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PROJECT ADMINISTRATION DATA SHEET

[x] ORIGINAL [ ] REVISION NO. ______

Project No. E-23-659 — DATE 9/21/81

Project Director: Dr. George J. Simitses — School/Lab ESM

Sponsor: National Science Foundation

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Sponsor Amount: $47,640 — Contracted through:

Cost Sharing: $ 5,101 E-23-347 — GTRI/~~GIT~~

Title: Nonlinear Stability Analysis of Elastic Unbraced Frames

ADMINISTRATIVE DATA

OCA Contact ______

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SPONSORED PROJECT TERMINATION/CLOSEOUT SHEET

Date 7/31/84

Project No. E-23-659

School/~~Lab~~ ESM

Includes Subproject No.(s)

Project Director(s) George J. Simitses GTRI / ~~GIT~~

Sponsor NSF

Title Non Linear Stability Analysis of Elastic Unbraced Frames

Effective Completion Date: 6/30/84 (Performance) 9/30/84 (Reports)

Grant/Contract Closeout Actions Remaining:

- [ ] None
- [x] Final ~~Invoice or Final Fiscal Report~~ FCTR
- [ ] Closing Documents
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Form OCA 60:1028

Progress Report No. I. on
NSF Grant CEE-81 1745 entitled
"NONLINEAR ANALYSIS OF ELASTIC UNBRACED FRAMES"
By George J. Simitses, Principal Investigator

The research deals with the nonlinear behavior, including buckling and postbuckling response, of linearly elastic, rigid-jointed and flexibly-jointed, unbraced, plane frameworks, subjected to distributed and eccentric concentrated loads.

In this investigation, there are two major areas to be dealt with. First, the assessment of the effect of flexible connections on the response characteristics of frames that are either subject to bifurcational buckling (sway-buckling, portal frames) or, subject to limit-point instability (snap-through buckling two bar-frames). Second, the development of a solution scheme (including a computer code) for the analysis of multibay multistory, unbraced plane frames.

The first task has been accomplished and the findings of the investigation have been reported on two papers (copies attached to this report as Attachments I and II), which have been submitted for publication.

The first paper deals with simple one-bay unbraced portal frames. This configuration is subject to bifurcational buckling (sway-buckling) with stable postbuckling behavior. It is found that flexible connections of the type reported by DeFalco and Marino [1] have negligibly small effect on the sway-buckling load, when one exists. On the other hand, in the case for which the frame is loaded asymmetrically, the effect of flexible connections is negligibly small for loads, which are small by comparison to the sway-buckling load of the corresponding symmetrically-loaded configuration (see Fig. 6 of Attachment 1), but its effect becomes increasingly non-neglible as the load approaches the sway-buckling load.

Of course, these observations are for the assumed cubic nonlinearity in the joint bending moment - relative rotation curve and for the tried values of the coefficient of the cubic term (A). It is the opinion of the investigator that for a realistic good connection, it is reasonable to expect that the magnitude of the bending moment tends to increase with increasing relative rotation prior to sway-buckling. This being the case, the observations hold for good real-world frame connections. (Good here implies that the connection is strong and it can transfer bending moment with increasing loading).

Since the two-bar frame is subject to limit-point instability rather than bifurcational buckling (sway-buckling), and since the effect of flexible connections on the response of an asymmetrically-loaded portal frame (for loads near the corresponding sway-buckling load) is not negligible, one should investigate the flexibly-connected and eccentrically loaded two-bar frame. This is done and reported on Attachment II. The procedure employed is similar to that of Attachment I, and the observations for this configuration are similar to those for the portal frame. The effect of realistic (good) flexible connection on the limit point load (whenever one exists) of the two-bar frame is negligibly small.

Moreover, for both configurations, several parametric studies are performed in order to assess the effects of various structural and geometric parameters on the critical loads, in the presence of flexible connections.

The conclusions are similar to the ones drawn for the corresponding rigid-jointed frames and are listed below:

(1) For all geometries and load cases considered, the effect of bar slenderness ratio on the nondimensionalized response characteristics, including critical loads, is negligibly small.

(2) The effect of load eccentricity on the nondimensionalized critical loads (limit point loads for the two-bar frames, bifurcation loads [sway-buckling] for the symmetrically loaded portal frames) is the same as in the case of rigid-jointed frames. For instance, the sway-buckling load decreases (slightly) with increasing load eccentricity. The limit point load (two-bar frame) decreases with increasing eccentricity (in a direction away from the support of the horizontal bar- outside the frame). Note that for eccentricities inside the frame there is no buckling. The frame simply bends in stable equilibrium.

On the basis of the above investigations and related observations, it is decided to abandon (for the present) any further investigations concerning the effect of flexible connections and concentrate on the second major task (in progress) which is: To develop a solution methodology for the response of a multi-bay multi-story, elastic, plane, unbraced, rigid-jointed framework, subjected to, in general, eccentric concentrated loads (near the joints) and uniformly distributed transverse loads.

The proposed solution, methodology (which is being developed presently) contains the following features (refer to Fig. 1 for the geometry and sign convention).

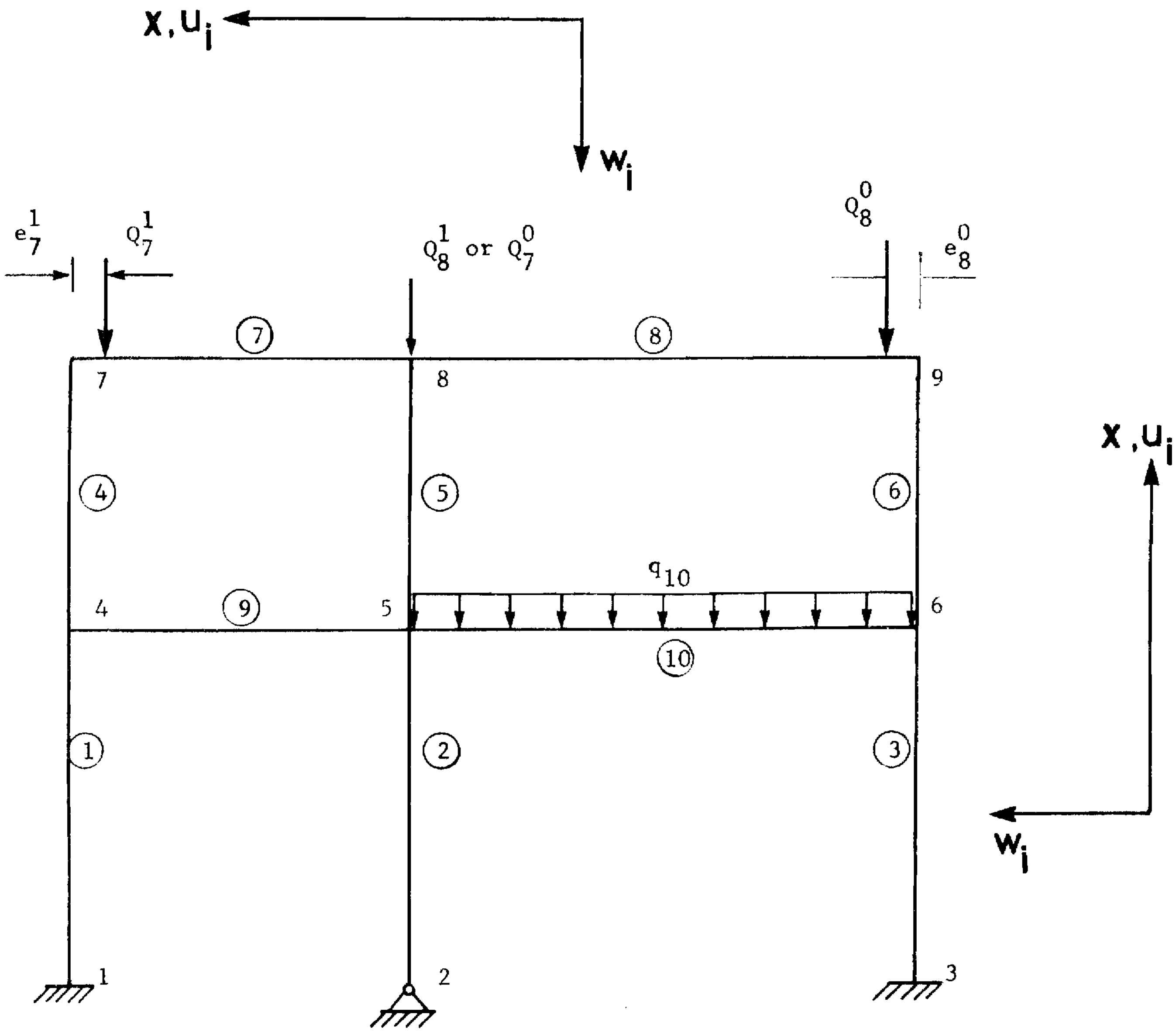

Figure 1. Geometry and Sign Convention

(a) The general solution to the equilibrium equations for the ith bar is given by

$$U_i(X) = A_{i5} \pm \frac{k_i^2}{\lambda_i^2} X - \frac{1}{2}\int_0^X W_{i,X}^2 \, dX \tag{1}$$

and

$$W(X) = A_{i1} \binom{\sin k_i X}{\sinh k_i X} + A_{i2} \binom{\cos k_i X}{\cosh k_i X} + A_{i3} X + A_{i4} \pm \frac{\bar{q}_i X^2}{2 k_i^2} \tag{2}$$

where the "upper" sign and terms are used, when the ith bar is in compression and the lower, when in tension. Also note that for the vertical bars, usually, the transverse loading $\bar{q}$ is zero. Furthermore, the displacement component U_i and W_i are in nondimensionalized form. A list of the nondimensional parameters is given below:

$$U_i = u_i/\ell_i \quad ; \quad W_i = w_i/\ell_i \quad ; \quad X = x_i/\ell_i \quad ;$$

$$\mu_i = \ell_i/\ell_1 \quad ; \quad r_i = (EI)_i/(EI)_1 \quad ; \quad \rho_i^2 = I_i/A_i \quad ;$$

$$\lambda_i = \ell_i/\rho_i \quad ; \quad \bar{Q}_i = (Q_i \ell_i^2)/(EI)_1 \quad ; \quad \bar{q}_i = q_i \ell_i^3/(EI)_i \quad ;$$

$$q^* = q_i \ell_1^3/(EI)_1 \quad ; \quad k_i^2 = \pm P_i \ell_i^2/(EI)_i$$

where P_i is the axial force in the ith bar ($\pm P_i$ is a positive number; P_i is positive when tensile and negative when compressive).

(b) Given an n-member frame, there are 6 n - unknowns that one must solve for [see Eqs. (1) and (2)] (5 n A_{ij}'s and n k_i's).

In order to solve for these unknowns, 6 n boundary and joint conditions are needed. At each boundary, there are three boundary conditions. At each joint, there are three force and moment conditions. Moreover, at each two-member joint there exist three kinematic continuity conditions, at each three-member six conditions, and finally at each four-member joint nine conditions.

From Fig. 1, it is clearly seen that the two-bay, two-story plane frame has ten members; therefore, the number of unknowns is 60 (50 A_{ij}'s and 10 k_i's).

There are three boundary conditions at each of points 1, 2, and 3.

$$\text{Pt. 1} \qquad U_1(0) = W_1(0) = W_{1,X}(0) = 0$$

$$\text{Pt. 2} \qquad U_2(0) = W_2(0) = W_{2,XX}(0) = 0 \tag{3}$$

$$\text{Pt. 3} \qquad U_3(0) = W_3(0) = W_{3,X}(0) = 0$$

Moreover, there are three force and moment equilibrium conditions at each of joints 4,5,6,7,8, and 9, or 18 such conditions.

As far as the kinematic continuity conditions are concerned, there are three at joint 7, three at joint 9 [see Eqs. (4) below], six at joint 4, six at joint 6, six at joint 6, and nine at joint 5, for a total of 33 such conditions.

Thus, the total number of conditions are 60 (33 kinematic continuity at joints, 18 force and moment equilibrium at joints and 9 boundary conditions).

at joint 7

$$U_4(1) = - W_7(1)$$

$$W_4(1) = U_7(1) \qquad (4)$$

$$W_{4,x}(1) = W_{7,x}(1)$$

(c) Satisfaction of all of the conditions in (b) lends to a system of 6 n equations in 6n unknowns. Of these, 5 n equations are linear in the 5 n A_{ij}'s. These can be written in matrix form, or

$$[C] \quad \{A\} \quad = \{D\} \qquad (5)$$

where $[C]$ is a 5 n by 5 n matrix in terms of geometric parameters and in general trigonometric functions of the k_i's, $\{A\}$ is a column matrix of the 5 n A_{ij}'s and $\{D\}$ is a column matrix containing, primarily load and load eccentricity parameters.

The remaining n equations are nonlinear in the A_{ij}'s and k_i's.

$$f_i(k_i, A_{ij}) = 0 \qquad i = 1, \dots n \qquad (6)$$

(d) For small values of the load (linear theory should hold), an estimate of the k_i's is obtained (different schemes are under consideration).

This estimate is used in Eq. (5) to solve for the A_{ij}. Then, these A_{ij} values are used in Eqs (6) to solve for the k_i's (a nonlinear solution code has been tried successfully for this purpose). If the k_i's are different from initial estimates, the new values are used in Eqs (5) to obtain new values for A_{ij}, which in turn are used in Eqs (6). The interation continues until a desired accuracy is obtained.

Then the load level is increased by a small amount, and the final k_i-values at the previous load level are used as initial estimates. The iteration procedure is again employed, and then the load parameters are incremented again. This procedure leads to the primary path response.

For finding critical loads (sway-buckling) and post-buckling path (whenever they exist) a number of approaches are under consideration. Finally, efforts are exerted into mechanizing the information pertaining to the recording of the geometry and assembly of the necessary equations.

References

1. DeFalco, F., and Marino, F. J., "Column Stability in Type 2 Construction", AISC Engineering Journal, Vol. 3, No. 2, April 1966.

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Washington, D.C. 20550

SUMMARY OF COMPLETED PROJECT

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1. INSTITUTION AND ADDRESS		2. NSF PROGRAM	3. PRINCIPAL INVESTIGATOR(S)
Georgia Institute of Technology 225 North Avenue Atlanta, Georgia 30332		Structional Mechanics	G. J. Simitses Professor of Engineering Science & Mechanics
4. AWARD NUMBER CEE-8117450	5. DURATION (MOS) 27	6. AWARD PERIOD from: 10/1/81-6/30/83	7. AWARDEE ACCOUNT NUMBER E-23-659

8. PROJECT TITLE
Nonlinear Stability Analysis of Elastic Unbraced Frames.

9. SUMMARY (ATTACH LIST OF PUBLICATIONS TO FORM)

The nonlinear analysis of plane elastic and orthogonal frameworks is presented. The static loading consists of both eccentric concentrated loads and uniformly distributed loads on all or few members. The joints can be either rigid or flexible. The flexible joint connection is characterized by connecting one member on an adjoining one through a rotational spring (with linear or nonlinear stiffness). The supports are immovable but are also characterized with rotational restraint by employing linear rotational springs. The mathematical formulation is presented in detail and the solution methodology is outlined and demonstrated through several examples. These examples include two-bar frames, portal frames as well as multibay multistorey frames. The emphasis is placed on obtaining sway buckling loads, prebuckling and post-buckling behaviors, whenever applicable.

The most important conclusions of the investigation are:

(i) The effect of flexible joint connection (bolted, riveted and welded) on the frame response (especially sway-buckling loads) is small.

(ii) Multistory, multibay orthogonal frames are subject to bifurcational (sway) buckling with stable post-buckling behavior. Sway-buckling takes place, when the frame and loads are symmetric.

(iii) The effect of slenderness ratio on the nondimensionalized response characteristics is negligibly small (except for the two-bar frame).

(iv) Starting with a portal frame, addition of bays increases appreciately the total sway-buckling load, while addition of storeys has a very small effect.

9. SIGNATURE OF PRINCIPAL INVESTIGATOR/ PROJECT DIRECTOR	TYPED OR PRINTED NAME	DATE
	George J. Simitses	7/20/84

PUBLICATIONS

1) "Stability Analysis of Semi-Rigidly Connected Simple Frames" Journal of Constructional Steel Research, Vol. 2, No. 3, September 1982, pp. 29-32.

2) "Flexibly-Jointed Unbraced Portal Frames" Journal of Constructional Steel Research, Vol. 4, No. 1, January 1984, pp. 27-44.

3) "Asymmetrically Loaded Portal Frames" Computers and Structures Vol. 18, (to appear in the September or October issue of 1984).

4) "A Nonlinear Solution Scheme for Multistory, Multibay Plane Frames" Computers and Structures, submitted for publication.

5) "Elastic stability of Rigidly and Semi-Rigidly Connected Unbraced Frames" Chapter IV in FRAMED STRUCTURES: Stability and Strength, Ed. by R. Narayanan, Applied Science Publishers, England (to appear in late 1984).

Moreover, three M. S. Theses and one Ph.D Dissertation have been written in connection with this project. From the material contained in these, at least four more publications (in refereed journal) will be prepared.

M.S. Students:
1) Mr. Jerry Swisshelm
2) Mr. John G. Simitses
3) Mr. Jordan Peck

Ph.D. Student:
1) Dr. Andreas S. Vlahinos (Currently Assistant Professor of Civil Engineering, University of Colorado, Denver, Colorado)

FINAL REPORT

NONLINEAR STABILITY ANALYSIS OF ELASTIC UNBRACED FRAMES

by G. J. Simitses
Georgia Institute of Technology, Atlanta, Georgia

ABSTRACT

The nonlinear analysis of plane elastic and orthogonal frameworks is presented. The static loading consists of both eccentric concentrated loads and uniformly distributed loads on all or few members. The joints can be either rigid or flexible. The flexible joint connection is characterized by connecting one member on an adjoining one through a rotational spring (with linear or nonlinear stiffness). The supports are immovable but are also characterized with rotational restraint by employing linear rotational springs. The mathematical formulation is presented in detail and the solution methodology is outlined and demonstrated through several examples. These examples include two-bar frames, portal frames as well as multibay multistorey frames. The emphasis is placed on obtaining sway buckling loads, prebuckling and postbuckling behaviors, whenever applicable.

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(iii) The effect of slenderness ratio on the nondimensionalized response characteristics in negligibly small (except for the two-bar frame).

(iv) Starting with a portal frame, addition of bays increases appreciably the total sway-buckling load, while addition of storeys has a very small effect.

1. INTRODUCTION

Plane frameworks, composed of straight slender bars, have been widely used as primary structures in several configurations. These include one- or multi-storey buildings, storage racks, factory cranes, off-shore platforms and others. Depending on characteristics of geometry (symmetric or asymmetric, and various support conditions) and loading (symmetric or asymmetric transverse and horizontal), plane frames may fail by general instability (in a sidesway mode or a symmetric mode) or they may fail by a mechanism or a criterion other than stability (excessive deformations and/or stresses etc). For example, a symmetric portal frame subjected to a uniformly distributed transverse load is subject to sway buckling. On the other hand if, in addition to the transverse load, a concentrate horizontal load is applied, excessive deformations and stresses will occur without the system being subject to instability (buckling).

The various frame responses, associated with the various geometries and loadings, have been the subject of many studies, both in analysis and in synthesis (design). A brief description and critique of these studies is presented in the ensuing articles.

1.1 Rigid-Jointed Frames - Linear Analyses

The first stability analyses of rigid-jointed plane frameworks may be traced to Zimmerman (1909, 1910 and 1925), Muller-Breslau (1910) and Bleich (1919). They only treated the problem for which a momentless primary state (membrane) exists and bifurcational buckling takes place through the existence of an adjacent bent equilibrium state (linear eigenvalue problem). Prager (1936) developed a method which utilizes the stability condition of a column with elastic end restraints. The first investigation of a problem for which the primary state includes bending moments (primary moments) is due to Chwalla (1938). He studied the sway buckling of a rigid-jointed, one-storey, symmetric, portal frame, under symmetric concentrated transverse loads, not applied at the joints of the horizontal bar. In obtaining both the primary path and the bifurcation load, Chwalla employed linear equilibrium equations and he assumed linearly elastic behavior. In more recent years, similar problems have been studied by Baker et al. (1949), Merchant (1954, 1955), Chilver (1956), Livesley (1956), Goldberg (1960), Masur et al. (1961), and Horne (1962). The last two consider the effect of primary moments which cause small deflections prior to instability in their buckling analysis of portal frames. Many of the aforementioned analyses have been incorporated into textbooks, such as those of Bleich (1952), McMinn (1962), Horne and Merchant (1965), and Simitses (1976). Other investigations of this category include the studies of Holldorsoon and Wang (1968) and Zweig and Kahn (1968). It is also worth mentioning the work of Switzky and Wang (1964), who outlined a simple procedure for designing rectangular, rigid frames against stability. Their procedure employs linear theory and is applicable to load cases for which the primary state is a membrane state (free of primary moments).

1.2 Rigid-Jointed Frames-Nonlinear Analyses.

The effects of finite displacements on the critical load and on the postbuckling behaviour of frameworks have only been investigated in the last 20 years or so. Saafan (1963) considered the effects of large deformations on the symmetric buckling of a gable frame. Similar effects were also considered by Britvec and Chilver (1963) in their studies of the buckling and postbuckling behaviour of triangulated frames and rigid-jointed trusses. The nonlinear behaviour of the two-bar frame was studied by Williams (1964), Roorda (1965), Koiter (1966), Huddleston (1967) and more recently by Kounadis et al. (1977) and by Simitses et al. (1977). Roorda's work contains experimental results, while Koiter's contribution employs his (1945) rigorous nonlinear theory for initial postbuckling behavior, applicable to structures that exhibit bifurcational buckling. The studies of Kounadis and Simitses employ nonlinear kinematic relations (corresponding to moderate rotations) and assume linearly elastic material behaviour. Huddleston's nonlinear analysis is based on equations of the Elastica. A similar approach (Elastica-type of equations) was outlined by Lee et al. (1968) for studying the large deflection buckling and postbuckling behaviour of rigid plane frameworks loaded by concentrated loads. They demonstrated their procedure by analyzing a two-bar frame and a portal frame, and they used a modified Newton-Raphson procedure to solve the nonlinear equations. More recently, Elastica-type of equations were employed by Qashu and DaDeppo (1983) for the analysis of elastic plane frames. They used numerical integration of the differential equations and their examples include one- and two-storey elastic, rigid frames. Besides the inherent assumptions of Elastica-type of equations that make then applicable to very slender members, the difficulty of solving the highly nonlinear equations in a straight forward method further limits the applicability of this approach to frames with a relatively small number of members. On the other hand, the nonlinear methodology, described herein, as developed by Simitses and his collaborators (1977, 1978, 1981, 1982) employs first order nonlinear kinematic relations (moderate rotations) but can be used, with relative ease, in analyzing the large deformation behaviour (including buckling and postbuckling) of multi-storey, multi-bay, of elastic, rigid-jointed, orthogonal, plane frameworks, with a large number of members.

The interested reader is referred to the book of Britvec (1973), which presents some of the nonlinear analyses of frames. Moreover, those who are interested in the design of elastic frames are referred to the Design Guide of the Structural Stability Research Council; see Johnston (1976).

1.3 Semi-Rigidly Connected Frames

All of the previously discussed analyses, are based on the assumption that the bars are rigidly connected at the frame joints. This means that the angle between connected members at the joints remains unchanged, during deformations.

Since the 1930's, there has been considerable interest and research into the behaviour of beam structural connections. A number of experimental and analytical studies have been carried out to measure the moment-relative rotation characteristics of various types of metal (primarily steel) framing connections. Various methods of (moment distribution, slope-deflection, elastic line) of analysis have been employed in order to account for the flexibility of the connections by Batho and Rowan (1934), Rathbun (1936) and Sourochnikoff (1946). Moreover, some efforts have been made, recently, to account for the effect of flexible connections in frame design. DeFalco and Marino (1966) modified the effective column length, used in frame design, by obtaining and employing a modified beam stiffness, which is a function of the semi-rigid connection factor (slope of the relative rotation to moment curve at the origin), Z, proposed by Lothers (1960). Fry and Morris (1975) presented an iterative procedure which incorporates the effects of nonlinear connection characteristics. They assumed linearly elastic material behaviour, and they developed equations that depict moment-relative rotation relations for a wide range of frame connections. More recently Moncarz and Gerstle (1981) presented a matrix displacement method for analyzing frame with flexible (nonlinear) connections. The effect of flexible joints on the response characteristic of simple two-bar frames, which are subject to limit point instability (violent buckling) has been reported by Simitses and Vlahinos (1982). This subject is further explored, herein, in a later article. Finally, a brief summary of recent research of the effect of end restrains on column stability is presented by Lui and Chen (1983).

In closing, it is worth mentioning that the analysis of plane frameworks, including stability studies, postbuckling behaviour and the study of the effect of flexible connections, has been the subject of several Ph.D. theses, especially in the United States. Of particular interest, and related to the objective of the present chapter are those of Ackroyd (1981), and Vlahinos (1983). Moreover, there exist a few reported investigations, in which the frame has been used as an object of demonstration. In these studies, the real interest lies in some nonlinear numerical scheme, especially the use of finite elements. Some of these works, but not limited to, are those of Argyris & Dunne (1975), Olesen & Byskov (1982), and Obrecht et al. (1982).

2. MATHEMATICAL FORMULATION

2.1 Geometry and Basic Assumptions

Consider a plane, orthogonal, rigid-jointed frame composed of N straight slender bars of constant cross-sectional area. A typical ten bar frame is shown on Fig. 1. Each bar, identified by the subscript "i", is of length L_i, cross-sectional area A_i, cross-sectional second moment of area I_i, and it subscribes to a local coordinate system, x & z, with displacement components u_i and w_i as shown. The frame is subjected to eccentric concentrated loads Q_i^0 and Q_i^1 and/or uniformly distributed loadings q_i. For the concentrated loads, the superscript "0" implies that the load is near the origin of the ith bar (x = 0), while the superscript "1" implies that the load is near the other end of the ith bar ($x = L_i$). The concentrated load eccentricities are also denoted in the same manner as the concentrated loads (e_i^0 and e_i^1). Moreover, these eccentricities are positive if the loads are inside the x-interval of the corresponding bar and negative if outside the interval. For example, on Fig. 1 e_7^0 is a positive number. But this same eccentricty (and therefore the corresponding load too) can be identified as e^1_8 in which case its value is negative. This is used primarily for corner overhangs (joint 7 or 9 with concentrated loads off the frame). The supports are such that translation is completely constrained, but rotation could be free. For this purpose rotational linear springs are used at the supports (see Fig. 1, support "3"). When the spring stiffness, β, is zero, we have an immovable simple support (pin). On the other hand, when β is a very large number ($\rightarrow \infty$) we have an immovable fixed support (clamped, built-in).

For clarity, all the limitation of the mathematical formulation are compiled below in form of assumptions. These are:

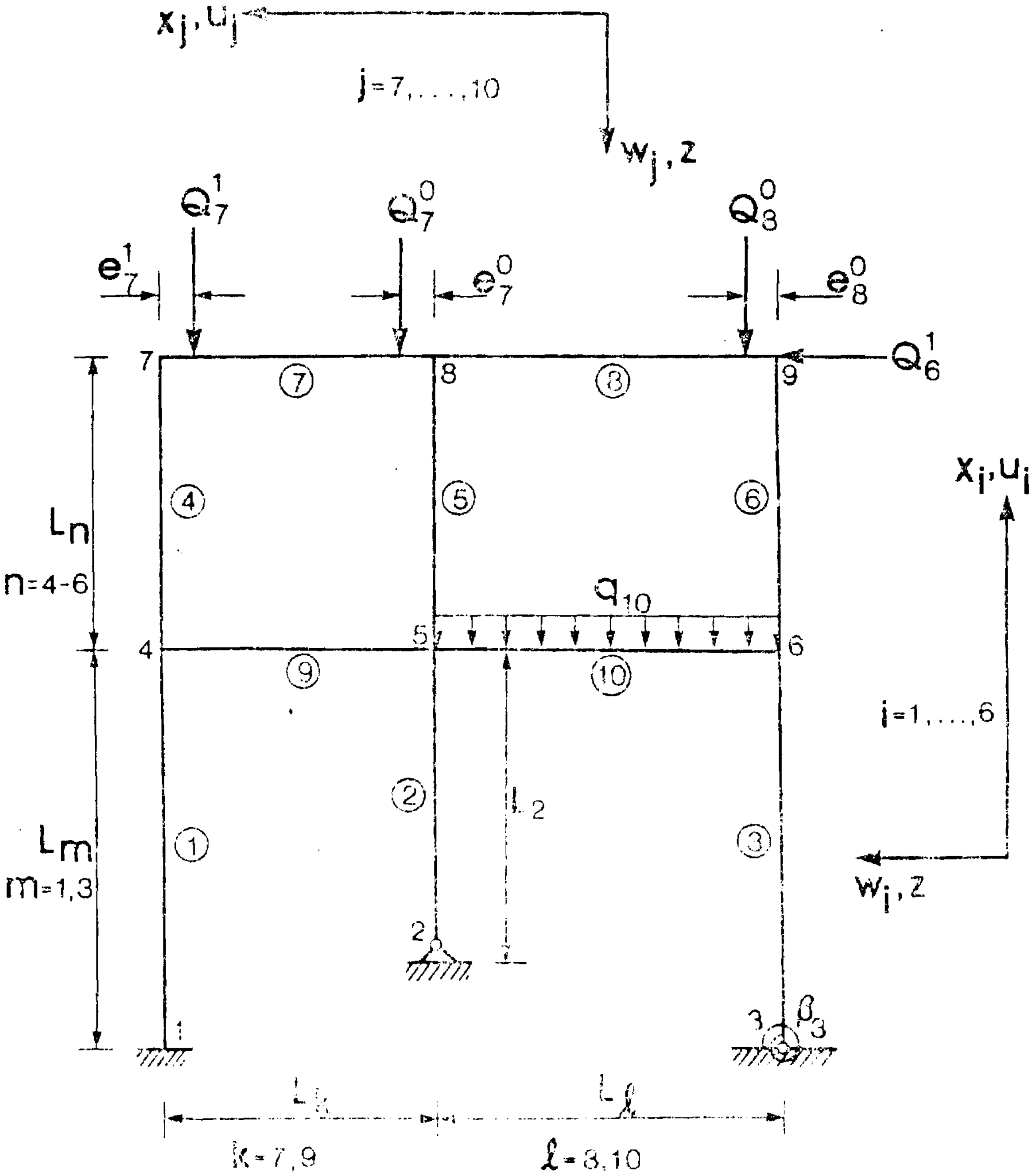


Fig. 1 Geometry and Sign Convention of a Multibay, Multistorey Frame.

(1) The frame members are initially straight, piecewise prismatic and joined together orthogonally and rigidly (this assumption can be and is relaxed later on).

(2) The material is homogeneous and isotropic and the material behaviour is linearly elastic with constant elastic constant, regardless of tension or compression.

(3) Normals remain normal to the elastic member axis and inextensional (the usual Euler-Bernoulli assumptions)

(4) Deformations and loads are confined to the plane of the frame.

(5) The concentrated loads are applied near the joints (small eccentricities). This assumption can easily be relaxed, but it will lead to an increase in the number of bars. A concentrated load at the midpoint of a bar is treated by considering two bars and an additional join at or near the location of the concentrated load.

(6) The effect of residual stresses on the system response (critical) load is neglected.

(7) The nonlinear kinematic relations correspond to small strains but moderate rotations for points on the elastic axes (first order nonlinearity)

On the basis of the above, the kinematic relations are:

$$\epsilon_{xx_i} = \epsilon^o_{xx_i} + zk_{xx_i}$$

$$\text{where} \quad \epsilon^o_{xx} = u_{i,x} + \frac{1}{2} w^2_{i,x} \quad \text{and} \quad k_{xx_i} = -w_{i,xx} \tag{1}$$

Furthermore, the axial force, P_i and bending moment, M_i, in terms of the displacement gradients are:

$$P_i = (EA_i)\,[u_{i,x} + \frac{1}{2} w^2_{i,x}]$$

$$M_i = EI_i\, w_{i,xx} \tag{2}$$

where E is the material Young's modulus of elasticity. Similarly, the expression for the transverse shear force is

$$V_i(x) = -EIw_{i,xxx} + P_i w_{i,x} \tag{3}$$

2.2 Equilibrium Equations; Boundary and Joint Conditions

Before writing the equilibrium equations and the associated boundary and joint conditions, the following nondimensionalized parameters are introduced:

$$X = x_i/L_i;\ U_i = u_i/L_i;\ W_i = w_i/L_i;$$

$$\bar{e}_i = e_i/L_i;\ \bar{q}_i = q_i L_i^3/EI_i;\ q_i^* = q_i L_1^3/EI_1$$

$$\bar{Q}_i = Q_i L_1^2/EI_1\ ;\ \bar{\beta} = \beta L_1/EI_1\ ;\ \lambda_i = L^i/\sqrt{I_i/A_i}\ ; \tag{4}$$

$$k_i^2 = \mp P_i L_i^2/EI_i;\ S_i = EI_i L_1/EI_1 L_i\ ;\ R_i = L_i/L_1$$

The expression for the internal forces, in terms of the nondimensionalized parameters are:

$$P_i = \mp k_i^2 (EI_i/L_i^2);\ M_i = W_{i,XX}(EI_i/L_i)$$

$$V_i = [\mp k_i^2 W_{i,X} - W_{i,XXX}](EI_i/L_i^2) \tag{5}$$

where the top sign holds for the case of compression in the bar, and the lower for the case of tension (the axial force P_i is positive for tension and negative for compression; thus k_i^2 is always positive).

The equilibrium equations for the frame are (in terms of the nondimensionalized parameters):

$$U_{i,X} + \frac{1}{2}\left(W_{i,X}\right)^2 = \mp\ k_i^2/\lambda_i^2 \qquad (6)$$

$$W_{i,XXXX} \pm k_i^2 W_{i,XX} = \bar{q}_i \qquad i = 1, 2 \ldots N$$

where N is the number of bars and the top sign holds for the compression case. The general solution to the equilibrium equations is given by:

$$U_i(X) = A_{i5} \pm \left(\frac{k_i}{\lambda_i}\right)^2 X - \frac{1}{2}\int_0^X \left[W_{i,X}(X)\right]^2 dX \qquad (7)$$

$$W_i(X) = A_{i1}\begin{pmatrix}\sin k_i X\\ \sinh k_i X\end{pmatrix} + A_{i2}\begin{pmatrix}\cos k_i X\\ \cosh k_i X\end{pmatrix} + A_{i3}X + A_{i4} \pm \frac{\bar{q}_i}{2k_i^2} X^2$$

where A_{ij} and k_i ($i = 1, 2 \ldots N$, $j = 1, 2, \ldots 5$) are constants (for a given level of the applied loads), to be determined from the boundary and joint conditions. For an N-member frame, the number of unknowns is <u>6N</u>. Therefore, 6N equations are needed for their evaluation.

These equations are provided by the boundary conditions and the joint conditions. At each boundary, three conditions must be satisfied (kinematic, natural or mixed; typical conditions are listed below). At each joint, three force and moment equations must be satisfied (equilibrium of a joint taken as a particle), and a number of kinematic continuity equations must also be satisfied. This number depends on the number of members coming into a joint and they represent continuity in displacement and continuity in rotation (typical conditions are listed below). For a two-member joint, we have three kinematic continuity condition, two in displacement and one in rotation. For a three-member joint the number is six, and for a four-member (largest possible) joint the number is nine.

A quick accounting of equilibrium equations, and boundary and joint conditions for the ten-bar frame, shown on Fig 1, yields the following:

(i) the number of equilibrium equations is <u>60</u> (6 x 10).

(ii) the number of boundary conditions is <u>nine</u> (three at each of boundaries 1, 2, and 3).

(iii) The number of joint conditions is <u>51</u> of these, 18 are force and moment equilibrium conditions (three at each of the six joints 4, 5, 6, 7, 8 and 9), and 33 kinematic continuity conditions (three at each of joints 7 and 9, six at each of joints 4, 6, and 8 and nine at joint 5).

Therefore, the total number of available equations is 60. Here, it is implied that the loading is of known magnitude.

For clarity, typical boundary and joint conditions are shown below, with reference to the frame of Fig. 1 (in nondimensionalized form).

<u>Boundary 3</u>

$$U_3(0) = 0; \quad W_3(0) = 0$$
$$S_3 W_{3,XX}(0) - \bar{\beta}_3 W_{3,X}(0) = 0 \qquad (8)$$

<u>Joint 5</u>

$$\left[\mp k_9^2 W_{9,X}(0) - W_{9,XXX}(0)\right] \frac{S_9}{R_9} + \left(\mp k_2^2\right) \frac{S_2}{R_2} - \left[\mp k_{10}^2 W_{10,X}(1)\right.$$
$$\left. - W_{10,XXX}(1)\right] \frac{S_{10}}{R_{10}} - \left(\mp k_5^2\right) \frac{S_5}{R_5} = 0 \qquad (9a)$$

$$\left[\mp k_5^2\, W_{5,x}(0) - W_{5,xxx}(0)\right] \frac{S_5}{R_5} + \left(\mp k_9^2\right) \frac{S_9}{R_9} - \left[\mp k_2^2\, W_{2,x}(1) - W_{2,xxx}(1)\right] \frac{S_2}{R_2} - \left(\mp k_{10}^2\right) \frac{S_{10}}{R_{10}} = 0 \qquad (9b)$$

$$S_5 W_{5,xx}(0) + S_9 W_{9,xx}(0) - S_2 W_{2,xx}(1) - S_{10} W_{10,xx}(1) = 0 \qquad (9c)$$

$$R_2 U_2(1) = - R_{10} W_{10}(1) = R_5 U_5(0) = - R_9 W_9(0)$$

$$R_2 W_2(1) = R_{10} U_{10}(1) = R_5 W_5(0) = R\ U\ (0) \qquad (10)$$

$$W_{2,x}(1) = W_{10,x}(1) = W_{5,x}(0) = W_{9,x}(0)$$

Joint 7

$$\bar{Q}_7^{1} + \left(\mp k_4^2\right) \frac{S_4}{R_4} - \left[\mp k_7^2\, W_{7,x}(1) - W_{7,xxx}(1)\right] \frac{S_7}{R_7} = 0 \qquad (11a)$$

$$\left[\mp k_4^2\, W_{4,x}(1) - W_{4,xxx}(1)\right] \frac{S_4}{R_4} + \left(\mp k_7^2\right) \frac{S_7}{R_7} = 0 \qquad (11b)$$

$$W_{4,xx}(1)\ S_4 + W_{7,xx}(1)\ S_7 + \bar{Q}_7^{-1}\, \bar{e}_7^{-1} = 0 \qquad (11c)$$

$$R_4 U_4(1) = - R_7 W_7(1);\ R_4 W_4(1) = R_7 U_7(1);$$

$$W_{4,x}(1) = W_{7,x}(1) \qquad (12)$$

Note that, in these expressions as well, the top sign corresponds to the compression case and the bottom to the tension.

2.3 Buckling Equations

The buckling equations and the associated boundary and joint conditions are derived by employing a perturbation method [Bellman (1969) and Sewell (1965)]. This derivation is based on the concept of the existence of an adjacent equilibrium position at either a bifurcation point or a limit point. In the derivation, the following steps are followed: (i) start with the equilibrium equations, Eqs. (6), and related boundary and joint conditions, expressed in terms of the displacements, (ii) purturb them by allowing small kinematically admissible changes in the displacement functions and a small change in the bar axial force, (iii) make use of equilibrium at a point at which an adjacent equilibrium path is possible and retain first order terms in the admissible variations. The resulting inhomogeneous differential equations are linear in the small changes. Replace U_i and W_i in Eqs.(6), by $\bar{U}_i + U_i^*$ and $\bar{W}_i + W_i^*$, respectively. Moreover, replace $\mp k_i^2$ by $\pm\bar{k}_i^2 + \sigma_i^*$, where σ_i^* is the change in the nondimensionalized axial force ($= P_i^* L_i^2 / EI_i$) and it can be either positive or negative, regardless of tension or compression in the bar at an equlibrium position. The bar quantities denote parameters at a static primary equilbrium position, and the star quantities denote the small changes.

The buckling equations are:

$$U^*_{i,X} + \bar{W}_{i,X} W^*_{i,X} = \sigma^*_i/\lambda^2_i$$
$$W^*_{i,XXXX} \mp \bar{k}^2_i W^*_{i,XX} = \sigma^*_i \bar{W}_{i,XX} \tag{13}$$

The related boundary and joint conditions are presented, herein, only for the same boundaries and joints as those related to the equilibrium equations, Eqs. (9)-(12).

Boundary 3

$$U_3^*(0) = W_3^*(0) = 0$$

$$S_3 W^*_{3,XX}(0) - \bar{\beta}_3 W^*_{3,X}(0) = 0 \tag{14}$$

Joint 5

$$\left[\mp \bar{k}_9^2 W^*_{9,X}(0) - W^*_{9,XXX} + \sigma_9^* \bar{W}_{9,X}\right] \frac{S_9}{R_9} + \sigma_2^* \frac{S_2}{R_2} - \left[\mp \bar{k}_{10}^2 W^*_{10,X}(1)\right.$$

$$\left. - W^*_{10,XXX}(1) + \sigma^*_{10} \bar{W}_{10,X}(1)\right] \frac{S_{10}}{R_{10}} - \sigma_5^* \frac{S_5}{R_5} = 0 \tag{15a}$$

$$\left[\mp \bar{k}_5^2 W^*_{5,X}(0) - W^*_{5,XXX}(0) + \sigma_5^* \bar{W}_{5,X}(0)\right] \frac{S_5}{R_5} + \sigma_9^* \frac{S_9}{R_9} -$$

$$\left[\mp \bar{k}_2^2 W^*_{2,X}(1) - W^*_{2,XXX}(1) + \sigma_2^* \bar{W}_{2,X}(1)\right] - \sigma^*_{10} \frac{S_{10}}{R_{10}} = 0 \tag{15b}$$

$$S_5 W^*_{5,XX}(0) + S_9 W^*_{9,XX}(0) - S_2 W^*_{2,XX}(1) - S_{10} W^*_{10,XX}(1) = 0 \tag{15c}$$

$$R_2 U_2^*(1) = -R_{10} W_{10}^*(1) = R_5 U_5^*(0) = -R_9 W_9^*(0)$$

$$R_2 W_2^*(1) = R_{10} U_{10}^*(1) = R_5 W_5^*(0) = R_9 U_9^*(0) \tag{16}$$

$$W^*_{2,X}(1) = W^*_{10,X}(1) = W^*_{5,X}(0) = W^*_{9,X}(0)$$

Joint 7

$$\sigma_4^* \frac{S_4}{R_4} - \left[\mp k_7^2 W_{7,X}^*(1) - W_{7,XXX}^*(1) + \sigma_7^* \overline{W}_{7,X}(1)\right] \frac{S_7}{R_7} = 0 \tag{17a}$$

$$\left[\mp \overline{k}_4^2 W_{4,X}^*(1) - W_{4,XXX}^*(1) + \sigma_4^* \overline{W}_{4,X}(1)\right] \frac{S_4}{R_4} + \sigma_7^* \frac{S_7}{R_7} = 0 \tag{17b}$$

$$S_4 W_{4,XX}^*(1) + S_7 W_{7,XX}^*(1) = 0 \tag{17c}$$

$$R_4 U_4^*(1) = - R_7 W_7^*(1); \quad R_4 W_4^*(1) = R_7 U_7^*(1); \quad W_{4,X}^*(1) = W_{7,X}^*(1) \tag{18}$$

The solution to the buckling equations is given by

$$U_i^*(X) = A_{i5}^* + \frac{\sigma_i^*}{\lambda_i^2} X - \int_0^X \overline{W}_{i,X} W_{i,X}^* dX$$

$$W_i^*(X) = A_{i1}^* \begin{pmatrix} \sin \overline{k}_i X \\ \sinh \overline{k}_i X \end{pmatrix} + A_{i2}^* \begin{pmatrix} \cos \overline{k}_i X \\ \cosh \overline{k}_i X \end{pmatrix} + A_{i3}^* X + A_{i4}^* + \frac{\sigma_i^* X}{2\overline{k}_i}\left[A_{i2} \begin{pmatrix} \sin \overline{k}_i X \\ \sinh \overline{k}_i X \end{pmatrix} + A_{i1} \begin{pmatrix} -\cos \overline{k}_i X \\ \cosh \overline{k}_i X \end{pmatrix} + \frac{\overline{q}_i X}{\overline{k}_i^3}\right] \tag{19}$$

Here also the top sign and expression correspond to the compression case (the ith bar is in compression at equilibrium) and the bottom to the tension case. Note that $\overline{k}_i, A_{i1}$ and A_{i2} are the values of the constants [see Eqs. (7)] on the primary path (equilibrium). On the other hand, the <u>star</u> parameters are 6N in number (60 for the ten bar frame). Moreover, the boundary and joint conditions associated with the buckling equations are also 6N in number and they are linear, homogeneous, algebraic equations in the 6N star parameters. Thus, the characteristic equation, which leads to the estimation of the critical load condition, is obtained by requiring a nontrivial (all A_{ij}^* and σ_i^* are not equal to zero) solution of the buckling equations to exist.

2.4. Semirigid Joint Connections

The mathematical formulation, presented so far, is based on the assumption of rigid-jointed connections. In the case of semirigid connections, the only difference lies in some of the joint conditions. Two types of non-rigid connections are treated herein. Both come under the general but vague term of semirigid connections. The first corresponds to the case where a member, at a given joint, is connected to the remaining through a linear rotational spring (Type A). The second corresponds to the case of realistic flexible connections at frame joints (Type B). In this latter case, especially for steel frame construction, the connections are usually bolted with the use of various connecting elements (top and bottom clip angles, end plates, web framing, etc.). In this case the bending moment-relative rotation curve (for a member connected to a group of members at a joint) is nonlinear. Initially, the slope is not infinite, as assumed in the case of rigid joints, but a very large number, which primarily depends on the beam depth and the type of connection [see Tables I-IV of DeFalco and Marino (1966)], but the slope decreases as the moment increases. In this latter case, we may still employ the idea of a rotational spring, but with nonlinear stiffness.

The needed modification in the mathematical formulation is treated separately for each case (Types A and B).

Type A

The only difference, from the case of rigid connections, is to modify the condition of kinematic continuity in rotation. For example, if member "7" is connected to member "4" through a rotational spring of linear stiffness β_7 (see Fig. 1), then the last of Eqs. (12) need be modified. Instead of

$$W_{4,X}(1) = W_{7,X}(1) \tag{12c}$$

one must use

$$W_{4,XX}(1) + \bar{\beta}_4^1 W_{4,X}(1) - W_{7,X}(1) = 0 \tag{20}$$

where β_i^m is the stiffness of the rotational spring that connects number "4" to joint 7 (see Fig. 1) in a nondimensionalized form, or

$$\bar{\beta}_i^m = \beta_i^m \frac{L_i}{EI_i} \qquad i = 1,2...10; \; m = 0, 1 \tag{21}$$

Note that β_i^m is the rotational stiffness associated with member i. If m = 1 the spring is at X = 1 of the member, while, if m = 0 the spring is at X = 0. Furthermore, note that Eq. (20) relates the member "4" end moment to the relative rotation (of member "4" to member 7). Moreover, for a rigid-jointed frame $\bar{\beta}_i^m$ tends to infinity (for calculations a very large number is used), on the other hand, when $\bar{\beta}_i^m$ tends to zero (pin connection), Eq. (20) implies that no moment is transferred through the pin.

Type B

For the case of realistic flexible connections the member end moment, $M_i(1 \text{ or } 0)$, is related to the relative rotation curve in a nonlinear fashion.

Again if the same example is used as for Type A, then

$$\bar{M}_4 = \frac{M_4 \, L_4}{EI_4} = f(\varphi_4) \tag{22}$$

where $f(\varphi_4)$ is a nonlinear function of φ_4, and φ_4 is the relative rotation of member "4" to member "7" at their joint (for a multimember joint, one member is considered immovable and φ_i is the relative rotation of the other members with respect to the immovable one)

$$\varphi_4 = W_{4,X}(1) - W_{7,X}(1) \tag{23}$$

One possible selection for the nonlinear function $f(\varphi_4)$ is a cubic relation, or

$$-\bar{M}_4 = {}_0\bar{\beta}_4^1 \, \varphi_4 - \bar{A}_4^1 \, \varphi_4^3 \tag{24}$$

where ${}_0\bar{\beta}_4^1$ denotes the slope of the member end moment to the relative rotation curve at the origin (or before the external loads are applied) and $\bar{A}_4^1$ a constant, which can be obtained from experimental data.

In order to employ the same equations as for type A (linear spring) connections and therefore the same solution methodology (instead of increasing the nonlinearity of the problem), the following concept is introduced. First, solutions for the frame response are obtained by starting with small levels for the applied loads and by using small increments. Then, Eq. (24) at load step (m + 1) can be written as

$$\left(-\bar{M}_4\right)_{m+1} = \left[{}_0\bar{\beta}_4^1 - \bar{A}_4^1\left(\varphi_4\right)_m^2\right]\left(\varphi_4\right)_{m+1} \tag{25}$$

This implies that for small steps in the load, the relative rotation experiences small changes. Thus, the required joint condition, Eq. (24), becomes

$$W_{4,XX}(1) + \left[W_{4,X}(1) - W_{7,X}(1)\right]\left(\bar{\beta}_4^1\right)_{m+1} = 0 \tag{26}$$

where $(\bar{\beta}_4^1)_{m+1}$ is evaluated at the previous load step by

$$\left(\bar{\beta}_4^1\right)_{m+1} = {}_0\bar{\beta}_4^1 - \bar{A}_4^1\left[W_{4,X}(1) - W_{7,X}(1)\right]_m^2 \tag{27}$$

Clearly then, the solution scheme for Type B connections is the same as the one for Type A connections and the nonlinearity of the problem is not increased.

3. SOLUTION PROCEDURE

The complete response of an N-member frame is known, for a given geometry and level of the applied loads, if one can estimate the values of the 6N unknowns that characterize the two displacement functions U(X) and W(X), Eqs (7). The needed 6N equations are provided from the satisfaction of the boundary and joint conditions. Furthermore, the estimation of the critical load condition requires the use of one more equation. This is provided by the solution to the buckling equations, Eqs. (13). As already mentioned, the satisfaction of the boundary and support conditions, for the buckling solution, leads to a system of 6N linear, homogeneous, algebraic equations in σ_i^* and A^*_{ij} ($i = 1, 2, \ldots N; j = 1, 2, \ldots 5$; these constants characterize the buckling modes). For a nontrivial solution to exist the determinant of the coefficients must vanish. This step provides the needed additional equation, which is one more equation in $\bar{k}_i$ and some of the A_{ij}, and it holds true only at the critical equilibrium point (either bifurcation or limit point).

A solution methodology has been developed (including a computer algorithm) for estimating critical conditions, prebuckling response and postbuckling behaviour. The scheme makes use of the following steps:

(1) Through a simple and linear frame analysis program, the values of the internal axial load parameters, k_i, are estimated, for some low level of the applied loads. This can be used as an initial estimate for the nonlinear analysis, but most importantly it tells us which members are in tension and which in compression. Note that the solution expressions, Eqs (7), differ for the two cases (compression versus tension). Such a subroutine is outlined in the text by Weaver and Gere (1980).

(2) Once the form of the solution has been established (from step 1 we know which members are in tension and which in compression), then through the use of the boundary and joint conditions one can establish the 6N equations that signify equilibrium states, for the load level of step 1.

In so doing, it is observed that 5N, out of the 6N equations, are linear in A_{ij} and nonlinear in k_i. Two important consequences are directly related to this observation. First, through matrix algebra the 5N equations are used to express the A_{ij} in terms of the k_i, and substitution into the remaining equations yields a system of N nonlinear equations in k_i.

Secondly, if the k_i's are (somehow) known, then the 5N equations (linear in A_{ij}) can be used to solve for A_{ij}.

(3) The N nonlinear equations are solved by employing one of several possible nonlinear solvers. There exist several candidates for this.

For the two-bar frame and for the portal frame (small number of nonlinear equations), the nonlinear equations, $f_j = 0 (j \leq 3)$ can be solved by first defining a new function by

$$F = \sum_{j=1}^{N} f_j^2 \tag{28}$$

Then, one recognizes that the set of k_i that minimizes F (note that the minimum value of F is zero) is the set that satisfies the nonlinear equations, $f_j = 0$. The mathematical search technique of Nelder and Mead (1964) can be used for finding this minimum. This nonlinear solver was employed by Simitses et al. (1976, 1977, 1978, 1981, 1982, 1983) for the two-bar and portal frame problems.

For multibay multistorey frames ($N \geq 5$), the nonlinear equations, $f_j = 0$ ($j \geq 5$), can be solved by Brown's (1969) method [see also Reinholdt (1974)]. This method was employed by Vlahinos (1983) in generating results,for all frames.

Regardless of the nonlinear solver, the k_i-values obtained from <u>step 1</u> are used as initial estimates.

Note that through <u>steps 1-3</u>, one obtains the complete nonlinear response of the system at the low level of the applied loads. Furthermore, note that <u>low</u> here means not necessarily small loads, but loads for which the linear analysis yields good estimates for k_i, to be used as initial points in the nonlinear solver.

(4) The load level is step-increased and the solution procedure of steps 1-3 is repeated. Another possibility is to use small increments in the load and employ the values of k_i of the previous load level as initial points for the nonlinear solver. In this case, step 1 is used only once for a truly low level of the applied loads.

(5) At each load level, the stability determinant (see section 2.3) is evaluated. If there is a sign change for two consecutive load levels, then a bifurcation point exists in this load interval. Note that the bifurcation point can be located, with any desired accuracy, by adjusting the size of the load increment. In the case of a limit point, the procedure is the same, but the establishment of the limit point requires special care. First, if the load level is higher than the limit point, the outlined solution steps either yield no solution or the solution does not belong to the primary path (usually this is a physically unacceptable solution for deadweight loading). If this is so, the load level is decreased until an acceptable solution is obtained. At the same time, as the load approaches the limit point the value of the determinant approaches zero. These two observations suffice to locate the limit point. Note that, when a non-primary path solution is obtained, the value of the buckling determinant does not tend to zero.

(6) Step 4 is employed to find post-critical point behaviour. The establishment of equilibrium points on the postbuckling branch is numerically difficult. The difficulty exists in finding a point, which then can serve as an initial estimate for finding other neighboring equilibrium points.

(7) The complete behaviour of the frame at each load level, regardless of whether the equilibrium point lies on the primary path or postbuckling

branch, has been established if one has evaluated all A_{ij} and k_i. Equilibrium positions can be presented, graphically, as plots of load or load parameter versus some characteristic displacement or rotation of the frame (of a chosen member at a chosen location).

Before closing this section, it should be noted that the procedure for the analysis of flexibly jointed frames is the same, with one small exception. The load increments must be small and the needed spring stiffness at the (m+1)st load step is evaluated from the solution of the mth load step [see Eq. (27)].

4. EXAMPLES AND DISCUSSION

The results for several geometries are presented and discussed in this section. The geometries include two-bar frames, which can be subject to limit point instability, as well as portal and multibay, multistorey frames, which for linearly elastic behavior are subject to bifurcational (sway-) buckling with stable postbuckling branch. The results are presented both in graphical and tabular form and they include certain important parametric studies. Each geometry is treated separately.

4.1 Two-bar Frames

Consider the two-bar frame shown on Fig. 2. For simplicity, the two bars are of equal length and stiffness and the eccentric load is constant-directional (always vertical). Results are presented for both rigid and flexible connections. These results are presented and discussed separately.

4.1.1. Rigid Joint Connection

Results are discussed for the case of an immovable pin support at the right hand end of the horizontal bar. For this geometry there are two important parameters that one must consider in generating results; first is the load eccentricity $\bar{e}$, and next the member slenderness ratio, λ. Note that for this geometry $L_1 = L_2 = L$ and λ.

$$-0.01 \leq \bar{e} \leq 0.01$$
$$\lambda = 40,\ 80,\ 120,\ \infty \tag{29}$$

Note that the positive eccentricities correspond to loads applied to the right of the elastic axis of the vertical bar, while the negative ones to the left (load applied, if needed, through a hypothetical rigid overhung).

For this configuration, it is clear from the physical system that, as the load increases (statically) from zero, with or without eccentricity, the response includes bending of both bars and a "membrane state only" primary path does not exist. Therefore, there cannot exist a bifurcation point from a primary path that is free of bending. The classical (linear theory) approach, for this simple frame, assumes that the vertical bar experiences a contraction without bending in the primary state, while the horizontal bar remains unloaded (zero eccentricity is assumed). Then a bifurcation exists and a bent state (buckling) is possible at the bifurcation load $P_{c\ell}$, which is the critical load [see Simitses (1976) for analytical details]

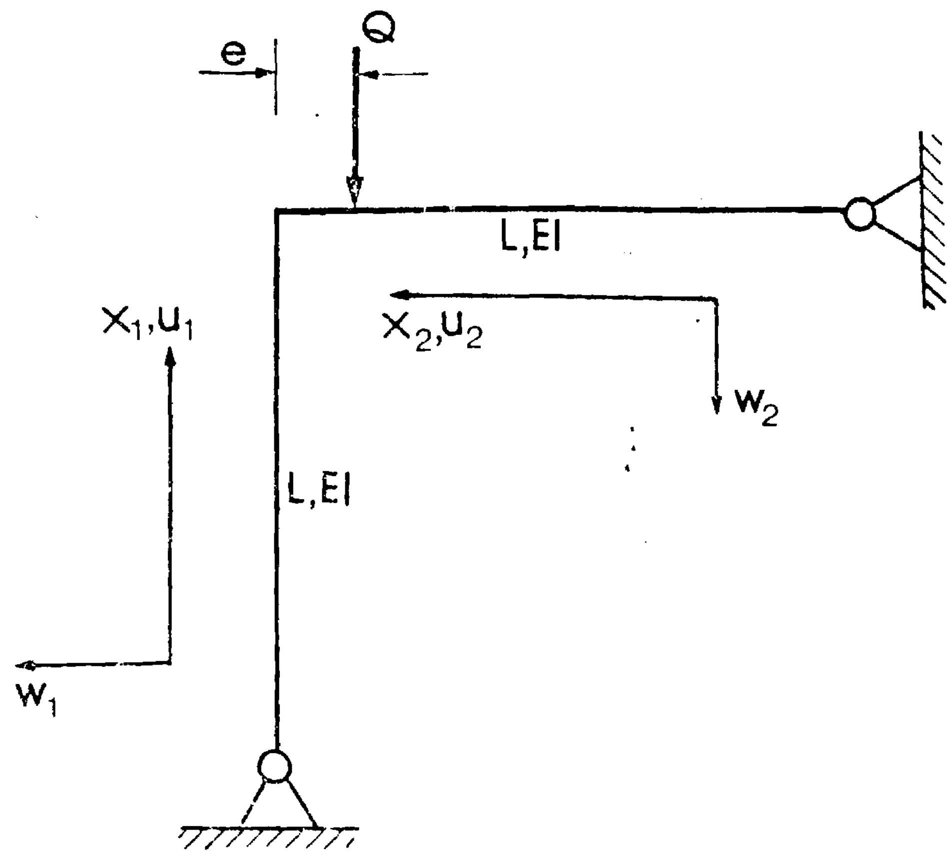


Fig. 2 Geometry of a Two-Bar Frame.

$$Q_{c\ell} = 13.89 \frac{EI}{L^2} \tag{30}$$

Results are presented graphically on Figs. 3 and 4. On Fig. 3, the load parameter $\lambda_c (= Q/Q_{c\ell})$ is plotted versus the joint rotation, $W_{1,X}$ (1), for several eccentricities and $\lambda = 80$ (slenderness ratio). The response for different values of λ is similar, and thus no other load - (characteristic) displacement curves are shown. It is seen from Fig. 3 that the response, regardless of whether it is stable (to the right) or subject to limit point instability (to the left), seems to be approaching asymptotically a line (almost straight) that makes an angle with the vertical and it intersects it at $\lambda_c = 1.00$. Moreover, the horizontal bar could be either in tension or in compression, regardless of the character of the response. Not shown on Fig 3, are equilibrium points which belong to curves above the asymptote. These equilibrium paths cannot be attained physically under deadweight loading. On Fig 4, plots of limit point (critical) loads are plotted versus eccentricity for various λ-values. Also, the experimental results of Roorda (1965), corresponding to $\lambda = 1275$ and the analytical results of Koiter (1966), based on his initial postbuckling theory, are shown for comparison. On the basis of the generated results, a few important observations and conclusions are offered. Depending on the value for the slenderness ratio, there exists a critical eccentricity which divides the response of the frame into two parts; on one side (see Fig 3; on the right) the response is characterized by stable bent equilibrium positions for all loads (within the limitations of the theory), while on the other side the response exhibits limit point instability. The maximum limit point load, for each slenderness ratio value corresponds to a specific eccentricity value (see Fig. 4) and it is identical in value to that predicted by linear theory. The results also show that this two-bar frame is sensitive to load

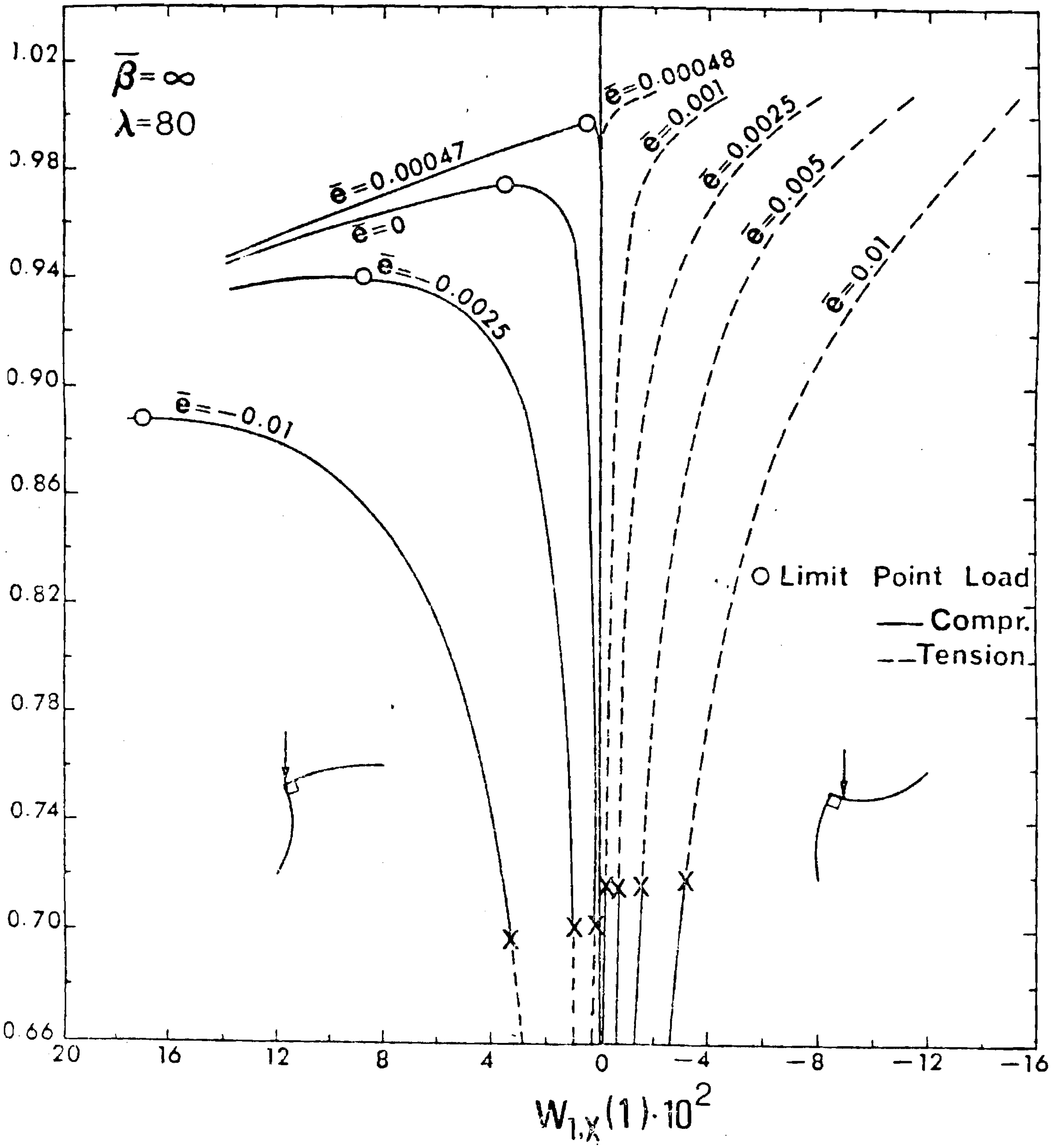


Fig. 3 Load-Deflection Curve, Hinged Two-Bar Frame with Rigid Joint Connection.

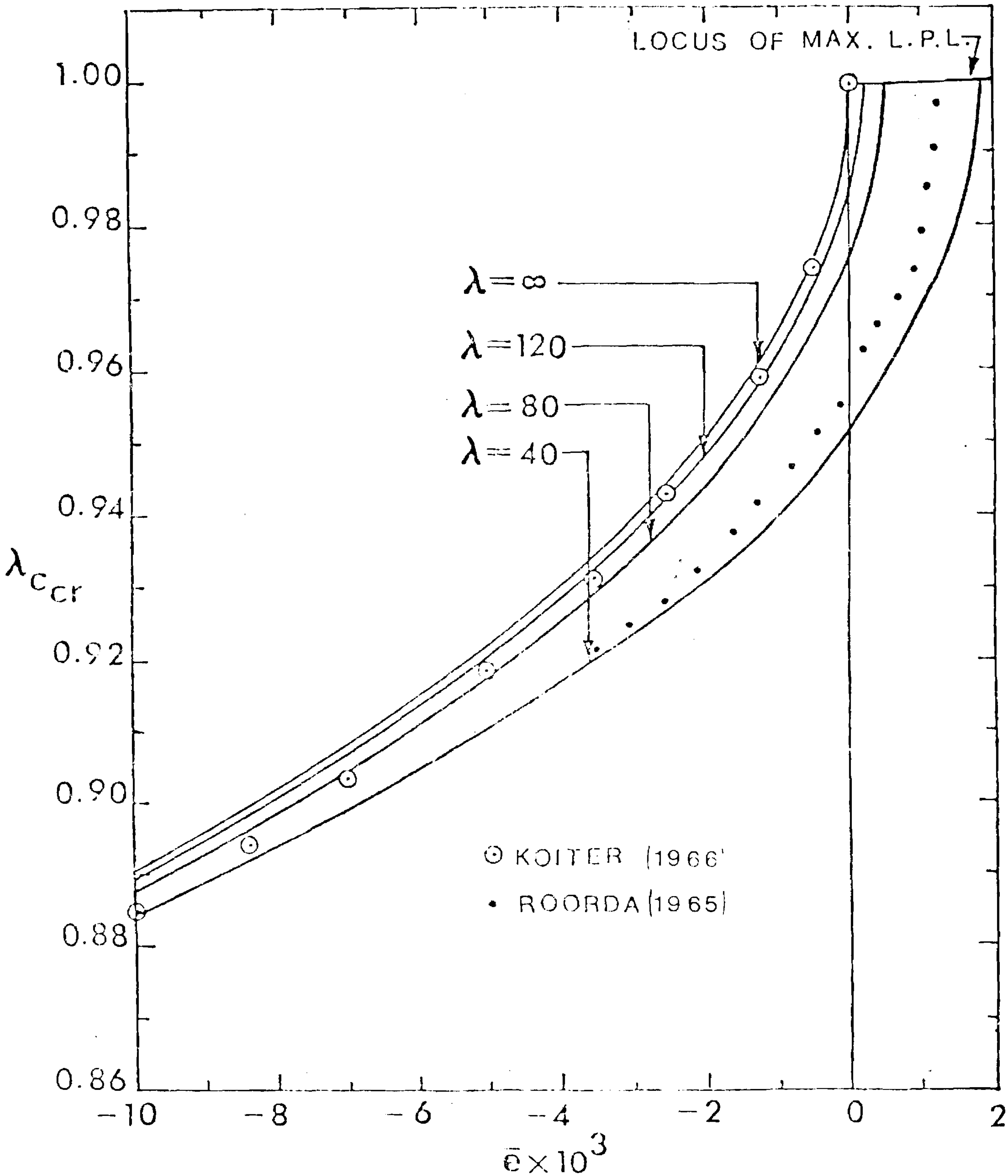


Fig. 4 Effects at Eccentricity and Slenderness Ratio on Critical Loads (Two-Bar Frame).

eccentricities (for $\bar{e} = -.01$, $\lambda_c \approx 0.89$) and it might be sensitive to initial geometric imperfections. Details and more results (depicting the effect of the right hand support, movable along a vertical plane or a horizontal plane versus immovable, on the response) are found in Kounadis et al. (1977) and in Simitses et al. (1978).

4.1.2 Semirigid Joint Connection

Consider that the two members connected at the joint through a rotational spring (Fig. 2). First, a linear spring is used at the joint and the nondimensionalized spring stiffness, $\bar{\beta}$, is varied from zero (pin connection) to 10^5 (rigid connection). Partial results are presented in graphical and tabular form, but the conclusions and observations are based on all generated data (a wide range of eccentricities and slenderness ratios were used). Fig. 5 depicts the response of the two-bar frame for $\bar{\beta} = 10$ and $\lambda = 80$. For the sake of economy and brevity, no attempt was made to find the critical eccentricity value for each $\bar{\beta}$ and λ. It is seen from Fig. 5 that the response for $\bar{\beta} = 10$ is similar to that for $\bar{\beta} = \infty$ (Fig. 3). Fig. 6 is a plot of $\bar{Q}_{cr}$ (limit point load) versus $\bar{\beta}$ for $\bar{e} = -0.01$. For very small values of $\bar{\beta}$, $\bar{Q}_{cr} \simeq \pi^2$ which is the critical load of a column pinned at both ends (Euler load), while for very large values it approaches the value corresponding to $\lambda_{c_{cr}} = 0.888$ [see Fig. 3; $\bar{Q}_{cr} = 0.888\ (13.89) = 12.34$]. Note that for $\bar{e} > -0.01$, similar curves can be obtained. For instance, for $\bar{e} = 0$ the curve would start from the value of π^2 for extremely small values of $\bar{\beta}$, and approach the value of 13.54 for $\bar{\beta} = 10^5$ ($\simeq \infty$). The influence of the slenderness ratio, for various $\bar{\beta}$- values, on the critical load is shown on Table 1.

For the case of realistic flexible connections, three depths of type II connections are considered (see Table 2). The required values are taken

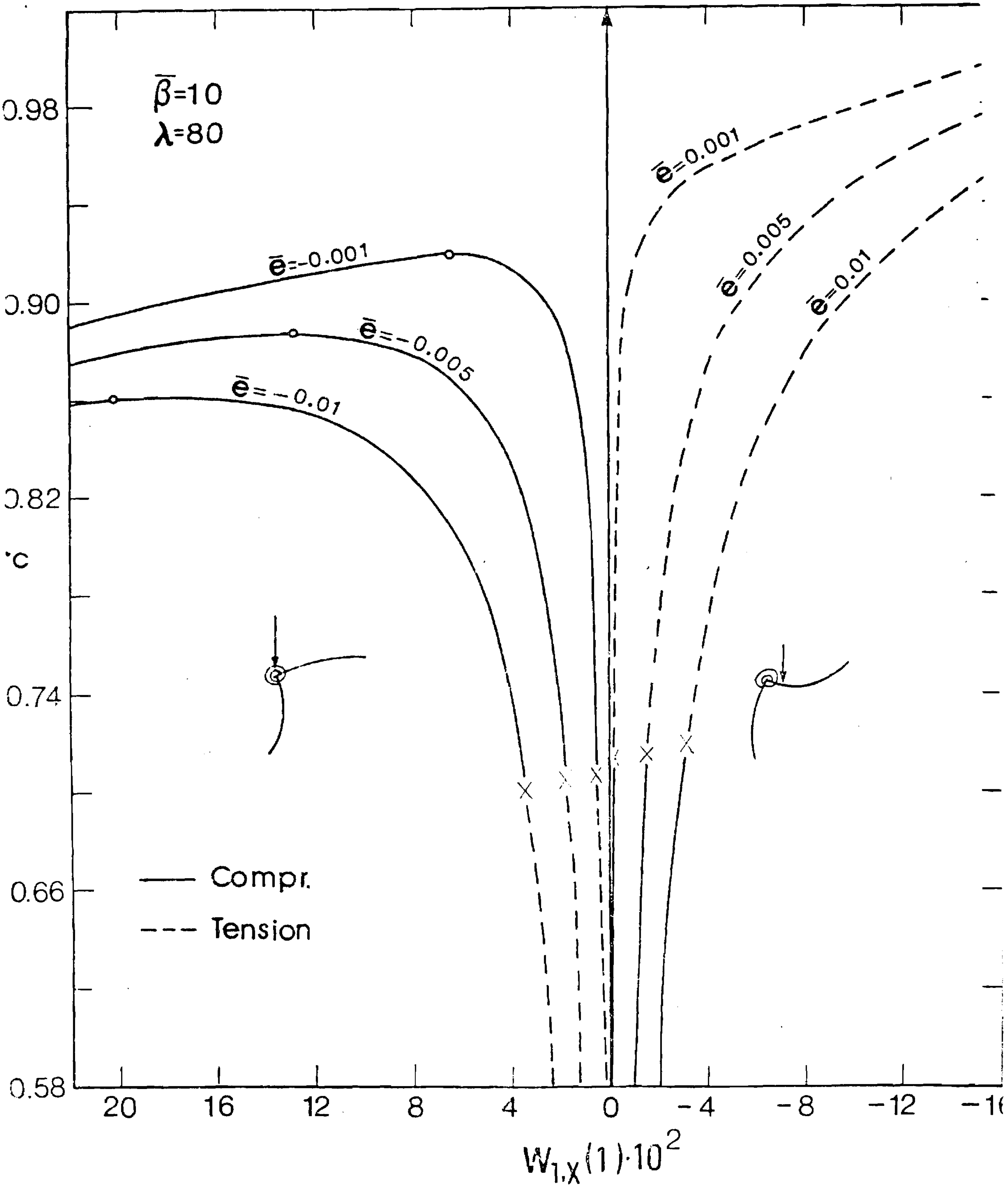


Fig. 5 Typical Load-Deflection Curve; Hinged Two-Bar Frame with Flexible Joint Connection.

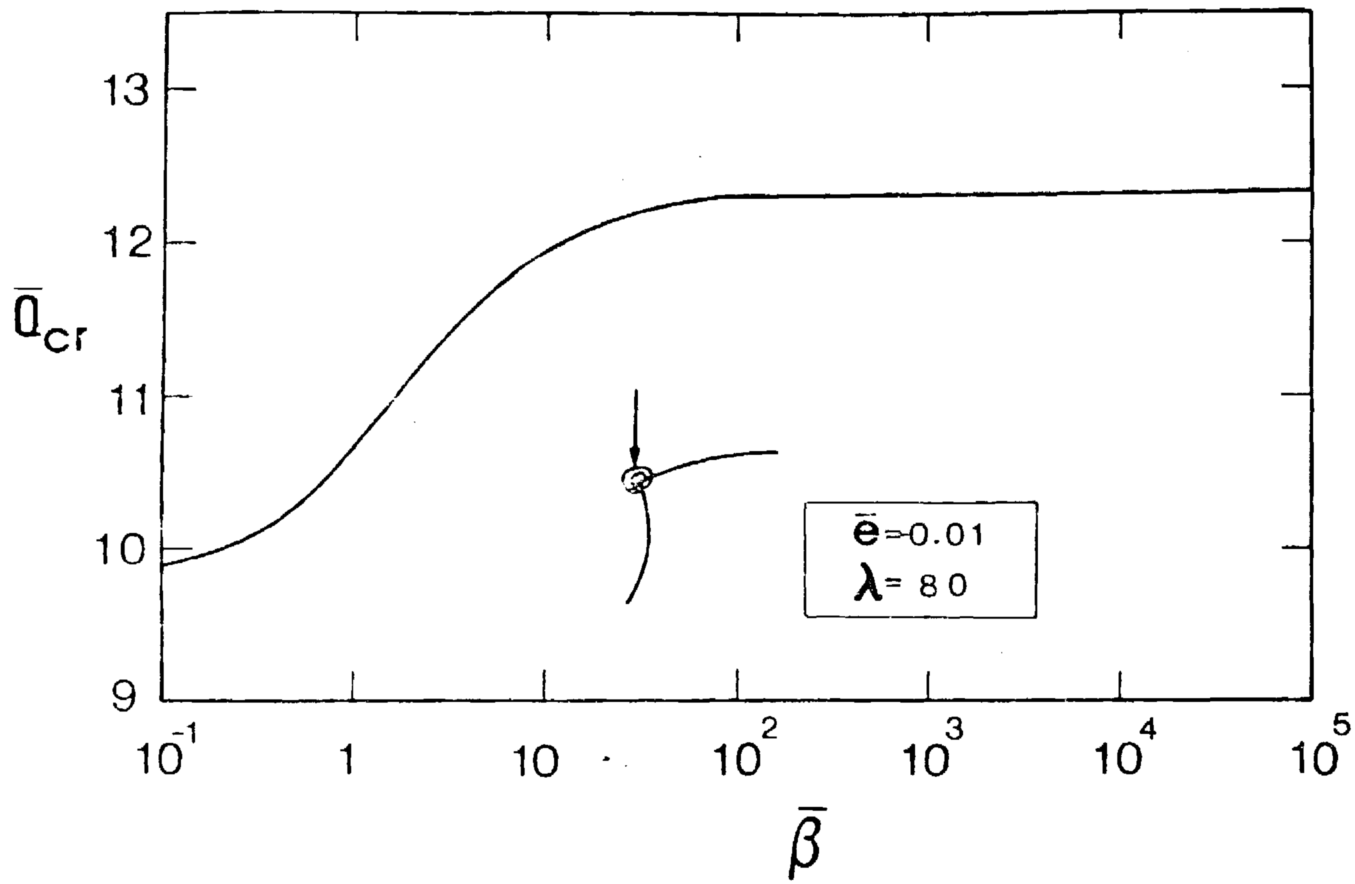


Fig. 6 Effect of Joint Spring Constant on the Critical Load (Hinged Two-Bar Frame).

Table 1: INFLUENCE OF SLENDERNESS RATIO ON THE CRITICAL LOADS OF THE TWO BAR FRAME

($\bar{e}$ = -0.01)

$\bar{Q}_{cr}$

$\beta\downarrow$ $\lambda\rightarrow$	80	120	1000
0.1	9.9028	9.9045	9.9051
1.0	10.6817	10.6868	10.6908
10	11.9504	11.9638	11.9744
100	12.2931	12.3089	12.3216
∞	12.3376	12.3538	12.3667

Table 2: DEPTH AND STIFFNESS OF FLEXIBLE CONNECTIONS (TYPE II)

Geom.	Depth in.	Z x 10^5 Rad/kip-in.	$_o\beta$ x 10^{-8} lb-in/Rad.	A_i in^2	I_i in^4	$_o\bar{\beta}$	$\bar{A}$-Range
1	8	0.0460	21.739	6.71	64.20	361.17	$\bar{A} \leq (7.5) \times 10^{10}$
2	18	0.0150	66.667	20.46	917.70	167.79	$\bar{A} \leq (6) \times 10^{9}$
3	36	0.0054	185.185	39.80	7833.65	114.36	$\bar{A} \leq (2.1) \times 10^{9}$

Table 3: EFFECT OF $\bar{A}$ (NON-LINEAR FLEXIBLE CONNECTION) ON THE CRITICAL LOADS ($\bar{e}$ = -0.01, λ = 100)

Geometry 1 $_o\bar{\beta}$ = 361.17		Geometry 2 $_o\bar{\beta}$ = 167.79		Geometry 3 $_o\bar{\beta}$ = 114.36	
$\bar{A}$	$\bar{Q}_{cr}$	$\bar{A}$	$\bar{Q}_{cr}$	$\bar{A}$	$\bar{Q}_{cr}$
0	12.7529	0	12.7631	0	12.7216
1.0 x 10^6	12.7529	1.0 x 10^5	12.7361	1.0 x 10^3	12.7216
1.0 x 10^7	12.7527	5.0 x 10^5	12.7359	1.0 x 10^4	12.7216
5.0 x 10^7	12.7515	1.0 x 10^6	12.7357	1.0 x 10^5	12.7214
1.0 x 10^8	12.7494	1.0 x 10^7	12.7298	1.0 x 10^6	12.7193
1.0 x 10^9	12.7456	1.0 x 10^8	12.7206	1.0 x 10^7	12.6991

from DeFalco and Marino (1966) and the bars are assumed to be steel I-beams. The value of $\bar{A}$ (nonlinear flexible connection) is varied in accordance with the limitations presented in the mathematical formulation, and its effect, for all three cases, on the limit point loads for $\bar{e} = -0.01$ and $\lambda = 100$ is shown on Table 3. An important conclusion here is that, for type II connections the degree of nonlinearity of the rotational spring has negligibly small effect on limit point loads for a fixed eccentricity and bar slenderness ratio.

For more details see Simitses and Vlahinos (1982).

4.2 Portal Frames

Consider the portal frame shown on Fig. 7. The loading consists of both eccentric concentrated loads near the joints and of a uniformly distributed load on bar "3".

When vertical concentrated loads are applied at joints "3" and "4" without eccentricity, and the geometry is symmetric ($EI_1 = EI_2 = EI$, $L_1 = L_2 = L$, $\beta_1 = \beta_2 = \beta$ but $\beta = 0$ or ∞), a primary state exists and beam-column theory can be employed to find critical loads for sway buckling, or for symmetric buckling (sidesway prevented) and for antisymmetric buckling. Such analyses can be found in texts [see Bleich (1952) and Simitses (1976)].

For example, if the horizontal bar has the same structural geometry as the other two members ($EI_3 = EI$ and $L_3 = L$), then the critical load for sway buckling (referred to herein as classical) is given by

simply supported ($\beta = 0$)
$$Q_{c\ell} = 1.82 \frac{EI}{L^2} \qquad (31)$$

clamped ($\beta \to \infty$)
$$Q_{c\ell} = 7.38 \frac{EI}{L^2} \qquad (32)$$

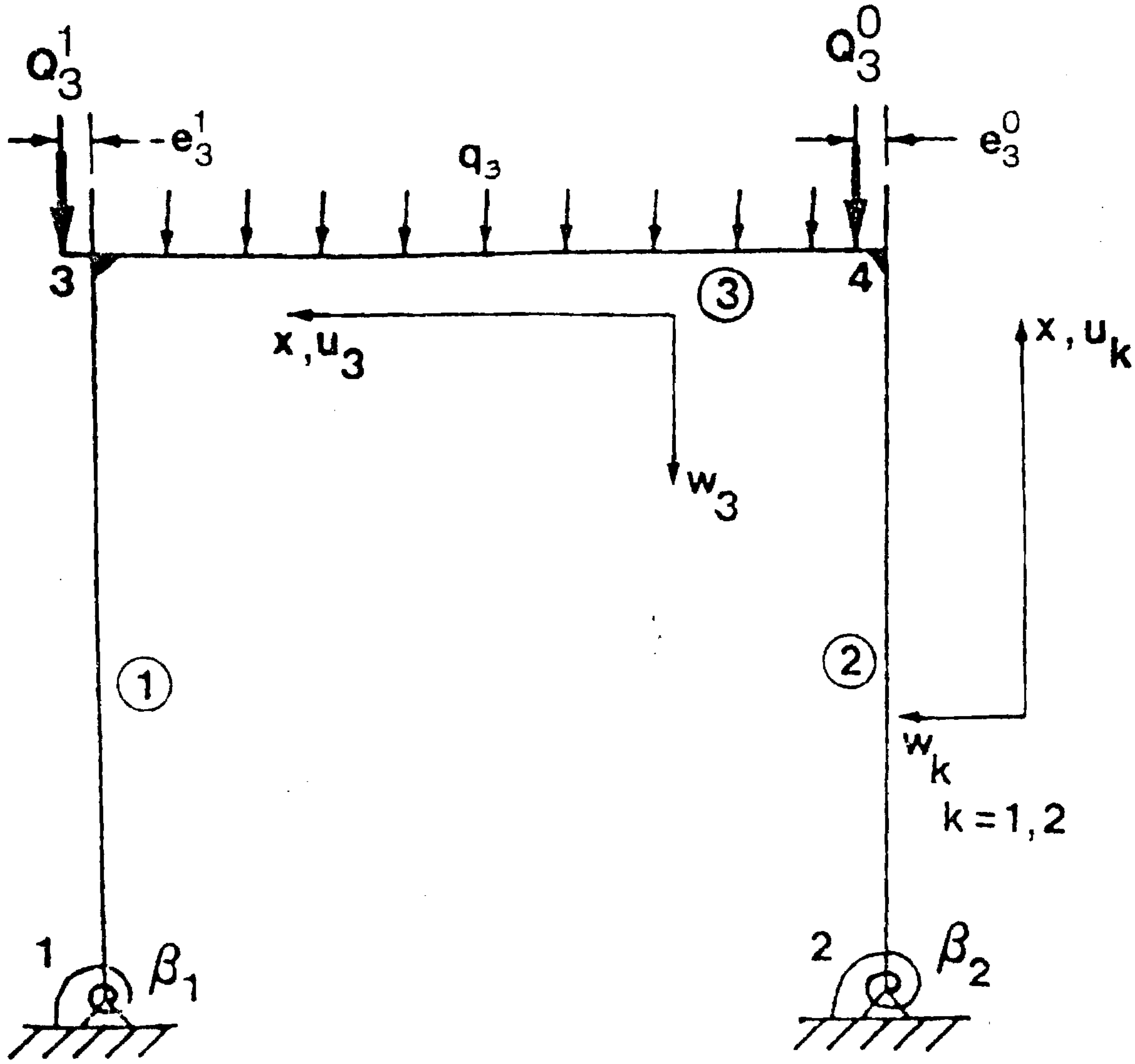


Fig. 7 Portal Frames; Geometry and Loading.

Results for loading that induces primary bending and parametric studies associated with the effect of various structural parameters on the frame response are presented below for rigidly connected portal frames. Moreover, some results corresponding to semi-rigidly connected portal frames are also presented.

4.2.1 Rigid Joint Connection

Partial results are presented both in graphical and in tabular form, but the conclusions are based on all available results.

Figs. 8 and 9 deal with the effect of load eccentricity on the response characteristics of a square (structurally; $EI_i = EI$, $L_i = L$), symmetric ($\bar{\beta}_1 = \bar{\beta}_2 = 0$), rigid-jointed frame. Fig. 8 shows primary path and postbuckling equilibrium positions for two symmetric eccentricities ($\bar{e}_3^1 = \bar{e}_3^0 = \bar{e}$). The value of the slenderness ratio ($\lambda_i = \lambda$) is taken as 1,000, but the effect of slenderness ratio on the nondimensionalized response characteristics is negligibly small. The rotation of bar "1" at joint "3" is chosen as the characteristic displacement for characterizing equilibrium states on this figure. As seen from Fig 8, bar "3" is in compression in the postbuckled branches and initially in the primary paths. As the eccentricity increases the sway buckling load decreases, but only slightly. This observation is in agreement with Chwalla's (1938) [see also Bleich (1952)] result, who found that the critical load when the eccentricity is one third ($\bar{e} = 0.333$) is equal to 1.78 EI/L^2. It is also observed that the primary path curves approach asymptotically the value of $\bar{Q}_{cr}$ corresponding to symmetric buckling of the portal frame [see Eq (66) of Ch, 4 in Simitses (1976)]. This value, as computed from said reference, is equal to 12.91 EI/L^2. Fig. 9 shows similar results but with antisymmetric eccentricity ($-\bar{e}_3^1 = \bar{e}_3^0 = \bar{e}$). Clearly for this case ($\bar{e} \neq 0$), there is a stable response that includes

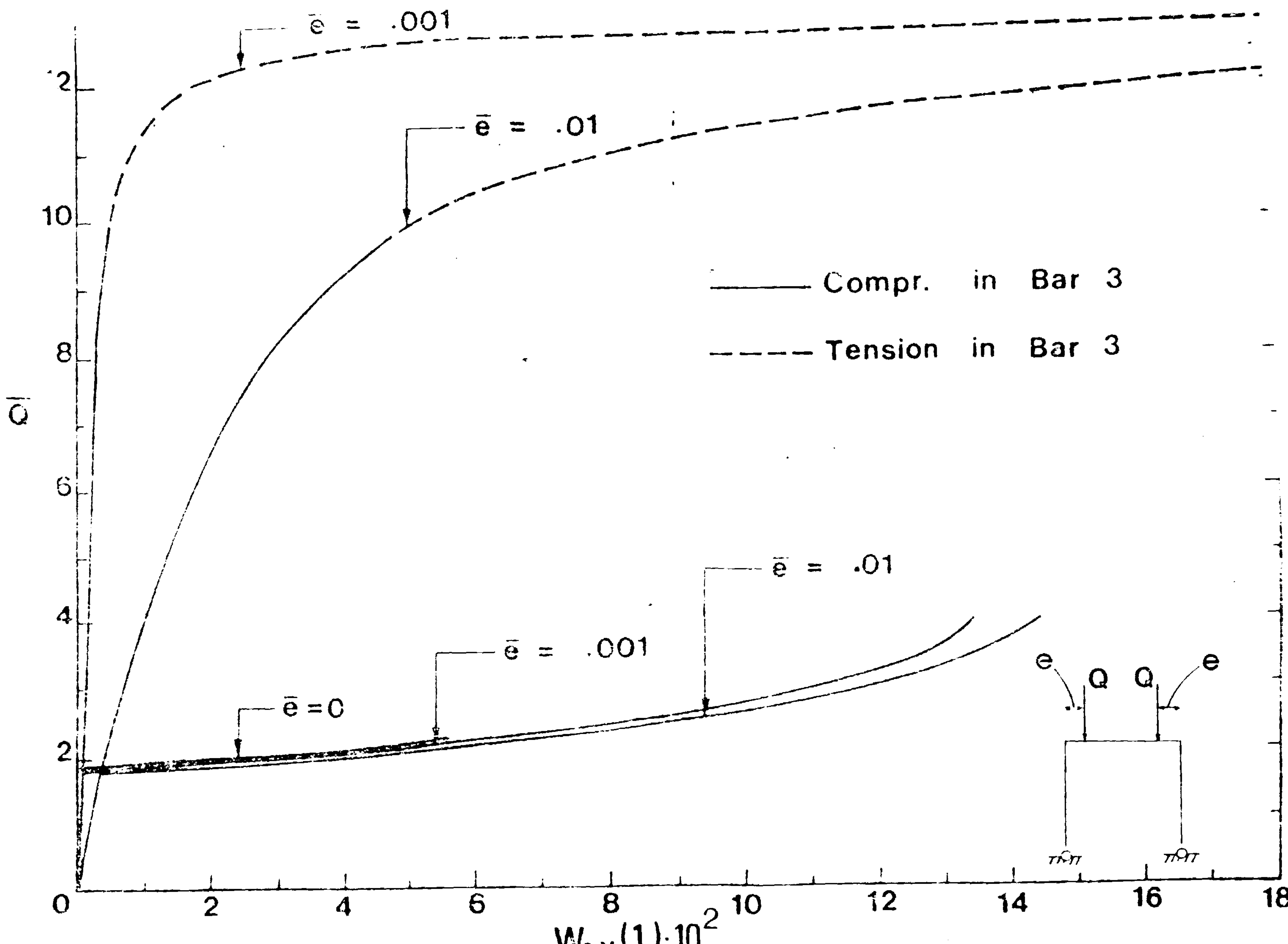


Fig. 8 Symmetrically and Eccentrically Loaded Symmetric Hinged Portal Frames ($S_i = R_i = 1$).

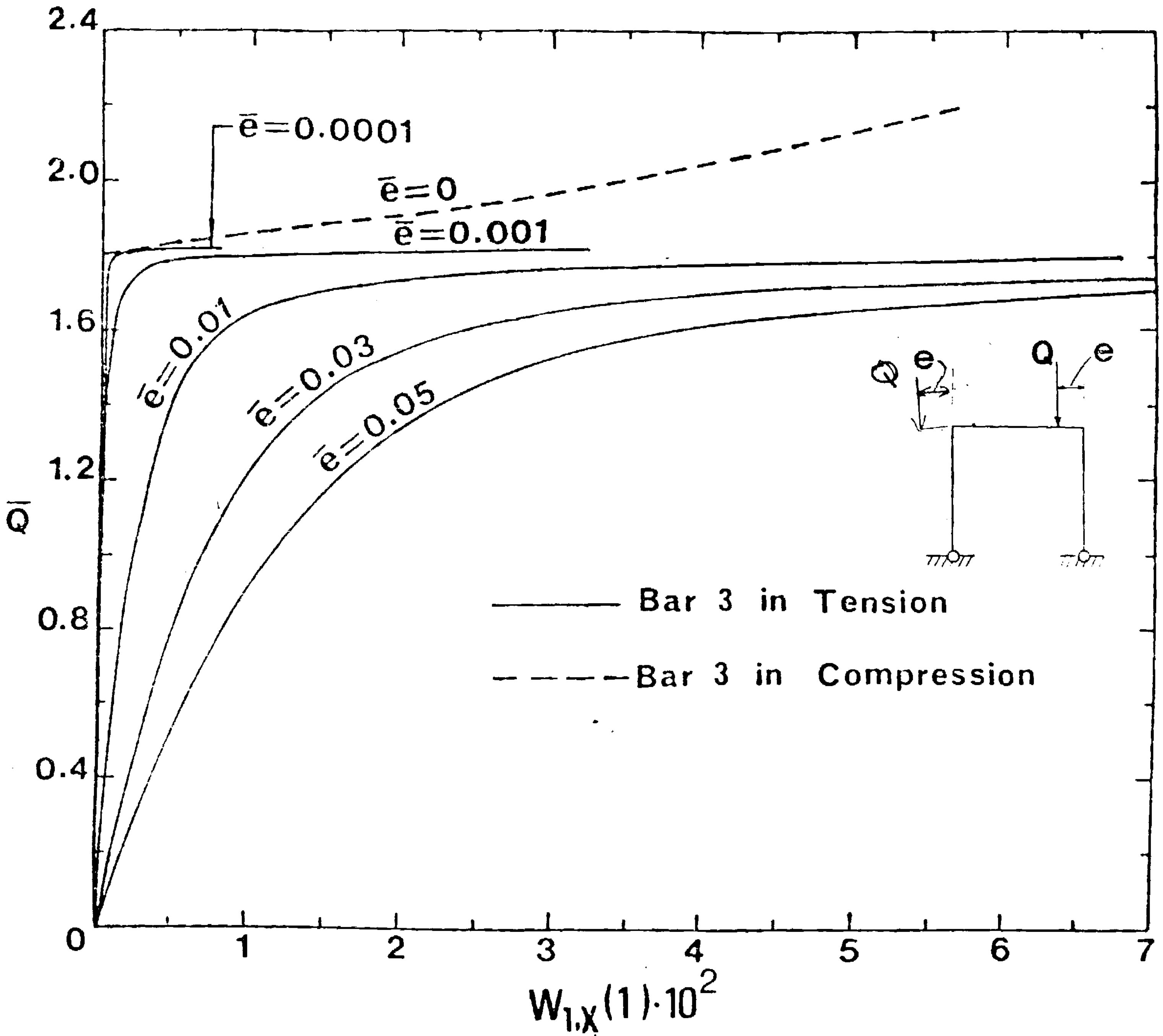


Fig. 9 Asymmetrically and Eccentrically Loaded Symmetric Hinged Portal Frames ($S_i = R_i = 1$).

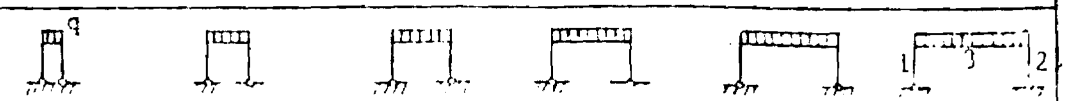
bending from the onset of loading. Moreover, this response approaches asymptotically a horizontal line corresponding to $\overline{Q} = \overline{Q}_{c\ell}$, Eq. (30), and not the postbuckling branch ($\overline{e} = 0$). Furthermore, for asymmetric eccentricity bar "3" is in tension.

Table 4 presents sway buckling loads of a symmetric simply supported portal frame loaded by a uniformly distributed load on bar "3", for a wide range of horizontal bar ("3") geometries. The value of $\lambda_1 = \lambda_2$ is taken to be 1000 and the value of λ_3 varies according to the changes in I_3 and L_3 by keeping the cross-sectional area, A_3, constant. This results into $50 \leq \lambda_3 \leq 4242$. Note that q^* is given on Table 4, instead of $\overline{q}$. This is done because L_3 is a variable. Moreover, if one is interested in comparing total load, q^* must be multiplied by L_3/L_1. Thus, the first row becomes 3.52 ($L_3/L_1 = 0.5$) 2.77, 2.27, 1.92, 1.65 and finally 1.44. Note also that the last row becomes 4.93, 4.91, 4.90, 4.89, 4.88 and 4.87, or all of them approximately equal to $2(\pi^2/4)$. This load is the buckling load of the two vertical bars, which are pinned at the bottom and clamped at the top to a very rigid bar that can move horizontally. Finally, k_1 and k_3 are measures of the axial compressive force in the vertical bars ($k_1 = k_2$) and the horizontal bar, respectively.

The final result shown, herein, is on Fig 10. This figure shows the effect of small variations in the length of bar "2" on the response characteristics of a uniformly loaded frame. Clearly, the change in L_2 provides a geometric imperfection and the response, accordingly, approaches asymptotically the "perfect geometry" response. The same can be said, if an imperfection in bending stiffness exists, such that the resulting geometry becomes asymmetric.

Details and more results can be found in Simitses et al. (1981, 1982).

TABLE 4. EFFECT OF HORIZONTAL BAR GEOMETRY ON CRITICAL LOADS (HINGED PORTAL FRAMES).

	L_3/L_1	0.5	1.0	1.5	2.0	2.5	3.0
EI_3/EI_1							
0.5	q^*_{cr}	7.035	2.772	1.518	0.9600	.6598	0.48093
	k_1	1.326144	1.177312	1.066970	0.979798	0.908143	0.849350
	k_3	0.204301	0.586181	1.040636	1.548835	2.128032	2.922292
1.0	q^*_{cr}	8.142	3.522	2.075	1.394	1.011	0.7769
	k_1	1.426682	1.327027	1.247465	1.180678	1.123931	1.079532
	k_3	0.128840	0.410684	0.778291	1.212997	1.725474	2.412621
2.0	q^*_{cr}	8.879	4.075	2.523	1.772	1.338	1.064
	k_1	1.489896	1.427337	1.375482	1.331290	1.293368	1.263309
	k_3	0.074499	0.258365	0.517961	0.840184	1.227140	1.709590
3.0	q^*_{cr}	9.166	4.309	2.721	1.945	1.491	1.200
	k_1	1.513758	1.467748	1.428456	1.394528	1.365357	1.341829
	k_3	0.052459	0.189001	0.389313	0.643696	0.951338	1.324863
10.0	q^*_{cr}	9.640	4.714	3.079	2.266	1.782	1.462
	k_1	1.552238	1.535271	1.519604	1.505210	1.492379	1.481047
	k_3	0.017124	0.066005	0.143481	0.247198	0.375752	0.528868
100.0	q^*_{cr}	9.865	4.909	3.266	2.444	1.951	1.622
	k_1	1.570430	1.566634	1.564618	1.563342	1.561408	1.559621
	k_3	0.002500	0.007062	0.015835	0.028044	0.043648	0.062619

$L_1 = L_2$, $EI_1 = EI_2$

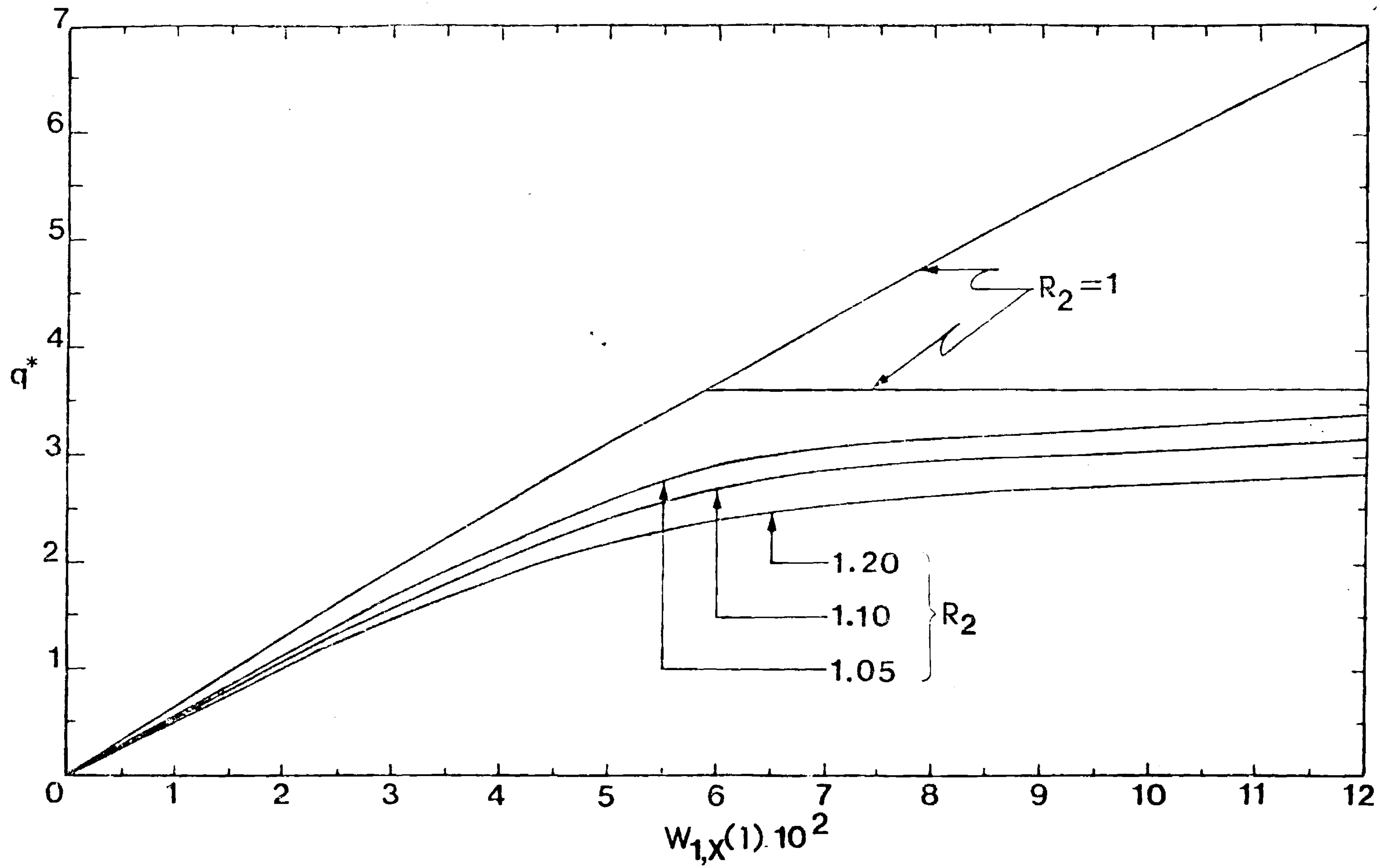


Fig. 10 Effect of Variable Vertical Column Length on the Portal Frame Response ($R_3 = 1$; $S_i = 1$).

4.2.2. Semirigid Joint Connection

As in the case of the two-bar frame (4.1.2), the horizontal bar is connected to the vertical bars through rotational springs. First, a linear spring is used, and its stiffness, $\bar{\beta}$, is varied from zero (10^{-1}) to infinity (10^{5}). Results are presented in tabular and graphical form for symmetric eccentric loading. Table 5, shows the effect of slenderness ratio for a square symmetric portal frame on the sway buckling load ($\bar{e}$ = 0.001) for various values of rotational spring stiffness (same at both joints). It is seen from Table 5, that this effect is neglibly small, as is in the case of rigid connections. Fig. 11, shows the effect of spring stiffness on the sway buckling load for various load eccentricities. For very small $\bar{\beta}$-values, the frame becomes unstable at very low load levels. Note that for $\bar{\beta} = 0$ the frame becomes a mechanism. As the rotational stiffness increases, the critical load approaches that of a rigid-jointed portal frame ($\bar{Q}_{cr}$ = 1.82 EI/L^2).

Next, results are presented for flexibly connected portal frames using the same type II connections as for the two-bar frame (see Table 2). For the portal frame also it is concluded that the degree of nonlinearity of the rotational springs has negligibly small effect on sway buckling loads, for each specified geometry (see Table 6). From these and other studies [Vlahinos (1983)], it is concluded that the effect of nonlinearity in the rotational spring stiffness (variations in $\bar{A}$) has negligibly small effect on the response characteristics of portal frames. In all generated results, it is required that the slope to the moment-relative rotation curve, for the flexible connection, be positive. This requirement is not only reasonable, it is also necessary for a good and efficient connection.

Table 5: EFFECT OF SLENDERNESS RATIO, λ, ON SWAY-BUCKLING LOAD (SYMMETRIC LOADS, $\overline{e}$ = 0.001)

$\overline{Q}_{cr}$

$\overline{\beta}$ ↓ / λ →	40	100	1000
1	.659	.659	.660
5	1.355	1.355	1.360
100	1.781	1.787	1.790
1000	1.807	1.813	1.814

TABLE 6: EFFECT OF $\overline{A}$ (NONLINEAR FLEXIBLE CONNECTIONS) ON CRITICAL LOADS, $\overline{Q}_{cr}$ (SYMMETRIC CASE; $\overline{e}$ = 0.01).

Geometry 1 $_o\overline{\beta}$ = 361.17		Geometry 2 $_o\overline{\beta}$ = 67.79		Geometry 3 $_o\overline{\beta}$ = 114.36	
$\overline{A}$	$\overline{Q}_{cr}$	$\overline{A}$	$\overline{Q}_{cr}$	$\overline{A}$	$\overline{Q}_{cr}$
0	1.807	0	1.798	0	1.790
1×10^5	1.807	1×10^5	1.798	1×10^5	1.790
1×10^8	1.807	1×10^8	1.798	1×10^8	1.790
3×10^{10}	1.807	1×10^9	1.798	1×10^9	1.788
5×10^{10}	1.806	3×10^9	1.797	1.75×10^9	1.785
7×10^{10}	1.803	5×10^9	1.795	2×10^9	1.782
7.5×10^{10}	1.801	6×10^9	1.793	2.1×10^9	1.781

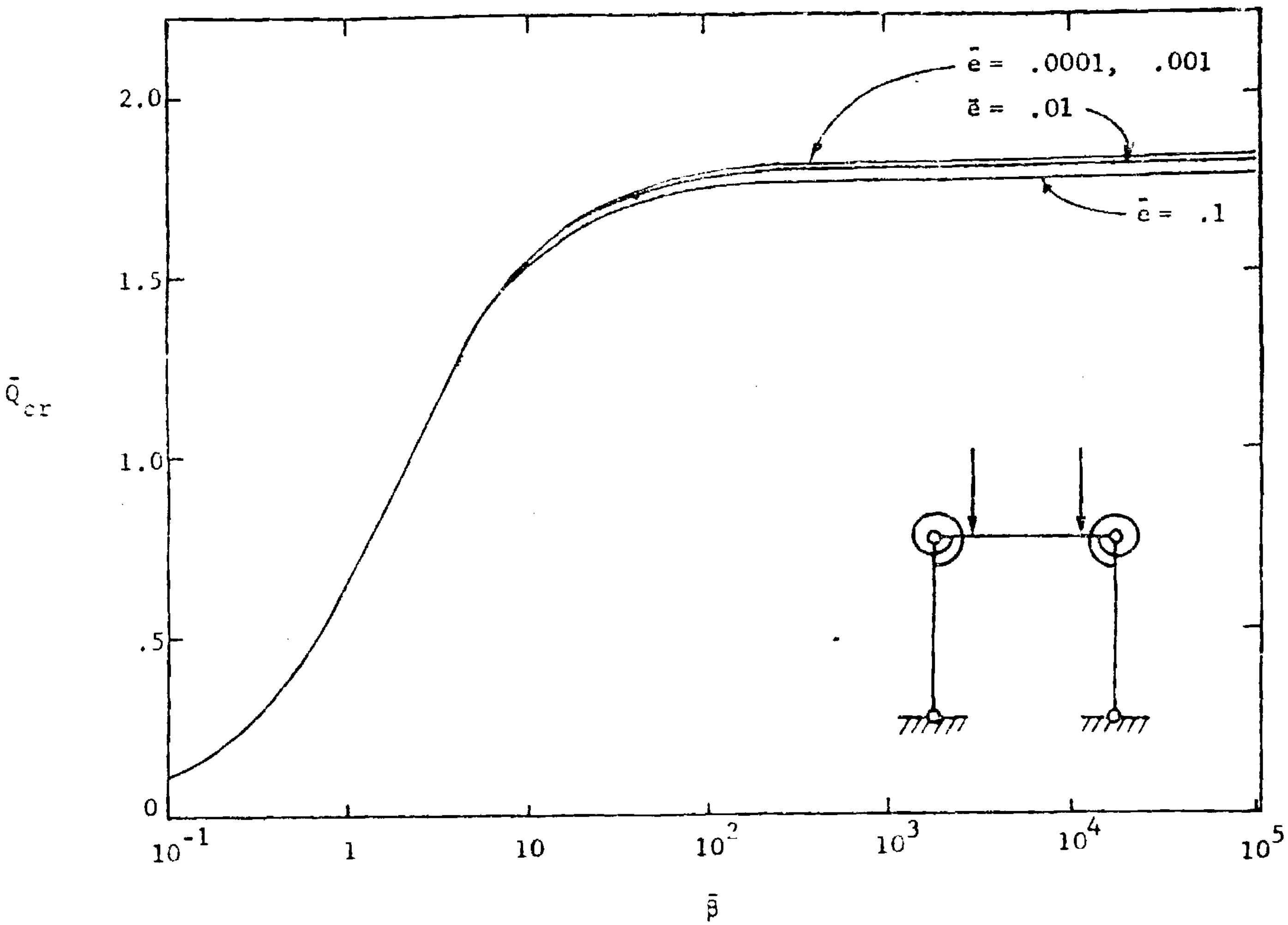


Fig. 11 Effect of Joint Rotational Stiffness on Critical Loads (Eccentrically Loaded Symmetric Portal Frame).

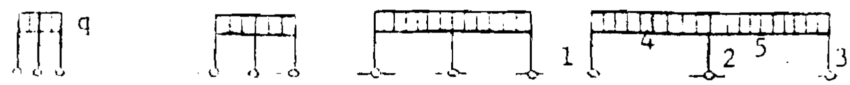
Because of the above observations and those associated with the two-bar frame (4.1.2), no further results are generated for flexibly connected frames.

4.3 Multibay, Multistorey Rigid-Jointed Frames

Several results are presented and discussed here.

First, results are presented, for symmetric two-bay frames loaded transversly by uniformly distributed loads, on Table 7. On this table, the length of the horizontal bars is varied ($L_4 = L_5 = L_h$; $L_1 = L_2 = L_3 = L_v$) as well as the stiffness. Here also, as in the case of portal frames, the slenderness ratio for the vertical bars is taken as 1000 ($\lambda_1 = \lambda_2 = \lambda_3 = 1000$) and the value of $\lambda_h (= \lambda_4 = \lambda_5)$ is varied accordingly, as I_h and L_h vary, but the cross-sectional area is kept constant. The critical loads, q^*_{cr}, represent sway buckling loads. The total load for the two-bay frame is obtained by multiplying q^* by 2 L_h/L_v. The factor of two is needed because of the two bays. In comparing the results of this table with those for the portal frame (Table 4), one observes that, by adding one bay (two bars; bars "5" and "3"), the total sway buckling load is increased by 50% or more, depending on the two ratios. The increase is larger with larger values for L_h/L_v and smaller values for EI_h/EI_v. The values for $k_1 (k_1 = k_3)$ k_2 and $k_4 (k_4 = k_5)$ are measures of the axial loads (compressive for this case) in the five bars. Because of the distribution, the middle vertical bar carries more load than the other two (expected). In spite of this, as the bending stiffness of the horizontal bars approaches infinity, the total sway buckling load approaches $3(\pi^2/4)$. Note that for the portal frame the total load is $2(\pi^2/4)$. Thus, for this particular case ($EI_h \to \infty$), the increase in buckling load from a single bay to a two-bay frame, is 50%, regardless of the ratio of L_h/L_v.

TABLE 7. EFFECT OF HORIZONTAL BAR GEOMETRY ON CRITICAL LOADS (HINGED, SYMMETRIC, ONE-STOREY TWO-BAY FRAMES).

	L_h/L_v	.5	1	2	3
$\frac{EI_h}{EI_v}$					
0.5	q^*_{cr}	5.474	2.243	0.822	0.425
	k_1	1.079615	1.001068	0.872701	0.768667
	k_2	1.773156	1.575093	1.328265	1.169728
	k_4	0.152456	0.467452	1.302599	2.342564
1	q^*_{cr}	6.190	2.739	1.124	0.635
	k_1	1.121867	1.072490	0.999675	0.926334
	k_2	1.916465	1.774246	1.580063	1.447954
	k_4	0.090564	0.304997	0.944516	1.787473
3	q^*_{cr}	6.887	3.258	1.487	0.921
	k_1	1.155458	1.138553	1.108115	1.079077
	k_2	2.049744	1.980715	1.868985	1.787660
	k_4	0.034894	0.129310	0.455144	0.925647
10	q^*_{cr}	7.221	3.530	1.703	1.101
	k_1	1.167638	1.162379	1.151948	1.142422
	k_2	2.120247	2.087248	2.039065	1.998682
	k_4	0.011611	0.043027	0.164391	0.353992

$L_h = L_4 = L_5$, $L_v = L_1 = L_2 = L_3$, $q^*_{cr} = q^*_{4\,cr} = q^*_{5\,cr}$

$EI_h = EI_4 = EI_5$, $EI_v = EI_1 = EI_2 = EI_3$

Limited results are also presented for a single bay multistory frame and a two-bay two-storey frame. These results are generated only for special geometries. All lengths and all stiffnesses are taken to be equal, and the loading is a uniformly distributed load of the same magnitude on every horizontal bar. The boundaries are simple supports and the bar slenderness ratio is taken to be 1,000. Note that for portal frames the effect of slenderness ratio on the nondimensionalized response is found to be negligibly small. This is found to be also true for two-bay, one storey, and multistorey one-bay frames, that were checked randomly. The value of λ_i was changed for a few geometries and this change did not affect the response appreciably. The results for the additional geometries are presented schematically on Fig. 12, by giving the total sway buckling load next to a sketch of the frame. From this figure it is clearly seen that the sway-buckling load is increased appreciably by adding bays but the change is insignificant, when storeys are added.

Another important result is related to the following study. A two-storey one-bay frame, with $L_i = L$ and $EI_i = EI$ (for all i), is loaded with uniformly distributed loads on the horizontal bars. The uniform loading is distributed in various amounts over the two horizontal bars. It is found that the total sway buckling load does not change appreciably with this variation. When only the top horizontal bar is loaded (top 100%, bottom 0%), the total sway buckling load is 3.677. When the top and bottom are loaded by the same amount, the total sway buckling load is 3.688 (see Fig. 11). Finally, when the top is loaded by an amount which is much smaller than the bottom (top 5%, bottom 95%) the total sway buckling load is 3.696.

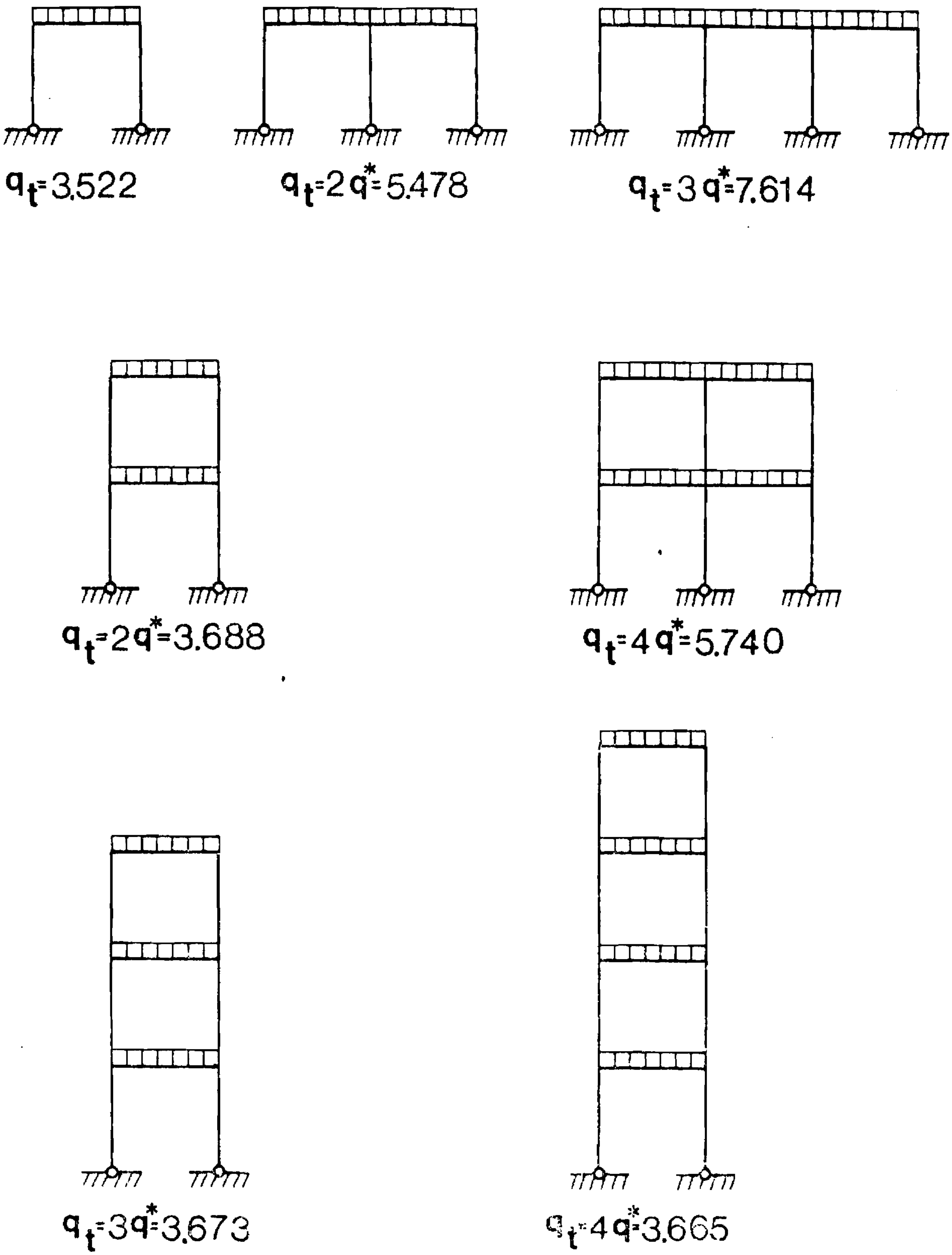


Fig. 12 Critical Loads for Hinged Multi-bay, Multi-storey Frames ($R_i = S_i = 1$).

When designing two-bay (or multibay) frames to carry uniformly distributed loads, inside columns must carry more load than outside columns. Because of this, inside columns are usually made stiffer. One possible design is to make the inside column(s) twice as stiff (in bending) as the outside one(s). Sway-buckling results for such a two-bay geometry are presented on Table 8. The lengths of all five members are the same, but the bending stiffness of the horizontal bars is varied. Axial load coefficients for all five bars are also reported on Table 8 ($k_3 = k_1$ and $k_5 = k_4$). Moreover, the total (nondimensionalized) sway-buckling load is given for each case. It is seen from Table 8 that as the stiffness of the horizontal bars increases the total load increases. Moreover, a comparison with the results of Table 7, corresponding to $L_h/L_v = 1$, reveals that by doubling the bending stiffness of the middle column the total sway-buckling load is increased by approximately 33%, regardless of the relative stiffness of the horizontal bars. Another important observation is that, the ratio of axial forces (inside to outside, P_2/P_1; $P_i = k_i^2 \, EI_i/L_i^2$) is not affected appreciably by the doubling of the bending stiffness of the middle column. This ratio varies (increases) with increasing bending stiffness of the horizontal bars.

TABLE 8. EFFECT OF HORIZONTAL BAR STIFFNESS ON CRITICAL LOADS FOR HINGED ONE-STOREY TWO-BAY FRAMES (WITH MIDDLE COLUMN STIFFNESS DOUBLED).

EI_h/EI_v	1	2	3	10
q^*_{cr}	3.599900	4.164400	4.391500	4.655000
k_1	1.235737	1.299518	1.320376	1.334522
k_2	1.439725	1.573468	1.627115	1.695136
k_4	0.346890	0.207330	0.147837	0.048834
q_t	7.199800	8.329880	8.783000	9.310000

All of the above observations point out that there exists an optimum distribution of bending stiffness, in multibay multistorey orthogonal frames which are subject to sway-buckling, for maximizing their load carrying capacity.

5. CONCLUDING REMARKS

From the several studies performed on elastic orthogonal plane frameworks, some of which are reported herein, one may draw the following general conclusions:

1. The effect of flexible joint connections (bolted, riveted and or welded connections are flexible rather than rigid) on the frame response characteristics is negligibly small. Thus, assuming rigid connections in analyzing elastic plane frameworks, leads to accurate predictions.

2. Eccentrically loaded two-bar frames lose stability through the existence of a limit point and do not experience bifurcational buckling. For these frames, the slenderness ratio of the bars has a small but finite effect on the critical load. Moreover, depending on the value for the slenderness ratio, there exists a critical eccentricity which divides the response of the frame into two parts. On one side the response is characterized by stable equilibrium positions and on the other hand it exhibits limit point instability (within the limitations of the theory, $w_{i,x}^2 \ll 1$).

3. Unbraced multibay multistory frames (including portal frames) are subject to bifurcational (sway) buckling with stable postbuckling behaviour. Sway buckling takes place, when the frame is structurally symmetric and the load is symmetric. Because of this, the frame is insensitive to geometric imperfections regardless of the type (load eccentricity, variation in geometry - length, stiffness, etc). In many respects, the behaviour of

these frames is similar to the behaviour of columns, especially cantilever columns.

4. The effect of slenderness ratio on the nondimensionalized response characteristics of plane frameworks (except the two-bar frame) is negligibly small.

5. Starting with a portal frame, addition of bays increases appreciably the total sway-buckling load, while addition of storeys has a very small effect.

6. For multistorey frames, distributing the load in various amounts among the different floors does not alter appreciably the total sway-buckling load. In all cases, the first storey vertical bars (columns) carry the total load.

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