Parameter estimation of short-time multi-component signals using damped-amplitude & polynomial-frequency model

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PARAMETER ESTIMATION OF SHORT-TIME MULTI-COMPONENT SIGNALS USING DAMPED-AMPLITUDE & POLYNOMIAL-FREQUENCY MODEL

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ABSTRACT

This paper concerns the parameter estimation of multi-component damped oscillations having non-linear frequency. In this paper, the instantaneous frequency is approximated by polynomials while the amplitude is characterized by damped exponentials to connect directly to its physical interpretations. A maximum likelihood procedure is developed via an adaptive simulated annealing technique which helps to speed up the convergence. Results on simulated signals show that the proposed algorithm is more efficient than the algorithm based on polynomial amplitude models, and allows the estimation of damping coefficients over a very short time duration. Finally, the proposed algorithm is applied for characterizing the ambient vibrations of a building.

Index Terms— Damped amplitude, Polynomial phase signal, Time-frequency, Maximum likelihood, Adaptive simulated annealing.

1. INTRODUCTION

This paper deals with multi-component signals contaminated with a white Gaussian noise. The signal is represented as a sum of components with time-varying frequency and exponentially damped amplitude. Similar signals had been investigated in [1], where the parameters are estimated by analytical techniques, such as Fourier-transform-based methods, high-resolution Kumaresan-Tufts, MUSIC, and matrix pencil methods. Classical estimation methods are often simple and fast by providing analytical solutions [2][3], but they fail to correctly estimate such signals.

In this paper, we address the issue of modeling short-time signals having strong non-stationarities both in amplitude and frequency, and then a Maximum Likelihood approach is investigated. The non-linearity of the likelihood function compels the use of stochastic optimization techniques such as simulated annealing, implemented by Monte-Carlo random sampling combined with a Metropolis acceptance rule. In [4][5][6], this optimization has been proved to be quite efficient as a solution for short-time polynomial models.

In this paper, we consider the adaptive simulated annealing. Each multi-dimensional search must take into account the varying sensitivities of different parameters. At any given annealing-time, the adaptive simulated annealing described in [7] stretches out the range over which the relatively insensitive parameters are being searched, with respect to the ranges of the more sensitive parameters. In addition, this way of doing induces a gain in computing time.

Therefore, a damped-amplitude model is proposed with the interest of exploring its pros and cons in the context of signals studied. This model is of great interest thanks to a faster computation and a direct extraction of the damping factor. For the purpose of comparison, referred to as an indirect approach in this paper, the polynomial amplitude model we proposed in [4] will be applied as well, the parameters being transformed in order to finally model a damped amplitude.

In section 2, the model is defined and the constraints of the modulation functions are discussed. Section 3 is focused on the parameter estimation as well as the extraction of components. The new Cramer-Rao bounds are given in section 4. In section 5, the performance is analyzed on simulated signals and versus the indirect approach. Section 6 presents the application on real-world signals. Finally, the conclusion is drawn in section 7.

2. DAMPED-AMPLITUDE & POLYNOMIAL-FREQUENCY MODEL

Let $y[n]$ be a discrete time process consisting of a deterministic multi-component process $s[n]$ embedded in an additive white Gaussian noise $e[n]$ with zero mean and variance σ^2.

$$y[n] = s[n] + e[n] \quad with \quad s[n] = \sum_{i=1}^{K} A_i[n] e^{j \cdot \Phi_i[n]} \quad (1)$$

$$\Phi_i[n] = 2\pi \left(\sum_{k=-N/2}^{n} F_i[k] - \sum_{k=-N/2}^{0} F_i[k] \right) + \phi_{0,i} \quad (2)$$

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$$and \quad F_i[n] = \sum_{m=0}^{M_f} f_{m,i} \cdot g_m[n] \quad M_f \leq 3 \tag{3}$$

where $-\frac{N}{2} \leq n \leq \frac{N}{2}$ with N even, K is the number of components. $A_i[n]$ is the time-varying amplitude and $\Phi_i[n]$ is the instantaneous phase of the i^{th} component. As in [8], this model is centered in the middle of the observation window to minimize the error of estimation, thus $\phi_{0,i} = \Phi_i[0]$. The instantaneous frequency $F_i[n]$ is approximated by discrete orthonormal polynomial functions at maximum third order. At m^{th} order, $f_{m,i}$ and g_m are the parameter and the orthonormal polynomial respectively.

In this paper, we intend to study a new model for the amplitude in correspondence with many real-world signals where $A_i[n] = \beta_i e^{-\alpha_i n}$. The initial amplitude β_i and the damping coefficient α_i characterize the amplitude of the i^{th} component. In order to proceed with estimation of $\beta_i, \alpha_i, \phi_{0,i}$ and the M_f+1 frequency parameters $f_{m,i}$, the following constraints are imposed: $0 < F_i[n] < \frac{F_s}{2}$ with F_s the sampling frequency, $N+1 > M_f+4$ and $\Phi_i[n]$ does not include any discontinuities. With regard to real-world data, β_i and α_i are both constrained to be strictly positive.

An intrinsic error of the complex model with damped amplitude has to be mentioned. The model defined in (1) does not always satisfy Bedrosian conditions because the exponential amplitude can present a wide-band spectrum. However, in actual problems that we investigate, frequencies are at about 1Hz, and damping coefficients α_i are usually trivial ($< 10^{-1}$)[9], then the -3 dB spectral bandwidth of the damped amplitude ($8.08\alpha_i$ Hz) is very narrow. This error is thus negligible even for the low frequency signal that we process.

3. PARAMETER ESTIMATION ALGORITHM

We discuss the parameter estimation of a low-order polynomial model which intends to track locally highly non-stationary modulations. The signals are of short time length of approximately 30 to 100 samples. Let us consider the instantaneous frequency (3) be approximated by an orthonormal polynomial at M_f^{th} order, the parameters of each component in (1) form a vector:

$$\boldsymbol{\theta}_i = [\boldsymbol{\theta}_{A_i}, \phi_{0,i}, \boldsymbol{\theta}_{F_i}] = \left[\beta_i, \alpha_i, \phi_{0,i}, f_{0,i}, \ldots, f_{M_f,i}\right], \tag{4}$$

where $1 \leq i \leq K$, so that the parameters of all the components are

$$\boldsymbol{\theta} = [\theta_{i,j}]_{K\times(M_f+4)} = \left[\boldsymbol{\theta}_1^{\mathbf{T}}, \ldots, \boldsymbol{\theta}_K^{\mathbf{T}}\right]^{\mathbf{T}}. \tag{5}$$

Each element $\theta_{i,j}$ in $\boldsymbol{\theta}$ corresponds to the j^{th} parameter of the i^{th} component. The estimation of $\boldsymbol{\theta}$ leads to a problem of Maximum Likelihood which corresponds to a Least Square approach under the hypothesis of a white Gaussian noise,

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}\in\mathbb{R}^{K\times(M_f+4)}}{\arg\min} \sum_{n=-\frac{N}{2}}^{\frac{N}{2}} |y[n] - s[n]|^2 . \tag{6}$$

In [6], optimal and sub-optimal algorithms were developed to balance between the precision and the computation cost. The sub-optimal approach can be affected when one component has distinctly weaker amplitude than the others. So thereafter, only the optimal approach is considered. We assume that the number of components K and the polynomial order of each component M_f are *a priori* known and remain unchanged. This assumption is justified by the short-time duration.

In this paper, the steps of the parameter initialization and of the estimation procedure, close to [4], take into account the damped amplitude model and use the adaptive simulated annealing proposed in [7]. Let N^{acc}, N^{gen}, c be factors *a priori* fixed, $\boldsymbol{\Delta} = [\Delta_{i,j}]_{K\times(M_f+4)}$ be a constant matrix of parameter search step, matrix $\boldsymbol{T}^{gen} = [T_{i,j}^{gen}]_{K\times(M_f+4)}$ and variable T^{acc} be the temperatures which control the generation of parameter candidates and the decision of acceptance respectively. In addition, the temperatures are regulated by a variable τ^{acc} and a matrix $\boldsymbol{\tau}^{gen} = [\tau_{i,j}^{gen}]_{K\times(M_f+4)}$. At the t^{th} iteration, the algorithm steps are:

i) Generation of parameter candidates: the candidate matrix $\hat{\boldsymbol{\theta}}^{can}$ is drawn element-wise from the parameter matrix at previous iteration $\hat{\boldsymbol{\theta}}^{t-1}$. The $(i,j)^{th}$ element of the candidate matrix is
$\hat{\theta}_{i,j}^{can} = \hat{\theta}_{i,j}^{t-1} + u\Delta_{i,j}T_{i,j}^{gen}\left(1 + \frac{1}{T_{i,j}^{gen}}\right)^{|u|-1}$, u being drawn randomly from uniform distribution $u \sim \mathcal{U}[-1,1]$.

ii) Acceptance of candidates by Metropolis rule: Let $\mathcal{MSE}(\hat{\boldsymbol{\theta}})$ denote the Mean Square Error of $\hat{\boldsymbol{\theta}}$, then $\mathcal{MSE}^t = \mathcal{MSE}(\hat{\boldsymbol{\theta}}^{can}) - \mathcal{MSE}(\hat{\boldsymbol{\theta}}^{t-1})$. The candidates are definitely accepted if $\mathcal{MSE}^t < 0$, otherwise they are accepted with the probability $\frac{exp(\mathcal{MSE}^t)}{T^{acc}}$;

iii) Update of temperatures:

iii-a) Cooling: Every N^{gen} number of loops, τ^{acc} and each element of $\boldsymbol{\tau}^{gen}$ are increased by 1. T^{acc} and each element of $\boldsymbol{T}^{gen}$ are decreased
$T^{acc} = exp\left(-c(\tau^{acc})^{1/K(M_f+4)}\right)$,
$T_{i,j}^{gen} = exp\left(-c(\tau_{i,j}^{gen})^{1/K(M_f+4)}\right)$;

iii-b) Re-annealing: When every N^{acc} candidate matrices are accepted, define $\hat{\boldsymbol{\theta}}^{best} = \underset{k\in 1,\ldots,t}{\arg\min}\,\mathcal{MSE}(\hat{\boldsymbol{\theta}}^k)$, and $q = \max_{(i,j)}(\boldsymbol{q}^{best})$ with $\boldsymbol{q}^{best} = \left[q_{i,j}^{best}\right]_{K\times(M_f+4)} = \left|\frac{\partial\mathcal{MSE}(\boldsymbol{\theta})}{\partial\boldsymbol{\theta}}\right|_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}^{best}}$ as its convergence speed matrix. T^{acc}, τ^{acc} and each element of $\boldsymbol{T}^{gen}$ and $\boldsymbol{\tau}^{gen}$ are updated:
$T_{i,j}^{gen} = \frac{qT_{i,j}^{gen}}{q_{i,j}^{best}}$, $\tau_{i,j}^{gen} = \frac{1}{c}\left|-log\left(T_{i,j}^{gen}\right)\right|^{K(M_f+4)}$,
$T^{acc} = \mathcal{MSE}(\hat{\boldsymbol{\theta}}^{best})$,

$\tau^{acc} = \frac{1}{c}\left|log\left(T^{acc}/\mathcal{MSE}(\hat{\boldsymbol{\theta}}^t)\right)\right|^{K(M_f+4)}$.

At this step, t is increased by 1. The algorithm stops when t reaches its pre-defined limit or when the estimation error is close enough to a white Gaussian noise.

4. CRAMER-RAO LOWER BOUNDS

The Cramer-Rao Bounds had been calculated in [10] for a damped polynomial phase signal. We propose to recalculate these bounds for the model defined in (1,2,3) under the discrete orthonormal polynomial base we used as in [6]. Further we derive these bounds not only for the parameters but also for the modulation functions. The Fisher information matrix of an arbitrary component defined by (1) is given as

$$\mathcal{I}_{\boldsymbol{\theta}_{A_i,F_i}} = \frac{1}{\sigma^2}\Re\left\{\begin{bmatrix} \boldsymbol{I}_{\boldsymbol{\theta}_{A_i}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}_{\boldsymbol{\theta}_{F_i}} \end{bmatrix}\right\} \quad (7)$$

$$\boldsymbol{I}_{\boldsymbol{\theta}_{A_i}} = \begin{bmatrix} \sum_{n=-N/2}^{N/2} 2e^{-2\alpha_i n} & -\sum_{n=-N/2}^{N/2} 2n\beta_i e^{-2\alpha_i n} \\ -\sum_{n=-N/2}^{N/2} 2n\beta_i e^{-2\alpha_i n} & \sum_{n=-N/2}^{N/2} 2n^2\beta_i^2 e^{-2\alpha_i n} \end{bmatrix}$$

Denote $\eta_m[n] = \sum_{k=-N/2}^{n} g_m[k]$ as the numerical integration of the orthonormal polynomial, $\boldsymbol{I}_{\boldsymbol{\theta}_{F_i}}$ is a block matrix of $(M_f+1)\times(M_f+1)$ dimension, with elements

$$I_{h,l} = \sum_{n=-N/2}^{N/2} 2\eta_h[n]\eta_l[n]\beta_i^2 e^{-2\alpha_i n}; \quad 1 \leq h, l \leq M_f + 1. \quad (8)$$

Then, in the non-biased case, CRBs of the instantaneous amplitude denoted as $\mathcal{CRB}_{A_i[n]}$ and of the instantaneous frequency denoted as $\mathcal{CRB}_{F_i[n]}$ are

$$\mathcal{CRB}_{A_i[n]} = \frac{\sigma^2}{2}\boldsymbol{d_i}^{\dagger}\{\boldsymbol{I}^{\dagger}_{\boldsymbol{\theta}_{A_i}}\boldsymbol{I}_{\boldsymbol{\theta}_{A_i}}\}\boldsymbol{d_i} \quad (9)$$

$$\mathcal{CRB}_{F_i[n]} = \frac{\sigma^2}{2}\boldsymbol{h_i}^{\dagger}\{\boldsymbol{I}^{\dagger}_{\boldsymbol{\theta}_{F_i}}\boldsymbol{I}_{\boldsymbol{\theta}_{F_i}}\}\boldsymbol{h_i} \quad (10)$$

where $\boldsymbol{d}_i = [e^{-\alpha_i n}, -\beta_i n e^{-\alpha_i n}]^{\boldsymbol{T}}$, $\boldsymbol{h}_i = [g_0[n], \ldots, g_{M_f}[n]]^{\boldsymbol{T}}$, $[\cdot]^{\dagger}$ denotes the conjugated transpose. These bounds will be used to compare the performance of the proposed algorithm to what we defined the indirect approach.

5. RESULTS ON SIMULATED SIGNALS

We consider the case of 2 components over 33 samples. In accordance with real-world data, the damping ratios are taken in the order of several percent. Fig.1 shows the results of the proposed algorithm with $K = M_f = 2$ and a signal-to-noise ratio (SNR) of 15 dB. It is important to notice that this global SNR can be drastically different from the local SNR due to the non-stationarity. In this example, the local SNR varies from 18.82 dB to 9.46 dB. Table 1 shows the normalized root mean square errors for the algorithm proposed compared to the indirect approach. The proposed algorithm gives better estimation results for all parameters, especially for the amplitude.

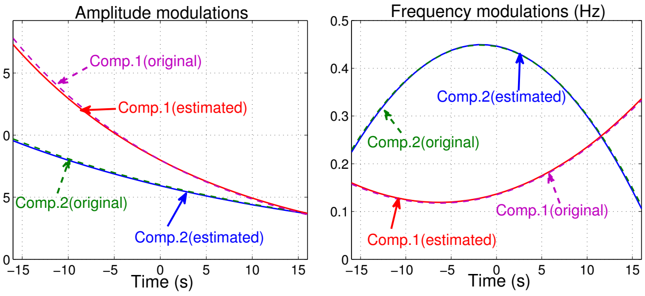
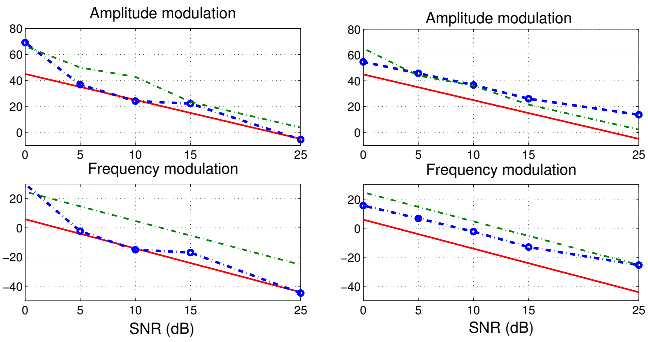


Fig. 1. Estimated modulations (–) and original modulations (- -) of the simulated signal sampled at 1 Hz - SNR=15 dB.

Table 1. Normalized root mean square errors in %

Algorithm	α_1	$A_1[n]$	$F_1[n]$	α_2	$A_2[n]$	$F_2[n]$
Proposed	0.67	1.38	0.66	3.4	1.92	1.07
Indirect	7	7.73	1.31	8.2	4.17	2.04

Fig.2 shows Cramer-Rao bounds of both components obtained by the proposed algorithm under the SNR varying from 0 to 25 dB, averaged among 100 noise realizations. We observe that these bounds are lower than those obtained by the indirect algorithm since the error caused by amplitude regression is avoided.



(a) first component (b) second component

Fig. 2. MSEs and CRBs in dB for the simulated signal. CRBs of the algorithm proposed (– red) and of the indirect algorithm (- - green), MSEs (-.o blue)

6. RESULTS ON BUILDING AMBIENT VIBRATIONS

The analyzed real-world signal is an ambient vibration recorded at the top of the Grenoble City Hall in France. The purpose is to evaluate the natural structural variation of the building, which is directly related to the damping coefficients and the frequencies of vibrations in all directions. The proposed algorithm permits a local analysis which has never been done before. Based on the priori knowledge that the signal has 3 main directions of observation: longitudinal, transverse and vertical, K is fixed to 3. Fig.3 shows the results of a segment of 12 s of the vertical measurement, sampled at 200 Hz and decimated by 40. The calculation is executed by both the proposed and the indirect algorithm with $N^{acc} = 60$, $N^{gen} = 80$, $c = 3$.

From Fig.3, we can conclude that these components and their variations are correctly identified using both algorithms [9][11]. Meanwhile, the proposed algorithm performs better than the indirect approach, indeed the normalized root mean square error of the reconstructed signal is about two times smaller than the indirect approach.

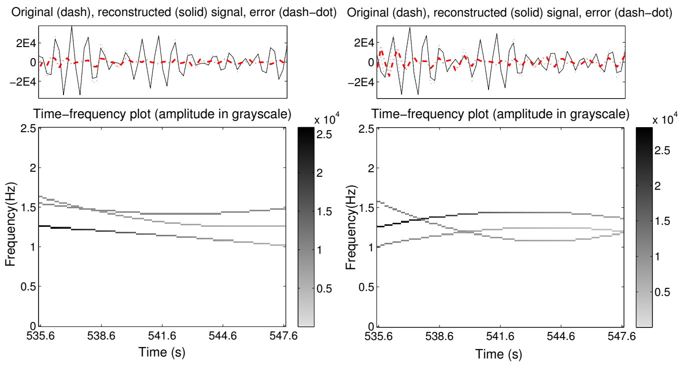


Fig. 3. Results for the building ambient vibration. Left: proposed algorithm. Right: indirect algorithm

As another advantage, the proposed algorithm requires smaller parameter space to track the amplitude variations, which induces a 25% saving in calculation time (Thinkpad x200 lap-top).

In this application, seeing that the lowest frequency is sufficiently far away from the null frequency as illustrated by Fig.3, and that all estimated damping coefficients are trivial ($< 2.5\%$), the intrinsic defect of the model mentioned in section 2 is therefore negligible.

7. CONCLUSION

In this paper, a damped-amplitude & polynomial-frequency model is proposed. This model is applied on short-time multi-component signals and the parameter estimation is based on the maximization of the likelihood function optimized by adaptive simulated annealing. By calculating and analyzing the Cramer-Rao bounds, it is shown that the estimation of both the amplitude and frequency modulation functions are improved compared to a polynomial-amplitude model. The proposed method is capable to track the variation of multi-component signals and directly identify the damping coefficient for each component. By that way, the ambient vibrations of a building and more particularly their damping coefficients have been characterized over a very short time of 12 s (60 samples), which has never been done before.

In future, we intend to study the performance of the proposed algorithm compared to other techniques. Moreover, the adaptability of this method could be enhanced by an automatic management of component births and deaths, and of the number of components. A long-time observation can thus be modeled by merging the estimation of short-time segments as in [6].

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