Superior Performance of Switched Fuzzy Control Systems: an Overview and Simulation Experiments

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***Abstract* — This article gives a brief overview of the basic notions of switched fuzzy systems, their model construction and stability analysis, followed by the proposed concepts for building simulation algorithms for this type of systems. Using these concepts we have shown the whole process of modelling, stability analysis and design of stabilizing switching fuzzy logic controllers for the hovercraft-vehicle as a typical nonholonomic system. MATLAB Simulator (Simulink model) and special MATLAB functions for the switching fuzzy controller of the hovercraft vehicle, along with the numerous simulation results, verify the correctness of the proposed concepts. Using this typical nonholonomic system we have also explored the good performance of switched fuzzy systems in comparison to the ordinary T-S fuzzy systems.**

Keywords – control, hybrid systems, fuzzy systems, switched systems, switching, simulation of switched fuzzy systems

I. INTRODUCTION

Switched systems are special class of hybrid dynamical systems, which consist of a set of continuous-time or discrete-time subsystems and a rule that coordinates the switching among them. The last couple of decades have witnessed an enormous growth of interest of the class of switched systems in combination with the even larger class of hybrid systems [1], [2], [3], [4], as these systems have a wide range of potential applications.

From the middle of the 1980's, there have appeared a number of analysis/synthesis problems for Takagi-Sugeno (T-S) fuzzy systems [5], [6], [7].

Succeeding the remarkable developments in theory, applications, and the industrial implementations of fuzzy control systems, recently switched systems have been extended further to encompass *switched fuzzy systems* [8].

In general, a switched fuzzy system involves fuzzy systems among its sub-systems or an alternative fuzzy-switching law, or the both (least explored case). Resent developments in this area promote a new direction in the control of dynamic systems [9], [10], [11], [12], [13], [14], [15], [16], [17], [18], [19] and it is clear that the field of switched fuzzy systems is becoming very popular.

To our best knowledge, it may well be found that up to know the most of the research in this area is focused on representation modelling, stability analysis and controller design, that guarantees stabilization and certain system performance. We are expecting that these results can be used as a good platform for solving real world control problems. Thus, in this paper we are proposing the simulation scheme algorithms that can be used in modelling and design of switching fuzzy controllers for real world systems and we continue with exploring the performance of switched fuzzy control systems. The model of a hovercraft vehicle (a typical nonholonomic system that can not be stabilized by any continuous feedback control law) is used to verify the proposed simulation schemes as well as to prove the good performance of switched fuzzy systems.

Since in [20] and [21] we have given a detailed overview of the achievements in the field of switched fuzzy systems, followed by the comparative study for this kind of systems (different approaches in their model construction and challenges associated with the stability analysis and stabilization), here, we have provided a discussion of some of the key principles of the joint concept. This paper is an extension of the study given in [22].

To begin with, in Section II we give a brief overview of the basic concepts of switched fuzzy systems and their representation modelling. Appropriate stability analyses for this type of systems are given in Section III. In Section IV we outline the proposed concept for building simulation algorithms for switched fuzzy systems. In Section V the dynamics of a hovercraft vehicle as a typical nonholonomic system is given. For the purpose of later comparison, in Section VI we have given the whole process for building normal (non-switched) T-S fuzzy controller for the hovercraft vehicle, showing that with this type of controllers the control purpose can not be achieved. Using the proposed simulation schemes from Section IV, in Section VII we present the whole process of modelling and design of the switching fuzzy controllers for the hovercraft vehicle. We conclude this section with numerous simulation results, proving the good performance of switched fuzzy control systems. In Section VIII we give the essential conclusions of this study.

II. CONCEPTS OF SWITCHED FUZZY SYSTEMS AND REPRESENTATION MODELLING

The idea for switched fuzzy systems was putted forward by Palm and Driankov in 1998 [8]. Tanaka et.al. according to their previous research in the field of T-S fuzzy systems, for the purpose of control of more complicated real systems such as multiple nonlinear systems, switched nonlinear hybrid systems, and second order nonholonomic systems, introduced new type of model-based fuzzy systems - switched fuzzy systems [9]-[11]. The main aim of this design approach was to lose the "curse of dimensionality" - the complexity of a system makes the number of rules of a fuzzy model exponentially increase. Differently from the ordinary T-S fuzzy model, the switching fuzzy model given in [9]-[11] has locally Takagi-Sugeno fuzzy models (local fuzzy rule level) and switches them according to the premise variables, i.e., states, measurable external variables and/or time (region rule level). In general, the switching fuzzy model has two key features. One is to switch local T-S fuzzy models represented in each region. The other is to decrease the number of rules which fire simultaneously in comparison with an ordinary fuzzy model. Moreover in [9] a stable fuzzy switching control design is presented, where the design conditions are based on "common" quadratic Lyapunov function (CQLF), expressed in linear matrix inequality (LMIs - [7], [24]) form. The same idea is extended further in [10] where the switching controller is constructed by naturally extending the idea of the parallel distributed compensation (PDC) [7], [25].

Parallel to these results, authors in [15] propose T-S fuzzy model which differs from existing ones in the literature and is based on the fuzzy controller switching. Switching PDC controller is designed for a linear fuzzy system based on the switched systems model, i.e. every sub-controller is a PDC controller. Continuous way and discrete way are adopted to establish the stability results. First, sufficient conditions for asymptotic stability are presented and then, stabilizing switching laws of the state-dependent form are designed. Different from [15], where the controller switching strategy is employed to the ordinary T-S fuzzy model, authors in [16]-[18] propose switched fuzzy model for continuous ([16], [17]) and discrete case ([16]-[18]). The switched fuzzy model is seemingly similar to the model proposed in [9]-[11], but there is a crucial difference (the details for these, along with the detailed comparative study for these two approaches, are given in [21]).

Even though there many different approaches on representation modelling of switched fuzzy systems, the typical design procedure, which is present in most of the works, consists of the following steps. First, the whole state-space R^n is partitioned on m regions, and every region is a switched sub system. While functioning, this is the level where via the appropriate switching law an appropriate sub system is chosen to be on. The sub system on every region Ω_i is represented by a suitable model. When the subsystems of the switched system are represented as T-S fuzzy systems the system is switched fuzzy system. T-S fuzzy system partitions the space in the corresponding region into fuzzy sub-regions. In that way, the whole state-space is partitioned into many fuzzy sub-regions, where every sub-region is represented by a local model. The local model can be linear or nonlinear, continuous time or discrete time.

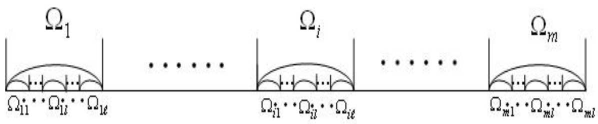


Figure 1. Sketch map of switched fuzzy system [19].

The sketch map of switched fuzzy system, regarding the state-space partitioning, is depicted on Fig. 1. In there, Ω_i denotes the state-space area of the i-th switched subsystem. Ω_{il} denotes the l-th fuzzy sub-region in the region Ω_i. In fact, the switched fuzzy systems partition the Ω_i region into l fuzzy sub-regions $\Omega_{i1},\cdots,\Omega_{il},\cdots,\Omega_{i\ell}$. There is a local linear/nonlinear model in every fuzzy sub-region. The model for every switched region $\Omega_1,\cdots,\Omega_m$, which consists of local linear/nonlinear models, is composed by linking the local models with fuzzy membership functions. When local model in fuzzy sub-region satisfies the switching law, switching to the Ω_i-th subsystem is made. In this way, stability of the switched fuzzy system, as a whole, is ensured.

For better understanding the concept of switched fuzzy systems, we will proceed by presenting one of the two basic analytical representations, analysed in [21], the so-called "**switched fuzzy system with levels of structure**", given in [9]-[12]. The purpose for choosing this type of representation modelling is that the analysis given in this paper is based on this type of modelling.

The model in [9]-[12] has local T-S fuzzy models and switches them according to the premise variables, i.e., states, measurable external variables and/or time. The switched fuzzy model from [9]-[12] is given with:

Region Rule i:

IF $z_1(t)$ **is** N_{i1} **and** $\cdots$ **and** $z_p(t)$ **is** N_{ip},

THEN

Local Plant Rule l: (1)

IF $z_1(t)$ **is** M_{il1} **and** $\cdots$ **and** $z_p(t)$ **is** M_{ilp},

$$\textbf{THEN}\begin{cases} \dot{x}(t) = A_{il}x(t) + B_{il}u(t), \\ y(t) = C_{il}x(t), \end{cases} \quad l = 1,2,\ldots,r \quad i = 1,2,\ldots,m$$

In (1), m is the number of regions partitioned on the premise parts space; $N_{ij}(z(t))$ is a crisp set; r is the number of rules of the local models; M_{ilj} is fuzzy set; $x(t)\in R^n$ is the state vector, $u(t)\in R^m$ is the input vector, $y(t)\in R^q$ is the output vector, $A_{il}\in R^{n\times n}$, $B_{il}\in R^{n\times m}$, and $C_{il}\in R^{q\times n}$; $z(t)=\lfloor z_1(t),\ldots,z_p(t)\rfloor$ are known premise variables that can be functions of the state variables, external disturbances, and/or time. $N_{ij}(z(t))$ is a crisp set, where:

$$N_{ij}(z(t))=\begin{cases}1 & ,z(t)\in N_{ij}\\ 0 & ,o.w\end{cases}. \tag{2}$$

From (1), it is clear that the switched fuzzy model has two levels of structure: region rule level and local fuzzy rule level. The region rule is crisply switched according to the premise variables. In other words, the membership functions $N_{ij}(z(t))$ in the premise parts of the region rules are crisp sets.

The switched fuzzy model (1) is inferred by fuzzily blending the linear system models $\dot{x}(t)=A_{il}x(t)+B_{il}u(t)$ and switching the global T-S fuzzy models, defined on every region.

Given a pair of $(x(t),u(t))$ and $z(t)$, the final output of the fuzzy system is inferred as follows:

$$\dot{x}(t)=\sum_{i=1}^{m}\sum_{l=1}^{r}\upsilon_i(z(t))h_{il}(z(t))(A_{il}x(t)+B_{il}u(t)) \tag{3}$$

$$y(t)=\sum_{i=1}^{m}\sum_{l=1}^{r}\upsilon_i(z(t))h_{il}(z(t))C_{il}x(t) \tag{4}$$

where

$$\upsilon_i(z(t))=\frac{\prod_{j=1}^{p}N_{ij}(z_j(t))}{\sum_{i=1}^{m}\prod_{j=1}^{p}N_{ij}(z_j(t))},\quad h_{il}(z(t))=\frac{\prod_{j=1}^{p}M_{ilj}(z_j(t))}{\sum_{l=1}^{r}\prod_{j=1}^{p}M_{ilj}(z_j(t))} \tag{5}$$

It is clear from (2) and (5) that $\upsilon_i(z(t))=\begin{cases}1 & ,z(t)\in \textbf{Region}\ i\\ 0 & ,o.w.\end{cases}$. This means that $\upsilon_i(z(t))=1$ if and only if $z(t)$ belongs to "*Region* i". The regions satisfy:

$$\textbf{Region}\ 1\cup\textbf{Region}\ 2\cup\ldots\cup\textbf{Region}\ m=\bigcup_{i=1}^{m}\textbf{Region}\ i=\mathrm{X} \tag{6}$$

$\textbf{Region}\ i_1\cap\textbf{Region}\ i_2=\varnothing, i_1\neq i_2,\ i_1=1,\ldots,m, i_2=1,\ldots,m$, where X denotes the universe of discourse.

In [9], authors propose a new PDC to design a stable switching fuzzy controller for the switched fuzzy system (1). The idea for designing a PDC controller is a natural extension of the same concept, used for the ordinary T-S fuzzy systems (see for example [7]).

Region Rule i:

IF $z_1(t)$ **is** N_{i1} **and** $\cdots$ **and** $z_p(t)$ **is** N_{ip},

THEN (7)

Local Control Rule l:

IF $z_1(t)$ **is** M_{il1} **and** $\cdots$ **and** $z_p(t)$ **is** M_{ilp},

THEN $u(t)=-F_{il}x(t),\ \ l=1,2,\ldots,r\ \ i=1,2,\ldots,m$

Finally, the overall fuzzy controller is represented by:

$$u(t)=-\sum_{i=1}^{m}\sum_{l=1}^{r}\upsilon_i(z(t))h_{il}(z(t))F_{il}x(t) \tag{8}$$

By substituting (8) into (3), the fuzzy control system can be represented as:

$$\begin{aligned}\dot{x}(t)&=\sum_{k=1}^{m}\sum_{l=1}^{m}\sum_{i=1}^{r}\sum_{j=1}^{r}\upsilon_k(z(t))\upsilon_l(z(t))h_{ki}(z(t))h_{lj}(z(t))\{A_{ki}-B_{ki}F_{lj}\}x(t)\\&=\sum_{k=1}^{m}\sum_{i=1}^{r}\sum_{j=1}^{r}\upsilon_k(z(t))h_{ki}(z(t))h_{kj}(z(t))\{A_{ki}-B_{ki}F_{kj}\}x(t)\end{aligned} \tag{9}$$

III. STABILITY ANALYSIS AND CONTROL DESIGN

Compared with the results on stability of switched systems and those of T-S fuzzy systems, the results on switched fuzzy systems are very few. Similar to the stability analysis for model based T-S fuzzy systems ([7]) and switched systems ([1]-[3]) alone, stability analysis of switched fuzzy systems has been pursued mainly based on Lyapunov stability theory but with different Lyapunov functions. One of them is the so-called *common* (or *global*) quadratic Lyapunov functions, another one is the so-called *piecewise* quadratic Lyapunov functions, and the third one is the so-called *fuzzy* (or *non-quadratic*) Lyapunov functions.

Regardless of the different Lyapunov functions that are used, the typical design procedure, common for most of the papers, concerning switched fuzzy systems, includes the following:

- Stability analysis and derivation of asymptotic stability conditions for autonomous system;
- Synthesis of the parameters of the predefined controllers that will stabilize the system [9]-[14], as well as (or) derivation of stabilizing switching laws (signals) that will stabilize the system [15]-[19].

As in Section II we outlined one of the two typical approaches for representation modelling of switched fuzzy systems, here we will summarise the conditions for asymptotic stability for this kinds of models, as well as the conditions for designing controllers for stabilizing this kind of systems. Detailed analyses on the stability results of switched fuzzy systems, based on the different types of Lyapunov functions, are given in [21].

Considering the literature related to T-S fuzzy model based systems ([5], [7]) it can be well found that the Linear Matrix Inequalities (LMIs - [7], [24]) play an important role in stability analysis of this kind of systems. Very natural, authors in [9]-[12], extend their previous works, considering stability of ordinary T-S fuzzy systems (expressed in the form of LMIs - see [7]), in the field of switched fuzzy systems. Bellow, we will address the stability conditions for the open-loop system (model (1)), as well as the stability and relaxed stability conditions for the closed-loop system (model (9)), given in [9]-[12].

A. *Stability Conditions of Switched Fuzzy Model with Levels of Structure*

To begin with, we show a stability condition (based on quadratic Lyapunov function (10)) for the switched fuzzy system (1), when $u = 0$.

$$V(x(t)) = x^T(t)Px(t),\ P > 0. \tag{10}$$

Note that Theorem 1 is taken from [13] and [14], although it addresses the stability problem given in [12].

Theorem 1 [12] ([13] and [14]): The switched fuzzy system (1), for $u = 0$, is asymptotic stable if there exist a positive definite matrix P, satisfying the following LMI conditions:

$$A_{il}^T P + PA_{il} < 0,\ \forall l, i. \tag{11}$$

The condition (11) is easily derived by taking the time derivative of equation (10) along the trajectories of the system (1).

B. *Stable Controller Design of Switched Fuzzy Model with Levels of Structure*

Here we will focus on the LMI stability conditions as well as LMI relaxed stability conditions for the closed-loop system (9) - system that consists of the switched fuzzy model (1) and the appropriate PDC controller, given with (7). The LMI stability conditions are obtained with respect to $X = P^{-1}$ and $M_{il} = F_{il}X$ (note that these LMI conditions correspond to the LMI (relaxed) stability conditions for the ordinary T-S fuzzy model-based systems given in [7]). The PDC fuzzy controller design is to determine the local feedback gains F_{il} in the consequent parts. As it is mentioned in [7] and [9], although the PDC fuzzy controller (8) is constructed using the local design sense, the feedback gains F_{il} should be determined using the global design sense (stability conditions for the global stabilization of the system).

Theorem 2 [9], [10] *(stability)*: The switched fuzzy model (3) can be stabilized via the PDC switching fuzzy controller (8), if there exist a common positive definite matrix X, such that:

$$-XA_{il}^T - A_{il}X + M_{il}^T B_{il}^T + B_{il}M_{il} > 0, \tag{12}$$

for $i = 1,2,\dots,m$, $l = 1,2,\dots,r$ and

$$\begin{aligned}&-XA_{il}^T - A_{il}X - XA_{ik}^T - A_{ik}X \\ &+ M_{ik}^T B_{il}^T + B_{il}M_{ik} + M_{il}^T B_{ik}^T + B_{ik}M_{il} \geq 0\end{aligned} \tag{13}$$

for all i and $l < k$ excepting all the pairs (l,k) such that $h_{il}(z(t))h_{ik}(z(t)) = 0$, $\forall t$, where $X = P^{-1} > 0$, $M_{il} = F_{il}X$.

Proof: See [9] and [10].

According to Theorem 2, stability analysis of the switched fuzzy control system is reduced to a problem of finding a common $\boldsymbol{P}$. If r, that is the number of IF-THEN rules, is large, it might be difficult to find a common $\boldsymbol{P}$ satisfying the conditions of Theorem 2. In Theorem 3, these conditions are relaxed.

Theorem 3 [9], [10] *(relaxed stability)*: Assume that the number of rules that fire for all t is less than or equal to ψ, where $1 < \psi \leq r$. The switched fuzzy model (3) can be stabilized via the switching PDC fuzzy controller (8), if there exist a common positive definite matrix X and a common positive semi-definite matrix Y_i, such that:

$$-XA_{il}^T - A_{il}X + M_{il}^T B_{il}^T + B_{il}M_{il} - (\psi - 1)Y_i > 0, \tag{14}$$

for $i = 1,2,\dots,m$, $l = 1,2,\dots,r$ and

$$\begin{aligned}&2Y_i - XA_{il}^T - A_{il}X - XA_{ik}^T - A_{ik}X \\ &+ M_{ik}^T B_{il}^T + B_{il}M_{ik} + M_{il}^T B_{ik}^T + B_{ik}M_{il} \geq 0,\end{aligned} \tag{15}$$

for all i and $l < k$ excepting all the pairs (l,k) such that $h_{il}(z(t))h_{ik}(z(t)) = 0$, $\forall t$ and $m > 1$, where $X = P^{-1} > 0$, $M_{il} = F_{il}X$, $Y_i = XQ_iX \geq 0$.

Proof: see [9], [10] and [12].

Except stability (relaxed stability) theorems for the switched fuzzy system (1), authors in [9]-[12] also present the conditions for the constraints on control inputs. These conditions match the corresponding ones for ordinary T-S fuzzy systems, given in [7].

Consider each input variable, i.e. [9]-[12]:

$$u_k(t) = E_k u(t) = -E_k \sum_{i=1}^{m}\sum_{l=1}^{r} \upsilon_i(z(t))h_{il}(z(t))F_{il}x(t) \tag{16}$$

where $E_k = \overset{1\ \cdots\ k\ \cdots\ f}{[0\cdots 1\cdots 0]}$.

Theorem 4 [11], [12] *(constraints on the control inputs)*: Assume that the initial condition $x(0)$ is known. The constraint $\|u_k(t)\|_2 \le \mu_k$ ($k = 1,2,\ldots,f$) is enforced at all times if the following LMIs hold:

$$\begin{bmatrix} 1 & x^T(0) \\ x(0) & X \end{bmatrix} \ge 0, \begin{bmatrix} X & M_{il}^T E_k \\ E_k M_{il} & \mu_k^2 I \end{bmatrix} \ge 0, \ \forall l, \ \forall i, \ \exists k, \qquad (17)$$

where $X = P^{-1} > 0$ and $M_{il} = F_{il} X$ are LMI variables.

Proof: See [12].

IV. PROPOSED SIMULATION ALGORITHMS

In this section we will present the proposed concepts for building simulation algorithms for switched fuzzy control systems which is the main contribution of the presented work.

First, we present the whole process for designing the switching fuzzy controller for the given nonlinear system. This is given on Fig. 2. As it is obvious from Fig. 2, first we have to find the switched fuzzy model for the nonlinear system, after what (using certain conditions) we will synthesize the appropriate switching PDC controllers. The final aim is to generate stable controllers which will satisfy certain stability conditions given in a form of LMIs (e.g. using Theorem 2 or Theorem 3).

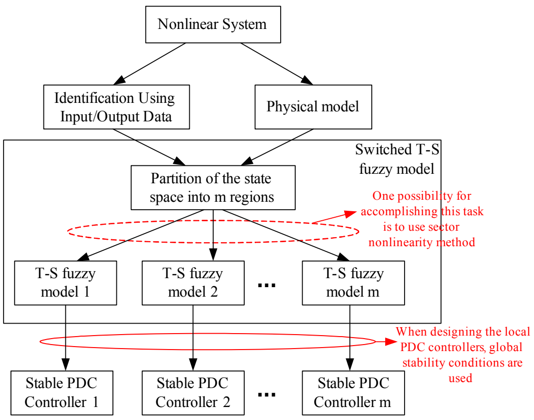


Figure 2. Switching fuzzy controllers design.

Next, we propose one possible way for finding this stable switching fuzzy logic PDC controllers (using MATLAB), after what, with the appropriate Simulink model, we can make the performance analyses simulations. The necessary steps are represented on Fig. 3. After the local switching controllers are generated (using the algorithm on Fig. 3), according and for the appropriate switched T-S fuzzy model (Fig. 2), they can be used for controlling the real nonlinear system. Choosing the appropriate controller in certain moment (switching among these controllers) is made according the conditions which define the certain region selection. This is shown on Fig. 4. In every region there is an appropriate stable PDC controller which is synthesized using the design procedure given on Fig. 3.

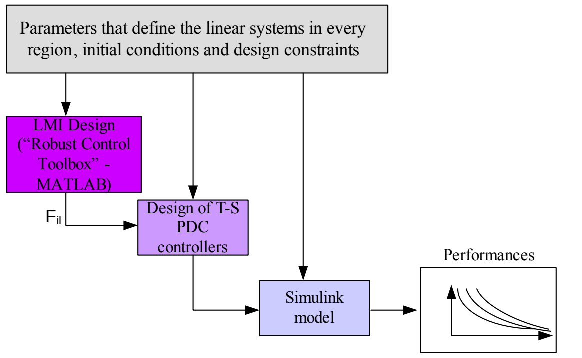


Figure 3. Steps for controller design and performance evaluation for the given switched fuzzy model.

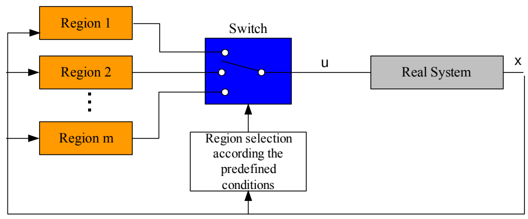


Figure 4. Control of a real system with previously designed controllers, according the procedure given on Fig. 3.

V. HOVERCRAFT VEHICLE (HV) AS A TYPICAL NONHOLONOMIC SYSTEMS

A hovercraft is a craft capable of travelling over surfaces while supported by a cushion of slow moving, high-pressure air which is ejected against the surface below and contained within a "skirt".

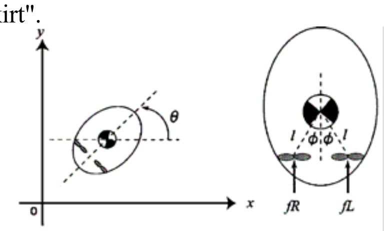


Figure 5. The model of a hovercraft vehicle [9].

From Fig. 5 it is obvious that if we apply the same force to both motors, the vehicle will go straight, and if different force is applied, the vehicle will turn.

According to hovercraft model of Fig. 5, the following state-space model of the hovercraft vehicle is given in [9]:

$$\begin{aligned} \dot{x}_1(t) &= \ddot{y}(t) = \frac{1}{M}\sin\theta(t) f_1(t), \quad \dot{x}_2(t) = \dot{y}(t) = x_1(t) \\ \dot{x}_3(t) &= \ddot{\theta}(t) = \frac{l\sin\phi}{I} f_2(t), \quad \dot{x}_4(t) = \dot{\theta}(t) = x_3(t) \end{aligned} \qquad (18)$$

where $f_1(t) = f_R(t) + f_L(t)$, $f_2(t) = f_R(t) - f_L(t)$, θ is the angle of the vehicle; l is the distance between the gravity and fans; ϕ is the angle between the gravity and fans; f_R and f_L are the forces generated by the right and left side fans, respectively; M and I are the mass and the inertia, respectively. The control purpose is $\lim_{t\to\infty} y(t) = 0$ and $\lim_{t\to\infty} \theta(t) = 0$, by manipulating $f_R(t)$ and $f_L(t)$.

Using the theory for nonholonomic systems in [1], it can be easily shown that the hovercraft vehicle, with the model (18), is a typical nonholonomic system. Nonholonomy means that the system is subject to constraints involving both, the position and velocity, whereupon it is shown that this systems can not be stabilized by any continuous feedback law [1].

Moreover, for proving this conclusion we will try to stabilize the system with the continuous control law (based on the T-S fazzy model), and then we will design a switching controller, based on the switched fuzzy-logic model.

VI. T-S Fuzzy Model Based Control for the HV

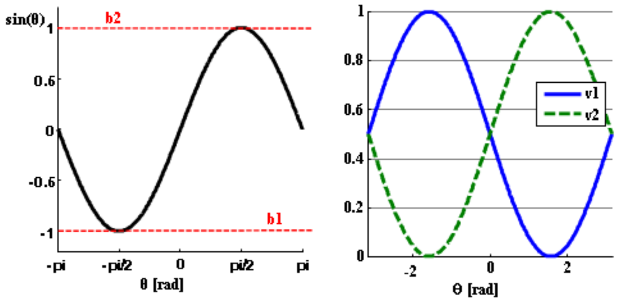
A. T-S Fuzzy Model for the Hovercraft Vehicle

The design of T-S fuzzy model for the hovercraft system will be made according to the presented design procedure for building T-S fuzzy models in [7].

The main feature of the T-S fuzzy model given in [7] (equation 2.1 – Chapter 2) is to express the join dynamics of each fuzzy implication (rule) by a linear system model.

We replace the equations (18) with an ordinary T-S fuzzy model. The idea of sector nonlinearity is employed in the T-S fuzzy model construction ([7] - Chapter 2).

To begin with, we replace $\sin(\theta(t))$ with a fuzzy model representation. By considering that $\theta(t) \in [-\pi \quad \pi]$, the range of $\sin(\theta(t))$ is given with $\sin(\theta(t)) \in [-1 \quad 1]$ (this is visible from Fig. 6-a):



a) b)

Figure 6. a) Nonlinearity $\sin(\theta(t))$, for $\theta(t) \in [-\pi \quad \pi]$; b) Membership functions $v_1(\theta)$ and $v_2(\theta)$.

Subsequently, the nonlinear function $\sin(\theta(t))$ can be converted into the following fuzzy model representation:

$$\sin\theta(t) = \sum_{i=1}^{2} v_i(\theta(t)) b_i \,, \tag{19}$$

where

$$v_1(\theta(t)) + v_2(\theta(t)) = 1 \,. \tag{20}$$

From (19) and (20) the membership functions $v_1(\theta(t))$ and $v_2(\theta(t))$ are calculated as:

$$v_1(\theta(t)) = \frac{1-\sin(\theta(t))}{2},\ v_2(\theta(t)) = \frac{1+\sin(\theta(t))}{2}, \tag{21}$$

and they are plotted on Fig. 6-b.

We construct the following fuzzy model by utilizing the representation (19) and the membership functions (21).

Rule 1 (R^1):

IF $\theta(t)$ **is** $v_1(\theta(t))$, $\left\langle \sin\theta(t) = \sum_{i=1}^{2} v_i(\theta(t)) b_i = 1 \cdot b_1 + 0 \cdot b_2 = -1 \right\rangle$

$$\textbf{THEN} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{M} f_1 \\ x_1 \\ \frac{l\sin\phi}{I} f_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} -\frac{1}{M} & 0 \\ 0 & 0 \\ 0 & \frac{l\sin\phi}{I} \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

Rule 2 (R^2): (22)

IF $\theta(t)$ **is** $v_2(\theta(t))$, $\left\langle \sin\theta(t) = \sum_{i=1}^{2} v_i(\theta(t)) b_i = 0 \cdot b_1 + 1 \cdot b_2 = 1 \right\rangle$

$$\textbf{THEN} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{M} f_1 \\ x_1 \\ \frac{l\sin\phi}{I} f_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \frac{1}{M} & 0 \\ 0 & 0 \\ 0 & \frac{l\sin\phi}{I} \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

The model (22) can be given in the following condense form:

Rule 1 (R^1):	**Rule** 2 (R^2):
IF $\theta(t)$ **is** $v_1(\theta(t))$, ,	**IF** $\theta(t)$ **is** $v_2(\theta(t))$, (23)
THEN $\dot{x}(t) = A_1 x(t) + B_1 u(t)$	**THEN** $\dot{x}(t) = A_2 x(t) + B_2 u(t)$,

where: $u = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$, $A_1 = A_2 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} -\frac{1}{M} & 0 \\ 0 & 0 \\ 0 & \frac{l \sin \phi}{I} \\ 0 & 0 \end{bmatrix}$,

$B_1 = \begin{bmatrix} \frac{1}{M} & 0 \\ 0 & 0 \\ 0 & \frac{l \sin \phi}{I} \\ 0 & 0 \end{bmatrix}$, and $x(t) = [x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)]$.

Following the design procedure in [7] (equation 2.1 – Chapter 2), the T-S fuzzy model is represented as: $\dot{x}(t) = \sum_{i=1}^{2} v_i(\theta(t))\{A_i x(t) + B_i u(t)\}$.

B. *Design of T-S Fuzzy Logic Controller*

The fuzzy logic PDC controller for the T-S fuzzy model (23) can be designed according the design procedure in [7] (equation 2.23 – Chapter 2), having the form:

Control rule 1: **Control rule** 2:

$$\textbf{IF } \theta(t) \textbf{ is } v_1, \qquad \textbf{IF } \theta(t) \textbf{ is } v_2, \tag{24}$$

$$\textbf{THEN } u(t) = -F_1 x(t) \qquad \textbf{THEN } u(t) = -F_2 x(t)$$

From (23), it is obvious that the design process depends on the local feedback gains F_1 and F_2, which can be obtained if there is a feasible solution to the LMI stability design conditions given in [7] (equations 3.15 and 3.16 – Chapter 3).

The linear matrix inequalities, adequate for the T-S fuzzy model (23) and the PDC control scheme (23), are given in the form:

$$X > 0$$
$$-XA_1^T - A_1 X + M_1^T B_1^T + B_1 M_1 > 0$$
$$-XA_2^T - A_2 X + M_2^T B_2^T + B_2 M_2 > 0$$
$$-XA_1^T - A_1 X - XA_2^T - A_2 X + M_2^T B_1^T + B_1 M_2 + M_1^T B_2^T + B_2 M_1 \geq 0$$

where the feedback gains F_i ($i = 1,2$) and the common matrix P can be derived according the equations:

$$P = X^{-1}, \qquad F_i = M_i X^{-1}, \tag{25}$$

using the resulting values of X and M_i.

Using the LMI toolbox in MATLAB the matrices P and F_i ($i = 1,2$) are calculated as:

$$P = 10^6 * \begin{bmatrix} 1.7171 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad F_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 12.1278 & 4.7399 \end{bmatrix} \quad F_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 12.1278 & 4.7399 \end{bmatrix}$$

Even though the solution seems to be feasible, this controller can not stabilize the system and achieve the control purpose. This was expected, as the hovercraft vehicle is typical nonholonomic system, and it can not be stabilised with any type of feedback continuous control law. Moreover, this is visible from the fact that:

$$\sum_{i=1}^{2} v_i(\theta(t)) B_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \frac{l \sin \phi}{I} \\ 0 & 0 \end{bmatrix}, \text{ for } \theta(t) = 0.$$

This means that the first control input $f_1(t)$ never contributes for achieving the control purpose. Given that, if the initial conditions are $x(0) = [0 \quad 1 \quad 0 \quad 1.5708]^T$, only control input $f_2(t)$ influence the system performance, which tries to turn the vehicle from state $\theta = \pi/2$ into the state $\theta = 0$, but $y = 1$, $\forall t$. This is shown on Fig. 7.

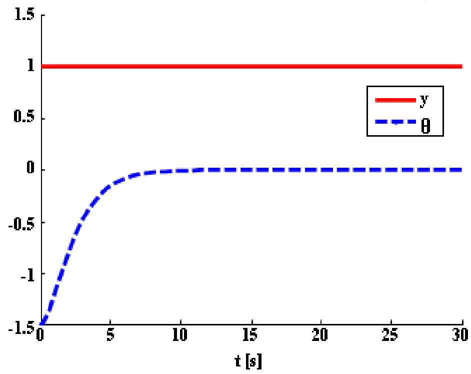


Figure 7. Simulation results using the PDC controller for the T-S fuzzy model, for the hovercraft vehicle, for initial conditions $x(0) = [0 \quad 1 \quad 0 \quad -1.5]^T$ and (M=0.1, $\phi = \pi/4$, I=0.5, l=0.1).

In the next Section we will show that the control purpose will be achieved by designing the switched fuzzy model based controller.

VII. T-S Switched Fuzzy Controller Design for a HV

In this section we will use the proposed simulation algorithms from Section IV (Fig. 2 and Fig. 3), in order to build switched fuzzy controller for the hovercraft-vehicle as a typical nonholonomic system. After synthesizing the appropriate local controllers we can build Simulink model (Fig. 4) and explore the performance of the given control system. This scheme for constructing the simulation environment can also be used for other nonlinear systems.

A. *Switched Fuzzy Model*

The switched fuzzy model that we will derive (according to the procedure represented on Fig. 2) will satisfy the form of the switched fuzzy model with levels of structure, given with the equations (1).

DOI 10.5013/IJSSST.a.12.02.03

ISSN: 1473-804x online, 1473-8031 print

To make a switched fuzzy model for the nonlinear system (18), assume that $\theta(t)\in[-179 \quad 179]$. We divide the premise variable space into three regions with nonnegative constant d (in [23] we have shown that the state-space partitioning into different regions has considerable influence to the control performance of the switched fuzzy system). Therefore, the switched fuzzy model has three regions (Region 1-3) according to the premise variable $\theta(t)$. The local nonlinear dynamics in each region is represented by a T-S fuzzy model. We will use sector nonlinearity concept ([7]) to determine the local linear models in every region. ()

Region 1 ***(*** $\theta(t)\geq d$ ***)***: In this region ($\theta(t)\in[d \quad 179]$), the nonlinear function $\sin(\theta(t))$ can be rewritten as $\sin\theta(t)=\sum_{i=1}^{2}h_{1i}(\theta(t))a_{1i}$. Since $\sum_{i=1}^{2}h_{1i}(\theta(t))=1$, the membership functions $h_{11}(\theta(t))=\frac{\sin(\theta(t))-a_{12}}{a_{11}-a_{12}}$ and $h_{12}(\theta(t))=\frac{a_{11}-\sin(\theta(t))}{a_{11}-a_{12}}$, where $a_{11}=1$ and $a_{12}=\sin(179^0)\approx 0.01$.

Region 2 ***(*** $-d<\theta(t)<d$ ***)***:

In this region, we fix the first input, i.e. $f_1(t)=C$, where C is a positive constant. This implies $\ddot{y}(t)=\frac{\sin\theta(t)}{M}C$. The nonlinear function $\sin(\theta(t))$ can be rewritten as $\sin\theta(t)=\left(\sum_{i=1}^{2}h_{2i}(\theta(t))a_{2i}\right)\theta(t)$. Since $\sum_{i=1}^{2}h_{2i}(\theta(t))=1$, the membership functions $h_{21}(\theta(t))=\frac{\frac{\sin(\theta(t))}{\theta(t)}-a_{22}}{a_{21}-a_{22}}$ and $h_{22}(\theta(t))=\frac{a_{21}-\frac{\sin(\theta(t))}{\theta(t)}}{a_{21}-a_{22}}$, where $a_{21}=1$ and $a_{22}=\sin(d)/d$.

Region 3 ***(*** $\theta(t)\leq -d$ ***)***: In this region ($\theta(t)\in[-179 \quad d]$)the nonlinear function $\sin(\theta(t))$ can be rewritten as $\sin\theta(t)=\sum_{i=1}^{2}h_{3i}(\theta(t))a_{3i}$. Since $\sum_{i=1}^{2}h_{3i}(\theta(t))=1$, the membership functions $h_{31}(\theta(t))=\frac{\sin(\theta(t))-a_{32}}{a_{31}-a_{32}}$ and $h_{32}(\theta(t))=\frac{a_{31}-\sin(\theta(t))}{a_{31}-a_{32}}$, where $a_{31}=-1$ and $a_{32}=\sin(-179^0)\approx -0.01$.

By aggregating the above results, according the relation (1), we construct the following switched fuzzy model for the hovercraft model (18):

Region Rule 1 : **IF** $\theta(t)\geq d$,

THEN

Local Plant Rule 1 : **IF** $\theta(t)$ **e** $h_{11}(\theta(t))$,

THEN $\dot{x}(t)=A_{11}x(t)+B_{11}u(t)$

Local Plant Rule 2 : **IF** $\theta(t)$ **e** $h_{12}(\theta(t))$,

THEN $\dot{x}(t)=A_{12}x(t)+B_{12}u(t)$

Region Rule 2 : **IF** - $d<\theta(t)<d$,

THEN

Local Plant Rule 1 : **IF** $\theta(t)$ **e** $h_{21}(\theta(t))$,

THEN $\dot{x}(t)=A_{21}x(t)+B_{21}u(t)$

Local Plant Rule 2 : **IF** $\theta(t)$ **e** $h_{22}(\theta(t))$,

THEN $\dot{x}(t)=A_{22}x(t)+B_{22}u(t)$

Region Rule 3 : **IF** $\theta(t)\leq -d$,

THEN

Local Plant Rule 1 : **IF** $\theta(t)$ **e** $h_{31}(\theta(t))$,

THEN $\dot{x}(t)=A_{31}x(t)+B_{31}u(t)$ (26)

Local Plant Rule 2 : **IF** $\theta(t)$ **e** $h_{32}(\theta(t))$,

THEN $\dot{x}(t)=A_{32}x(t)+B_{32}u(t)$

where $u(t)=\begin{bmatrix} f_1(t) \\ f_2(t)\end{bmatrix}$ and $x(t)=[x_1(t) \quad x_2(t) \quad x_3(t) \quad x_4(t)]$,

$$A_{11}=A_{12}=A_{31}=A_{32}=\begin{bmatrix}0&0&0&0\\1&0&0&0\\0&0&0&0\\0&0&1&0\end{bmatrix},\ A_{21}=\begin{bmatrix}0&0&0&\frac{C}{M}a_{21}\\1&0&0&0\\0&0&0&0\\0&0&1&0\end{bmatrix},$$

$$A_{22}=\begin{bmatrix}0&0&0&\frac{C}{M}a_{22}\\1&0&0&0\\0&0&0&0\\0&0&1&0\end{bmatrix},\quad B_{11}=\begin{bmatrix}\frac{1}{M}a_{11}&0\\0&0\\0&\frac{l\sin\phi}{I}\\0&0\end{bmatrix},$$

$$B_{12}=\begin{bmatrix}\frac{1}{M}a_{12}&0\\0&0\\0&\frac{l\sin\phi}{I}\\0&0\end{bmatrix},\quad B_{21}=B_{22}=\begin{bmatrix}0&0\\0&0\\0&\frac{l\sin\phi}{I}\\0&0\end{bmatrix},$$

$$B_{31}=\begin{bmatrix}\frac{1}{M}a_{31}&0\\0&0\\0&\frac{l\sin\phi}{I}\\0&0\end{bmatrix},\ B_{32}=\begin{bmatrix}\frac{1}{M}a_{32}&0\\0&0\\0&\frac{l\sin\phi}{I}\\0&0\end{bmatrix}.$$

The defuzzification is carried out according the relation (3), whereas for this case we get:

$$\dot{x}(t)=\sum_{i=1}^{3}\sum_{l=1}^{2}\upsilon_i(\theta(t))h_{il}(\theta(t))\{A_{il}x(t)+B_{il}u(t)\} \qquad (27)$$

$$\upsilon_1(t)=\begin{cases}1, & \theta(t)\geq 0\\ 0, & o.w.\end{cases},\ \upsilon_2(t)=\begin{cases}1, & -d<\theta(t)<d\\ 0, & o.w.\end{cases},\ \upsilon_3(t)=\begin{cases}1, & \theta(t)\leq -d\\ 0, & o.w.\end{cases}.$$

B. Controller Design via Switching PDC

The switching fuzzy controller of PDC type for the switched fuzzy model (26) can be designed according the relation (7), having the form:

Region Rule 1 : **IF** $\theta(t) \geq d$,

THEN

Local Control Rule 1 : **IF** $\theta(t)$ **is** $h_{11}(\theta(t))$,

THEN $u(t) = -F_{11}x(t)$

Local Control Rule 2 : **IF** $\theta(t)$ **is** $h_{12}(\theta(t))$,

THEN $u(t) = -F_{12}x(t)$

Region Rule 2 : **IF** $-d < \theta(t) < d$,

THEN

Local Control Rule 1 : **IF** $\theta(t)$ **is** $h_{21}(\theta(t))$,

THEN $u(t) = -F_{21}x(t)$

Local Control Rule 2 : **IF** $\theta(t)$ **is** $h_{22}(\theta(t))$,

THEN $u(t) = -F_{22}x(t)$

Region Rule 3 : **IF** $\theta(t) \leq -d$,

THEN

Local Control Rule 1 : **IF** $\theta(t)$ **is** $h_{31}(\theta(t))$,

THEN $u(t) = -F_{31}x(t)$ (28)

Local Control Rule 2 : **IF** $\theta(t)$ **is** $h_{32}(\theta(t))$,

THEN $u(t) = -F_{32}x(t)$

The overall fuzzy controller is obtained according the relation (8), and in this case it has the form:

$$u(t) = -\sum_{i=1}^{3}\sum_{l=1}^{2} \upsilon_i(z(t)) h_{il}(z(t)) F_{il}\, x(t) \tag{29}$$

From (29), it is obvious that the design process depends on the local feedback gains F_{il}, which can be obtained if there is a feasible solution to the LMI stability conditions given with Theorem 2.

The linear matrix inequalities, adequate for the switched fuzzy model (23) and the PDC control scheme (23), are given in the form:

$$\begin{aligned}
& X > 0 \\
& \%\,\mathrm{Region1} \\
& -XA_{11}^T - A_{11}X + M_{11}^T B_{11}^T + B_{11}M_{11} > 0 \\
& -XA_{12}^T - A_{12}X + M_{12}^T B_{12}^T + B_{12}M_{12} > 0 \\
& -XA_{11}^T - A_{11}X - XA_{12}^T - A_{12}X + M_{12}^T B_{11}^T + B_{11}M_{12} + M_{11}^T B_{12}^T + B_{12}M_{11} \geq 0 \\
& \%\,\mathrm{Region2} \\
& -XA_{21}^T - A_{21}X + M_{21}^T B_{21}^T + B_{21}M_{21} > 0 \\
& -XA_{22}^T - A_{22}X + M_{22}^T B_{22}^T + B_{22}M_{22} > 0 \\
& -XA_{21}^T - A_{21}X - XA_{22}^T - A_{22}X + M_{22}^T B_{21}^T + B_{21}M_{22} + M_{21}^T B_{22}^T + B_{22}M_{21} \geq 0 \\
& \%\,\mathrm{Region3} \\
& -XA_{31}^T - A_{31}X + M_{31}^T B_{31}^T + B_{31}M_{31} > 0 \\
& -XA_{32}^T - A_{32}X + M_{32}^T B_{32}^T + B_{32}M_{32} > 0 \\
& -XA_{31}^T - A_{31}X - XA_{32}^T - A_{32}X + M_{32}^T B_{31}^T + B_{31}M_{32} + M_{31}^T B_{32}^T + B_{32}M_{31} \geq 0
\end{aligned} \tag{30}$$

where the feedback gains F_{il} ($i = 1,2,3$, $l = 1,2$) and the common matrix P can be derived according the equations:

$$P = X^{-1}, \qquad F_{il} = M_{il} X^{-1}, \tag{31}$$

using the resulting values of X and M_i from (30).

Using the proposed simulation scheme on Fig. 3, i.e. using the LMI toolbox in MATLAB, we have got feasible solution for the values of matrices P and F_{il} ($i = 1,2,3$, $l = 1,2$), in the form:

$$P = \begin{bmatrix} 0.0866 & 0.0097 & 0.0083 & 0.0942 \\ 0.0097 & 0.0016 & 0.0009 & 0.0106 \\ 0.0083 & 0.0009 & 0.0016 & 0.0124 \\ 0.0942 & 0.0106 & 0.0124 & 0.1416 \end{bmatrix}$$

$$F_{11} = 10^3 * \begin{bmatrix} 0.0028 & 0.0003 & 0.0003 & 0.0031 \\ 1.4596 & 0.1645 & 0.1934 & 2.1942 \end{bmatrix}$$

$$F_{12} = 10^3 * \begin{bmatrix} 0.0085 & 0.0010 & 0.0008 & 0.0091 \\ 1.4591 & 0.1644 & 0.1933 & 2.1937 \end{bmatrix}$$

$$F_{21} = 10^3 * \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.8826 & 0.0908 & 0.1380 & 1.5663 \end{bmatrix}$$

$$F_{22} = 10^3 * \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.8830 & 0.0999 & 0.1381 & 1.5667 \end{bmatrix}$$

$$F_{31} = 10^3 * \begin{bmatrix} -0.0028 & -0.0003 & -0.0003 & -0.0030 \\ 1.4610 & 0.1646 & 0.1935 & 2.1958 \end{bmatrix}$$

$$F_{32} = 10^3 * \begin{bmatrix} -0.0085 & -0.0010 & -0.0008 & -0.0091 \\ 1.4592 & 0.1644 & 0.1933 & 2.1937 \end{bmatrix}$$

Fig. 8 shows the values of the control variables y and θ, using the switching PDC controller. In this case, unlike the case when ordinary T-S fuzzy controller is used, it is clear that the control purpose ($\lim_{t\to\infty} y(t) = 0$ and $\lim_{t\to\infty} \theta(t) = 0$) is achievable.

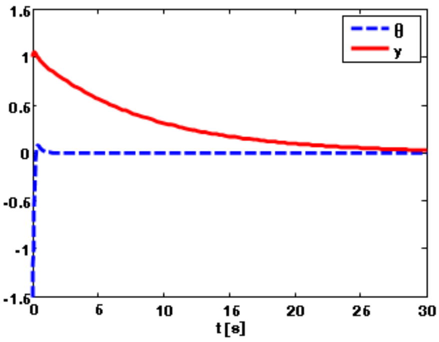
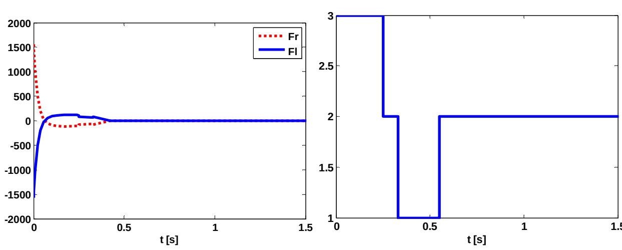


Figure 8. . Values for θ and y, when the LMI conditions according the Theorem 2 are used. Initial conditions are $x(0) = [0 \quad 1 \quad 0 \quad -1.5]^T$ and (M=0.1, $\phi = \pi/4$, I=0.5, l=0.1, C=0.5, $d = \pi/50$).



a) b)

Figure 9. . a) Control inputs $f_R(t)$ and $f_L(t)$; b) Region selection.

Showing this it is obvious that the use of switching fuzzy controllers is worthwhile in comparison of ordinary T-S fuzzy controllers which are continuous feedback control laws. However, if we view the control effort, i.e. the values of $f_R(t)$ and $f_L(t)$, (see Fig. 9-a), it is obvious that they have very high amplitudes. The respective switching signal that selects the appropriate region during the control period is shown on Fig. 9- b.

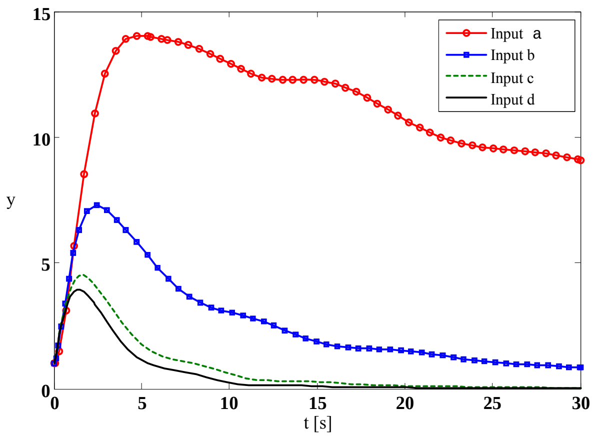


Figure 10. . Values for y, using the control signals, shown on Fig. 12.

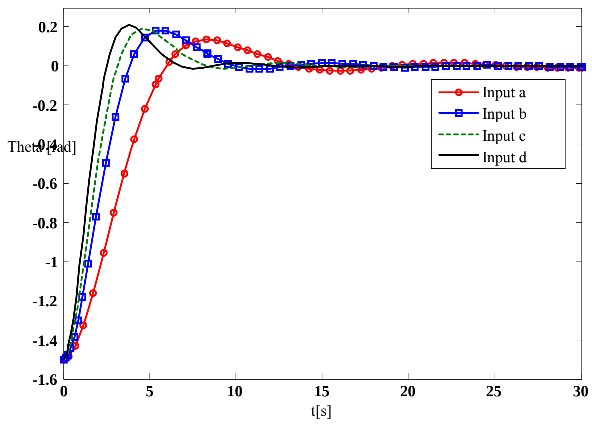


Figure 11. . Values for θ, using the control signals, shown on Fig. 12.

For constraining the control inputs $f_R(t)$ and $f_L(t)$, we can use the LMI conditions given with (17). Please note that the LMI conditions (17) depend on the initial conditions, so whenever we would like to change the initial conditions it would be necessary to recalculate the feedback control gains according the design process on Fig. 3.

Fig. 10 and Fig. 11 show the values of the control variables for the four different constraint inputs, given on Fig. 12-a,b,c,d. It is obvious that as much as the control inputs are constraint, as hard as the control purpose is achieved.

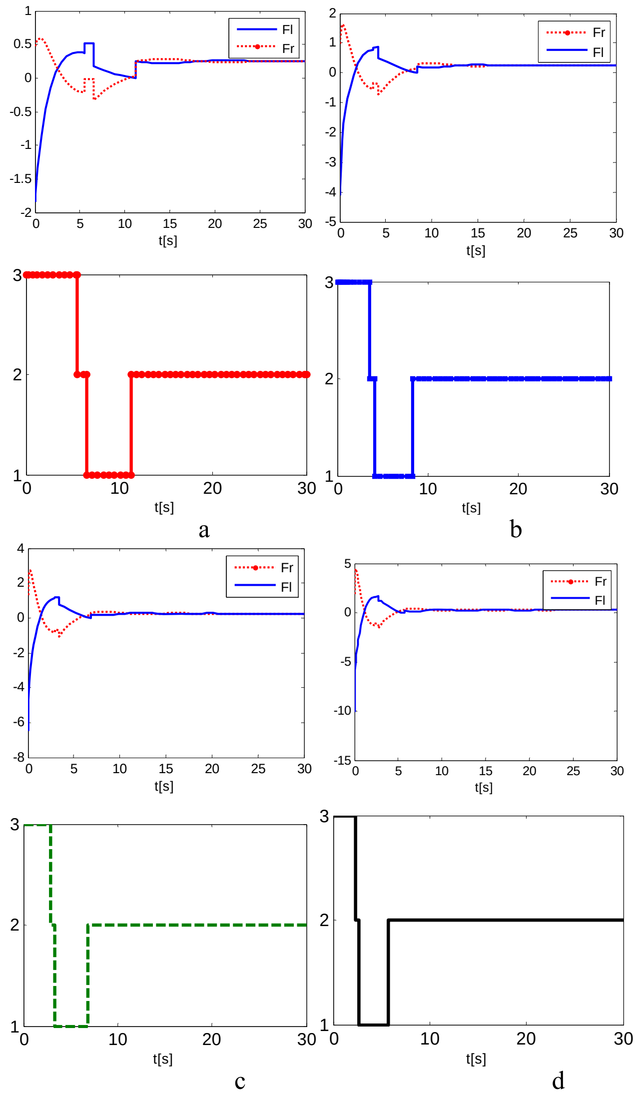


Figure 12. . Control inputs $f_R(t)$ and $f_L(t)$ and the appropriate switching signal, when the joined LMI conditions according the Theorem 2 and Theorem 4 are used. Initial conditions are $x(0)=\begin{bmatrix}0 & 1 & 0 & -1.5\end{bmatrix}^T$ and (M=0.1, $\phi=\pi/4$, I=0.5, l=0.1, C=0.5, $d=\pi/50$).

VIII. Conclusion

We have shown that the proposed mechanism for modelling and simulation of the performance of switched fuzzy systems is applicable for building and simulating the switched fuzzy control of the hovercraft vehicle, whereas we expect that these concepts can be used for building and simulating the switched fuzzy control of other nonlinear systems. Also, with the derived simulation results that show the feasibleness of the use of switching fuzzy controllers for the systems that are not stabilizable with any continuous feedback control laws, we believe that this type of control will be used in solving many control problems that were not

DOI 10.5013/IJSSST.a.12.02.03

ISSN: 1473-804x online, 1473-8031 print

solvable with other types of controllers. Among many questions that arise considering the performance of switched fuzzy systems is the state-space partitioning. In [23] we have shown that the state-space partitioning into different regions has considerable influence to the control performance of the switched fuzzy system.

ACKNOWLEDGMENT

This research was part of the FANEN Scientific Research Project, financed by the Ss Cyril and Methodius University in Skopje, R. Macedonia, Faculty of Electrical Engineering and Information Technologies (Tatjana Kolemishevska-Gugulovska, Principal investigator - 2011 – 2012, contracted on 20.1.2011).

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