$g_{D_sDK^{*0}}$ and $g_{B_sDK^{*0}}$ coupling constants in QCD sum rules

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Abstract. In the present study, we calculate the strong coupling constants $g_{D_sDK_0^*(800)}$ and $g_{B_sBK_0^*(800)}$ within the three-point QCD sum rules approach. We evaluate the correlation function of the considered vertices taking into account both D[B] and $K_0^*(800)$ mesons as off-shell states.

1. Introduction

In low energies, it is difficult to obtain confident theoretical results using the perturbation theory since the interaction between quarks and gluons becomes large in this scale. Therefore, we need some non-perturbative approaches to describe low energy dynamics of hadrons clearly. Among these dynamics, the strong coupling constants of mesons are closely related to their strong interactions. Using the QCD sum rules method as one of the powerful and applicable non-perturbative approaches [1], we can determine these coupling constants more accurately. In this work, we calculate the strong coupling constants of $D_sDK_0^*(800)$ and $B_sBK_0^*(800)$ vertices. Calculation of such coupling constants can help us in understanding the nature of the strong interaction among the participating particles.

2. QCD Sum Rules for the Strong Coupling Constants

In this section, we obtain QCD sum rules for the strong coupling constants associated with the $D_s - D - K_0^*(800)$ and $B_s - B - K_0^*(800)$ vertices. For this aim, the following three-point correlation function for $D(B)$ off-shell case is studied:

$$\Pi^{D(B)} = i^2 \int d^4x \, d^4y \, e^{ip' \cdot x} \, e^{iq \cdot y} \langle 0|\mathcal{T}\left(\eta^{K_0^*}(x) \, \eta^{D(B)}(y) \, \eta^{D_s(B_s)\dagger}(0)\right)|0\rangle, \tag{1}$$

where $\mathcal{T}$ indicates the time ordering product, $q = p - p'$ is the momentum of the off-shell state and p' is the momentum of the final on-shell state. The interpolating quark currents, which produce the considered mesons from the vacuum with the same quantum numbers as these currents can be written in terms of the quark field operators as shown in [2]. In the QCD sum rules method, we calculate the related correlation function in two different ways as outlined in [2]. The strong coupling constants are obtained equating these two different presentations via dispersion relation. To suppress contributions of the higher states and continuum, we will

apply double Borel transformation with respect to the momentum squared of the initial and final on-shell states to both sides of the obtained sum rules.

First, we start to the calculation of the physical side of the concerned correlation function. Inserting a complete set of intermediate hadronic states with the same quantum numbers as interpolating quark currents into the correlation function, we obtain:

$$
\Pi^{D(B)} = \frac{\langle 0|\eta^{K_0^*}|K_0^*(p')\rangle \langle 0|\eta^{D(B)}|D(B)(q)\rangle \langle K_0^*(p')D(B)(q)|D_s(B_s)(p)\rangle \langle D_s(B_s)(p)|\eta^{D_s(B_s)}|0\rangle}{(q^2 - m_{D(B)}^2)(p^2 - m_{D_s(B_s)}^2)(p'^2 - m_{K_0^*}^2)}
$$
$$
+ \ ...,
\tag{2}
$$

The matrix elements shown in the above equation can be parameterized in terms of leptonic decay constants and strong coupling constant $g_{D_s DK_0^*(B_s BK_0^*)}^{D(B)}(q^2)$ (see [2] for details). The final physical representation of the correlation function in the case of D(B) off-shell is obtained as:

$$
\Pi^{D(B)} = g_{D_s DK_0^*(B_s BK_0^*)}^{D(B)}(q^2) \frac{f_{K_0^*} m_{K_0^*} \frac{f_{D(B)} m_{D(B)}^2}{m_{c(b)}+m_u} \frac{f_{D_s(B_s)} m_{D_s(B_s)}^2}{m_{c(b)}+m_s}}{2\ (q^2 - m_{D(B)}^2)(p'^2 - m_{K_0^*}^2)(p^2 - m_{D_s(B_s)}^2)} (m_{D_s(B_s)}^2 + m_{K_0^*}^2 - q^2)
$$
$$
+ \ ....,
\tag{3}
$$

Now, we concentrate to calculate the QCD or theoretical side of the considered correlation function. The correlation function in QCD side is written in terms of the perturbative and non-perturbative parts. The perturbative part is defined in terms of double dispersion integral as shown in [2]. In order to obtain the spectral density, we need to calculate the bare loop diagram for $D(B)$ off-shell presented in [2]. As a result, the spectral density is obtained as follows:

$$
\rho^{D(B)}(s, s', q^2) = \frac{N_c}{2\ \lambda^{1/2}(s, s', q^2)} \Big\{ m_s \left(m_u(m_s + m_u) - q^2 \right) - sm_u
$$
$$
- m_{c(b)} \left((m_s + m_u)^2 - s' - m_{c(b)}(m_s + m_u) \right) \Big\},
\tag{4}
$$

Now, we proceed to calculate the nonperturbative contributions in QCD side. We consider the quark-quark and quark-gluon condensate diagrams presented in [2]. As a result, we obtain:

$$
\Pi_{nonper}^{D(B)} = \frac{\langle \bar{s}s \rangle}{2} \Big\{ \frac{2m_{c(b)}m_u - m_{c(b)}^2 + q^2}{rr'} - \frac{1}{r} - \frac{1}{r'} + \frac{m_0^2(4m_{c(b)}m_u - m_{c(b)}^2 + q^2)}{4r^2 r'}
$$
$$
- \frac{m_0^2}{4rr'} - \frac{m_0^2}{4r^2} + \frac{m_0^2(m_{c(b)}^2 - 4m_{c(b)}m_u - q^2)}{4rr'^2} + \frac{m_0^2}{4r^2} + \frac{m_0^2}{4rr'} \Big\},
\tag{5}
$$

Finally, after applying the double Borel transformation, the following sum rules for the considered coupling constant is obtained as:

$$
g_{D_s DK_0^*(B_s BK_0^*)}^{D(B)}(q^2) = \frac{2(q^2 - m_{D(B)}^2)(m_{c(b)}+m_u)(m_{c(b)}+m_s)}{m_{D_s(B_s)}^2 m_{D(B)}^2 m_{K_0^*} f_{D_s(B_s)} f_{D(B)} f_{K_0^*}(m_{D_s(B_s)}^2 + m_{K_0^*}^2 - q^2)} e^{\frac{m_{D_s(B_s)}^2}{M^2}}
$$
$$
\times\ e^{\frac{m_{K_0^*}^2}{M'^2}} \Big[-\frac{1}{4\pi^2} \int_{(m_{c(b)}+m_s)^2}^{s_0} ds \int_{(m_s+m_u)^2}^{s_0'} ds' \rho^{D(B)}(s, s', q^2)\theta[1 - (f^{D(B)}(s, s'))^2] e^{\frac{-s}{M^2}} e^{\frac{-s'}{M'^2}}
$$
$$
+\ \hat{B}\Pi_{nonper}^{D(B)} \Big],
\tag{6}
$$

Similarly, one can calculate the coupling constant for $K_0^*(800)$ off-shell case.

3. Numerical analysis

To obtain the behavior of the coupling constants in terms of q^2, we should look for working regions for Borel mass parameters (M^2 and M'^2)and continuum thresholds (s_0 and s_0') (see [2] for details). We use the following Boltzmann function to find the Q^2 behavior of the considered strong coupling constants:

$$g(Q^2) = \left[A_1 + \frac{A_2}{1 + \exp[\frac{Q^2 - x_0}{\Delta x}]} \right] [GeV^{-1}]. \tag{7}$$

where, $Q^2 = -q^2$ and the parameters A_1, A_2, x_0, Δx are given in Table 1.

Table 1. Parameters appearing in the fit function of the coupling constants for $D_s D K_0^*(800)$ and $B_s B K_0^*(800)$ vertices.

	A_1	A_1	x_0	Δx
$g_{D_s D K_0^*(800)}^{(D)}(Q^2)$	3.468	-2.741	8.067	4.995
$g_{D_s D K_0^*(800)}^{(K_0^*(800))}(Q^2)$	-0.024	0.772	5.723	1.257
$g_{B_s B K_0^*(800)}^{(B)}(Q^2)$	4.151	-1.932	13.842	12.149
$g_{B_s B K_0^*(800)}^{(K_0^*(800))}(Q^2)$	-0.017	0.547	5.431	1.121

Table 2. Value of the $g_{D_s D K_0^*(800)}$ and $g_{B_s B K_0^*(800)}$ coupling constants in GeV^{-1} unit.

	$Q^2 = -m_D^2$	$Q^2 = -m_{K_0^*(800)}^2$	Average
$g_{D_s D K_0^*(800)}$	0.97 ± 0.02	0.74 ± 0.05	0.85 ± 0.08
	$Q^2 = -m_B^2$	$Q^2 = -m_{K_0^*(800)}^2$	Average
$g_{B_s B K_0^*(800)}$	2.28 ± 0.18	0.53 ± 0.09	1.41 ± 0.21

The final result for each coupling constant is obtained taking the average of the coupling constants obtained from two different off-shell cases.

4. Acknowledgement

References

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