Measurement scale for colour perception

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Abstract – The colour, with the particularity to be defined simultaneously as a physical quantity and as a psychophysical quantity, is one of the concepts that can link hard sciences and behavioural sciences. From the viewpoint of behavioural sciences colours are basically measured with nominal scales, and in hard science colours are measured with interval scales. Our hypothesis is that the main relation that must be preserved during colour measurement is a metric. We suggest then that colours must be measured with metrical scales. The fuzzy metrical scale is preferred due to the possibility to define it like a nominal scale.

Keywords: Measurement theory, Colour, Scale, Distance, Fuzzy subsets theory.

1. INTRODUCTION

Some quantities like colour, odour, software complexity are usually measured with inappropriate scales. Indeed, the theories chosen to abstract such quantities usually define an affine space to represent measurement values even if this choice is not justified. For example, colours are represented in many different spaces like RGB, xyz, Luv, Lab, HSV and the transformation from one to each other is not always an affine transformation. We conclude from this situation that the empirical space of colours doesn't hold an affine structure and then cannot be represented by an affine space.

Conversely, the metric, defined with psychophysic experiments stays stable and is the most known relation on colours. The basis hypothesis of this paper is that the empirical space of some quantities manifestations, more particularly the colour, can be represented by a non-affine abstract space that holds a metric.

The determination of such metric depends on the theory that is used to perform calculus reasoning and decision on an abstract world where quantities are represented by their well known quantity value. Let describe the full process [1]. First the field of interest, i.e. the concrete world is identified. Then concrete objects and their properties are selected. A theory, made of entities, axioms and theorems, is chosen. Experiments are then performed to obtain observations of quantity manifestations. The representations of the manifestations are named *quantity values* [VIM] and are expressed into a space which structure depends on the theory chosen to infer conclusions. The choice of the theory is crucial and depends on the goal of the experiment. In the colour field, the goal can be to use the colour to identify the chemical components of a liquid. In this case the theory is defined on the field of molecular physics and colour manifestations are represented by spectral energy distributions. Each spectra are expressed into a n-dimensional vector space. If the goal is to check the quality of a manufactured colour, then the experiment is based on a theory of colour vision and colours are expressed into a metric space.

2. COLOUR VISION REPRESENTATION

This paper, will focus on psychophysical aspects of colours. This means that colour quantities are not considered exclusively into the context of physics but also into the context of human perception. From a pure physics consideration, a colour of an electromagnetic flow is defined by its spectral power distribution (SPD). As for any distribution, a general definition is never obtained due to the necessity to define the spectral resolution. Indeed the quantity that represent a colour is a vector such length depends on the chosen resolution, and the chosen range of the spectra. We can see that even with a given theory, the goal of the measured quantity has a strong incidence on its representation. As an example, the International Commission on Illumination (CIE) specified that for colour measurement of visible light the spectra range is from 360nm to 830nm with 1nm resolution. This institution gave also a first approximation of human colour perception with the definition of the tristimulus values XYZ. The X, Y and Z values are obtained with 3 colorimetric observers $\bar{x}(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$ (see (1) and Fig. 1.) that approximate the spectral sensitivity of human photosensors [2].

$$X = k \cdot \int_{380nm}^{780nm} (\phi(\lambda) \cdot \bar{x}(\lambda)) d\lambda \quad (1)$$

Where k is a constant, and $\phi(\lambda)$ is the acquired SPD.

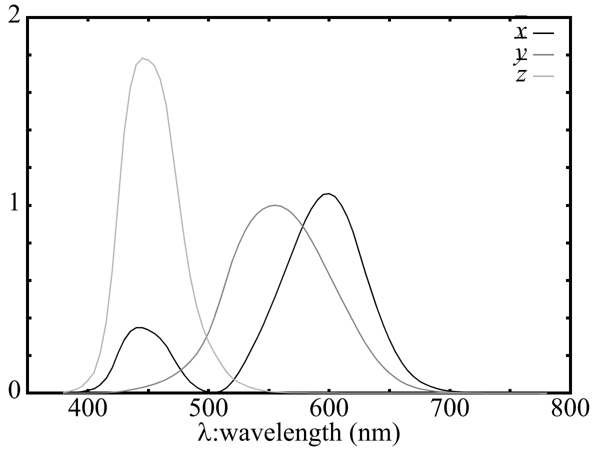

Fig. 1. colour-matching functions $\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$

This representation of colour into a 3D space is justified by the fact that 2 colours represented with the same XYZ tristimuli are considered as equal by any human. Its then easy to deduce that this colour measurement is based on a nominal scale, i.e. on a scale that links an equivalence relation on empirical quantities with an equality on measured values.

The colorimetric spaces are derived from the XYZ tristimuli by the use of bijective transformations. This confirms that colour measurement scales are equivalent when considered as nominal scales: bijections are admissible transformations for nominal scales. For a given colour classification, colour histogram methods do not depend on the colorimetric space and they use the classification as a nominal scale. Most other methods need a metric on the colorimetric space or at least a similarity relation between colours [3]. In the first case, a metrical scale, i.e. a scale that preserves a metric, is needed. With the main property of preservation of a similarity relation, a fuzzy nominal scale is a good candidate for the second case [4][5]. Some methods use colorimetric spaces as affine spaces [6]. These last approaches are questionable when the different colorimetric spaces must be considered as equivalent representation spaces of the same quantity.

From formal point of view, a method might not depends on the choice of a representation space that actually depends on the choice of scale. In this paper, we promote the hypothesis that the number of different colorimetric spaces proposed shows that the representation space of a scale for colour measurement may have a metric but not necessary be an affine space. The preservation of distance is then the generic relation of such scale known as metrical scales. The consequence is that all signal processing need to be defined on the basis of a distance.

3. COLOUR REPRESENTATION BY LEXICAL FUZZY SUBSETS

Fuzzy nominal scales where introduced in order to formalize an application to the measurement process of a mechanism of description of a quantity by a fuzzy subset of symbols [7]. With these scales, values in the representation space are fuzzy subsets of symbols also called in this paper *lexical fuzzy subsets* (LFS). The measurement is split into a measurement from the set of manifestations to a numerical space X, then a mapping D called fuzzy description or simply description translates a numerical scalar or vector into a lexical fuzzy subset. In the following example, a manifestation is represented by a scalar itself described by a LFS defined by its membership function μ on a lexical set $S = \{a,b,c,d\}$.

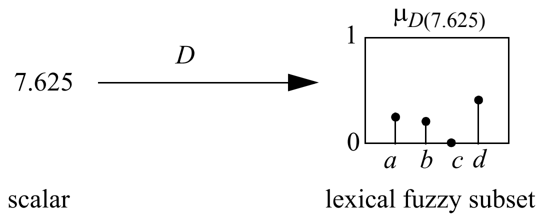

Fig. 2. Example of a fuzzy description mapping a scalar into a lexical fuzzy subset (LFS).

In this paper, we restrict our study to the fuzzy nominal scales that respect:

$$\forall A \in D(X),\ \sum_{s \in S} \mu_A(s) = 1 \tag{2}$$

Such scales define a fuzzy equivalence relation between LFSs like for example the simplest one (3).

$$\forall A, B \in D(X),\ \mu_{\sim}(A, B) = \sum_{s \in S} min(\mu_A(s), \mu_B(s)) \tag{3}$$

This relation also known as similarity relation is a representation of a relation between manifestations. It respects the reflexivity condition, expressed by (2), and a weak version of the transitivity condition. In our case $\mu_{\sim}$ is T_L-transitive:

$$T_L(\mu_{\sim}(x, y), \mu_{\sim}(y, z)) \leq \mu_{\sim}(x, z) \tag{4}$$

where

$$T_L(x, y) = max(0, x + y - 1) \tag{5}$$

Using fuzzy scales for colour measurement is justified by the fact that the existence of a similarity relation between colours even if such relation is not clearly known. On the basis of a lexical set $S = \{red, blue, yellow, green, ...\}$ a fuzzy nominal scale defines the meaning of each symbol with a fuzzy subset of manifestations, actually a fuzzy subset on a colorimetric space.

The first step to define such scale is to define the lexical set. A usual set will be $S = \{green, yellow, red, purple, blue, cyan, black, white\}$, the 8 colours of the RGB cube and of its affine transformations. In the paper we restrict to a chromatic plane then white, black and intermediate colours are represented with the same value. We choose the symbol *neutral* for this value. We propose also to use different symbols to represent real colours and colours that define the extremes of the chromatic plane. Finally we add the colour *orange* to the set in order to have a lexical set more representative of the human feeling. A possible lexical set is S = {*full_green, full_orange, full_yellow, full_red, full_purple, full_blue, full_cyan, neutral, green, orange, yellow, red, purple, blue, cyan*}.

The meaning are define by a piece wise interpolation based on a triangulation of the chromatic plane. First each symbol is associated with a chromatic coordinate that is characteristic to the symbol. Then the plane is split into triangles such that vertices are characteristic coordinates. The meaning of a symbol is a fuzzy subset which membership function is equal to 1 for the characteristic coordinate of the symbol and equal to 0 on the other characteristic coordinates, and interpolated on the triangles. The next figure shows the triangulation used to define the meaning of the lexical set on the ab chromatic plane.

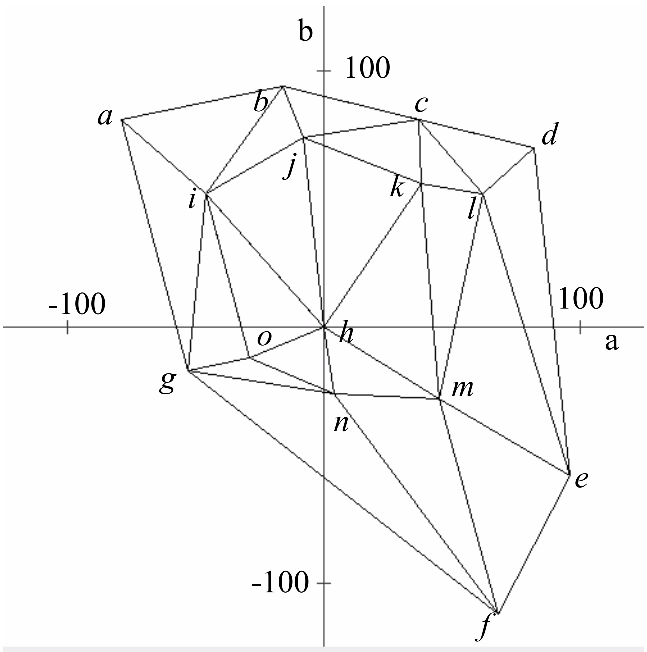

Fig. 3. Triangulation that defines the fuzzy meaning of *S* = {*full_green, full_orange, full_yellow, full_red, full_purple, full_blue, full_cyan, neutral, green, orange, yellow, red, purple, blue, cyan*}. Letters *a,b,c,..* replace *full_green, full_orange, full_yellow, ...*

Building a metrical scale from a fuzzy scale needs to define a distance d on the lexical set and to define a distance d' between LFSs that verifies:.

- the singleton coincidence: $d'(\{a\},\{b\}) = d(a,b)$
- the continuity property.
- the precision property that imposes that the distance between two LFSs must be a positive real number.
- the consistency property that is usually verified by distances on crisp subsets

The transportation distance, denoted d_{tp}, verifies all these properties [9][10]. It is computed as solution for a mass transportation problem [10] where the masses are membership degrees, sources and destinations are items of the lexical set and the unit cost from a source to a destination is given by the distance d on S. This distance can be defined relatively to the goal of the measurement process or relatively to the application. Another solution is to use the triangulation to compute a distance on the basis of the adjacency of the symbols provided by the graph of characteristic coordinates.

4. ADAPTATION OF THE SCALE

The crucial point of the scale definition is the location of the characteristic coordinates. The coordinates given in Fig. 3. are defined for a general use and are usually not usable for specific cases. For example, Van Gogh painting usually not fit with this generic scale.

Fig. 4. Van Gogh painting as a context for colour measurement

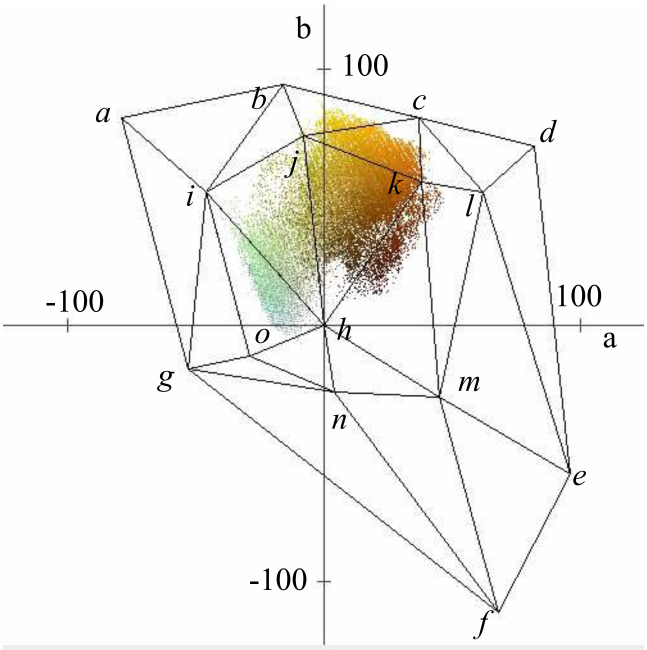

Fig. 5. Colour histogram of the painting in comparison with the characteristic coordinates of each symbol.

The proposal of this paper is to perform a Fuzzy C-Means clustering (FCM) to adapt the generic knowledge given by the initial characteristic coordinates. The idea is to fit each characteristic point with the center of its closest cluster. The difficulty is that the FCM algorithm is based on the minimization of an objective function based on the computation of the Euclidean distance between samples and cluster centers and cannot be directly used. Indeed, as seen before, nothing can justify that the colorimetric space, or the space of LFSs, holds an Euclidean metric.

In the original FCM algorithm a set of clusters is first defined. Each cluster is randomly defined by a fuzzy subset of samples. At each iteration, the FCM algorithm computes the center of each cluster. Then the membership degree of each sample to each cluster is re-evaluated relatively to its proximity to the associated cluster center.

In our approach, we propose several adaptations to this process.

Let M be a set of samples in X.

In the initial state, Each cluster C_s is identified by a symbol s and is defined by the fuzzy subset of X derived from the fuzzy description.

$$\mu_{C_s}(x) = E_\alpha(1 - d(\{s\}, D(x)))^2 \tag{6}$$

where

$$E_\alpha(u) = \begin{cases} u & if \quad u \geq \alpha \\ 0 & else \end{cases} \tag{7}$$

As the fuzzy equivalence relation that characterizes the scale defines a distance for short range LFSs, the eq. (6) can be simplified into:

$$\begin{aligned} \mu_{C_s}(x) &= E_\alpha(\mu_{\sim}(\{s\}, D(x)))^2 \\ &= E_\alpha(\mu_{D(x)}(s))^2 \end{aligned} \tag{8}$$

The main difference with the standard FCM is the inclusion of a basic knowledge at the initial step of the algorithm. This knowledge can be considered as an average knowledge about the representation of colours.

At each iteration, the cluster center is simply computed as the gravity center of the cluster.

$$c_s = \frac{\sum_{x \in M} x \cdot \mu_{C_s}(x)}{\sum_{x \in M} \mu_{C_s}(x)} \tag{9}$$

The scale is then transformed such that the center of the cluster C_s become the characteristic point of the symbol s.

As for the original algorithm the iterations stop when changes reach a termination criterion.

The α parameter, must be defined into [0,1]. It represents the inertia of the learning process. If $\alpha = 1$, each iterated cluster includes only its characteristic point as unique sample. The characteristic points never move during the algorithm. If $\alpha = 0$, each iterated cluster can include new samples far from the characteristic point.

The next figure shows the triangulation after the adaptation of the scale with this method. As the colours *full_green, full_orange, full_yellow, full_red, full_purple, full_blue, full_cyan, neutral* are synthetic colours defined by a norm, they are not supposed to be changed during the learning process. So $\alpha = 0$ for these colours.

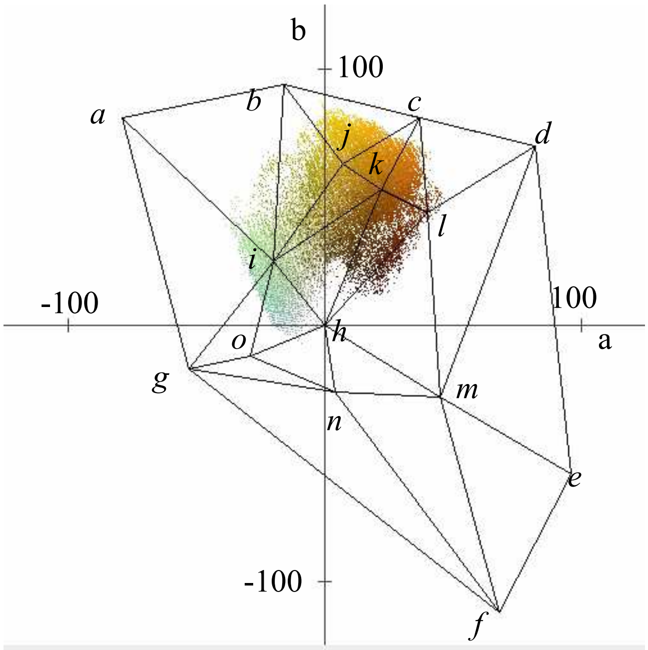

Fig. 6. Characteristic coordinates after the adaptation of the scale with the FCM like clustering ($\alpha = 1$ for *a* to *h*, $\alpha = 0.5$ for *i* to *o*).

DISCUSSION

The information of colour has the property on one side to be typically a psychophysic information, on the other side to be acquired with accurate measuring instrument and then to be accurately represented on a numerical space. Between the physical world, from where the colour entities are issued, and the abstract human mental world, where they are defined and represented, is the sensitive world that is a partial perception of the physical world. Colour entities, like the *orange colour* for example, cannot be considered as concrete physical entities of the concrete physical world. they are issued from a concrete entity, a spectra of a electromagnetic flow, and appear into the sensitive world. The difficulty of colour measurement is that the human sensitive world, i.e. the human perception of the concrete world, differs from the instrumental sensitive world, i.e. the instrumented perception of the world. The consequence is that the abstract worlds used to represent these sensitive worlds also differs. In particular, the structure of the colorimetric spaces of the instrumental abstract world are richer than the structure of the colorimetric spaces of the abstract human mental world.

Indeed, each colorimetric space of the instrumental abstract world holds a metric. Usually, but not necessary, an Euclidean metric. So it's legitimate to consider that the colorimetric space in instrumental sensitive world also holds this metric. A colorimetric space of the abstract human mental world is a metrizable space, but the associated metric is not defined. This fits with the general knowledge that a distance between colour exists but cannot be precisely defined. Within this context, the space of lexical fuzzy subsets gives an alternative to usual numerical colorimetric spaces. Indeed, this space is a metrizable space, and the metric depends on the goal of the colour process and not on the sensitive world. Furthermore, the distance is based on a fuzzy scale that can be adapted to colour process throw a learning algorithm.

CONCLUSION

Colour measurement does not lead to a unique theory and needs a scale for each application, or more precisely for each concept. We proposed in this paper to use scales that preserve a similarity relation or scales preserving a metric. The fuzzy scales, with the expression of measurement values on a non affine space give a good solution for colour measurement. The counter part is the necessity to adapt the scale according to a context or a colour process. This paper gave an algorithm to perform such adaptation. This adaptation can be compared with a calibration process where the calibration standards are colour entities. Finally that the colorimetric space associated to a fuzzy scale has a structure closer to the human representation than classical colorimetric spaces.

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