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E Veli Veliev[*1], **K Azizi**[2], **H Sundu**[1], **G Kaya**[1] and **A Türkan**[1]

[1] Department of Physics, Kocaeli University, 41380 Izmit, Turkey
[2] Department of Physics, Faculty of Arts and Sciences, Doğuş University, Acıbadem-Kadıköy, 34722 Istanbul, Turkey

E-mail: [*1]`elsen@kocaeli.edu.tr`

Abstract. This study deals with determination of the decay constants of heavy vector mesons in the framework of the thermal QCD sum rules. We calculate both thermal spectral density and non-perturbative contributions taking into account the traditionally existing operators at $T = 0$ and also additional operators appearing at finite temperature. Analysis of the obtained thermal sum rules shows that the decay constants almost remain unchanged with respect to the variation of temperature up to $T \cong 100 MeV$, however after this point, they start to decrease sharply with increasing temperature.

1. Introduction

It is believed that investigation of thermal behavior of hadronic properties in medium provides testing ground for the Standard Model and helps understanding the results of the heavy ion collision experiments. In particular, investigation of hadronic properties of J/ψ and Υ as heavy vector mesons in medium plays important role in the study of quark gluon plasma [1]. One of the most powerful and applicable approaches to calculate the hadronic parameters is the QCD sum rules [2]. This method was later extended to finite temperature and density [3]. In formulation of thermal sum rules, the complications are breakdown of Lorentz invariance in medium by the choice of the thermal rest frame and appearance of additional operators in operator product expansion (OPE) compared to the QCD sum rules in vacuum [4]-[6]. This method has been extensively used to predict the behavior of hadronic parameters such as masses, widths, decay constants at zero [7, 8] and finite temperature [9]-[13].

In the present work, we calculate the decay constants f_V of the heavy vector quarkonia $J/\psi(\bar{c}c)$ and $\Upsilon(\bar{b}b)$, which are defined by the matrix element of the vector current J_μ between the vacuum and the vector-meson state,

$$\langle 0|J_\mu|V(q,\lambda)\rangle = f_V m_V \varepsilon_\mu^{(\lambda)}. \tag{1}$$

In our calculations, we use the values of the energy density and gluon condensates obtained via Chiral perturbation theory [14] and lattice QCD [15]-[17]. We observe that the values of the decay constants decrease considerably near to the critical or deconfinement temperature comparing to their values in vacuum.

2. Thermal QCD sum rules for vector mesons

We consider the thermal correlation function,

$$\Pi_{\mu\nu}\Big(q,T\Big) = i \int d^4x \; e^{iq\cdot x} Tr\Big(\rho \; \mathcal{T}\Big(J_\mu(x)J_\nu^\dagger(0)\Big)\Big), \tag{2}$$

with $J_\mu(x) =: \overline{Q}(x)\gamma_\mu Q(x) :$. Here, $Q(x)$ is heavy (charm or bottom) quark field, $\mathcal{T}$ indicates the time ordered product and $\rho = e^{-\beta H}/Tre^{-\beta H}$ is the thermal density matrix of QCD at temperature $T = 1/\beta$. The correlation function in thermal field theory is given by Lorentz invariant functions, $\Pi_l\Big(q^2, \omega\Big) = \frac{1}{\overline{q}^2}\Pi_2$ and $\Pi_t\Big(q^2, \omega\Big) = -\frac{1}{2}(\Pi_1 + \frac{q^2}{\overline{q}^2}\Pi_2)$. Here $\Pi_1 = g^{\mu\nu}\Pi_{\mu\nu}$, $\Pi_2 = u^\mu\Pi_{\mu\nu}u^\nu$, $\overline{q}^2 = \omega^2 - q^2$, $\omega = u \cdot q$ and u_μ is four-velocity.

It can be shown that, in the fixed value of $|\mathbf{q}|$, the spectral representation of the thermal correlation function is given by [6]:

$$\Pi_{l,t}\Big(q_0^2, T\Big) = \int_0^\infty dq_0'^2 \; \frac{\rho_{l,t}\Big(q_0'^2, T\Big)}{q_0'^2 + Q_0^2}, \tag{3}$$

where $Q_0^2 = -q_0^2$, and

$$\rho_{l,t}\Big(q_0^2, T\Big) = \frac{1}{\pi}Im\Pi_{l,t}\Big(q_0^2, T\Big)\tanh\frac{\beta q_0}{2}. \tag{4}$$

In order to obtain thermal sum rules, we equate the spectral representation and results of operator product expansion for amplitudes $\Pi_l(q^2, \omega)$ or $\Pi_t(q^2, \omega)$ at sufficiently high Q_0^2 to the hadronic representation of the correlation function. When performing numerical results, we should exchange our reference to one at which the particle is at rest, i.e., we shall set $|\mathbf{q}| \to 0$. In this limit, the functions Π_l and Π_t are related to each other so it is enough to use one of them to acquire thermal sum rules. Here, we use the function Π_t. The fundamental assumption of the thermal QCD sum rule approach is the principle of duality, i.e., it is assumed that there is an interval over which this thermal correlation function may be equivalently described at the quark representation and at the hadronic representation. Therefore the hadronic spectral density is expressed by the ground state vector meson pole plus the contribution of the higher states and continuum,

$$\rho_t^{had}(s) = f_V^2(T)m_V^2(T)\delta(s - m_V^2) + \theta(s - s_0)\rho_t^{pert}(s). \tag{5}$$

Firstly, we consider $\Pi_2(q,T) = u^\mu\Pi_{\mu\nu}(q,T)u^\nu$ function. After some simplifications, we obtain the imaginary part of the $\Pi_2(q,T)$ in the form

$$\Pi_2(q,T) = -4iN_c \int \frac{d^4k}{(2\pi)^4}(k^2 - q\cdot k - m^2 + 2q_0k_0 - 2k_0^2)D(k)D(k - q), \tag{6}$$

where $D(k) = 1/(k^2 - m^2 + i\varepsilon) + 2\pi in(|k_0|)\delta(k^2 - m^2)$. Carrying out the integral over k_0 and angles we obtain annihilation and scattering parts of $Im\Pi_2(q,T)$ as follows:

$$Im\Pi_{2,a} = N_c \int_{\omega_-}^{\omega_+} \frac{d\omega_1}{8\pi|\mathbf{q}|}(4q_0\omega_1 - q^2 - 4\omega_1^2)F(\omega_1), \tag{7}$$

$$Im\Pi_{2,s} = N_c \int_{\omega_+}^\infty \frac{d\omega_1}{8\pi|\mathbf{q}|}(4q_0\omega_1 - q^2 - 4\omega_1^2)G(\omega_1). \tag{8}$$
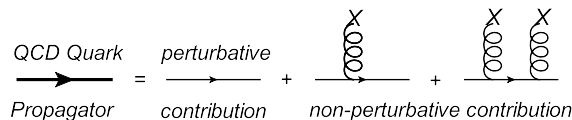
Figure 1. The quark propagator in the gluon background fields.

Here $F(\omega_1) = 1 - n(\omega_1) - n(q_0 - \omega_1) + 2n(\omega_1)n(q_0 - \omega_1)$, $G(\omega_1) = 2n(\omega_1)n(q_0 - \omega_1) - n(\omega_1) - n(q_0 - \omega_1)$ and $\omega_\pm = \frac{1}{2}(q_0 \pm |\mathbf{q}|v)$. From a similar way, the annihilation and scattering parts of $\Pi_1(q, T)$ are also calculated.

As we also previously mentioned, when doing numerical analysis, we will set $|\mathbf{q}| \to 0$ representing the rest frame of the particle. In this case, the annihilation part of spectral density is expressed as follows:

$$\rho_{t,a}(s) = \frac{1}{8\pi^2} sv(s)(3 - v^2(s))\left[1 - 2n\left(\frac{\sqrt{s}}{2}\right)\right],\tag{9}$$

for $4m^2 \leq s < \infty$ and $v(s) = \sqrt{1 - 4m^2/s}$. Note that the scattering cut shrinks to a point in the considered limit and this part of the spectral density does not contribute to the thermal QCD sum rule.

In our calculations, we also take into account the perturbative two-loop order α_s correction to the spectral density. This correction at zero temperature can be written as [7]:

$$\rho_{\alpha_s}(s) = \alpha_s \frac{s}{6\pi^2} v(s)\left(3 - v^2(s)\right)\left[\frac{\pi}{2v(s)} - \frac{1}{4}\left(3 + v(s)\right)\left(\frac{\pi}{2} - \frac{3}{4\pi}\right)\right],\tag{10}$$

where we replace the strong coupling α_s in Eq. (10) with its temperature dependent lattice improved expression [12, 17]. Now, we proceed to calculate the non-perturbative part in QCD side. Taking into account one and two gluon lines attached to the quark line as shown in Fig. 1, up to terms required for our calculations, the non-perturbative part of the massive quark propagator at finite temperature is obtained as:

$$
\begin{aligned}
S^{aa' \, nonpert}(k) &= -\frac{i}{4} g(t^c)^{aa'} G^c_{\kappa\lambda} \frac{1}{(k^2 - m^2)^2}\left[\sigma_{\kappa\lambda}(\slashed{k} + m) + (\slashed{k} + m)\sigma_{\kappa\lambda}\right] \\
&+ \frac{i\,g^2\,\delta^{aa'}}{9\,(k^2 - m^2)^4}\left\{\frac{3m(k^2 + m\,\slashed{k})}{4}\langle G^c_{\alpha\beta} G^{c\alpha\beta}\rangle + \left[m\left(k^2 - 4(k \cdot u)^2\right)\right.\right. \\
&+ \left.\left.\left(m^2 - 4(k \cdot u)^2\right)\slashed{k} + 4(k \cdot u)(k^2 - m^2)\,\slashed{u}\right]\langle u^\alpha \Theta^g_{\alpha\beta} u^\beta\rangle\right\}.
\end{aligned}
\tag{11}
$$

where $\Theta^g_{\alpha\beta}$ is the traceless gluonic part of the energy-momentum tensor of the QCD.

Applying Borel transformation with respect to Q_0^2 in Eq. (3) and taking into account hadronic representation in Eq. (5), we obtain (for details see [18])

$$f_V^2 m_V^2 \exp\left(-\frac{m_V^2}{M^2}\right) = \int_{4m^2}^{s_0} ds\, [\rho_{t,a}(s) + \rho_{\alpha_s}(s)] \exp\left(-\frac{s}{M^2}\right) + \widehat{B}\Pi_t^{nonpert}.\tag{12}$$

Here $\widehat{B}\Pi_t^{nonpert}$ shows the nonperturbative part of QCD side in Borel transformed scheme, which is given by:

$$\hat{B}\Pi_t^{nonpert} = \int_0^1 dx \, \frac{1}{144 \, \pi \, M^6 \, x^4 \, (-1+x)^4} \exp\left[\frac{m^2}{M^2 \, x \, (-1+x)}\right]\left\{\langle\alpha_s G^2\rangle\left[12 \, M^6 \, x^4 \, (-1+x)^4\right.\right.$$
$$- \quad m^6 \, (1-2x)^2(-1-x+x^2) - 12 \, m^2 \, M^4 \, x^2 \, (-1+x)^2 \, (1-3x+3x^2) + m^4 \, M^2 \, x \, (-2+19x$$
$$- \quad 32x^2 + 11x^3 + 6x^4 - 2x^5)\big] + 4 \, \alpha_s\langle\Theta^g\rangle\big[- 8 \, M^6 \, x^3 \, (1-2x)^2(-1+x)^3 + m^6 \, (1-2x)^2(-1-x$$
$$+ \quad x^2) - 2 \, m^2 \, M^4 \, x^2 \, (-1+x)^2(-1-6x+8x^2-4x^3+2x^4) + m^4 \, M^2 \, x \, (-2+3x-12x^2+31x^3$$
$$- \quad 30x^4 + 10x^5)\big]\big\}, \tag{13}$$

where $\Theta^g = \Theta_{00}^g$.

In further analysis, we use the values, $m_c = (1.3 \pm 0.05) \, GeV$, $m_b = (4.7 \pm 0.1) \, GeV$ and $\langle 0 \mid \frac{1}{\pi}\alpha_s G^2 \mid 0\rangle = (0.012 \pm 0.004) \, GeV^4$ for quarks masses and gluon condensate at zero temperature. We choose the values $s_0 = (11-12) \, GeV^2$ and $s_0 = (98-100) \, GeV^2$ for continuum threshold at J/ψ and Υ channels, respectively.

Our final task is to discuss the temperature dependence of the leptonic decay constant of the considered particles. For this aim, we plot this quantity in terms of temperature in Fig. 2 and Fig. 3 using the total energy density from both chiral perturbation theory [14] and lattice QCD (valid only for $T \geq 100 \, MeV$) [15]-[17] at different fixed values of s_0. As shown in these graphs, at $T = 0$, the values of the decay constants of the J/ψ and Υ are obtained as $f_{J/\psi} = (0.460 \pm 0.022) \, GeV$ and $f_\Upsilon = (0.715 \pm 0.032) \, GeV$. These results are in good consistency with the existing experimental data and predictions of other nonperturbative models[19]-[21]. Also, we observe that the decay constants remain insensitive to the variation of the temperature up to $T \cong 100 \, MeV$, however after this point, they start to diminish with increasing temperature. At deconfinement or critical temperature, the decay constants approach roughly to 45% of their values at zero temperature.
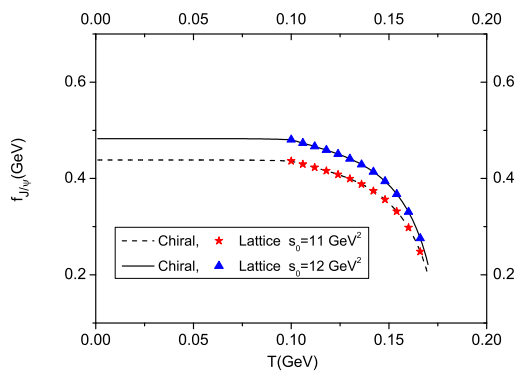

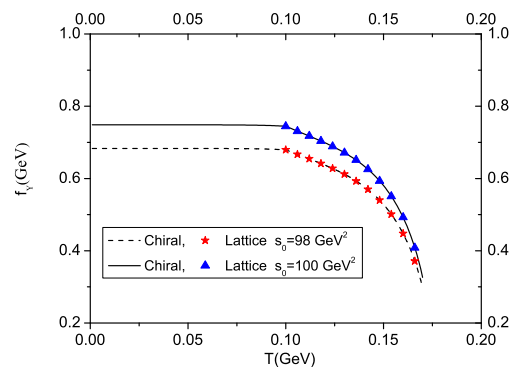**Figure 2.** The dependence of the leptonic decay constant of J/ψ in GeV on temperature at $M^2 = 10 \, GeV^2$

Figure 3. The dependence of the leptonic decay constant of Υ in GeV on temperature at $M^2 = 20 \, GeV^2$.

Our results at zero temperature as well as the behavior of the mass and decay constant with respect to the temperature can be checked in future experiments. Also the temperature dependence of the considered quantities can be used in analysis of heavy ion collision experiments.

Acknowledgments

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