methods. These waves travel at about 3 mm/minute and invade adjacent non-refractory regions. The accompanying depolarization of neurons and glial cells is due mainly to an accumulation of K^+ in the extracellular space with a concomitant influx of Cl^- and Na^+ into neurons. Cells in neighboring regions are affected by diffusion of ions and in turn become sources of K^+ and sinks for Na^+ and Cl^-. The fluxes into these cells are aided by depolarization-induced release of excitatory and inhibitory transmitters. These fluxes diminish due to a variety of mechanisms, one of which is probably depletion of external Ca^{++} which is necessary for the normal release of transmitter. Active transport mechanisms across cell membranes (pumps) are activated by the abnormal distribution of ions and eventually the ion concentrations are returned to their normal levels and the membranes are repolarized. We give a model of these phenomena which consists of a system of nonlinear diffusion equations coupled with other nonlinear differential equations. Some results for these model equations will be presented.

Analytic solutions are provided for the governing differential equations which involve the frequency and dipolar parameters. The behaviors of the instantaneous velocity field, sectional mean velocity and resistance coefficient are discussed for the case of flow with small and large frequencies of oscillation. The results for the case of a viscous fluid are deduced as a special case. Some interesting features are observed for the case of a dipolar fluid in comparison to that for a classical viscous fluid.

Tuesday, October 25 Afternoon Session II Chairman: S. S. Sternstein

3:00-3:30 (J1) *Polymer Structure and Morphology in Relation to Cold Drawing of Poly(Vinyl Chloride)*
E. R. Harrell, Jr., B.F.Goodrich Chemical Company, Technical Center, Independence, Ohio 44131; S. K. Lo, Owens Corning Fiberglass Company, Technical Center, Granville, Ohio 43023; R. P. Chartoff, Department of Chemical and Nuclear Engineering, University of Cincinnati, Cincinnati, Ohio 45221

Tensile stress-strain experiments were performed on thin film specimens of PVC with variations in crystallinity and molecular weight. The data obtained are consistent with the notion that yielding and subsequent cold drawing involve largely localized deformation and are not necessarily consecutive processes.

Visual observation and force variations recorded for deformation beyond the initial yield point indicate that further extension involves localized yielding and multiple neck formation.

While ultimate elongation for the samples decreased with increasing crystallinity and decreasing molecular weight, the suggested localized deformation mechanism is supported by several observations on the cold drawn material obtained from the neck region of highly elongated tensile specimens. Prior to elongation all specimens registered about 15% crystallinity, with correspondingly high densities. Necked specimens in all cases displayed densities comparable to amorphous PVC films. Also the infrared spectra of elongated neck regions indicated that absorption bands used as crystallinity indices, had reduced intensities characteristic of completely amorphous PVC. Further, dynamic mechanical spectra of the neck region material is similar to that for amorphous PVC prepared by quickly quenching from the melt state.

A discussion of the molecular mechanisms associated with cold drawing in PVC will be presented, emphasizing the localized nature of deformation and the role of crystallinity in the processes which occur.

Lecture 4

3:30-4:00 (J2) *An Anomaly in the Theory of Granular Media*
S. L. Passman, School of Engineering Science and Mechanics Georgia Institute of Technology, Atlanta, Georgia 30332; and J. P. Thomas, Jr., Wright-Patterson Air Force Base, Ohio 45433

An ingenious theory for the rheological behavior of granular media was introduced by Goodman and Cowin in 1972. The theory is unique in that it allows for independent variations of porosity and bulk density, with the Coulomb yield criterion emerging as a consequence of the theory. The equations of linear momentum of the theory in their linear and non-linear forms have been shown to describe qualitatively rheological phenomena which are indeed characteristic of granular materials in nature. For example, the linear momentum equation in the linear theory implies the existence of "plug flow" in a pipe. Also, consideration of wave propagation shows the existence of two types of acceleration waves. If the coupling effects are small, one propagates with a velocity determined by the elasticity of the grains, while the second is associated with the compaction process.

We show that, in the context of the linearized theory, if not only the linear momentum equation, but all of the equations of the theory are considered, they form an overdetermined system, for which solutions seldom exist. Indeed, rectilinear and helical flows, which form the members of the class of viscometric flows most easily accessible by experiment, are impossible according to the full set of equations.

Whether this anomaly is a general property of this theory of granular media, or is an artifact of the method of linearization, is a subject still open to investigation. Some approaches to modifying the theory to yield a consistent set of equations are discussed.

4:00-4:30 (J3) *Deformational Behavior of Composite Polymeric Particles*
S. K. Ahuja, Xerox Corporation, Webster, New York 14580

Composite polymer particles made up of brittle and ductile shell and cores were deformed using a Single Particle Crush Apparatus. This apparatus consisted of strain gauges mounted on a metallic strip through which passed a glass probe. This glass probe was used to deform particles through the aid of micromanipulator. The deformation of particles sitting on the stage of an inverted metallograph were viewed and the diameters of the particle determined before and after the deformation. The signal from the Wheatstone strain gauges was fed into a strain indicator and then into a recorder. The apparatus was calibrated with the help of different weights. Forces necessary to fracture or deform to a constant strain was monitored and the distributions computed.

Using Hashin's analysis for composite moduli and Poisson's ratios, Hertzian fracture and yield stresses were obtained and compared with the deformational behavior of the bulk cylindrical specimens of length to diameter ratio of 2/1 using an Instron.

Wednesday, October 26 Morning Session I Chairman: M. Shida

9:00-9:30 (K1) *Melt Rheology of Polyethylene, Polybutadiene and Hydrogenated Polybutadiene Samples with Narrow Molecular Weight Distributions*
V. R. Raju, W. E. Rochefort and W. W. Graessley, Chemical Engineering Department, Northwestern University, Evanston, Illinois 60201

Samples of linear and 4-arm star polybutadiene (90% 1,4 and 10% 1,2 linkages) with narrow molecular weight distributions were prepared by anionic polymerization. They were hydrogenated catalytically to provide narrow distribution models of polyethylene. The large scale molecular structure appears to remain essentially intact during hydrogenation. The hydrogenated polybutadienes (HPB) contain approximately 2 ethyl branches per 100 main chain atoms owing to the 1,2 linkages in the parent polybutadienes (PB). Dynamic moduli, $G'(\omega)$ and $G''(\omega)$, were measured in the melt state as functions of frequency (10^{-1} sec^{-1} – 10^{2} sec^{-1}) with the Rheometrics Mechanical Spectrometer – at 25°C and 50°C for PB; at 130°C and 190°C for HPB and some narrow distribution fractions of linear polyethylene (PE). The zero shear viscosity η_o was proportional to M [illegible] for linear samples of all three polymers. At 190°C η_o vs M (estimated from intrinsic viscosity) was virtually identical for linear HPB and PE, although HPB was found to have a

On the Linear Theory of Flow of Granular Media

By

S. L. Passman
School of Engineering Science and Mechanics
Georgia Institute of Technology
Atlanta, Georgia 30332

and

J. P. Thomas, Jr.
Air Force Flight Dynamics Laboratory
Wright-Patterson Air Force Base, Ohio 45433

Developments in Theoretical and Applied Mechanics
Volume 9, 325-332, 1978.

Abstract

It is shown that, within a commonly accepted theory for the flow of granular materials, Poiseuille flows are impossible.

I. Introduction

A novel mathematical model for flow of granular media has been introduced by Goodman and Cowin [1,2]. This model has been studied, exploited, generalized, and modified quite extensively by various authors [3,4,5,6,7,8]. Here, we consider the linearized version of the theory as given in Goodman and Cowin's first paper [1]. We note that, using their assumed form for the free energy, we are able to obtain an explicit equation for balance of director momentum. As a consequence of the theory of Goodman and Cowin, this equation must be satisfied in addition to the equations of balance of mass, linear momentum, and angular momentum. This additional condition strongly restricts the possible flows described by the theory.

II. The Linear Theory of Granular Media

We outline the theory as given by Goodman and Cowin. The theory takes into account the usual kinematical properties, i.e., velocity $\underset{\sim}{v}$, acceleration $\dot{\underset{\sim}{v}}$, velocity gradient $\underset{\sim}{G}$, and stretching $\underset{\sim}{D}$, with

$$\underset{\sim}{G} = \text{grad}\ \underset{\sim}{v}, \tag{2.1}$$

$$\underset{\sim}{D} = \frac{1}{2}(\underset{\sim}{G} + \underset{\sim}{G}^T), \tag{2.2}$$

where "grad" is with respect to spatial co-ordinates, and $\underset{\sim}{G}^T$, denotes the transpose of $\underset{\sim}{G}$. The material density, $\rho > 0$, is introduced in the usual fashion, but is decomposed into two parts

$$\rho = \gamma\nu, \tag{2.3}$$

where γ, the distributed density, denotes the mass per unit volume of the grains, and ν, the volume distribution function, denotes the portion of the volume occupied by grains, and is bounded thus*

$$0 < \nu \leq 1. \tag{2.4}$$

The stress $\underset{\sim}{T}$, body force $\underset{\sim}{b}$, specific internal energy ϵ, heat flux $\underset{\sim}{q}$, heat supply r, specific entropy η, and entropy flux $\underset{\sim}{\phi}$ have their usual meanings. Furthermore, the introduction of the volume distribution as a new kinematical quantity allows** the introduction of an equilibrated inertia k, equilibrated stress vector $\underset{\sim}{h}$, external equilibrated body force ℓ, and intrinsic equilibrated body force g. The appropriate local forms of the equations of balance of mass, linear momentum, angular momentum, director momentum, equilibrated inertia, energy, and entropy are, respectively

$$\dot{\rho} + \rho\ \text{div}\ \underset{\sim}{v} = 0\ , \tag{2.5}$$

$$\rho\dot{\underset{\sim}{v}} = \rho\underset{\sim}{b} + \text{div}\ \underset{\sim}{T}\ , \tag{2.6}$$

$$\underset{\sim}{T} = \underset{\sim}{T}^T\ , \tag{2.7}$$

$$\rho k\ddot{\nu} = \text{div}\ \underset{\sim}{h} + \rho(\ell + g)\ , \tag{2.8}$$

$$\dot{k} = 0, \tag{2.9}$$

$$\rho\dot{\epsilon} = \text{tr}\ \underset{\sim}{T}\underset{\sim}{D} + \underset{\sim}{h}\ .\ \text{grad}\ \dot{\nu} - \rho g\dot{\nu} - \text{div}\ \underset{\sim}{q} + \rho r, \tag{2.10}$$

$$\rho\dot{\eta} \geq \text{div}\ \underset{\sim}{\phi} + \frac{\rho r}{\theta}. \tag{2.11}$$

The constitutive theory allows Ψ, η, $\underset{\sim}{T}$, $\underset{\sim}{h}$, g and $\underset{\sim}{q}$ each to depend on ν_o (an initial value of ν), ν, grad ν, $\dot{\nu}$, γ, θ, and grad θ, where θ is the temperature. A set of standard thermodynamic arguments then yield the following results

$$\eta = -\frac{\partial\Psi}{\partial\theta}\ , \tag{2.12}$$

$$\underset{\sim}{h} = \rho\frac{\partial\Psi}{\partial(\text{grad}\ \nu)}\ , \tag{2.13}$$

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[*] Goodman and Cowin allow $\nu = 0$. By (2.3), however, this corresponds to $\rho = 0$, and is of no physical interest.

[**] See Jenkins [3] for a full discussion of this point.

and motivate the following definitions,

$$p = \rho\gamma \frac{\partial\Psi}{\partial\gamma} , \quad \hat{p} = \rho\nu \frac{\partial\Psi}{\partial\nu} . \qquad (2.14)$$

Goodman and Cowin [2], interpret "the pressure p as a material pressure related to the compressibility of granules" and the pressure $\hat{p}$ as a "configuration pressure related to the volume distribution of granules."

In order to linearize their equations, Goodman and Cowin assume that $\rho\Psi$ is a quadratic form in grad ν and $\nu - \nu_c$, where ν_c is a non-negative constant:

$$\rho\Psi = a_o + a_2(\nu - \nu_c)^2 + a_3 \operatorname{grad} \nu \, . \operatorname{grad} \nu. \qquad (2.15)$$

Here a_o, a_2, and a_3 are functions of γ. If $\rho\Psi$ is to be non-negative with a minimum at $\nu = \nu_c$, grad $\nu = 0$, then each of a_o, a_2, a_3 must be non-negative. In order to retain the equilibrated stress $\underset{\sim}{h}$ as a nonzero variable in the theory, by (2.13) we must make the further restriction that a_3 is strictly positive. Goodman and Cowin define

$$\alpha_o = a_o + a_2(\nu-\nu_c)^2 , \quad \alpha = a_3, \quad \beta_o = a_o + a_2\nu_c^2, \quad \beta = a_2 . \qquad (2.16)$$

We rewrite Ψ in terms of γ, ν, grad ν, and α, β_o and β, assumed to be material constants, thus obtaining

$$\Psi = \frac{\beta_o + \beta\nu^2 - 2\beta\nu_c\nu + \alpha \operatorname{grad} \nu . \operatorname{grad} \nu}{\gamma\nu} . \qquad (2.17)$$

It is then a straightforward task to compute p, $\hat{p}$ and $\underset{\sim}{h}$ from (2.13) and (2.14). We obtain

$$p = - (\beta_o + \beta\nu^2 - 2\beta\nu_c\nu + \alpha \operatorname{grad} \nu . \operatorname{grad} \nu) , \qquad (2.18)$$

$$\hat{p} = p + 2\beta\nu^2 - 2\beta\nu_c\nu = -\beta_o + \beta\nu^2 - \alpha \operatorname{grad} \nu . \operatorname{grad} \nu, \qquad (2.19)$$

$$\underset{\sim}{h} = 2\alpha \operatorname{grad} \nu . \qquad (2.20)$$

Generally, g has two parts: an equilibrium part for which Ψ is a potential and a viscous part whose form is assumed. Goodman and Cowin [2], in keeping with their assumption of a quadratic Ψ (thus giving a linear form for the equilibrium portions of the stresses) assume g linear in $\dot{\nu}$ and $\underset{\sim}{D}$, obtaining

$$g = \frac{p-\hat{p}}{\gamma\nu^2} - \zeta\dot{\nu} - \delta \operatorname{tr} \underset{\sim}{D}, \qquad (2.21)$$

where ζ and δ are constitutive constants. By (2.19) this becomes

$$g = \frac{2\beta(\nu_c-\nu)}{\gamma\nu} - \zeta\dot{\nu} - \delta \operatorname{tr} \underset{\sim}{D} . \qquad (2.22)$$

We substitute (2.20) and (2.21) into (2.8), obtaining

$$\gamma\nu k\ddot{\nu} = 2\alpha \nabla^2\nu + 2\beta(\nu_c - \nu) - \zeta\gamma\nu\dot{\nu} - \delta\gamma\nu \,\mathrm{tr}\underset{\sim}{D} + \gamma\nu\ell \, . \qquad (2.23)$$

The equation of linear momentum is obtained similarly. Goodman and Cowin [2] give for the stress

$$\underset{\sim}{T} = (-p + \xi\dot{\nu} + \lambda \mathrm{tr}\underset{\sim}{D})\underset{\sim}{1} - 2\alpha \,\mathrm{grad}\, \nu \otimes \mathrm{grad}\, \nu + 2\mu\underset{\sim}{D} \, , \qquad (2.24)$$

where ξ, λ and μ are material constants.

The divergence of this equation is

$$\mathrm{div}\, \underset{\sim}{T} = -\,\mathrm{grad}\, p + \xi \,\mathrm{grad}\, \dot{\nu} - 2\alpha[(\mathrm{gradgrad}\, \nu)\, \mathrm{grad}\, \nu$$

$$+ (\nabla^2 \nu)\, \mathrm{grad}\, \nu] + (\lambda + \mu)\, \mathrm{grad\, div}\, \underset{\sim}{v} + \mu\nabla^2\underset{\sim}{v} \, . \qquad (2.25)$$

By (2.18), the pressure gradient is

$$\mathrm{grad}\, p = 2\beta\nu \,\mathrm{grad}\, \nu - 2\beta\nu_c \mathrm{grad}\, \nu + 2\alpha(\mathrm{gradgrad}\, \nu)\, \mathrm{grad}\, \nu. \qquad (2.26)$$

By (2.25) and (2.26), the linear momentum equation (2.6) is

$$\gamma\nu\dot{\underset{\sim}{v}} = -\,2 \,\mathrm{grad}\, \nu[\alpha\nabla^2\nu - \beta(\nu - \nu_c)] + \xi \,\mathrm{grad}\, \dot{\nu} + (\lambda + \mu)\, \mathrm{grad\, div}\, \underset{\sim}{v}$$

$$+ \mu\nabla^2\underset{\sim}{v} + \gamma\nu\underset{\sim}{b} \, . \qquad (2.27)$$

An alternate form of (2.27), obtained by substituting $\nabla^2\nu$ from (2.23), is

$$\gamma\nu(\dot{\underset{\sim}{v}} + k\ddot{\nu}\, \mathrm{grad}\, \nu) = -\,\gamma\nu(\zeta\dot{\nu} + \delta\, \mathrm{div}\, \underset{\sim}{v})\, \mathrm{grad}\, \nu + \xi\, \mathrm{grad}\, \dot{\nu}$$

$$+ (\lambda + \mu)\, \mathrm{grad\, div}\, \underset{\sim}{v} + \mu\nabla^2\underset{\sim}{v} + \gamma\nu(\ell\, \mathrm{grad}\, \nu + \underset{\sim}{b}) \, . \qquad (2.28)$$

Equations (2.23) and (2.28) are the equations of motion of this theory. Boundary value problems in this theory are formulated in much the same fashion as boundary value problems for linearly viscous fluids, except that (2.23) and (2.28) replace the Navier-Stokes equations. The "extra" equation (2.23) is "needed" to specify the "extra" variable ν.

Although equations (2.23) and (2.28) appear here for the first time, they are direct consequences of the physical principles postulated by Goodman and Cowin. Their physical appropriateness is considered in Section 5.

III. A Note on Body Forces

In addition to terms which are easy to interpret physically, the equations of motion (2.23) and (2.28) contain the external equilibrated body force ℓ. Before we can reasonably expect to find solutions to these equations, we need some information on the nature of ℓ.

In many cases of physical interest, the body force per unit mass $\mathbf{b}$ has a potential. That is, there exists a scalar function of position in space only, whose gradient is the body force $\mathbf{b}$. Jenkins [3] shows that this idea extends naturally to the theory of granular media. We call a function $\Phi(\mathbf{x}, \nu(\mathbf{x}))$ a body force potential for a granular medium if

$$\mathbf{b} = - \nabla\Phi \ , \tag{3.1}$$

and

$$\ell = - \frac{\partial\Phi}{\partial\nu} \ . \tag{3.2}$$

Here $\nabla\Phi$ is the partial gradient of Φ with ν held constant. The total gradient of Φ, grad Φ, would be given by

$$\text{grad } \Phi = \nabla\Phi + \frac{\partial\Phi}{\partial\nu} \text{ grad } \nu, \tag{3.3}$$

so that, by (3.1) and (3.2)

$$- \text{grad } \Phi = \ell \text{ grad } \nu + \mathbf{b} \ ,$$

precisely the last term appearing in parentheses in (2.28). In physical situations where body forces are unimportant, as is the case in some flow problems, it is customary to make the idealization $\mathbf{b} = \mathbf{0}$, corresponding to constant Φ. The same assumption here leads to the conditions

$$\mathbf{b} = \mathbf{0}, \quad \ell = 0. \tag{3.4}$$

It is then possible that these two conditions occur simultaneously. When that is the case, we say there are zero body forces.

IV. Rectilinear Motions

Goodman and Cowin [1] consider a set of steady isochoric motions with $\dot{\nu} = 0$. We do also, but assume in addition zero body forces. The equations of motion then become

$$\gamma\nu\mathbf{v} \cdot \text{grad } \mathbf{v} = \mu\nabla^2\mathbf{v} \ , \tag{4.1}$$

$$\nabla^2\nu - \frac{\beta}{\alpha}\nu = - \frac{\beta}{\alpha}\nu_c \ . \tag{4.2}$$

Furthermore, the equation of conservation of mass may be written in this case, as

$$\underset{\sim}{v} \cdot \mathrm{grad}\, \nu = 0. \tag{4.3}$$

We choose Cartesian co-ordinates x, y, z and respective velocity components u, v, w and search for solutions of the form $u = u(y)$, $v = w = 0$, $\nu = \nu(x,y)$. Such flows are generalizations of the concept of <u>steady shearing flow</u>*. In most known theories of fluids, simple shearing and plane Poiseuille flows, both of which are steady shearing flows, constitute exact solutions of the equations of motion.

Our assumed forms of the velocity and volume distribution gives $\underset{\sim}{v} \cdot \mathrm{grad}\, \underset{\sim}{v} = 0$ and by (4.3)

$$u \frac{\partial \nu}{\partial x} = 0. \tag{4.4}$$

Thus, either these motions are static ($u = 0$), or $\partial\nu/\partial x = 0$.

In the former case (4.1) is of course identically satisfied, while (4.2) becomes

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - \frac{\beta}{\alpha}\right)(\nu - \nu_c) = 0, \tag{4.5}$$

so that a static configuration with non-homogeneous volume distribution is possible.

In the latter case $\nu = \nu(y)$, and (4.2) becomes

$$\left(\frac{d^2}{dy^2} - \frac{\beta}{\alpha}\right)(\nu - \nu_c) = 0 . \tag{4.6}$$

The form of the solution of this equation depends upon the sign of β/α. Our assumptions in Section 2 require $\beta/\alpha \geq 0$. We require the inequality hold, so that, say

$$\xi^2 = \frac{\beta}{\alpha} , \tag{4.7}$$

with ξ real. Then

$$\nu = \nu_c + A \sinh \xi y + B \cosh \xi y, \tag{4.8}$$

with A and B arbitrary, also in this case (4.1) may be solved to give

$$u = Cy + D. \tag{4.9}$$

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*See, e.g., [9].

Equations (4.8) and (4.9) represent all possible non-static volume distribution and velocity fields for steady shearing flows which satisfy the equations of motion of this theory. We concentrate on the velocity field (4.9).

Experimental studies of flows of granular media indicate that a "slip" type of boundary condition is appropriate at solid boundaries. That is, the component of the velocity of the granular medium orthogonal to the boundary is equal to the component of the velocity of the boundary orthogonal to itself, but this equality does not hold for tangential components.

We attempt to describe the fully developed velocity field between two parallel plates a distance L apart, one stationary, and one moving with a velocity parallel to itself. We then impose boundary conditions of the type, say $u = u_o$ at $y = 0$, $u = u_1$ at $y = L$. In that case (4.9) becomes

$$u = u_o\left(\frac{L-y}{L}\right) + u_1 \frac{y}{L} . \tag{4.10}$$

Such a flow is called a simple shearing.

A Poiseuille flow between two parallel plates would require, say $u = u_o$ at $y = \pm L$. It is seen from (4.9) that the only motion of this type allowed by this theory is a rigid motion. A similar set of calculations yields the result that Poiseuille flow in a circular tube is also impossible, but that rigid motion is possible. These conclusions appear to agree with some experimental results [10] and to disagree with others [11] where, typically a small shear layer near the wall is observed.

V. Conclusion

In this paper we have presented no new theory. Rather, we have concentrated on carrying a known theory to a logical conclusion. Whether the conclusion matches with experiment is unclear. However, it appears physically plausible that some, perhaps thin, shear layer should surround the rigid plug in Poiseuille flow, a phenomenon we have shown is not allowed by this theory. In a later paper, we will show that a slight modification of the theory presented here does allow this phenomenon.

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