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► To cite this version:

Thomas Pottier, Hervé Louche, Serge Samper, Hugues Favreliere, Franck Toussaint, et al.. A new filtering approach dedicated to heat sources computation from thermal field measurements. A. Chrysochoos, H. Louche, B. Wattrisse. PhotoMechanics Conference, May 2013, Montpellier, France. pp.175-178, 2013. <hal-00938117>

HAL Id: hal-00938117

<https://hal.archives-ouvertes.fr/hal-00938117>

Submitted on 29 Jan 2014

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A NEW FILTERING APPROACH DEDICATED TO HEAT SOURCES COMPUTATION FROM THERMAL FIELD MEASUREMENTS

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ABSTRACT: This paper presents a new approach in heat source evaluation from thermal field measurements. A modal projection based on dynamics (Discrete Modal Decomposition) is used to estimate the spatial term of a heat diffusion problem. A numerical example is presented in order to validate this approach and compare it to a more classical spectral decomposition (based on strictly thermal considerations). Results show that the proposed projection basis not only provides closer assessment of the heat sources but is also more stable to measurement noise and side effects.

1. INTRODUCTION

The study of calorific effects associated with material deformation has been widely investigated since the early works of Lord Kelvin. The thermoplasticity models now offer the ability to describe the heat generation during material loading from a phenomenological standpoint. Since the late 90s, experimenters have turned to the use of Infra-Red (IR) thermography in order to estimate, from temperature information, the heat sources related to plasticity, phase transformation and other dissipation related phenomenon [1, 2, 3].

Classically, the heat sources computation relies on performing both spatial and time derivation of the measured temperature fields. Under some classical assumptions [4], the heat diffusion equation is given by

$$\rho C \frac{\partial \theta}{\partial t} - k \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) = w'_{ch} \quad \text{with} \quad \theta(x, y, t) = T - T_0, \quad (1)$$

where ρ is the mass density, C the specific heat, k the conductivity coefficient, $T(x, y, t)$ the current temperature field, $T_0(x, y)$ the initial temperature field and w'_{ch} the internal calorific heat sources. Such a definition allows the assessment of w'_{ch} from the time-derivative and the second space-derivative of $\theta(x, y, t)$ and the *a priori* knowledge of the material constants (ρ , C and k).

In practice, the time resolution of modern IR camera (up to 380 Hz) often allows the computation of a proper time derivation of θ . On the other hand, the space-derivative cannot be estimated as straight-forwardly. The measurement noise, intrinsic to IR-thermography, is amplified by space-derivation operators and massively impairs the calculation of the Laplace terms in Eq.(1).

Many numerical approaches have been developed in the last decade in order to estimate as accurately as possible the spatial term of Eq.(1). These regularisation methods can be sorted in 3 main categories:

- Filtering methods aimed at suppressing the noise from the measurement using Gaussian [5] or low pass filters based on linear, Fourier or wavelet definitions [6, 7].
- Inverse methods aimed at obtaining a quasi-solution through the use of one or several optimization algorithm [8, 9].
- Projection methods are among classical means in inverse heat transfer analysis. They consist of decomposing the measurement within a spectral basis built from eigen function of the Laplace operator [4, 3]. These techniques have recently experienced various improvement through the use of wavelet decompositions [10] or branch modes decomposition [11]. This latter approach allows the use of generalized boundary conditions which usually represent the main disadvantage of projection methods.

The present paper aims to propose the implementation of this latter approach and to compare the results obtained using a decomposition built from structural dynamics and another from thermodynamics considerations. For the sake of comparison, a numerical example is built and the heat sources are reconstructed using (i) the proposed modal basis and (ii) the classical spectral basis. Hence the influence the measurement noise is investigated.

2. SETUP OF PROJECTION BASES

2.1 Modal basis

This first projection basis is classically defined as the harmonic solution of a rectangular plate structural equation:

$$M \cdot \ddot{q} + K \cdot q = 0 \quad \text{with} \quad q = q(x, y) \quad \forall k \in \mathbb{N}, \quad (2)$$

where M and K stand for the mass and the stiffness matrices respectively. Under such formalism, $q(x, y)$ stands for the displacements which characterize the modal shapes. Let's denote Q as the matrix whose columns are Q_0, \dots, Q_{n-1} : the eigen vectors of Eq.(2) and $\mathfrak{B}_D = (Q_0, Q_1, \dots, Q_{n-1})$ the associated modal basis. Thus, the set of modal coordinates α_k resulting from the projection within a non-orthonormal basis is given as follows [12]

$$\alpha_k = \{\alpha_{k,i} \in \mathbb{R}\} = (Q^T Q)^{-1} Q^T \cdot q_k \quad (3)$$

The point of the present paper is to associate the displacement fields $q_k(x, y)$ to the measured temperatures $\theta_k(x, y)$ through a projection operator as follows

$$\theta_k(x, y) = \sum_{i=0}^{n-1} \alpha_{k,i} \cdot Q_i(x, y) + R_n(x, y), \quad (4)$$

where n is the number of modes used for the projection (or truncation order). Even though \mathfrak{B}_D is far from describing the actual underlying physics, it may (in theory) lead to an exact solution if n tends toward infinity. In practice, the assumption of an infinite size is of course not relevant from a numerical standpoint, however the obtained basis exhibit several significant advantages. First of all, it naturally describes a space vector and thus allows a straight forward projection. In addition, the energy sorting of modes lead to an increasing complexity of shape (and frequency) when the truncation order n increase. Finally, it does not require the use of variable separation and therefore leads to a slow frequency increasing along i . In other words, the i^{th} mode of \mathfrak{B}_D has a smaller frequency than any other mode assessed assuming variable separation. This latter point is of the utmost interest since the computation of the second order space-derivative (finite differences) is well known for significant divergence when high frequencies are involved.

2.2 Spectral Basis

Let us now consider the two dimensional Laplace equation in the rectangle $U = [0; a] \times [0; b] \times [t_k; t_{k+1}]$, with boundary conditions prescribed on the four edges of the rectangle. The Laplace problem is solved, using Dirichlet conditions over ∂U , as:

$$\begin{cases} \Delta\theta := \frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = \frac{\rho C}{k} \frac{\partial\theta}{\partial t} & \text{over } U \\ \frac{\partial\theta}{\partial x}(0; y, t) = \frac{\partial\theta}{\partial x}(a; y, t) = 0 & \text{over } \partial U_x \\ \frac{\partial\theta}{\partial y}(x; 0, t) = \frac{\partial\theta}{\partial y}(x; b, t) = 0 & \text{over } \partial U_y, \end{cases} \quad (5)$$

Assuming a steady state case within $[t_k; t_{k+1}]$, it classically comes a harmonic solution related to the Laplace operator of Eq.(1) as

$$\theta_k(x, y) = \sum_{p=0}^{+\infty} \sum_{m=0}^{+\infty} A_p C_m \underbrace{\cos\left(\frac{p\pi x}{a}\right) \cos\left(\frac{m\pi y}{b}\right)}_{\text{mode vectors } Q_{p,m}}. \quad (6)$$

In order to sort the obtained basis in increasing order of frequencies, the modes are sorted by increasing products $p \times m$. Double occurrences are treated by placing the greater value of p ahead. One of the main consequence of the sorting operation is that it must be performed again if n (denoting the total number of modes or truncation order) varies. This disadvantage prevents from a single, *once and for all* calculation of \mathfrak{B}_Δ . Another disadvantage comes from the periodicity of the Q_i vectors which requires high truncation order to describe a non-periodic shape. However, this basis exhibits a major advantage: it is an hyperbolic solution of Eq.(1) and is therefore close-to-physics since it is built from heat diffusion considerations.

3. NUMERICAL EXAMPLE

3.1 Description

The comparison of the two presented projection bases is performed by a numerical example. A square plate exhibiting two distinct heat sources is modeled in Abaqus. The finite element diffusion problem is solved numerically over a time period of $t = 200$ s. The resulting temperature fields are output to a matlab environment and interpolated over a 300×300 monotonic grid. A random white noise is added (standard deviation is denoted σ) and the projector Eq.(4) is used to approximate the original thermal fields. Finally the space derivation of modes and a numerical time derivation of the obtained temperature fields (finite differences) allows the use of the heat diffusion equation Eq.(1) and provides with an approximation of the imposed heat sources.

The chosen geometry is a 3D plane-parallel plate with dimensions (100; 100; 5). As shown in Fig.1, two zones are chosen to be the active heat sources of the model. The first one is rectangular and host a pyramid-shaped distribution of heat. The second one is circular and exhibits a peak-shaped distribution of heat. The boundary conditions over the six sides of the model imposes a zero output heat flux.

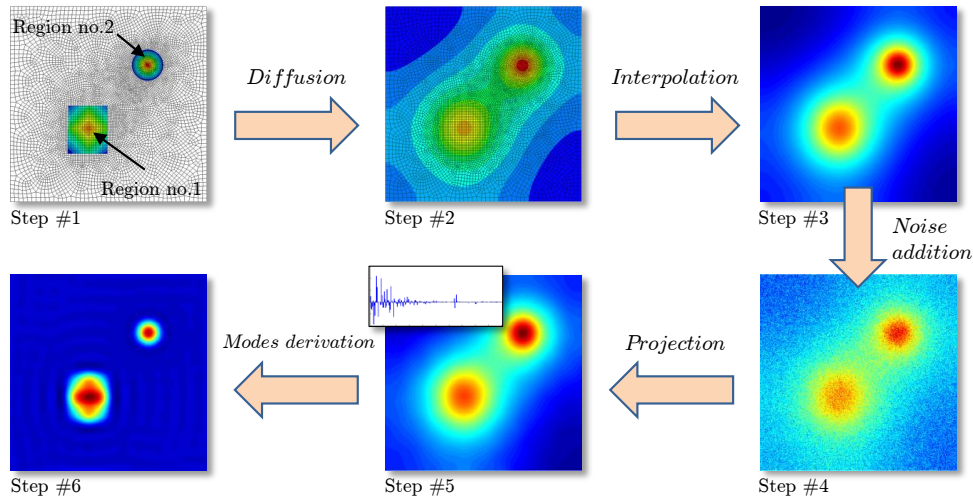


Figure 1 - The six steps solving flowchart of the proposed validation example.

3.2 Comparison and results

The measurement noise generated by IR-camera capture may lead to significantly small signal-to-noise ratio if the recorded phenomenon involve small temperature rises. Indeed, the difficulties encountered in inverse heat transfer analysis mainly rely on the measurement noise which lead to a ill-posed problem which becomes very unstable to the double space derivation operator. Hence, the stability of the presented approach to noise is therefore highly relevant.

Accordingly, a random white noise was artificially added to the numerical temperature fields with 30 increasing values of the σ . Fig.2 shows the reconstructed heat sources profiles along the two lines that cross regions no.1 and no.2. Moreover, the global Root of mean square errors (*RMS*) are computed and plotted in Fig.3.

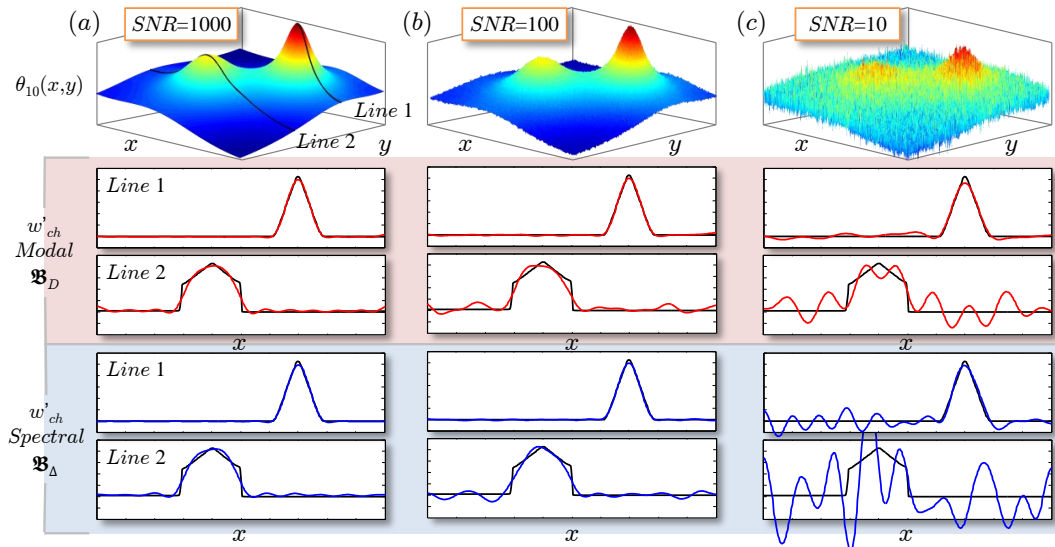


Figure 2 - Heat sources (w'_{ch}) reconstruction using modal and spectral projection for increasing noises (decreasing signal-to-noise ratio : *SNR*). Graphs depict the evolution of the reconstructed heat sources along two lines that cross the main heat sources (region no.1 and no.2 presented in Fig.1).

Two conclusions can be drawn from Fig.2 and Fig.3: firstly, as expected, the reconstruction error increases rapidly when the *SNR* decreases. Secondly, the modal projection presents an improvement from the spectral approach since the *RMS* error is smaller for every noise magnitude. Though, the gap between these two reconstructions becomes particularly visible for highly noised measurements (see Fig.3), the relative improvement is actually about 10% for all the studied *SNR*. This specific point constitutes a major improvement of the proposed projection method since it allows a better heat source assesment when temperatures rises are small: e.g. small strain rates, low conductivity, early localization, vicinity of peaks...

Another relevant point can be seen in Fig.2-(c). In this case, the heat sources in region no.1 (pyramid-shaped) is actually very poorly estimated while the heat sources in region no.2 is far better approximated (especially with \mathfrak{B}_D). This emphasizes the shape influence on the reconstruction capability. It is well known that square-like shape leads to the use of high frequency modes and therefore a good approximation is disabled by a finite truncation order n . However, as proposed in [13], basis enrichment performed with tailor made *sharp modes* can be helpful in such case.

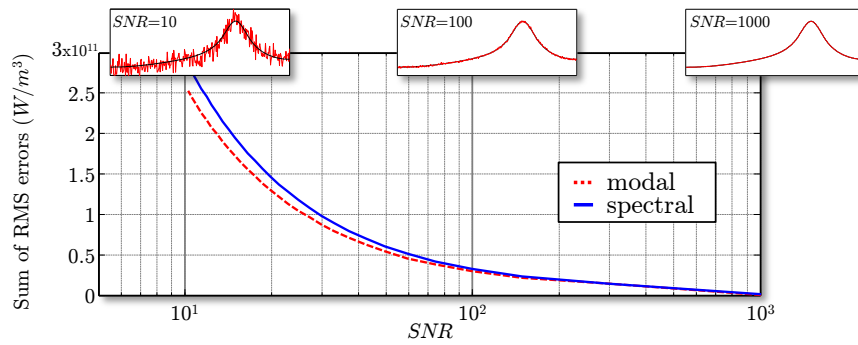


Figure 3 - Comparison of the reconstruction error for increasing values of the signal-to-noise ratio (SNR).

4. CONCLUSIONS

The presented study shows that regardless of how far from physics the modal projection is, it allows significant improvements in heat source estimation. It appears to be less sensitive to noise and thus allows a more accurate characterisation of calorific effects from temperature full-field measurements. It is also noticeable that building the modes from structural dynamics allows to account various geometry such as disk, cylinder, hollowed plates, notched specimen, etc since a very simple FE calculation is required.

Boundary conditions constitute a classical problem of thermal field projection methods. The presented approach allows to impose a wide range of them. Indeed the mechanical formulation of the problem leads to generate the modal basis from Finite Element Analysis and hence allows to impose either displacement, forces or velocity which all have thermal equivalences.

The spectral basis, is built from thermal considerations and thus is closer to physics. However, it appears to be less efficient in the presented numerical tests. The formulation of this basis tested in this paper is the classical one and not an enhanced one such as proposed by [3, 13]. Indeed, it should be accounted that several flaws of such a decomposition have already been pointed out in many former studies and improvement procedure has been proposed and implemented [11].

Though, such a complex filtering approach is probably unnecessary when measurements are clear from noise and exhibit sufficient space resolution, it constitutes a robust alternative in ill-posed inverse thermal problems. It must also be pointed that the presented numerical implementation leads to quick calculation since the modal basis is inverted once and for all. Processing a dozen images require less than a minute with an up-to-date computer.

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